Sigma J Eng Nat Sci, Vol. 42, No. 1, pp. 225–234, February, 2024

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2024.00019

Research Article

# New exact solutions for the Boiti-Leon-Manna-Pempinelli equation

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## ARTICLE INFO

*Article history*
Received: 23 December 2021
Revised: 08 April 2022
Accepted: 28 June 2022

**Keywords:**
Evolution Equations; Exact Solution; $(G'/G, 1/G)$-Expansion Method

## ABSTRACT

Phenomena in physics, plasma physics, optical fibers, chemical physics, fluid mechanics, and many fields are often described by the nonlinear evolution equations. The analytical solutions of these equations are very important to understand the evaluation of the physical models. In this paper, the Boiti-Leon-Manna-Pempinelli (BLMP) nonlinear partial differential equation, which can be used to describe the incompressible fluid flow, is analytically studied by using the five different techniques which are direct integration, $(G'/G)$-expansion method, different form of the $(G'/G)$-expansion method, two variable $(G'/G, 1/G)$-expansion method, and $(1/G')$- expansion method. Hyperbolic, trigonometric and rotational forms of solutions are obtained. Our solutions are reduced to the well-known solutions found in the literature by assigning the some special values to the constants appeared in the analytic solutions. Moreover, we have also obtained the new analytic solutions of the BLMP equation.

**Cite this article as:** Yılmaz RK, Dağhan D. New exact solutions for the Boiti-Leon-Manna-Pempinelli equation. Sigma J Eng Nat Sci 2024;42(1):225–234.

## INTRODUCTION

Certain physical systems can be explained by the nonlinear evolution equations (NLEEs) and have the chaotic structures due to their nature [1, 2]. It is usually very difficult to understand these chaotic structures. Hence, there are many mathematical models to solve the NLEEs analytically or numerically [3-30]. One of the these types of equations is Korteweg-de Vries (KdV) equation

$$U_t + 6UU_{xx} + U_{xxx} = 0, \qquad (1)$$

which plays an important role in mathematical physics [31]. The other one is the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation which is a different form of the KdV equation given bellow [32]:

$$U_{yt} + U_{xxxy} - 3U_{xx}U_y - 3U_xU_{xy} = 0, \qquad (2)$$

where $U = U(x,y,t), U_t = \frac{\partial U}{\partial t}, U_x = \frac{\partial U}{\partial x}, U_y = \frac{\partial U}{\partial y}, \ldots$ etc. BLMP equation reduces the KdV equation in the case of $y = x$ [33, 34]. The solution of Cauchy problem for the BLMP equation was improved by using an inverse scattering scheme in Boiti et al. [33, 34]. Recently, some analytic solutions of the BLMP equation were obtained by Arbabi and Najafi using the semi-inverse variational principle given in Ref. [2]. The other analytic solutions of the BLMP equation can be seen in Refs. [1, 35-45].

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*This paper was recommended for publication in revised form by Regional Editor Ahmet Selim Dalkilic*

Published by Yıldız Technical University Press, İstanbul, Turkey

In this paper, our aim is to present the new analytic solutions of the (2+1)- dimensional BLMP equation. Therefore, we have found the analytic solutions of the BLMP equation by using the five different techniques which are direct integration, ($G'$ / $G$)-expansion [3], different form of the ($G'$ / $G$) -expansion [4], two-variable ($G'$ / $G$, 1 / $G$)-expansion [5], and (1 / $G'$) -expansion methods [6]. Using these different methods we have not only produced the same solutions found in the literature, but also derived new solutions of this equation. We believe that our solutions are more general than the ones obtained in the literature.

The paper covers the following sections: in Section 2, the ($G'$ / $G$)-expansion methods to obtain analytic solutions of the BLMP equation are introduced. In Section 3, the analytic solutions of BLMP equation are given for five different techniques. Finally, Section 4 is devoted to the conclusion of the study.

## MAERIALS AND METHODS

The general form of the partial differential equation (PDE) can be given

$$P(u,\, u_t,\, u_x,\, u_{tt},\, u_{xt},\, u_{xx}, \ldots) = 0, \tag{3}$$

where $u = u(x,t)$ is an unknown function, $P$ is a polynomial depending on $u$. By using transformation $u(x,t) = u(\eta),\, \eta = x - Vt$ in Eq. 3, we get an ordinary differential equation (ODE) as follows

$$P(u, -Vu',\, u',\, V^2u'', -Vu'', u'', \ldots) = 0, \tag{4}$$

where $u' = \frac{du}{d\eta}$. Now, used methods, which will be applied to non-linear partial differential equation, are summarized in the following subsections.

### $\left(\frac{G'}{G}\right)$- Expansion Method

The most general form of the solution of Eq. 4 by using the $\left(\frac{G'}{G}\right)$-expansion method is given in [3]

$$u(\eta) = \alpha_m \left(\frac{G'}{G}\right)^m + \ldots, \tag{5}$$

where $\alpha_m$ contains sets of unknown coefficients and $G = G(\eta)$ satisfies the second order linear ODE given as

$$G'' + \lambda G' + \mu G = 0, \tag{6}$$

where $\mu$ and $L$ are arbitrary constants which have to be defined later. The more detailed discussions can be found in Refs.[3, 46, 47]. From the solution, $\left(\frac{G'}{G}\right)$ term is represented respect to hyperbolic ($\lambda^2 - 4\mu > 0$), trigonometric ($\lambda^2 - 4\mu < 0$) and rational functions ($\lambda^2 - 4\mu = 0$), and they are

$$\begin{aligned}
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{c_1 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta + c_2 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta}{c_1 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta + c_2 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta}\right) - \frac{\lambda}{2},\\
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \frac{\sqrt{4\mu - \lambda^2}}{2}\left(\frac{-c_1 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta + c_2 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta}{c_1 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta + c_2 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta}\right) - \frac{\lambda}{2},\\
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \frac{c_2}{c_1 + c_2\eta} - \frac{\lambda}{2},
\end{aligned} \tag{7}$$

where $c_1$ and $c_2$ are arbitrary integration constants.

