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# **Research Article**

# Using systematic review and meta-analysis in order to obtain different empirical-informative prior for bayesian binomial proportion

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#### ABSTRACT

The Bayesian approach provides a direct and useful inference about parameters better so than the frequentist (likelihood-only based) approach. This is because Bayesian approach uses both sources of information: prior information and likelihood. The eliciting of prior information is important because of a visible impact on the posterior inference. The motivation of this study is to avoid the subjectivity in obtaining informative prior. In order to elicit informative priors, this study proposed using systematic reviews, and the meta-analysis which is a statistical synthesis of the results from a series of empirical studies. Even though the systematic review and meta-analysis may include publication bias, may give more objective information from expert opinion due to the publishing process. This study also aimed to present the impact of domestic information obtained from domestic systematic reviews and meta-analysis on estimation proportion. Systematic reviews and meta-analysis of proportion used in order to obtain discrete, histogram, and conjugate (Beta) informative priors. The effectiveness of the Bayesian inference of proposed different informative prior distributions compared within and between (all-domestic) prior distribution. The results revealed that the discrete and histogram priors were more effective than the conjugate and non-informative priors. On the other hand, the importance of using systematic reviews and meta-analysis for domestic studies was observed.

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# INTRODUCTION

The Bayesian approach was considered as an alternative to the frequentist approach until the 1980s. At the beginning of the 21<sup>st</sup> century, Bayesian-based approaches became more popular in science. Statistical inference in the frequentist approach is based solely on the information contained in data (likelihood) and neglecting prior knowledge that is expressed through a prior distribution, and thus considers parameters as fixed variables. In the Bayesian approach, statistical inference is based on the posterior distribution of the parameters obtained by combining both the prior distribution and likelihood using Bayes' theorem. In

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recent years, calculating posterior distribution was solved with using asymptotic methods via computer algorithms (e.g. Gibbs sampler, Metropolis-Hasting algorithm, and MCMC) in order to draw a random sample from the posterior distribution [1-2].

The determination of the prior distribution is important because it impacts the posterior inference. The importance of prior distribution is revealed when the sample size is small, or when the data supply contains only indirect information about the parameters [3].

More reliable methods aimed at synthesizing data and findings are systematic reviews and meta-analyses. As the name implies, a systematic review is a detailed and comprehensive search strategy of published literature on a particular topic. Meta-analysis collects information from multiple independent studies in order to obtain an average estimate. Different meta-analysis method exist depending on the statistic being reported [4].

The objective of this study is to propose to use systematic reviews and meta-analyses of proportion in order to obtain discrete, Beta, and histogram informative prior distributions. The effectiveness of the Bayesian inference of proposed different informative prior distributions compared within and between (all-domestic) prior distribution.

#### **BAYESIAN INFERENCE**

The Bayesian approach was derived from the application of Bayes' theorem, as developed by Thomas Bayes in the 1700s. The principle of Bayesian approach is using probabilities that are conditional on data in order to explain beliefs about given parameters. The Bayesian approach is a method that updates the beliefs about the parameters, given the data.

For  $\theta$  parameters and data, Bayes' theorem is expressed as.

 $g(\theta|data) = \frac{g(\theta) \times f(data|\theta)}{\int g(\theta) \times f(data|\theta)d\theta}$ 

where  $g(\theta)$ ,  $f(data|\theta)$ ,  $g(\theta|data)$  and

 $\int g(\theta) \times f(data|\theta) d\theta$  represents prior, likelihood,

posterior and normalising factor, respectively. The prior must not be evaluated from the data. Depending on the selecting prior, a closed form of Eq. (1) is known to exist only in a handful of simple cases. In other cases, either the numerical solution or computer algorithms (e.g. Gibbs sampler, Metropolis-Hasting algorithm, and MCMC) are required for integration [1].

The posterior distribution can either be strongly (subjective or informative prior) or minimally (objective or non-informative prior) impacted based on choosing the prior [2].

