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Research Article

MINIMAL INTUITIONISTIC OPEN AND MAXIMAL INTUITIONISTIC OPEN SETS

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ABSTRACT

In intuitionistic topological space, we introduce the concepts of minimal intuitionistic open and maximal intuitionistic open sets. Also, we give some of their basic properties. Regarding these sets, we introduce and study some generalizations of intuitionistic continuous functions.
Keywords: Minimal intuitionistic open set, maximal intuitionistic open sets, minimal intuitionistic continuity, maximal intuitionistic continuity.

1. INTRODUCTION

Minimal open and maximal open sets were introduced and some applications of these sets were studied by Nakaoka and Oda in [9, 10]. Afterwards, these notions were handled on various topological spaces such as generalized topological spaces and soft topological spaces. Maximal μ-open and minimal μ-closed sets in generalized topological spaces were introduced by Roy and Sen in [11]. Also, soft minimal open and soft maximal open sets were defined and some types of continuity via these sets were obtained in [2, 5, 8].

The concept of intuitionistic set which is a generalization of an ordinary set and the specialization of an intuitionistic fuzzy set was given by Çoker in [3]. After that time, intuitionistic topological spaces were introduced in [4]. The subject like separation axioms, continuity, homeomorphism etc. were investigated on these spaces [1, 7, 12].

The aim of this paper is to introduce minimal open and maximal open sets in intuitionistic topological spaces. For this purpose, we investigate some fundamental properties of these sets. Then, we also introduce minimal intuitionistic continuous and maximal intuitionistic continuous functions and obtain several characterizations and properties of such functions. In addition, we give some examples and counterexamples to support this work.

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2. PRELIMINARIES

Definition 2.1. [3] Let X be a nonempty fixed set. An intuitionistic set (briefly, IS) S is an object having the form

$$S = (S_T, S_F)$$

where S_T and S_F are subsets of X such that $S_T \cap S_F = \emptyset$.

The intuitionistic empty set and intuitionistic whole set in X are defined by $\emptyset_I = (\emptyset, X)$ and $X_I = (X, \emptyset)$, respectively. $IS(X)$ is the set of all ISs in X.

Definition 2.2. [3] Let X be a nonempty set and let $S, T \in IS(X)$ and $(S_i)_{i\in I} \subset IS(X)$.

(1) $S \subset T$ if and only if $S_T \subset T_T$ and $S_F \supset T_F$.
(2) $S = T$ if and only if $S \subset T$ and $T \subset S$.
(3) $X_I \setminus S = (S_F, S_T)$.
(4) $S \cup T = (S_T \cup T_T, S_F \cap T_F)$.
(5) $S \cap T = (S_T \cap T_T, S_F \cup T_F)$.
(6) $\bigcup_{i\in I} S_i = (\bigcup_{i\in I}(S_i)_T, \bigcap_{i\in I}(S_i)_F)$.
(7) $\bigcap_{i\in I} S_i = (\bigcap_{i\in I}(S_i)_T, \bigcup_{i\in I}(S_i)_F)$.
(8) $S \setminus T = S \cap (X_I \setminus T)$.

Proposition 2.1. [6] Let $S, T, P \in IS(X)$. Then,

(1) $S \cup S = S, S \cap S = S$.
(2) $S \cup T = T \cup S, S \cap T = T \cap S$.
(3) $S \cup (T \cup P) = (S \cup T) \cup P, S \cap (T \cap P) = (S \cap T) \cap P$.
(4) $S \cup (T \cap P) = (S \cup T) \cap (S \cup P), S \cap (T \cup P) = (S \cap T) \cup (S \cap P)$.
(5) $S \cup (S \cap T) = S, S \cap (S \cup T) = S$.
(6) $X_I \setminus (S \cup T) = (X_I \setminus S) \cap (X_I \setminus T), X_I \setminus (S \cap T) = (X_I \setminus S) \cup (X_I \setminus T)$.
(7) $X_I \setminus (X_I \setminus S) = S$.
(8) i. $S \cup \emptyset_I = S, S \cap \emptyset_I = \emptyset_I$.
ii. $S \cup X_I = X_I, S \cap X_I = S$.
iii. $X_I \setminus X_I = \emptyset_I, X_I \setminus \emptyset_I = X_I$.