### Different Form of the $\left(\frac{G'}{G}\right)$- Expansion Method

Li and Wang used the $\left(\frac{G'}{G}\right)$-expansion method in Ref. [4] which is different than the one described in the original case given in Ref. [3]. By using of differentform of $\left(\frac{G'}{G}\right)$ -expansion method, the solution of Eq. 4 can be expressed in terms of $\left(\frac{G'}{G}\right)$ as following

$$u(\eta) = \alpha_m \left(\frac{G'}{G}\right)^m + \alpha_{-m}\left(\frac{G'}{G}\right)^{-m}, \tag{8}$$

where $\alpha_m$ and $\alpha_{-m}$ are unknown coefficients and $G = G(\eta)$ satisfies the following ODE,

$$G'' + \mu G = 0, \tag{9}$$

where $\mu$ is an arbitrary constant. Unknown coefficients and constant can be determined by similar techniques from algebraic equation given in $\left(\frac{G'}{G}\right)$-expansion method. $\left(\frac{G'}{G}\right)$ term can be written as functions of hyperbolic ($-\mu > 0$), trigonometric ($-\mu < 0$) and rational functions ($\mu = 0$), and they are given

$$\begin{aligned}
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \sqrt{-\mu}\left(\frac{c_1 sinh\sqrt{-\mu}\eta + c_2 cosh\sqrt{-\mu}\eta}{c_1 cosh\sqrt{-\mu}\eta + c_2 sinh\sqrt{-\mu}\eta}\right),\\
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \sqrt{\mu}\left(\frac{-c_1 sin\sqrt{\mu}\eta + c_2 cos\sqrt{\mu}\eta}{c_1 cos\sqrt{\mu}\eta + c_2 sin\sqrt{\mu}\eta}\right),\\
\left[\frac{G'(\eta)}{G(\eta)}\right] &= \frac{c_2}{c_1 + c_2\eta},
\end{aligned} \tag{10}$$

where $c_1$ and $c_2$ are arbitrary integration constants. The more detailed discussions can be found in Refs.[4, 47].

### $\left(\frac{G'}{G}, \frac{1}{G}\right)$- Expansion Method

The third method is called the $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion method which is considered as the generalization of the original $\left(\frac{G'}{G}\right)$-expansion method [3]. As a pioneer work, Ref. [5] has applied the two-variable $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion method and found the analytic solutions of Zakharov equations. Now, we describe the main steps of the $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion method for finding the travelling wave solutions of nonlinear evolution equations. First of all, we consider the following second order ordinary linear differential equation

$$G''(\eta) + \lambda G(\eta) = \mu, \tag{11}$$

and to have a simplicity we use the following new definitions

$$\varphi = G'/G, \quad \psi = 1/G. \tag{12}$$

Combining Eq. 11 with Eq. 12, it yields

$$\varphi' = -\varphi^2 + \mu\psi - \lambda, \quad \psi' = -\varphi\psi. \tag{13}$$

Now, we have three different forms of general solutions of Eq. 11 which are

**Case I:** When $\lambda < 0$,

The first form of the general solution of the Eq. 11 is

$$G(\eta) = c_1 sinh\left(\sqrt{-\lambda}\eta\right) + c_2 cosh\left(\sqrt{-\lambda}\eta\right) + \frac{\mu}{\lambda}, \tag{14}$$

and we have

$$\psi^2 = -\frac{\lambda}{\lambda^2\nu + \mu^2}\left(\varphi^2 - 2\mu\psi + \lambda\right),$$

where $c_1$ and $c_2$ are arbitrary integration constants and $\nu = {c_1}^2 - {c_2}^2$.

**Case II:** When $\lambda > 0$,

The second form of the general solution of the Eq. 11 is

$$G(\eta) = c_1 sin\left(\sqrt{\lambda}\eta\right) + c_2 cos\left(\sqrt{\lambda}\eta\right) + \frac{\mu}{\lambda}, \tag{15}$$

and we have

$$\psi^2 = \frac{\lambda}{\lambda^2\nu - \mu^2}\left(\varphi^2 - 2\mu\psi + \lambda\right),$$

where $c_1$ and $c_2$ are arbitrary integration constants and $\nu = {c_1}^2 + {c_2}^2$.

**Case III:** When $\lambda = 0$,

The third form of the general solution of the Eq. 11 is

$$G(\eta) = \frac{\mu}{2}\eta^2 + c_1\eta + c_2, \tag{16}$$

and we have

$$\psi^2 = \frac{\lambda}{{c_1}^2 - 2\mu c_2}\left(\varphi^2 - 2\mu\psi\right),$$

where $c_1$ and $c_2$ are arbitrary integration constants. The more detailed discussions about the $\left(\frac{G'}{G}, \frac{1}{G}\right)$- expansion method can be seen in Refs. [5, 48, 49].