#### **Bayesian Inference for Binomial Proportion**

The binomial proportion is defined as a total number of successes in an *n* independent trail. Each trail has two possible outcomes. The conditional distribution of the observation *y*, the total number of successes in *n* trails given the parameter  $\theta$ , is binomial(*n*, $\theta$ ). The conditional probability function for *y* given  $\theta$  is given by,

$$f(y|\theta) = \binom{n}{y} \theta^{y} (1-\theta)^{n-y} \quad \text{for } y=1,...,n$$
(2)

Hold *y* fixed at the observed number of success, and let  $\theta$  vary over its possible values. The likelihood function of  $\theta$  is also given by Eq. (2).

In order to evaluate Bayesian inference for binomial proportion, previous knowledge or belief had needed to be assessed before the data observed from the survey. Given that the proportion ranged between (0,1), some values are much more likely than others. The knowledge or belief about proportion is quantified using prior distribution. Depending on the degree of knowledge or belief, there are different choices of priors for the proportion [5].

## **Informative Priors**

An informative prior dominates likelihood, and thus has a visible impact on the posterior distribution. Two main informative priors can be described: informative-empirical, which is based on using data from related experiments, and informative-non-empirical data, which is based on inherent reason in order to prefer certain values over others. This information should be appropriately summarized using prior distribution [2,6].

## a. Discrete prior

(1)

One of the forms of informative prior is a discrete prior. A discrete prior has two main advantages: one of which is the ease of specifying a prior probability distribution, the other is being straightforwardness in terms of computing the posterior and predictive distribution.

A list of plausible k values of  $\theta$ ,  $\theta_1, ..., \theta_k$  with respective

 $p(\theta_1),...,p(\theta_k)$ , where  $\sum_{j=1}^k p(\theta_j) = 1$  The posterior probabilities for  $\theta_j$  is given by;

$$p(\theta_j | y) = \frac{p(\theta_j) \times \sum_{i=1}^n f(y_i | \theta_j)}{\sum_{m=1}^k p(\theta_m) \times \sum_{i=1}^n f(y_i | \theta_m)}$$
(3)

The posterior probabilities are used for the inference of the parameter. The normalizing factor (denominator of Eq. (3)) ensures that the probabilities add to 1[7]. A discrete prior always yields a discrete posterior distribution [5].

#### b. Histogram prior

Another easily specified and well understood informative prior is the histogram prior. Other advantages of this prior include its not requiring any parametric assumption and its flexibility when it comes to quantified prior beliefs [5].

In order to evaluate a histogram prior for success probability, first the interval (0,1) is divided into predefined, non-overlapping or equal subintervals, and then the probability is assigned to each interval in accordance with researchers prior belief. The posterior distribution is calculated by multiplying these prior probabilities by the binomial likelihood, and normalizing the result [5].

## c. Conjugate prior

In case prior and posterior distributions stem from the same family, such priors are hence called conjugate prior. Conjugate priors result in closed-form solutions for the posterior distribution that enable either direct inference or the construction of efficient Markov chain Monte Carlo sampling algorithms [2].

The shape of binomial likelihood function of  $\theta$  is the same form of the beta(*a*,*b*). Since the proportion is a continuous parameter, a convenient prior distribution is the beta distribution.

$$g(\theta) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \theta^{a-1} (1-\theta)^{b-1} \text{ for } 0 \ " \ \theta \ " \ 1 \tag{4}$$

where *a* and *b* are the hyperparameters selected in order to express the researcher's prior beliefs about  $\theta$ . An appropriate choice of beta distribution may be based on the histogram plot. If the histogram plot has two or three peaks, instead of one then researchers might need a mixture of Betas in order to fit the data. The maximum likelihood, which method of moments that used the mean and variance or the "fitdistr" function from MASS could be used in estimating the hyperparameters. For a given mean (*m*) and standard deviation (*s*) the methods of moments are estimated *a* and *b* given in Eq.(5) and Eq. (6).

$$m = \frac{a}{a+b} \tag{5}$$

$$s^{2} = m(1-m)/(a+b+1)$$
(6)

The posterior density given by,

$$g(\theta | y) = \frac{\Gamma(n+a+b)}{\Gamma(y+a)\Gamma(n-y+b)} \theta^{y+a-1} (1-\theta)^{n-y+b-1}$$
(7)

The posterior obtained without integration [2, 8-9].