Remark 2.1. [6] In general, $S \cup (X_I \setminus S) \neq X_I$ and $S \cap (X_I \setminus S) \neq \emptyset_I$.

Let $IS_*(X) = \{S \in IS(X): S_T \cup S_F = X\}$ and $S \in IS_*(X)$. Then, $S \cup (X_I \setminus S) = X_I$ and $S \cap (X_I \setminus S) = \emptyset_I$.

Definition 2.3. [3] Let X be a nonempty set, $x \in X$ and let $S \in IS(X)$.

(1) The IS $x_I = (\{x\}, \{x\}^c)$ is called an intuitionistic point (briefly, IP) in X. $x_I \in S$ if and only if $x \in S_T$.

(2) The IS $x_{IV} = (\emptyset, \{x\}^c)$ is called a vanishing intuitionistic point (briefly, VIP) in X. $x_{IV} \in S$ if and only if $x \notin S_F$.

$IP(X)$ is the set of all intuitionistic points and vanishing intuitionistic points in X.

Proposition 2.2. [3] Let $(S_i)_{i\in I} \subset IS(X)$ and let $x \in X$.

(1) $x_I \in \bigcap_{i\in I} S_i$ (resp. $x_{IV} \in \bigcap_{i\in I} S_i$) if and only if $x_I \in S_i$ (resp. $x_{IV} \in S_i$) for each $i \in I$.

(2) $x_I \in \bigcup_{i\in I} S_i$ (resp. $x_{IV} \in \bigcup_{i\in I} S_i$) if and only if there exists $i \in I$ such that $x_I \in S_i$ (resp. $x_{IV} \in S_i$).

Proposition 2.3. [6] Le t $S, T \in IS(X)$. Then,

(1) $S \subset T$ if and only if $x_I \in S$ (resp. $x_{IV} \in S$) implies $x_I \in T$ (resp. $x_{IV} \in T$) for each $x \in X$.
(2) $S = T$ if and only if $x_I \in S$ (resp. $x_{IV} \in S$) if and only if $x_I \in T$ (resp. $x_{IV} \in T$) for each $x \in X$.

Definition 2.4. [4] Let X be a nonempty set and let $\tau \subset IS(X)$. Then, τ is called an intuitionistic topology (briefly, IT) on X if it satisfies the following axioms:

(1) $\emptyset_I, X_I \in \tau$,
(2) $G \cap H \in \tau$ for any $G, H \in \tau$,
(3) $\bigcup_{i \in I} G_i \in \tau$ for each $(G_i)_{i \in I} \subset \tau$.

Also, (X, τ) is called an intuitionistic topological space (briefly, ITS) and each element of τ is called an intuitionistic open set in X. The complement of any intuitionistic open set is called intuitionistic closed.

We will denote the set of all intuitionistic open sets in X containing x_I (resp. x_{IV}) by $\mathcal{U}(x_I)$ (resp. $\mathcal{U}(x_{IV})$).

Definition 2.5. [4] Let (X, τ) be an ITS and let $S \in IS(X)$. Then,

(1) $Icl(S) = \bigcap\{F : X_I \setminus F \in \tau$ and $S \subset F\}$ is an intuitionistic closure of S with respect to τ.
(2) $Iint(S) = \bigcup\{G : G \in \tau$ and $G \subset S\}$ is an intuitionistic interior of S with respect to τ.

Proposition 2.4. [4] Let (X, τ) be an ITS and let $S \in IS(X)$. Then,

(1) $Iint(X_I \setminus S) = X_I \setminus Icl(S)$ and $Icl(X_I \setminus S) = X_I \setminus Iint(S)$.
(2) S is an intuitionistic closed if and only if $S = Icl(S)$.
(3) S is an intuitionistic open if and only if $S = Iint(S)$.

Definition 2.6. [6] Let (X, τ) be an ITS, $x \in X$ and let $S \in IS(X)$. Then,

(1) x_I is called a τ_I-closure point of S if $S \cap G \neq \emptyset_I$ for each $G \in \mathcal{U}(x_I)$. Also $\tau_I\text{-}cl(S) = \bigcup\{x_I : S \cap G \neq \emptyset_I$ for each $G \in \mathcal{U}(x_I)\}$.
(2) x_{IV} is called a τ_{IV}-closure point of S if $S \cap G' \neq \emptyset_I$ for each $G' \in \mathcal{U}(x_{IV})$. Also $\tau_{IV}\text{-}cl(S) = \bigcup\{x_{IV} : S \cap G' \neq \emptyset_I$ for each $G' \in \mathcal{U}(x_{IV})\}$.

