



**Research Article**

**EVALUATION OF TOOTH PROFILE CHANGES OF SYMMETRIC AND ASYMMETRIC SPUR GEAR FORGING DIES DUE TO SHRINK FIT**

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**ABSTRACT**

The compressive pre-stress due to the shrink fitting cause dimensional changes on the tooth profile of the precision gear forging dies. The accuracy of the gear die is directly affecting the accuracy of the final product. Therefore, the dimensional variations due to shrink fit must be pre-determined and the gear tooth profile on the die insert modified accordingly. In this study, the change of tooth profile of the symmetric and asymmetric precision spur gear forging dies because of shrink fitting are analyzed by finite element (FE) method and the results are compared with the experimental ones. Both the (CMM) measurements and the finite element results of gear dies predict much higher radial displacements than the results of the cylindrical approach (Lame Equations). The amount of radial displacements along the tooth profiles of both symmetric and asymmetric gear dies are not uniform and changing from root to tip of the tooth. The radial displacements of the asymmetric gear forging die are higher than the symmetric one.

**Keywords:** Gear die, asymmetric gear, shrink fit, forging, involute profile.

**1. INTRODUCTION**

The precision gear forging technology has realized great commercial success in recent years. In the high volume commercial sectors such as automotive [1], the process has been found as an economical and reliable manufacturing method [2]. The straight and spiral bevel gears were manufactured at first then spur and helical gears have been forged with functional surfaces which can be finished in one operation [3-4]. The researchers are now mainly focused on the quality of the precision forged gears for high quality gear transmission applications such as turboprop gearboxes [5]. The pre-determination of the dimensional change of the die components during forging is one of the important problems in the process. During precision forging, very high loads are applied to the components of the die, so that they must withstand to high stresses and both thermal and mechanical fatigue [6]. The strength of inner die is generally increased by shrink fitting the inner die with one or more outer rings. By this way, a compressive pre-load on the inner die is created by interference between mating diameters of inner die and the outer ring. The

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compressive tangential stress applied by the outer ring can substantially reduce the tangential tensile stress imposed by forging load to the inner die

In the calculation of the required amount of interference between the die components (the inner die and the outer ring), thick wall cylinder approach can be used [7]. The equations of the thick wall cylinder approach were derived by Lamé [8]. Most of the previous studies used this approach by considering the gear forging die design by accepting “the die assembly as a short cylinder” [9]. In this approach, the gear profile is neglected and “the die is considered as a hollow cylinder with an inner diameter equal to the pitch diameter of the gear” [2, 6]. It was shown that the stresses exposed during gear forging is higher than the simple cylindrical shape and the cylindrical approach is not capable to determine the gear forging die dimensions and the amount of interference [10]. A new design procedure was developed for precision spur gear forging die considering the gear tooth profile and easy to use nomograms and a set of formula to calculate the sizes of die components and radial interference were recommended by Eyercioglu [11].

The compressive pre-stress exposed by the shrink fitting results in dimensional variations on the gear profile of the inner die. The accuracy of the gear die is directly affecting the dimensional accuracy of the final product. Therefore, the change of gear tooth profile because of should be pre-determined and it must be modified accordingly. Due to this, many researchers have worked on the dimensional accuracy of the gear forging dies [11-14].

Behrens [15] studied the influence of material characteristics on the shrinking characteristics hot forged helical gears. Kang [16-17] introduced a new tool design procedure to facilitate the surface finish operation and to increase the accuracy of the corner radii of gear forging die. The dimensional deviations of the gear tooth were analyzed by finite element simulations for cold extrusion gear die by Silveria [18-19] and the effect of different levels of pre-stressing was investigated. Lee [20-22] carried out finite element simulations and experimental studies on closed die upsetting of net-shape components to predict precisely the dimensions of the forged part and to determine the die dimensions.

Zuo [23] presented a theoretical model to predict involute profile deflection in hot precision forging of gears. Pederson [24] suggests two methods (classical plane analysis and a super element technique) to get a straight forward solution without iteration to determine the shrink fit surface shape due to contact pressure distribution. The dimensional variations of symmetric spur gear forging die due to shrink fit depending on the thick-wall cylinder approach were analyzed by Eyercioglu [25] and it was shown that the cylindrical approach is inadequate to predict the amount of radial displacement of the gear die.

In this study, the changes of tooth profiles of the symmetric and asymmetric precision spur gear forging dies because of shrink fitting are analyzed by finite element (FE) method and the results are compared with the experimental ones. The results are compared with the mostly used thick-wall approach and the design procedure suggested by Eyercioglu [11].