### $\left(\frac{1}{G'}\right)$- Expansion Method

$\left(\frac{1}{G'}\right)$- expansion method is firstly introduced by Yokus [6]. Suppose that the solution of Eq. 4 can be expressed in terms of $\left(\frac{1}{G'}\right)$

$$u(\eta) = \sum_{i=1}^{N} a_i \left(\frac{1}{G'}\right)^i, \tag{17}$$

where $G = G(\eta)$ satisfies the following linear ordinary differential equation.

$$G''(\eta) + \lambda G'(\eta) + \mu = 0, \tag{18}$$

where $a_i (i = 1, \ldots, N), c, \lambda$, and $\mu$ are constants to be determined later, and the positive integer $N$ can be determined by using the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. 11. Additionally, the solution of the differential equation given in Eq. 18 is

$$G(\eta) = c_1 e^{-\lambda\eta} - \frac{\mu}{\lambda}\eta + c_2, \tag{19}$$

where $c_1$ and $c_2$ are arbitrary integration constants. $\left(\frac{1}{G'}\right)$ term can be expressed as

$$\left(\frac{1}{G'}\right) = \frac{\lambda}{-\mu + \lambda c_1[cosh(\lambda\eta) - sinh(\lambda\eta)]}. \tag{20}$$

The more detailed discussions about the $\left(\frac{1}{G'}\right)$- expansion method can be seen in Refs.[6, 49].

## ANALYTIC SOLUTIONS OF THE BLMP EQUATION

In this section, we present the analytic solutions of the BLMP equation given in Eq. 2 by using the different approaches. To do that, Eq. 2 can be converted into the following ODEs by using the transformation of $\eta = k_1 x + k_2 y + V_p t,\ U = U(\eta)$:

$$V_p U'' + {k_1}^3 U^{(4)} - 6{k_1}^2 U'' U' = 0, \tag{21}$$

where $U' = \frac{\partial U}{\partial \eta}$. By integrating, we get

$$V_p U' + {k_1}^3 U''' - 3{k_1}^2 {U'}^2 + c = 0, \tag{22}$$

where $c$ is an arbitrary integration constant. Defining the transformation $U' = W$, we get

$$V_p W + {k_1}^3 W'' - 3{k_1}^2 W^2 + c = 0. \tag{23}$$

Eq. 23 contains an extra integration constant $c$, different from the literature. In literature, some authors omit the arbitrary integration constant after integrating of the nonlinear ordinary differential equations [50]. Hence, it is believed that the solutions of this equation are more general than the ones found in the literature. Additionally, we substitute the integration constant $c = 0$ in our solution to compare them with the literature.

**Solutions of the BLMP Equation by Using Direct Integration**

Let us multiply right hand side of Eq. 23 with $W'$, hence we have

$$V_p W W' + {k_1}^3 W' W'' - 3{k_1}^2 W^2 W' + cW' = 0. \quad (24)$$

Eq. 24 is an integrable equation. We have obtained the following first-order equation after integrating it once.

$$W'^2 = \frac{2}{k_1}W^3 - \frac{V_p}{{k_1}^3}W^2 - \frac{2c}{{k_1}^3}W - \frac{2d}{{k_1}^3}, \quad (25)$$

where $c$ and $d$ are integration constants. Thus, we have

$$W' = \pm\sqrt{\frac{2}{k_1}W^3 - \frac{V_p}{{k_1}^3}W^2 - \frac{2c}{{k_1}^3}W - \frac{2d}{{k_1}^3}}, \quad (26)$$

Eq. 26 can be written for the plus sign as follows (similar calculation can be also done for negative sign):

$$\int \frac{dW}{\sqrt{\frac{2}{k_1}W^3 - \frac{V_p}{{k_1}^3}W^2 - \frac{2c}{{k_1}^3}W - \frac{2d}{{k_1}^3}}} = \int d\eta\,. \quad (27)$$

By the integrating right side of this equation, we get

$$\int \frac{dW}{\sqrt{\frac{2}{k_1}W^3 - \frac{V_p}{{k_1}^3}W^2 - \frac{2c}{{k_1}^3}W - \frac{2d}{{k_1}^3}}} = \eta + e\,. \quad (28)$$

where $e$ is a new arbitrary integration constant. By the integrating left side of this equation, we choose integration constants $c = d = e = 0$. In this case Eq. 28 can be written as

$$\int \frac{dW}{\sqrt{\frac{2}{k_1}W^3 - \frac{V_p}{{k_1}^3}W^2}} = \eta\,. \quad (29)$$

By rearranging Eq. 29 we get

$$k_1\sqrt{k_1}\int \frac{dW}{W\sqrt{2{k_1}^2 W - V_p}} = \eta\,. \quad (30)$$

**Case I:** Changing the variable $2{k_1}^2 W - V_p = X^2$ in Eq. 30 and integrating once and then solving $W$, we get

$$W(\eta) = \frac{V_p\left(1 + tan^2\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right]\right)}{2{k_1}^2}. \quad (31)$$

Finally, by the integrating Eq. 31, we get the analytic solution of the BLMP equation given below

$$U(\eta) = \sqrt{\frac{V_p}{k_1}}\, tan\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right] \quad (32)$$

where $\eta = k_1 x + k_2 y + V_p t$. The solution given in Eq. 32 is same as Eq. 33 in Ref.[2].

**Case II:** Changing the variable $W = Sinx$ in Eq 30 and integrating once and then solving $W$, we have

$$W(\eta) = \frac{V_p\left(csc^2\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right]\right)}{2{k_1}^2}. \quad (33)$$

Finally, we integrate $W$ to obtain the analytic solutions of the BLMP equation given in Eq. 2.

$$U(\eta) = -\sqrt{\frac{V_p}{k_1}}\, cot\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right], \quad (34)$$

where $\eta = k_1 x + k_2 y + V_p t$. The solution found here Eq. 34 is same as Eq.34 in Ref.[2].