Proposition 2.5. [6] Let (X, τ) be an ITS, $x \in X$ and let $S \in IS(X)$. Then $\tau_I\text{-}cl(S) \subset Icl(S)$ and $\tau_{IV}\text{-}cl(S) \subset Icl(S)$.

Definition 2.7. [3] Let $f : X \to Y$ be a function, and let $S \in IS(X)$ and $P \in IS(Y)$. Then,

(1) $f(S) = (f(S)_T, f(S)_F)$ where $f(S)_T = f(S_T)$ and $f(S)_F = Y \setminus (f(X \setminus S_F))$.
(2) $f^{-1}(P) = (f^{-1}(P)_T, f^{-1}(P)_F)$ where $f^{-1}(P)_T = f^{-1}(P_T)$ and $f^{-1}(P)_F = f^{-1}(P_F)$.

Proposition 2.6. [3] Let (X, τ) and (Y, τ') be two ITSs, $f : (X, \tau) \to (Y, \tau')$ be a function and let $S \in IS(X)$. Then the following hold:

(1) $f(S) = \emptyset_I$ if and only if $S = \emptyset_I$ and $f(\emptyset_I) = \emptyset_I$.
(2) $f^{-1}(Y_I) = X_I$ and $f^{-1}(\emptyset_I) = \emptyset_I$.

Definition 2.8. [4] Let (X, τ) and (Y, τ') be two ITSs. Then a function, $f : (X, \tau) \to (Y, \tau')$ is said to be intuitionistic continuous if $f^{-1}(G)$ is an intuitionistic open set in X for each intuitionistic open set G in Y.

Definition 2.9. [7] Let (X, τ) be an ITS and let $S \in IS(X)$. Then, $\tau_S = \{G \cap S : G \in \tau\}$ is called the subspace topology on S and τ_S is an IT on S. The pair (S, τ_S) is called a subspace of (X, τ) and each member of τ_S is called an intuitionistic open set in S.

3. MINIMAL INTUITIONISTIC OPEN SETS

Definition 3.1. Let (X,τ) be an ITS. A proper intuitionistic nonempty open set G in X is said to be a minimal intuitionistic open set if any intuitionistic open set which is contained in G is $\emptyset_I$ or G.

Example 3.1.

(1) Let $X = \{1,2,3,4,5\}$ and consider IT τ on X given by

$$\tau = \{\emptyset_I, X_I, G_1, G_2, G_3, G_4\}$$

where $G_1 = (\{2,3,4\},\{5\})$, $G_2 = (\{4\},\{1\})$, $G_3 = (\{4\},\{1,5\})$, $G_4 = (\{2,3,4\},\emptyset)$. It is easy to observe that G_3 is a minimal intuitionistic open set in X.

(2) Let $X = \mathbb{N}^+$ and consider IT τ on X given by

$$\tau = \{\emptyset_I, X_I\} \cup \{S_n : n = 1,2,3,\dots\}$$

where $S_1 = (\emptyset,\{2,3,4,\dots\})$, $S_2 = (\{1\},\{3,4,5,\dots\})$, $S_3 = (\{1,2\},\{4,5,6,\dots\})$, $S_n = (\{1,2,3,\dots,n-1\},\{n+1,n+2,n+3,\dots\})$ $(n \geq 2)$. Then, S_1 is a minimal intuitionistic open set in X.

Lemma 3.1. Let (X,τ) be an ITS and $G,H \in IS(X)$. Then

(1) Let G be a minimal intuitionistic open set and H be an intuitionistic open set. Then, $G \cap H = \emptyset_I$ or $G \subset H$.

(2) Let G and H be minimal intuitionistic open sets. Then, $G \cap H = \emptyset_I$ or $G = H$.

Proof.

(1) Let H be an intuitionistic open set and $G \cap H \neq \emptyset_I$. Since $G \cap H \subset G$ and G is minimal intuitionistic open, we obtain $G \cap H = G$. That is $G \subset H$.

(2) Let $G \cap H \neq \emptyset_I$. Since G and H are minimal intuitionistic open sets, we have $H \subset G$ and $G \subset H$ by (1). Hence, $G = H$.