## 2. SHRINK FIT DESIGN

The general method of shrink fit design is using the thick-wall cylinder theory which is described by Lamé [8]. In this method, the interference pressure ( $p$ ) between inner die and the outer ring because of shrink fitting in terms of radial interference ( $z$ ), is calculated as:

$$p = \frac{Ez(b^2 - a^2)(c^2 - b^2)}{2b^3(c^2 - a^2)} \quad (1)$$

where ( $E$ ) is Young's Modulus, ( $a$ ) and ( $b$ ) are the inner and outer diameters of the die insert, respectively, and ( $c$ ) is the outer diameter of the ring. The die insert that is shrunk by the ring is subjected to the same pressure ( $p$ ) externally because of the shrink-fit between the die components (see Figure 1). The tangential (hoop) ( $\sigma_t$ ) and the radial stresses ( $\sigma_r$ ) on the die insert can be calculated as:

$$\sigma_t = -\frac{pb^2}{b^2 - a^2} \left( 1 + \frac{a^2}{r^2} \right) \tag{2}$$

$$\sigma_r = -\frac{pb^2}{b^2 - a^2} \left( 1 - \frac{a^2}{r^2} \right) \tag{3}$$

and the radial displacement ( $u_d$ ) is given as:

$$u_d = -\frac{pb^2}{E(b^2 - a^2)} \left[ (1 - \nu)r + (1 + \nu) \frac{a^2}{r} \right] \tag{4}$$

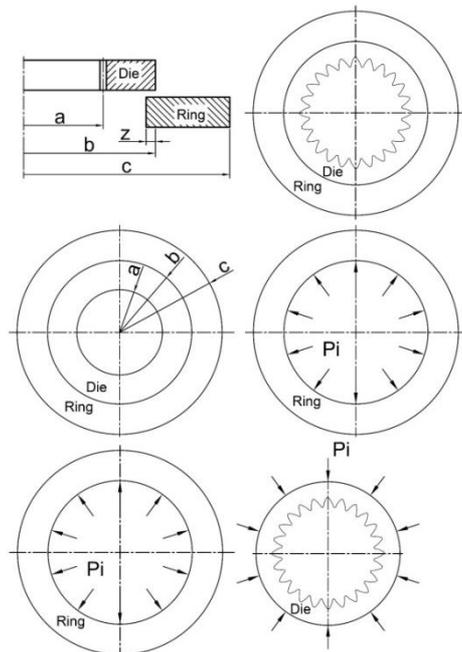
where ( $\nu$ ) is Poisson's ratio and the limits of the variable  $r$  changes between  $a$  and  $b$ .

For the outer ring similar equations can be used to calculate the stresses and the radial deflections ( $u_r$ ), here the limits of  $r$  varies between  $b$  and  $c$ :

$$\sigma_t = \frac{pb^2}{c^2 - b^2} \left( 1 + \frac{c^2}{r^2} \right) \tag{5}$$

$$\sigma_r = \frac{pb^2}{c^2 - b^2} \left( 1 - \frac{c^2}{r^2} \right) \tag{6}$$

$$u_r = \frac{pb^2}{E(c^2 - b^2)} \left[ (1 - \nu)r + (1 + \nu) \frac{c^2}{r} \right] \tag{7}$$



**Figure 1.** Shrink fit models for cylindrical and gear dies

The optimized values of (b, c, and z) can be calculated based on the procedures given by Lange in Handbook of Metal Forming [26]. For a given inner die radius, the outside radii of die and ring (b and c) and the radial interference z, can be found as:

$$z = \frac{b \cdot S_y}{E} \left( \frac{1}{K_1} - Q_1^2 \right) \quad (8)$$

$$b = a / Q_1 \quad (9)$$

$$c = a / Q \quad (10)$$

where;

$$Q_1 = \sqrt{\frac{1}{2} \left( 1 + \frac{1}{K_1} \right) - p'} \quad (11)$$

$$p' = \pi / S_y \quad (12)$$

$$K_1 = S_y(\text{ring}) / S_y(\text{die}) \quad (13)$$

here ( $S_y$ ) is the yield strength.