**Solutions of the BLMP Equation by Using $\left(\frac{G'}{G}\right)$-expansion Method**

Eq. 2 was studied in Ref. [43] by using the $\left(\frac{G'}{G}\right)$-expansion method with the integration constant $c = 0$. In this section, we do not only study Eq. 2 by using the $\left(\frac{G'}{G}\right)$-expansion method in the case $c \neq 0$, but we will also solve Eq. 2 by using three different forms of the $\left(\frac{G'}{G}\right)$-expansion methods. Finally, we compare our results with the existing ones in the literature.

Considering the homogeneous balance between the terms $W''$ and $W^2$ in Eq. 33, we have reached the following form of solution

$$W(\eta) = \alpha_2\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) + \alpha_0\,, \quad \alpha_2 \neq 0. \quad (35)$$

Substituting of the Eq. 35 and its derivatives in Eq. 33 yields a set of simultaneous algebraic equations for $\alpha_2, \alpha_1, \alpha_0, c, V_p, k_1, \lambda$ and $\mu$ as follows

$$\left(\frac{G'}{G}\right)^4: \alpha_2(-\alpha_2+2k_1)=0,$$

$$\left(\frac{G'}{G}\right)^3: (-3\alpha_2\alpha_1+5\alpha_2k_1\lambda+\alpha_1k_1)=0,$$

$$\left(\frac{G'}{G}\right)^2: \left(-6\alpha_2\alpha_0{k_1}^2+4\alpha_2{k_1}^3\lambda^2+8\alpha_2{k_1}^3\mu+\alpha_2V_p-3{\alpha_1}^2{k_1}^2+3\alpha_1{k_1}^3\lambda\right)=0,$$

$$\left(\frac{G'}{G}\right)^1: \left(6\alpha_2{k_1}^3\lambda\mu-6\alpha_1\alpha_0{k_1}^2+\alpha_1{k_1}^3\lambda^2+2\alpha_1{k_1}^3\mu+\alpha_1V_p\right)=0,$$

$$\left(\frac{G'}{G}\right)^0: \left(2\alpha_2{k_1}^3\mu^2+\alpha_1{k_1}^3\lambda\mu-3{\alpha_0}^2{k_1}^2+\alpha_0V_p+c\right)=0.$$

By solving the algebraic equations given above, we get following set of solutions

$$\alpha_2=2k_1,\ \ \alpha_1=2k_1\lambda,\ \ \alpha_0=\frac{{k_1}^3\lambda^2+8{k_1}^3\mu+V_p}{6{k_1}^2},$$
$$\lambda^2-4\mu=\pm\frac{\sqrt{12c{k_1}^2+{V_p}^2}}{{k_1}^3} \tag{36}$$

where $c$, $\lambda$ and $\mu$ are arbitrary constants. By using the Eq. 36 in Eq. 35, we get the following analytic solutions of Eq 23

$$W(\eta)=2k_1\left(\frac{G'}{G}\right)^2+2k_1\lambda\left(\frac{G'}{G}\right)+\frac{{k_1}^3\lambda^2+8{k_1}^3\mu+V_p}{6{k_1}^2} \tag{37}$$

where $\eta=k_1x+k_2y+V_pt$, $\lambda^2-4\mu=\pm\frac{\sqrt{12c{k_1}^2+{V_p}^2}}{{k_1}^3}$, $c_1$ and $c_2$ are arbitrary constants. Finally, we integrate the Eq. 37 to obtain the analytic solutions of the BLMP equation and find the hyperbolic, trigonometric and rational function solutions of Eq.2 given as follows

**First Type:** $\lambda^2-4\mu>0,$

$$U(\eta)=\frac{A(-{c_1}^2+{c_2}^2)+sinh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right]}{{c_1}^2cosh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right]+c_1c_2sinh\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right]}+B\eta, \tag{38}$$

where $A=k_1\sqrt{\lambda^2-4\mu}, B=\frac{1}{6{k_1}^2}\left[V_p\pm\sqrt{12c{k_1}^2+{V_p}^2}\right]$, $\lambda^2-4\mu=\pm\frac{\sqrt{12c{k_1}^2+{V_p}^2}}{{k_1}^3}$, $\eta=k_1x+k_2y+V_pt$, $c_1$ and $c_2$ are arbitrary constants.

If we choose the constants as $c=0$, $c_2=0$, and $c_1\neq 0$ in Eq. 38, the following solutions are obtained

$$U_1(\eta)=\mp\sqrt{-\frac{V_p}{k_1}}\,tanh\left[\frac{1}{2}\sqrt{-\frac{V_p}{{k_1}^3}}\eta\right], \tag{39}$$

$$U_2(\eta)=\mp\sqrt{\frac{V_p}{k_1}}\,tanh\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right]+\frac{V_p}{3{k_1}^2}\eta, \tag{40}$$

where $\mp\frac{V_p}{k_1}>0$ and $\eta=k_1x+k_2y+V_pt$. Eq. 39 is the same with the solution given in Ref. [2] as Eq. 21. But the solution given in Eq. 40 is a new solution.

**Second Type:** $\lambda^2-4\mu<0,$

$$U(\eta)=\frac{A({c_1}^2+{c_2}^2)+sin\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right]}{{c_1}^2cos\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right]+c_1c_2sin\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right]}+B\eta, \tag{41}$$

where $A=k_1\sqrt{4\mu-\lambda^2}, B=\frac{1}{6{k_1}^2}\left[V_p\pm\sqrt{12c{k_1}^2+{V_p}^2}\right]$, $4\mu-\lambda^2=\mp\frac{\sqrt{12c{k_1}^2+{V_p}^2}}{{k_1}^3}$, $\eta=k_1x+k_2y+V_pt$, $c_1$ and $c_2$ are arbitrary constants.