Proposition 3.1. Let (X,τ) be an ITS and $G,H,H' \in IS(X)$ and let G be a minimal intuitionistic open set. If $x_I \in G$ (resp. $x_{IV} \in G$), then $G \subset H$ (resp. $G \subset H'$) for each $H \in \mathcal{U}(x_I)$ (resp. $H' \in \mathcal{U}(x_{IV})$).

Proof. Let $H \in \mathcal{U}(x_I)$ and $G \not\subset H$. Then, we have $G \cap H \neq \emptyset_I$. This contradicts with Lemma 3.1 (1). The proof for x_{IV} can be done by similar way.

Proposition 3.2. Let (X,τ) be an ITS and $G,H,H' \in IS(X)$. Let G be a minimal intuitionistic open set. Then

$$G = \bigcap\{H : H \in \mathcal{U}(x_I)\} \text{ (resp. } G = \bigcap\{H' : H' \in \mathcal{U}(x_{IV})\}\text{)}$$

for each $x_I \in G$ (resp. $x_{IV} \in G$).

Proof. It is obvious by Proposition 3.1.

Theorem 3.1. Let (X,τ) be an ITS and let G,H,U be three minimal intuitionistic open sets in X such that $G \neq H$. If $U \subset G \cup H$, then either $U = G$ or $U = H$.

Proof. If $U = G$, then the proof is clear. Let $U \neq G$. By Lemma 3.1(2), we have $G \cap U = \emptyset_I$. Therefore, we obtain $U \cup H = U \cup (H \cup \emptyset_I) = U \cup \big(H \cup (G \cap U)\big) = (H \cup G) \cap (U \cup H) =$

$H \cup (G \cap U) = H$. Then $U \subset H$. Since U and H are minimal intuitionistic open sets, we get $U = H$.

Theorem 3.2. Let (X, τ) be an ITS and let G, H, U be three minimal intuitionistic open sets in X such that they are different from each other. Then, $G \cup H \not\subset G \cup U$.

Proof. Suppose that $G \cup H \subset G \cup U$. Then, $(G \cup H) \cap (H \cup U) \subset (G \cup U) \cap (H \cup U)$ implies $H \cup (U \cap G) \subset U \cup (H \cap G)$. By Lemma 3.1(2), we obtain $H \subset U$. Thus $H = U$ since H and U are minimal intuitionistic open sets. This contradicts our assumption. Hence, $G \cup H \not\subset G \cup U$.

Theorem 3.3. Let (X, τ) be an ITS and $G, S \in IS(X)$. If G is a minimal intuitionistic open set and $\emptyset_I \neq S \subset G$, then $Icl(G) = Icl(S)$.

Proof. Let G be a minimal intuitionistic open set and $\emptyset_I \neq S \subset G$. Then we have $Icl(S) \subset Icl(G)$. We must show that $Icl(G) \subset Icl(S)$. Let $x_I \in G$. By Proposition 3.1, we obtain $S = S \cap G \subset S \cap H$ for each $H \in \mathcal{U}(x_I)$. Then, $S \cap H \neq \emptyset_I$ and hence, $x_I \in \tau_I\text{-}cl(S)$. Thus, we get $x_I \in Icl(S)$. Similarly, we obtain $x_{IV} \in Icl(S)$ for $x_{IV} \in G$. Therefore, $G \subset Icl(S)$ implies $Icl(G) \subset Icl(S)$. Hence, we have $Icl(G) = Icl(S)$.

The following example shows that converse of Theorem 3.3 is not true, in general.

Example 3.2. Consider Example 3.1(1). For $G = (\{2,3,4\}, \{5\})$ and for each intuitionistic nonempty set S such that $S \subset G$, we obtain $Icl(G) = Icl(S) = X_I$. However G is not a minimal intuitionistic open set.

Theorem 3.4. Let (X, τ) be an ITS such that $\tau \subset IS_*(X)$ and G be an intuitionistic open set in X. G is a minimal intuitionistic open set if and only if $Icl(G) = Icl(S)$ for any $\emptyset_I \neq S \subset G$.

Proof.

($\Rightarrow$) It is obvious by Theorem 3.3.