#### Non-informative Priors

Non-informative prior used when relatively little is known about the data. Non-informative priors have a minimal impact on the posterior distribution.

The most used non-informative priors are uniform (U(0,1) or Beta(1,1)) and Jeffery (Beta(0.5,0.5)). The uniform prior gives equal weights to all possible values of success probability. The Jeffrey prior is preferred to make

inferences about some transformation of  $\theta$  instead of  $\theta$  itself because the uniform prior is not "invariant under transformations". The posterior is a another member of the beta family (Beta(*y*+1,*n*-*y*+1) and Beta(*y*+0.5,*n*-*y*+0.5), respectively) and does not require integration [2, 5].

# **META-ANALYSIS OF BINOMIAL PROPORTION**

Meta-analysis can be useful in order to obtain a more accurate estimate of the success proportion. Based on the inverse variance method, the pooled proportion is an average that is calculated by weighting the variance of each study using a fixed/random effects model [10]. The selection between fixed and random effects models is based on whether the heterogeneity test is significant or not. If the heterogeneity test significant then a random effect model should be selected. Because the heterogeneity commonly arises when the studies obtained from the published literature, the selection of random effects model is more appropriate [4]. The individual study weights:

$$Var(p) = \frac{p(1-p)}{N}$$
(8)

where *p* is the proportion and *N* the population size.

Based on inverse variance method the pooled proportion estimate is given by,

$$P = \frac{\sum_{i} \frac{p_i}{\operatorname{var}(p_i)}}{\sum_{i} \frac{1}{\operatorname{var}(p_i)}}$$
(9)

with standard error,

$$SE(P) = \sum_{i} \frac{1}{\operatorname{var}(p_i)} \tag{10}$$

Assuming the  $p_i$  is a random sample from beta distribution with parameters a and b, the mean and variance of p are given in Eq. (5) and (6). The homogeneity among studies is measured by a+b. If a+b is large then the difference among studies will be slight enough that the distribution of the  $p_i$ will concentrate around the mean [11,12].

# PROPOSED METHOD: USING SYSTEMATIC REVIEWS AND META-ANALYSIS IN ORDER TO ELICIT EMPIRICAL-INFORMATIVE PRIOR

The motivation of this study is to avoid the subjectivity in obtaining informative prior. This study proposed using systematic reviews and meta-analysis to synthesize results from a series of empirical studies. Even though the systematic review and meta-analysis may include publication bias, due to the publication process it may give more objective information from expert opinion. Studies were identified by systematically searching electronic databases using relevant search terms and pre-specified criteria. The overarching aim of these methods is to limit bias or assumptions on particular topics, thus allowing researchers to have greater confidence in drawing conclusions [13]. This study also aimed to present the impact of domestic information obtained from domestic systematic reviews and meta-analysis on estimation proportion.

After implementing the systematic reviews and meta-analysis, the combined studies and these frequencies were used in order to obtain three informative priors mentioned before as follows:

## Discrete prior

- i. Using the studies as a list of plausible k values of  $\theta$ ,  $(\theta_1,...,\theta_k)$
- ii. Calculating the frequency of studies
- iii. Obtaining the probability of each value of  $\theta$ ,  $p(\theta_1),...,p(\theta_k)$  by dividing each frequency by the sum
- iv. Calculating the posterior probabilities

## Histogram prior

- i. Using the studies to determine the interval
- ii. Dividing the interval into subintervals
- iii. Calculating the frequency of studies in each subintervals
- iv. Obtaining the probability of each value by dividing each frequency by the sum
- v. Calculating the posterior probabilities

#### **Conjugate Prior-1**

- i. Applying meta-analysis to the studies
- ii. Obtaining mean and standard deviation for pooled proportion
- iii. Estimating the hyperparameters *a* and *b* by methods of moments
- iv. Calculating the posterior probabilities