If we choose the constants as $c=0$, $c_2=0$ and $c_1\neq 0$ in Eq. 41, the following solutions are obtained

$$U_1(\eta)=\mp\sqrt{\frac{V_p}{k_1}}\,tan\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right], \tag{42}$$

$$U_2(\eta)=\mp\sqrt{-\frac{V_p}{k_1}}\,tan\left[\frac{1}{2}\sqrt{-\frac{V_p}{{k_1}^3}}\eta\right]+\frac{V_p}{3{k_1}^2}\eta, \tag{43}$$

where $\mp\frac{V_p}{k_1}>0$ and $\eta=k_1x+k_2y+V_pt$. Eq. 42 is the same with the solution given in Ref. [2] as Eq. 21. But we find a new solution given in Eq. 43.

**Third Type:** $\lambda^2-4\mu=0,$

$$U(\eta)=\frac{V_p}{6{k_1}^2}\left(\frac{c_1}{c_2}+\eta\right)-2k_1\left(\frac{c_2}{c_1+c_2\eta}\right), \tag{44}$$

where $\eta=k_1x+k_2y+V_pt$, $V_p=\pm 2k_1\sqrt{-3c}$, $c$, $c_1$ and $c_2$ are arbitrary constants.

If we choose the constants as $c=0$, $c_1=0$ and $c_2\neq 0$ in Eq. 44, the following solution are obtained

$$U_1(\eta)=-\frac{2k_1}{\eta}, \tag{45}$$

where $\eta=k_1x+k_2y$.

**Solutions of the BLMP Equation by Using Different Form of the $\left(\frac{G'}{G}\right)$-expansion Method**

Considering the homogeneous balance between the terms, $W''$ and $W^2$, in Eq. 23, we have reached the following form of solution

$$W(\eta)=\alpha_2\left(\frac{G'}{G}\right)^2+\alpha_1\left(\frac{G'}{G}\right)+\alpha_{-1}\left(\frac{G'}{G}\right)^{-1}+\ \alpha_{-2}\left(\frac{G'}{G}\right)^{-2}. \tag{46}$$

Substituting of the Eq. 46 and its derivatives in Eq. 23 yields a set of simultaneous algebraic equations for $\alpha_2, \alpha_1, \alpha_{-1}, \alpha_{-2}, c,\ V_p, k_1$ and $\mu$. Solving the algebraic equations, the following results can be found

$$\begin{aligned} &\textbf{Case 1}:\ \alpha_2 = 2k_1, \alpha_1 = 0, \alpha_{-1} = 0, \alpha_{-2} = \frac{V_p^{\,2}}{32{k_1}^5}, \mu = -\frac{V_p}{8{k_1}^3}, c = \frac{V_p^{\,2}}{4{k_1}^2}, \\ &\textbf{Case 2}:\ \alpha_2 = 2k_1, \alpha_1 = 0, \alpha_{-1} = 0, \alpha_{-2} = 0, \mu = -\frac{V_p}{8{k_1}^3}, c = -\frac{V_p^{\,2}}{16{k_1}^2}. \end{aligned} \quad (47)$$

Substituting Eq. 47 in Eq. 46 and the corresponding solution of ODE given in Eq. 10, we obtain the analytic solutions of the Eq. 23 given below.

$$\begin{aligned} W_1(\eta) &= 2k_1\left(\frac{G'}{G}\right)^2 + \frac{c}{32{k_1}^5}\left(\frac{G'}{G}\right)^{-2},\ \mu = -\frac{V_p}{8{k_1}^3}, c = \frac{V_p^{\,2}}{4{k_1}^2}, \\ W_2(\eta) &= 2k_1\left(\frac{G'}{G}\right)^2,\ \mu = -\frac{V_p}{8{k_1}^3}, c = -\frac{V_p^{\,2}}{16{k_1}^2}, \end{aligned} \quad (48)$$

where $W_1(\eta)$ and $W_2(\eta)$ are solutions of the Case1 and Case2, respectively. Then, integrating $W_1$ and $W_2$ (using the transformation $U' = W$), we get the following three types of solutions of Eq. 2.

**Type I:** $-\mu < 0$,

$$\begin{aligned} U_1(\eta) &= \frac{A\, sinh\left[\sqrt{\frac{V_p}{2{k_1}^3}}\eta\right] + B\, cosh\left[\sqrt{\frac{V_p}{2{k_1}^3}}\eta\right]}{C\, cosh\left[\sqrt{\frac{V_p}{2{k_1}^3}}\eta\right] + D\, sinh\left[\sqrt{\frac{V_p}{2{k_1}^3}}\eta\right]}, \\ U_2(\eta) &= \sqrt{\frac{V_p}{2k_1}}\left(\frac{(-{c_1}^2 + {c_2}^2)\, sinh\left[\sqrt{\frac{V_p}{8{k_1}^3}}\eta\right]}{{c_1}^2\, cosh\left[\sqrt{\frac{V_p}{8{k_1}^3}}\eta\right] + c_1c_2\, sinh\left[\sqrt{\frac{V_p}{8{k_1}^3}}\eta\right]}\right) + \frac{V_p}{4{k_1}^2}\eta, \end{aligned} \quad (49)$$

where $A = \sqrt{\frac{V_p}{k_1}}{k_1}^2\left(\sqrt{2}{c_1}^4 - 2\sqrt{2}{c_1}^2{c_2}^2 + \sqrt{2}{c_2}^4 + {c_1}^3c_2\sqrt{\frac{V_p}{{k_1}^3}}\eta + c_1{c_2}^3\sqrt{\frac{V_p}{{k_1}^3}}\eta\right)$, $B = 2{c_1}^2{c_2}^2V_p\eta$, $C = 4{c_1}^2{c_2}^2{k_1}^2$, $D = 2c_1c_2{k_1}^2({c_1}^2 + {c_2}^2), V_p, c_1, c_2$ and $k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt$.