($\Leftarrow$) Assume that G is not a minimal intuitionistic open set. Then, there exists a nonempty intuitionistic open set H such that $H \subset G$ and hence, there exists an element $x_I \in G$ such that $x_I \notin H$. Since $\tau \subset IS_*(X)$, we have $x_I \in X_I \setminus H$ and $IcI(\{x_I\}) \subset X_I \setminus H$. Then, we obtain $IcI(\{x_I\}) \neq Icl(G)$.

4. MAXIMAL INTUITIONISTIC OPEN SETS

Definition 4.1. Let (X, τ) be an ITS. A proper intuitionistic nonempty open set G in X is said to be a maximal intuitionistic open set if any intuitionistic open set which contains G is X_I or G.

Example 4.1.

(1) Consider Example 3.1(1), G_4 is a maximal intuitionistic open set in X.
(2) Let $X = \mathbb{N}^+$ and consider the IT τ on X in Counterexample 3.10[1] given by

$$\tau = \{X_I, \emptyset_I\} \cup \{S_n : n = 1, 2, 3, \dots\}$$

Where $S_1 = (\{2, 3, 4, \dots\}, \emptyset), S_2 = (\{3, 4, 5, \dots\}, \{1\}), S_3 = (\{4, 5, 6, \dots\}, \{1, 2\}), S_n = (\{n+1, n+2, n+3, \dots\}, \{1, 2, 3, \dots, n-1\})$ $(n \geq 2)$. Then, S_1 is a maximal intuitionistic open set in X.

Lemma 4.1. Let (X, τ) be an ITS and $G, H \in IS(X)$.

(1) Let G be a maximal intuitionistic open set and H be an intuitionistic open set. Then $G \cup H = X_I$ or $H \subset G$.

(2) Let G and H be maximal intuitionistic open sets. Then, $G \cup H = X_I$ or $G = H$.

Proof.

(1) Let H be an intuitionistic open set and $G \cup H \neq X_I$. Since G is a maximal intuitionistic open and $G \subset G \cup H$, then we get $G \cup H = G$. Hence $H \subset G$.

(2) Suppose that $G \cup H \neq X_I$. Since G and H are maximal intuitionistic open sets, they are also intuitionistic open sets. By (1), we have $G \subset H$ and $H \subset G$. Therefore, $G = H$.

Proposition 4.1. Let (X, τ) be an ITS and $G, H, H' \in IS(X)$. Let G be a maximal intuitionistic open set. If $x_I \in G$ (resp. $x_{IV} \in G$), then $G \cup H = X_I$ (resp. $G \cup H' = X_I$) or $H \subset G$ (resp. $H' \subset G$) for each $H \in \mathcal{U}(x_I)$ (resp. $H' \in \mathcal{U}(x_{IV})$).

Proof. It is clear from Lemma 4.1(1).

Proposition 4.2. Let (X, τ) be an ITS and $G, H, H' \in IS(X)$. Let G be a maximal intuitionistic open set. Then,

$$G = \bigcup\{H : H \in \mathcal{U}(x_I) \text{ such that } G \cup H \neq X_I\}$$
$$(\text{resp. } G = \bigcup\{H' : H' \in \mathcal{U}(x_{IV}) \text{ such that } G \cup H' \neq X_I\})$$

for each $x_I \in G$ (resp. $x_{IV} \in G$).

Proof. Since G is a maximal intuitionistic open set, the proof is obvious from Proposition 4.1.

Theorem 4.1. Let (X, τ) be an ITS and G, H, U be three maximal intuitionistic open sets in X such that $G \neq H$. If $G \cap H \subset U$, then either $G = U$ or $H = U$.

Proof. If $G = U$, then the proof is completed. Let $G \neq U$. Then, we have $H \cap U = H \cap (U \cap X_I) = H \cap (U \cap (G \cup H)) = H \cap ((U \cap G) \cup (U \cap H)) = H \cap (G \cup U) = H \cap X_I = H$ by Lemma 4.1(2). Thus, $H \subset U$. Since H and U are maximal intuitionistic open sets, we obtain $H = U$.

Theorem 4.2. Let (X, τ) be an ITS and G, H, U be three maximal intuitionistic open sets in X such that they are different from each other. Then, $G \cap H \not\subset G \cap U$.

Proof. Assume that $G \cap H \subset G \cap U$. Then, $(G \cap H) \cup (H \cap U) \subset (G \cap U) \cup (H \cap U)$ implies that $H \cap (G \cup H) \subset U \cap (G \cup H)$. By Lemma 4.1(2), we have $H \subset U$. Since H and U are maximal intuitionistic open sets, we obtain $H = U$. This is a contradiction. Therefore, $G \cap H \not\subset G \cap U$.