## **Conjugate Prior-2**

- i. Using the studies to plot histogram
- ii. Fitting Beta or mixture of Betas distribution based on the peak number and obtaining the hyperparameters *a* and *b* via the "fitdistr" function from MASS
- iii. Calculating the posterior probabilities

## **DATA ANALYSIS**

Energy drinks (ED) are highly caffeinated beverages that often include a high level of sugar (or a sugar substitute) as well as herbal ingredients such as guarana (a naturally occurring form of caffeine). The consumption of energy drinks has increased over the past several decades. Recent studies suggest that young adults under age 30 purchase and consume energy drinks the most [14].

The data was obtained by conducting a survey among the high school students in the province of Giresun, Turkey. The questionnaire was applied in December 2015 with the permission of the Provincial Directorate of National Education. In order to observe the energy drink consumption and opinions of high school students, a questionnaire consisting of 29 questions was applied face to face on students from each grade level as much as possible from 8 high schools. Eight of the questionnaire questions are open-ended and the rest are closed-ended questions. Before the questionnaire was applied, a pilot study was conducted and necessary arrangements were made. A total of 581 students voluntarily participated as subjects for the study from eight separate high schools 360 students had stated having consumed energy drink at least once.

In order to determine the proportion of energy drink consumption, the Bayesian approach was considered. Informative-empirical priors were obtained using systematic reviews and meta-analysis.

Using the keyword "energy drink" and "stimulant drink", nine electronic bibliographic databases, reference lists of relevant studies, and internet searches were performed with the intent of identifying articles related to energy drinks. Cross-sectional design published in both English and Turkish language studies were considered for inclusion criteria provided that they reported raw data or proportion available for (ED) consumption. Repeated, studies that used same survey results, as well as case-control and experimental designed studies were excluded. A total of 629 studies were examined, 135 studies meeting the inclusion criteria. Eight studies were conducted in Turkey, while the remaining 127 studies were conducted in other countries. Most of the energy drink studies were conducted in the USA (56 studies).

Data analysis was carried out using R software. In order to implement discrete and histogram priors "pdisc", and "histprior" functions in the LearnBayes package was used; for meta-analysis of the proportion, the "metaprop" function in the meta package was used; and for fitting beta distribution "fitdistr" function from MASS, respectively.

## RESULTS

The survey revealed that 360 out of 581 high school students consume energy drinks with common  $\theta$ . The likelihood function of  $\theta$  is given by,

$$L(\theta) \propto \theta^{360} (1 - \theta)^{221} \tag{11}$$

The frequentist estimates of proportion and 95% confidence interval was 0.62 (95% CI, 0.58 to 0.66). The difference between the upper and the lower bound (range) was 0.08.

The Bayesian approach of the proportion of energy drink consumption was obtained for non-informative and informative priors, respectively. Table 1 summarized the results of Bayesian inference for non-informative priors (U(0,1) or Beta(1,1)) and Jeffery (Beta(0.5,0.5)).

The proportion of energy drink consumption was the same for both frequentist and non-informative Bayesian approaches.

To demonstrate the usefulness of the proposed method, 135 studies that obtained from Systematic reviews and meta-analysis, were used to construct four informative (discrete, histogram, conjugate-1, and conjugate-2) prior mentioned before. Eight of 135 studies were conducted in Turkey. Due to eight of the 135 studies and the survey data gathered from Turkey, in addition to comparing the effectiveness of proposed different informative priors, this study also aimed to present the impact of domestic information on estimation proportion. Hence, firstly four informative-empirical priors (discrete, histogram, conjugate-1, and conjugate-2) were constructed for 135 studies (all) and then for 8 studies (domestic). The results of four priors based on 135 and 8 studies are summarized in Table 2 and 3.

According to Table 2, while the common proportion of energy drink consumption for all (135) of the studies was 32%, it was 57% for the domestic (8) studies. Eight of the studies generated approximately 4% of the sample size. For the 135 studies, the proportion of energy drink consumption was ranged between 1% and 99%, with the highest frequency (6) being observed at 47%.