If we choose the constants as $c_1 = c_2 \neq 0$ in the first equation ($U_1(\eta)$) of Eq. 49, the following solution is obtained

$$U_1(\eta) = \frac{V_p}{2{k_1}^2}\eta, \quad (50)$$

where $\eta = k_1x + k_2y + V_pt.$ It is a new solution.

If we choose the constants as $c_1 \neq 0$ and $c_2 = 0$ in the second equation ($U_2(\eta)$) of Eq. 49, the following new solution is obtained

$$U_2(\eta) = -\sqrt{\frac{V_p}{2k_1}}\ tanh\left[\frac{1}{2}\sqrt{\frac{V_p}{2{k_1}^3}}\eta\right] + \frac{V_p}{4{k_1}^2}\eta, \quad (51)$$

where $\eta = k_1x + k_2y + V_pt.$

**Type II:** $-\mu < 0,$

$$\begin{aligned} U_3(\eta) &= \frac{A\, sin\left[\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right] + B\, cos\left[\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right]}{C\, cos\left[\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right] + D\, sin\left[\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right]}, \\ U_4(\eta) &= \sqrt{-\frac{V_p}{2k_1}}\left(-\frac{({c_1}^2 + {c_2}^2)\, sin\left[\sqrt{-\frac{V_p}{8{k_1}^3}}\eta\right]}{{c_1}^2\, cos\left[\sqrt{-\frac{V_p}{8{k_1}^3}}\eta\right] + c_1c_2\, sin\left[\sqrt{-\frac{V_p}{8{k_1}^3}}\eta\right]}\right) + \frac{V_p}{4{k_1}^2}\eta, \end{aligned} \quad (52)$$

where $A = \sqrt{-\frac{V_p}{k_1}}{k_1}^2\left(\sqrt{2}{c_1}^4 + 2\sqrt{2}{c_1}^2{c_2}^2 + \sqrt{2}{c_2}^4 + {c_1}^3c_2\sqrt{-\frac{V_p}{{k_1}^3}}\eta - c_1{c_2}^3\sqrt{-\frac{V_p}{{k_1}^3}}\eta\right)$, $B = -2{c_1}^2{c_2}^2V_p\eta$, $C = -4{c_1}^2{c_2}^2{k_1}^2$, $D = 2c_1c_2{k_1}^2({c_1}^2 - {c_2}^2), V_p, c_1, c_2$ and $k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt.$

If we choose the constants as $c_1 = c_2 \neq 0$ in the first equation ($U_3(\eta)$) of Eq. 52, the following new solution is obtained

$$U_3(\eta) = -\sqrt{-\frac{2V_p}{k_1}}\ tan\left[\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right] + \frac{V_p}{2{k_1}^2}\eta, \quad (53)$$

where $\eta = k_1x + k_2y + V_pt.$

If we choose the constants as $c_1 \neq 0$ and $c_2 = 0$ in the second equation ($U_4(\eta)$) of Eq. 52, the following new solution is obtained

$$U_4(\eta) = -\sqrt{-\frac{V_p}{2k_1}}\ tan\left[\frac{1}{2}\sqrt{-\frac{V_p}{2{k_1}^3}}\eta\right] + \frac{V_p}{4{k_1}^2}\eta, \quad (54)$$

where $\eta = k_1x + k_2y + V_pt.$

**Type III:** $-\mu = 0,$

$$U_5(\eta) = -\frac{2k_1c_2}{c_1 + c_2\eta}, \quad (55)$$

where $c_1$, $c_2$, $k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt.$

If we choose the constants as $c_1 = 0$ in equation Eq. 55, we reach the following solution

$$U_5(\eta) = -\frac{2k_1}{\eta}, \quad (56)$$

where $\eta = k_1x + k_2y + V_pt.$

**Solutions of the BLMP Equation by Using $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion Method**

Balancing the terms, $W''$ and $W^2$ in Eq. 23, we have the following form of solution

$$W(\eta) = a_0 + a_1\varphi + a_2\varphi^2 + b_1\psi + b_2\psi\varphi. \quad (57)$$

Substituting the Eq. 57 and its derivatives into Eq. 23 and then using the Eq. 13 with Eqs. 14, 15, 16, we get the simultaneous algebraic equations for $a_0$, $a_1$, $a_2$, $b_1$, $b_2$, $k_1$, $c$, $\lambda$, $\mu$. $\nu$, $c_1$ and $c_2$ in the cases $\lambda < 0$, $\lambda > 0$ and $\lambda = 0$. Then, by solving the algebraic equations for each case, we can obtain the following solutions.