Theorem 4.3. Let (X, τ) be an ITS and $G, H, H' \in IS(X)$. Let G be a maximal intuitionistic open set and $x_I \in X_I \backslash G$. Then, $X_I \backslash G \subset H$ for each $H \in \mathcal{U}(x_I)$. If $\tau \subset IS_*(X)$ and $x_{IV} \in X_I \backslash G$, then $X_I \backslash G \subset H'$ for each $H' \in \mathcal{U}(x_{IV})$.

Proof. Let $x_I \in X_I \backslash G$. For each $H \in \mathcal{U}(x_I)$, we have $H \not\subset G$. By Lemma 4.1(1), we get $H \cup G = X_I$. Hence, $X_I \backslash G \subset H$. Let $x_{IV} \in X_I \backslash G$ and $\tau \subset IS_*(X)$. Similarly, we have $X_I \backslash G \subset H'$ by using Lemma 4.1(1).

Theorem 4.4. Let (X, τ) be an ITS such that $\tau \subset IS_*(X)$ and let G be a maximal intuitionistic open set in X. Then, $Icl(G) = X_I$ or $Icl(G) = G$.

Proof. Let G be a maximal intuitionistic open set in X. By Theorem 4.3, for each $x_I \in X_I \backslash G$ and $H \in \mathcal{U}(x_I)$, we have $X_I \backslash G \subset H$. Assume that $X_I \backslash G \subsetneq H$. Thus, $X_I \backslash G \neq H$ implies that $G \cap H \neq \emptyset_I$. Therefore, $x_I \in \tau_I\text{-}cl(G)$. Hence, $X_I \backslash G \subset \tau_I\text{-}cl(G) \subset Icl(G)$. Similarly, we have $x_{IV} \in \tau_{IV}\text{-}cl(G)$ implies that $X_I \backslash G \subset \tau_{IV}\text{-}cl(G) \subset Icl(G)$ for each $x_{IV} \in X_I \backslash G$. Since $X_I = G \cup (X_I \backslash G) \subset G \cup Icl(G) = Icl(G) \subset X_I$, then $Icl(G) = X_I$. Suppose $X_I \backslash G = H \neq X_I$. This implies that G is intuitionistic closed. So, $Icl(G) = G$.

Theorem 4.5. Let (X, τ) be an ITS such that $\tau \subset IS_*(X)$ and let G be a maximal intuitionistic open set in X. Then, $Iint(X_I \backslash G) = X_I \backslash G$ or $Iint(X_I \backslash G) = \emptyset_I$.

Proof. It is clear from Theorem 4.4.

Theorem 4.6. Let (X, τ) be an ITS and $\tau \subset IS_*(X)$. Let G be a maximal intuitionistic open set in X and S be an intuitionistic nonempty set in X such that $S \subset X_I \backslash G$. Then, $Icl(S) = X_I \backslash G$.

Proof. From Theorem 4.3, for each $x_I \in X_I \backslash G$ and for each $H \in \mathcal{U}(x_I)$, we have $X_I \backslash G \subset H$. Since $\emptyset_I \neq S \subset X_I \backslash G$, we get $S \cap H \neq \emptyset_I$. Thus, $x_I \in Icl(S)$. Hence, we obtain $X_I \backslash G \subset Icl(S)$. On the other hand, since $S \subset X_I \backslash G$ and $X_I \backslash G$ is intuitionistic closed, we have $Icl(S) \subset Icl(X_I \backslash G) = X_I \backslash G$. Therefore, $X_I \backslash G = Icl(S)$.

Corollary 4.1. Let (X, τ) be an ITS and $\tau \subset IS_*(X)$. Let G be a maximal intuitionistic open set in X and P be an intuitionistic set in X such that $G \subsetneq P$. Then, $Icl(P) = X_I$.

Proof. Assume $G \subsetneq P$. Then, there exists an intuitionistic nonempty set S in X such that $S \subset X_I \backslash G$ and $P = S \cup G$. So, $Icl(P) = Icl(S) \cup Icl(G) \supset (X_I \backslash G) \cup G = X_I$ by Theorem 4.6. Hence, $Icl(P) = X_I$.