The discrete prior can easily be specified by determining the prior probability of the 135 plausible proportion values Fig (1 (a) upper). The prior probability was ranged between 0.7% and 4%. By dividing the same interval into ten equal subintervals, the frequency and the probability of each subinterval had increased. Hence, the probability of histogram prior ranged between 2% to 19% Fig (1 (c) upper). For the 8 domestic studies, the proportion of energy drink consumption had ranged between 33% and 78%, and each value of proportion was observed on the same frequency. The discrete prior was easily specified by determining the prior probability of eight plausible proportion values. The prior probability was equal (12.5%) Fig (1 (b) upper). Dividing the same interval into ten equal subintervals leads to the frequency of each subinterval and the probability of each subinterval increasing. Hence, the probability of histogram prior was ranged from 12.5% to 37.5% Fig (1 (d) upper). Figure 1 illustrates the plots of prior and posterior of discrete (a and b) and histogram priors (c and d) for 135 and 8 studies, respectively.

For the 135 studies, based on discrete prior, most of the posterior probability was concentrated on the values between p=0.58 and p=0.65 (Fig 1 (a) bottom). By combining the most likely values, the posterior probability was equal to 0.978. Based on the histogram, prior the posterior probability was concentrated on the value p=0.65 (Fig 1 (c) bottom). The posterior probability was equal to 0.986.

For the other 8 studies, based on discrete prior, most of the posterior probability was concentrated on the values between p=0.60 and p=0.62 (Fig 1 (b) bottom). Combining the most likely values, the posterior probability was equal to 0.998. Based on the histogram prior, the posterior probability was concentrated on the values p=0.65 (Fig 1 (d) bottom). The posterior probability was equal to 0.980.

 Table 1. Results of non-informative priors

| Sample Size | Number of Success | Prior               | Proportion | LB   | UB   | Range |
|-------------|-------------------|---------------------|------------|------|------|-------|
| 581         | 360               | U(0,1) or Beta(1,1) | 0.62       | 0.58 | 0.66 | 0.08  |
|             |                   | Beta(0.5,0.5)       | 0.62       | 0.58 | 0.66 | 0.08  |

 Table 2. Results of discrete and histogram priors obtained via systematic reviews and meta-analysis based on 135 and 8 studies

| Number of<br>Study | Sample Size | Number<br>of Success | Common<br>Proportion | Prior     | Interval of<br>Proportion | Interval of<br>Frequency |
|--------------------|-------------|----------------------|----------------------|-----------|---------------------------|--------------------------|
| 135                | 311220      | 99955                | 0.32                 | Discrete  | (0.01-0.99)               | (1-6)                    |
|                    |             |                      |                      | Histogram | (0.01-0.99)               | (3-26)                   |
| 8                  | 10982       | 6274                 | 0.57                 | Discrete  | (0.33-0.78)               | (1-1)                    |
|                    |             |                      |                      | Histogram | (0.33-0.78)               | (1-3)                    |

| Tab | le 3 | . Resu | lts of | conjugate | e priors o | btained | l via s | systemation | : reviews | and | meta-ana | lysis | based | l on 1 | 35 and | 18 s | tudi | es |
|-----|------|--------|--------|-----------|------------|---------|---------|-------------|-----------|-----|----------|-------|-------|--------|--------|------|------|----|
|     |      |        |        | 1 11      |            |         |         | /           |           |     |          |       |       |        |        |      |      |    |

| Number of<br>Study | Sample Size | Number<br>of Success | Common<br>Proportion | Prior       | Estimated a | Estimated b |
|--------------------|-------------|----------------------|----------------------|-------------|-------------|-------------|
| 135                | 311220      | 99955                | 0.32                 | conjugate-1 | 84.72       | 99.51       |
|                    |             |                      |                      | conjugate-2 | 1.57        | 1.83        |
| 8                  | 10982       | 6274                 | 0.57                 | conjugate-1 | 91.22       | 81.06       |
|                    |             |                      |                      | conjugate-2 | 7.15        | 6.35        |



**Figure 1.** Plots of prior and posterior distributions for discrete and histogram priors: (a) 135 studies for discrete prior and posterior, (b) 8 studies for discrete prior and posterior, (c) 135 studies for histogram prior and posterior and (d) 8 studies for histogram prior and posterior.