**Case I:** $\lambda < 0$

$$a_0 = \frac{5k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1}, a_1 = 0, a_2 = k_1,$$
$$b_1 = -k_1\mu, b_2 = \mp\sqrt{\frac{-\lambda^2\nu - \mu^2}{\lambda}}k_1,\ V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1. \quad (58)$$

Substituting Eq. 58 into Eq. 57 we have the solution of the Eg. 23.

$$W(\eta) = \frac{5k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1} + k_1\varphi^2 - k_1\mu\psi \mp \sqrt{\frac{-\lambda^2\nu - \mu^2}{\lambda}}k_1\psi\varphi \quad (59)$$

where $V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1$. Integrating Eq. 59, we get the following analytic solution of the BLMP equation as given in Eq.2.

$$U(\eta) = \frac{A\,sinh[\sqrt{-\lambda}\eta] + B}{\mu c_2 + \lambda c_2^2\,cosh[\sqrt{-\lambda}\eta] + \lambda c_1 c_2 sinh[\sqrt{-\lambda}\eta]} + C\eta, \quad (60)$$

where $A = k_1\lambda\sqrt{-\lambda}\,(c_2^2 - c_1^2)$, $B = \mp k_1\sqrt{-\lambda}\left[c_2\sqrt{\mu^2 + \lambda^2(c_1^2 - c_2^2)} + \mu c_1\right]$, $C = \frac{-k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1}$, $V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1$, $c, c_1, c_2, k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt$.

If we choose the constants as $c = 0$ and $c_1 = c_2 \neq 0$ in Eq. 60, we get the following new solutions

$$U_1(\eta) = \frac{2\mu\sqrt{-\frac{V_p}{k_1}}}{\mu \mp c_1\frac{V_p}{k_1^3}\left(cosh\left[\sqrt{-\frac{V_p}{k_1^3}}\eta\right] + sinh\left[\sqrt{-\frac{V_p}{k_1^3}}\eta\right]\right)}, \quad (61)$$

where $V_p < 0, k_1 > 0$ and $\eta = k_1x + k_2y + V_pt$.

$$U_2(\eta) = \frac{-2\mu\sqrt{\frac{V_p}{k_1}}}{\mu \mp c_1\frac{V_p}{k_1^3}\left(cosh\left[\sqrt{\frac{V_p}{k_1^3}}\eta\right] + sinh\left[\sqrt{\frac{V_p}{k_1^3}}\eta\right]\right)} + \frac{V_p}{3k_1^2}\eta, \quad (62)$$

where $V_p < 0, k_1 < 0$ and $\eta = k_1x + k_2y + V_pt$.

**Case II:** $\lambda > 0$

$$a_0 = \frac{5k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1}, a_1 = 0, a_2 = k_1,$$
$$b_1 = -k_1\mu,\ b_2 = \mp\sqrt{\frac{\lambda^2\nu - \mu^2}{\lambda}}k_1, \quad (63)$$
$$V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1.$$

Substituting Eq. 63 into Eq. 67 we have the solution of the Eq. 23.

$$W(\eta) = \frac{5k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1} + k_1\varphi^2 - k_1\mu\psi \mp \sqrt{\frac{\lambda^2\nu - \mu^2}{\lambda}}k_1\psi\varphi \quad (64)$$

where $V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1$. Integrating Eq. 64, we get the following analytic solution of the BLMP equation as given in Eq. 2.

$$U(\eta) = \frac{A\,sin[\sqrt{\lambda}\eta] + B}{\mu c_2 + \lambda c_2^2\,cos[\sqrt{\lambda}\eta] + \lambda c_1 c_2 sin[\sqrt{\lambda}\eta]} + C\eta, \quad (65)$$

where $A = k_1\lambda\sqrt{\lambda}\,(c_1^2 + c_2^2)$, $B = \pm k_1\sqrt{\lambda}\left[c_2\sqrt{-\mu^2 + \lambda^2(c_1^2 + c_2^2)} + \mu c_1\right]$, $C = \frac{-k_1^2\lambda \mp \sqrt{-12c + k_1^4\lambda^2}}{6k_1}$, $V_p = \mp\sqrt{-12c + k_1^4\lambda^2}\,k_1$, $c, c_1, c_2, k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt$.

If we choose the constants as $c = 0$ and $c_1 = 0$, $c_2 \neq 0$ and $\mu = \lambda c_2$ in Eq. 65, we get the following new solutions

$$U_1(\eta) = -\frac{\sqrt{\frac{V_p}{k_1}}\,sin\left[\sqrt{\frac{V_p}{k_1^3}}\eta\right]}{1 + cos\left[\sqrt{\frac{V_p}{k_1^3}}\eta\right]}, \quad (66)$$

$$U_2(\eta) = \frac{\sqrt{-\frac{V_p}{k_1}}\,sin\left[\sqrt{-\frac{V_p}{k_1^3}}\eta\right]}{1 + cos\left[\sqrt{-\frac{V_p}{k_1^3}}\eta\right]} + \frac{V_p}{3k_1^2}\eta, \quad (67)$$

where $\eta = k_1x + k_2y + V_pt$.

**Case III:** $\lambda = 0$

$$a_0 = \mp\frac{\sqrt{-3c}}{3k_1}, a_1 = 0, a_2 = k_1,$$
$$b_1 = -k_1\mu,\ b_2 = \mp\sqrt{-2c_2\mu + c_1^2}k_1,\ V_p = \mp 2\sqrt{-3c}k_1. \quad (68)$$

Substituting Eq. 68 into Eq. 57 we have the solution of the Eq. 23.

$$W(\eta) = \mp\frac{\sqrt{-3c}}{3k_1} + k_1\varphi^2 - k_1\mu\psi \mp \sqrt{-2c_2\mu + {c_1}^2}k_1\psi\varphi, \quad (69)$$

where $V_p = \mp 2\sqrt{-3c}k_1$. Integrating Eq. 69, and then if we choose the integrating constants as $c = c_1 = c_2 = 0$, we get the following analytic solution of the BLMP equation given in Eq. 2.

$$U(\eta) = -\frac{2k_1}{\eta}, \quad (70)$$

where $\eta = k_1x + k_2y + V_pt$.