Theorem 4.7. Let (X, τ) be an ITS and $G, P \in IS(X)$. Let G be a maximal intuitionistic open set and P be a proper intuitionistic set such that $G \subset P$. Then, $Iint\,(P) = G$.

Proof. Assume that $G = P$. Then, $Iint(G) = G = Iint(P)$. Otherwise, if $G \neq P$, then $G = Iint(G) \subset Iint(P)$. Since G is maximal intuitionistic open, $Iint(P) \subset G$. Thus, $Iint(P) = G$.

5. MINIMAL INTUITIONISTIC CONTINUOUS AND MAXIMAL INTUITIONISTIC CONTINUOUS FUNCTIONS

Definition 5.1.

(1) An ITS (X, τ) is said to be IT_{min} space if every proper intuitionistic nonempty open set in X is a minimal intuitionistic open set.

(2) An ITS (X, τ) is said to be IT_{max} space if every proper intuitionistic nonempty open set in X is a maximal intuitionistic open set.

Definition 5.2. Let (X, τ) and (Y, τ') be two ITSs. Then, a function $f: (X, \tau) \to (Y, \tau')$ is said to be

(1) minimal intuitionistic continuous if $f^{-1}(G)$ is an intuitionistic open set in X for each minimal intuitionistic open set G in Y.

(2) maximal intuitionistic continuous if $f^{-1}(H)$ is an intuitionistic open set in X for each maximal intuitionistic open set H in Y.

Theorem 5.1. Each intuitionistic continuous function is minimal intuitionistic continuous and maximal intuitionistic continuous.

Proof. Let (X, τ) and (Y, τ') be two ITSs and $f: (X, \tau) \to (Y, \tau')$ be intuitionistic continuous function. Assume that G is a minimal intuitionistic open set in Y. Hence, G is also an intuitionistic open set in Y. Since f is intuitionistic continuous, $f^{-1}(G)$ is an intuitionistic open set in X. Thus, f is minimal intuitionistic continuous. It is proved that every intuitionistic continuous function is maximal intuitionistic continuous by similar way.

The following example shows that the converse implications of Theorem 5.1 are not true, in general.

Example 5.1. Let $X = \mathbb{R}$ and τ be the usual intuitionistic topology on $\mathbb{R}$ which is generated by the family $\upsilon = \{((x, y), (-\infty, x'] \cup [y', \infty)): x, y, x', y' \in \mathbb{R}, x' \leq x, y' \leq y\}$ and let a function $f: (\mathbb{R}, \tau) \to (\mathbb{R}, \tau')$ be defined by

$$f(x) = \begin{cases} x + 4 & if \ x \geq 0 \\ x & if \ x < 0 \end{cases}$$

(1) Consider the intuitionistic topology $\tau' = \{G \subset \mathbb{R}: 3_I \in G \ or \ G = \emptyset_I\}$ on $\mathbb{R}$. Then, f is minimal intuitionistic continuous but it is not intuitionistic continuous.

(2) Consider the intuitionistic topology $\tau' = \{G \subset \mathbb{R}: 3_I \notin G \ or \ G = \mathbb{R}_I\}$ on $\mathbb{R}$. Then, f is maximal intuitionistic continuous but it is not intuitionistic continuous.

Theorem 5.2. Let (X, τ) and (Y, τ') be two ITSs and let $f: (X, \tau) \to (Y, \tau')$ be a function.

(1) If f is minimal intuitionistic continuous and Y is IT_{min} space, then f is intuitionistic continuous.

(2) If f is maximal intuitionistic continuous and Y is IT_{max} space, then f is intuitionistic continuous.

Proof.

(1) Assume that f is minimal intuitionistic continuous. Let G be a proper intuitionistic nonempty open set in Y. Since Y is IT_{min}, G is also a minimal intuitionistic open set. By hypothesis, $f^{-1}(G)$ is an intuitionistic open set in X. Furthermore, we have $f^{-1}(\emptyset_I) = \emptyset_I$, $f^{-1}(Y_I) = X_I$ and $\emptyset_I, X_I$ are intuitionistic open in X. Thus, f is intuitionistic continuous.

(2) The proof is similar to that of (1).

The following example shows that the composition of minimal intuitionistic continuous (resp. maximal intuitionistic continuous) functions need not to be a minimal intuitionistic continuous (resp. maximal intuitionistic continuous).

Example 5.2.