Table 3 summarized the estimated hyperparameters of beta distribution prior as based on conjugate-1 and conjugate-2. The estimated hyperparameters from the meta-analysis (conjugate-1) were larger than the fitted methods (conjugate-2). Because a+b was large within the meta-analysis, the difference among studies was slight, thus indicating homogeneity among studies.

The posterior distribution of  $\theta$  for conjugate priors is given by;

$$g(\theta|y) \propto \theta^{360+a-1} (1-\theta)^{221+b-1}$$
 (12)

Figure 2 illustrates the prior, likelihood, and posterior distributions for conjugate priors as based on 135 (a and b) and 8 (c and d) studies, respectively. It was observed that the priors obtained from the mean and standard deviation of pooled proportion by meta-analysis (a and c) were similar in shape to the posteriors. The fitted priors from the combined studies had higher kurtosis distribution than that of the meta-analysis.

For the 135 studies, the posterior probability was 0.58 (95% CI, 0.55 to 0.62) as based on conjugate-1 prior. Based on the conjugate-2 prior, the posterior probability was 0.62 (95% CI, 0.58 to 0.66).

For the 8 studies, the posterior probability was 0.60 (95% CI, 0.56 to 0.63) as based on conjugate-1 prior. Based on conjugate-2 prior, the posterior probability was 0.62 (95% CI, 0.58 to 0.66).

In order to compare the effectiveness of four priors, the results of the credible interval were summarized in Table 4 based on 135 (all) and 8 (domestic) studies.

Upon looking at Table 4 for the 135 (all) studies, one sees that even though the range of the credible interval was approximately the same for discrete prior and conjugate-1 and 2, the discrete prior gave a more accurate credible probability (0.978). The histogram prior was concentrated on point value (p=0.65) estimation with the largest credible probability (0.986). For the 8 (domestic) studies, besides the discrete prior gave narrower credible interval than conjugate-1 and 2, it was also a more accurate credible probability (0.998). Similar to the 135 (all) studies, the histogram prior was also concentrated on point value (p=0.65) estimation but with the second largest credible probability (0.98).

When the priors were compared between the 135 (all) studies and the 8 (domestic) studies, the conjugate-1 and 2 gave approximately the same credible interval range with the same credible probability (0.95). The discrete prior for the 8 (domestic) studies, gave a narrower credible interval range and larger credible probability than the 135 (all) studies. For both study number, the histogram prior was concentrated on point value (p=0.65) but gave a bit smaller credible probability for the 8 (domestic) studies.

Finally, when the proposed four priors (discrete, histogram, conjugate-1, and conjugate-2) were compared with



**Figure 2.** Plot of prior, likelihood, and posterior distributions for conjugate priors (a) 135 studies conjugate-1 (Beta(84.72, 99.51)), (b) 135 studies conjugate-2 (Beta(1.57, 1.83)), (c) 8 studies conjugate-1 (Beta(91.22, 81.06)) and (d) 8 studies conjugate-2 (Beta(7.15, 6.35)).

**Table 4.** Results of the credible interval of four priorsbased on 135 and 8 studies

| Studies<br>number | Prior Type                          | Credible<br>Interval | Credible<br>Probability |
|-------------------|-------------------------------------|----------------------|-------------------------|
| 135 (All)         | Discrete                            | (0.58-0.65)          | 0.978                   |
|                   | Histogram                           | (0.65-0.65)          | 0.986                   |
|                   | Conjugate-1<br>(Beta(84.72, 99.51)) | (0.55-0.62)          | 0.95                    |
|                   | Conjugate-2<br>(Beta(1.57, 1.83))   | (0.58-0.66)          | 0.95                    |
| 8                 | Discrete                            | (0.60-0.62)          | 0.998                   |
| (Domestic)        | Histogram                           | (0.65-0.65)          | 0.98                    |
|                   | Conjugate-1<br>(Beta(91.22, 81.06)) | (0.56-0.63)          | 0.95                    |
|                   | Conjugate-2<br>(Beta(7.15, 6.35))   | (0.58-0.66)          | 0.95                    |