Eq. 70, Eq. 45 and Eq. 56 are the same solutions and they are compatible with the literatures.

**Solutions of the BLMP Equation by Using $\left(\frac{1}{G'}\right)$-expansion Method**

Balancing the terms, $W''$ and $W^2$ in Eq. 23, we get the following form of solution

$$W(\eta) = a_0 + a_1\left(\frac{1}{G'}\right) + a_2\left(\frac{1}{G'}\right)^2. \quad (71)$$

Substituting the Eq. 71 in Eq. 23 and collecting all coefficients with respect to $\left(\frac{1}{G'}\right)$ and equating them to zero we get the following system of equations.

$$\left(\frac{1}{G'}\right)^0 : \ -3{a_0}^2{k_1}^2 + a_0V_p + c = 0,$$

$$\left(\frac{1}{G'}\right)^1 : \ a_1(-6a_0{k_1}^2 + V_p + {k_1}^3\lambda^2) = 0,$$

$$\left(\frac{1}{G'}\right)^2 : \ -6a_0a_2{k_1}^2 + a_2V_p + 4a_2{k_1}^3\lambda^2 - 3{a_1}^2{k_1}^2 + 3a_1{k_1}^3\lambda\mu = 0,$$

$$\left(\frac{1}{G'}\right)^3 : \ 2{k_1}^2(-3a_2a_1 + 5a_2k_1\lambda\mu + a_1k_1\mu^2) = 0,$$

$$\left(\frac{1}{G'}\right)^4 : \ 3a_2{k_1}^2(-a_2 + 2k_1\mu^2) = 0.$$

Solving the system of equations given above, we get

$$a_0 = \frac{V_p + {k_1}^3\lambda^2}{6{k_1}^2}, \ a_1 = 2k_1\lambda\mu, \ a_2 = 2k_1\mu^2, \ \lambda = \pm\left(\pm\frac{12c{k_1}^2 + {V_p}^2}{{k_1}^6}\right)^{1/4}. \quad (72)$$

Substituting these solutions into Eq. 71, we obtain the following solution

$$W(\eta) = \frac{V_p + {k_1}^3\lambda^2}{6{k_1}^2} + 2k_1\lambda\mu\left(\frac{1}{G'}\right) + 2k_1\mu^2\left(\frac{1}{G'}\right)^2. \quad (73)$$

where $\left(\frac{1}{G'}\right) = \frac{\lambda}{-\mu + \lambda c_1\,[cosh(\lambda\eta) - sinh(\lambda\eta)]}, \lambda = \pm\left(\pm\frac{12c{k_1}^2 + {V_p}^2}{{k_1}^6}\right)^{1/4}$ and $c_1$ is an arbitrary integration constant.

Integrating Eq. 73, we get the following analytic solution of the BLMP equation given in Eq. 2.

$$U(\eta) = \frac{A\,sinh\left[\frac{\lambda}{2}\eta\right]}{B\,cosh\left[\frac{\lambda}{2}\eta\right] + C sinh\left[\frac{\lambda}{2}\eta\right]} + D\eta, \quad (74)$$

where $A = 4k_1\lambda^2\mu c_1, B = (\mu - \lambda c_1)^2, C = \mu^2 - \lambda^2{c_1}^2$, $D = \frac{V_p + {k_1}^3\lambda^2}{6{k_1}^2}, \ \lambda = \pm\left(\pm\frac{12c{k_1}^2 + {V_p}^2}{{k_1}^6}\right)^{\frac{1}{4}}, \ c, V_p, \mu, c_1, k_1$ are arbitrary constants and $\eta = k_1x + k_2y + V_pt$.

If we choose the integration constants as $c = 0$ and $\mu = -\lambda c_1$ in Eq. 74, we get the following solutions

$$U_1(\eta) = -\sqrt{\frac{V_p}{k_1}}\,tanh\left[\frac{1}{2}\sqrt{\frac{V_p}{{k_1}^3}}\eta\right] + \frac{V_p}{3{k_1}^2}\eta, \quad (75)$$

$$U_2(\eta) = \sqrt{-\frac{V_p}{k_1}}\,tanh\left[\frac{1}{2}\sqrt{-\frac{V_p}{{k_1}^3}}\eta\right]. \quad (76)$$

Eq. 75 is the same as with given in Eq. 40 and is a new solution. Eq. 76 is the same as with given in Eq. 39 and Eq. 21 in Ref. [2].

## CONCLUSION

The Boiti-Leon-Manna-Pempinelli nonlinear partial differential equation is analytically studied by using the five different techniques which are direct integration, (*G'* / *G*)-expansion, different form of the (*G'* / *G*)-expansion, two variable (*G'* / *G*, 1 / *G*)-expansion and (1 / *G'*)-expansion methods. Hyperbolic, trigonometric and rotational forms of solutions are obtained. Using these different methods we have not only produced the same solutions found in the literature, but also derived the new exact solutions of the differential equation.

Using the different methods we have found two new solutions from (*G'* / *G*)-expansion method, four new solutions from different form of the (*G'* / *G*)-expansion method, four new solutions from two variable (*G'* / *G*, 1 / *G*)-expansion method, and one new solution from (1 / *G'*)-expansion method.

Finally, it is important to mention here that the analytic solutions found from the different techniques can be used to understand the dynamics of the travelling waves and their nonlinear features seen in different physical systems.

## AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

## DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

## CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## ETHICS

There are no ethical issues with the publication of this manuscript.

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