(1) Let $X = \{1, 2, 3, 4, 5\}$ and let $\tau_1 = \{\emptyset_I, X_I, H_1\}$, $\tau_2 = \{\emptyset_I, X_I, H_1, H_2\}$ and $\tau_3 = \{\emptyset_I, X_I, H_2, H_3, H_4, H_5\}$ be three ITs on X where $H_1 = (\emptyset, \{1, 3, 4, 5\})$, $H_2 = (\{4\}, \{1,5\})$, $H_3 = (\{2, 3, 4\}, \{5\})$, $H_4 = (\{4\}, \{1\})$ and $H_5 = (\{2, 3, 4\}, \emptyset)$. Consider two identity functions $f: (X, \tau_1) \to (X, \tau_2)$ and $g: (X, \tau_2) \to (X, \tau_3)$. Then, f and g are minimal intuitionistic continuous but $g \circ f$ is not minimal intuitionistic continuous.

(2) Let $X = \{1, 2, 3, 4\}$ and let $\tau_1 = \{\emptyset_I, X_I, U_1\}$, $\tau_2 = \{\emptyset_I, X_I, U_1, U_2\}$ and $\tau_3 = \{\emptyset_I, X_I, U_2, U_3\}$ be three ITs on X where $U_1 = (\{1\}, \{2, 3\})$, $U_2 = (\{1\}, \{2, 3, 4\})$, $U_3 = (\emptyset, \{2, 3, 4, 5\})$. Consider two identity functions $f: (X, \tau_1) \to (X, \tau_2)$ and $g: (X, \tau_2) \to (X, \tau_3)$. Then, f and g are maximal intuitionistic continuous but $g \circ f$ is not maximal intuitionistic continuous.

Theorem 5.3. Let (X, τ), (Y, τ') and (Z, τ'') be three ITSs and $f: (X, \tau) \to (Y, \tau')$ be intuitionistic continuous function.

(1) If $g: (Y, \tau') \to (Z, \tau'')$ is a minimal intuitionistic continuous function, then $g \circ f: (X, \tau) \to (Z, \tau'')$ is a minimal intuitionistic continuous function.

(2) If $g: (Y, \tau') \to (Z, \tau'')$ is a maximal intuitionistic continuous function, then $g \circ f: (X, \tau) \to (Z, \tau'')$ is a maximal intuitionistic continuous function.

Proof.

(1) Let G be a minimal intuitionistic open set in Z. Since g is minimal intuitionistic continuous, then $g^{-1}(G)$ is intuitionistic open in Y. Since f is intuitionistic continuous, then $f^{-1}((g^{-1}(G)) = (g \circ f)^{-1}(G)$ is intuitionistic open in X. Thus, $g \circ f$ is minimal intuitionistic continuous.

(2) It is similar to that of (1).

The following example shows that minimal intuitionistic continuity and maximal intuitionistic continuity are independent each other.

Example 5.3.

(1) Consider Example 5.2(1). Then, f is minimal intuitionistic continuous but it is not maximal intuitionistic continuous.

(2) Consider Example 5.2(2). Then, f is maximal intuitionistic continuous but it is not minimal intuitionistic continuous.

Theorem 5.4. Let (X, τ) and (Y, τ') be two ITSs and let S be an intuitionistic nonempty set in X.

(1) If $f: (X, \tau) \to (Y, \tau')$ is minimal intuitionistic continuous function, then $f|_S: (S, \tau_S) \to (Y, \tau')$ is minimal intuitionistic continuous function.

(2) If $f: (X, \tau) \to (Y, \tau')$ is maximal intuitionistic continuous function, then $f|_S: (S, \tau_S) \to (Y, \tau')$ is maximal intuitionistic continuous function.

Proof.

(1) Let f be minimal intuitionistic continuous and G be minimal intuitionistic open in Y. Then, $f^{-1}(G)$ is intuitionistic open in X. This implies that $f^{-1}|_S(G) = f^{-1}(G) \cap S$ is intuitionistic open in S. Thus, $f|_S$ is minimal intuitionistic continuous.

(2) The proof is similar to that of (1).

6. CONCLUSION

We introduced two types intuitionistic open sets and gave their basic properties. In addition, we defined two types intuitionistic continuities via these open sets and investigated their features. We hope that several properties of these concepts would be studied or new types of intuitionistic continuities would be defined.

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