the non-informative prior results, the conjugate-1 and 2 gave approximately the same credible interval range with the same credible probability (0.95). According to the discrete prior, while the credible interval range was approximately

the same for the 135 (all) studies, was the larger for the 8 (domestic) studies, and with a smaller credible probability. The histogram prior gave the largest credible probability.

As a result, the discrete and histogram priors gave more largest credible probability than the conjugate-1 and 2 and non-informative priors. There was anecdote evidence for big study number for histogram prior. The histogram prior gave a more accurate result for the 135 (all) studies. The discrete prior gave a more accurate result for the 8 (domestic) studies. The importance of using systematic reviews and meta-analysis for domestic studies was observed.

## CONCLUSION

One of the advantages of the Bayesian approach on the frequentist approach is taking into account both the source of information: the prior information, and the data information. The knowledge or belief is quantified by the prior distribution, the data include information about the process. In the case that no prior information is available, the prior distribution or non-informative prior ought to be defined as a minimal impact on the posterior distribution. If prior information is available on the other hand, it should be appropriately summarized by the prior distribution, known as the informative prior.

In general, informative prior is depending on the degree of knowledge or belief of the expert. Especially the probability prior to discrete and histogram priors is assigned arbitrarily. The proposed approach is aimed to obtain powerfully and consistency the informative prior to published studies. In order to achieve this aim and to avoid subjectivity in the assessment of the published studies, systematic reviews and meta-analysis were conducted. By conducting systematic reviews and meta-analysis, the combined studies and these frequencies were used in order to obtain four informative priors (discrete, histogram, conjugate-1, and conjugate-2).

The proportion of energy drink consumption was considered. The informative prior was obtained via systematic reviews and meta-analysis for proportion. The data was obtained by conducting a survey on 581 high school students in Giresun, Turkey. It was revealed that 360 students had consumed an energy drink at least once in their lifetime.

The Bayesian approach was applied to different numbers of studies in order to reveal the effectiveness of informative prior based on all and domestic systematic reviews and meta-analysis. The effectiveness of priors was compared within and between the study numbers (all-domestic).

The results showed that for both numbers of studies (135 (all) and 8 (domestic)) the discrete and histogram priors were more effective than the conjugate-1 and 2, and non-informative priors. There was anecdotal evidence for the discrete and histogram priors when compared according to study numbers. The histogram prior was observed that to be less effective for a small number of the studies (8 (domestic)) than for large numbers of studies (135 (all)). There were two reasons for this: First, as the number of studies increased, the numerical range of the proportion of energy drinks obtained from the studies changed, and thus affected the discrete and histogram prior distributions because they were obtained depending on the intervals. Second, because of the credible probability obtained by combining the most probable values; even though the histogram prior distribution gives a high probability value for a single value, while the discrete prior distribution performs better for the interval.

It was concluded that domestic systematic reviews and meta-analysis prior was more effective than all.

#### **AUTHORSHIP CONTRIBUTIONS**

Concept: E.A.; Design: E.A.; Supervision: E.A.; Materials: E.A.; Data: E.A.; Analysis: E.A.; Literature search: E.A.; Writing: E.A.; Critical revision: E.A.

## DATA AVAILABILITY STATEMENT

The published publication includes all graphics and data collected or developed during the study. Ana başlık olarak büyük harfle yazılıp, satır başından başlamalıdır.

# **CONFLICT OF INTEREST**

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. Ana başlık olarak büyük harfle yazılıp, satır başından başlamalıdır.

## ETHICS

There are no ethical issues with the publication of this manuscript. Ana başlık olarak büyük harfle yazılıp, satır başından başlamalıdır.

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