For gear forging die, the shrink fitting was designed based on thick-wall cylindrical approach by considering the gear pitch diameter as the inner diameter of the die insert (a) and the equations 8-13 were used [9]. However, the gear tooth profile causes stress concentrations on the inner die surface. Eyercioglu [11] modified the thick-wall cylinder approach using FEA and suggested the following set of formula to shrink fit design of spur gear forging dies;

$$z = \frac{b S_y}{E} \left( \frac{1.17}{K_1} - 0.7 Q_1^2 \right) \quad (14)$$

$$a = \frac{m N}{2} \quad (15)$$

$$b = a / Q_1 \quad (16)$$

$$c = \frac{1.8a}{\left[ 0.5 \left( 1 + 2.62 \left( \frac{1}{K_1} \right) \right) - 2.24 p' \right] \sqrt{K_1}} \quad (17)$$

where, m and N are module and the tooth number of the gear, respectively. The study was for symmetric spur gear dies.

### 3. EXPERIMENTAL STUDY

#### 3.1. Die Material, Manufacturing and Shrink Fitting (This section should be given in 3.1)

For inner die and the outer ring materials AISI H13 hot work tool steel was used. The chemical composition and the mechanical properties of the die material are given in Table 1 and Table 2, respectively. The die components are hardened and tempered to obtain 52-55 HRC. The outer rings for the symmetric and asymmetric gear dies are machined to required diameters using a CNC lathe and ground to the accuracy of  $\pm 0.01$  mm. The gear tooth profiles were cut by using wire electro discharge machine (WEDM). The inner die and the outer ring were shrunk by cooling the inner die in liquid nitrogen and heating the outer ring to a temperature of about 200°C.

**Table 1.** Chemical composition of H13 hot work steel [27]

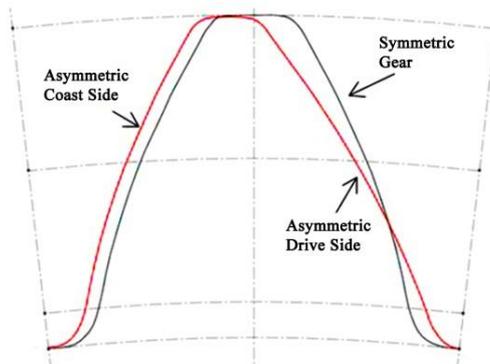
Cr	Mo	Si	V	C	Ni	Cu	Mn	P	S
4.75-5.50	1.10-1.75	0.8-1.2	0.8-1.2	0.32-0.45	0.3	0.25	0.2-0.5	0.03	0.03

**Table 2.** Mechanical properties of H13 hot work steel [28]

Property	Value
Tensile strength, ultimate (@20°C, varies with heat treatment)	1200 - 1590 MPa
Tensile strength, yield (@20°C, varies with heat treatment)	1000 - 1380 MPa
Reduction of area (@20°C)	50.00%
Modulus of elasticity (@20°C)	210 GPa
Poisson's ratio	0.27-0.30
Thermal expansion	10.4 x 10 <sup>-6</sup> /°C
Thermal conductivity	28.6 W/mK

### 3.2. Gear Forging Dies

Two dies for symmetric and asymmetric spur gear forging are designed and manufactured for this study. The symmetric gear forging die is a 3 mm module (m), 28 teeth (N) and 20° pressure angle standard spur gear die (i.e. addendum is equal to module and the dedendum is 1.25 times the module). The corresponding pitch radius (a) is equal to 42 mm. The height of the inner die and the outer ring is selected as 50 mm. The asymmetric gear die has the same module and tooth number, however, it has 22° and 33° pressure angles for coast side and drive side respectively. The single tooth profiles of symmetric and asymmetric gear dies are shown in Figure 2.



**Figure 2.** Symmetric and asymmetric gear tooth profiles

The internal pressure experienced during precision forging of the gear used in this study was taken as 620 MPa from a previous study [6]. The corresponding die geometry parameters (b, c and z) were calculated by using Equations 14-17 and shown in Table 1. The gear die assembly is given in Figure 3.

**Table 3.** The gear die parameters

m (mm)	N	E (Gpa)	S <sub>y</sub> (Mpa)	v	p (Mpa)	a (mm)	b (mm)	c (mm)	z (mm)
3	28	210	1030	0.3	133	42	66.42	162.38	0.29



**Figure 3.** The gear die assembly.

### 3.3. Profile Measurements

The dimensions of the die components and the gear profiles before and after shrink fitting were measured by Kemco 3D Coordinate Measuring Machine (CMM) using a 1 mm ruby touch probe of Renishaw. The measurements repeated at least three times to ensure the results.

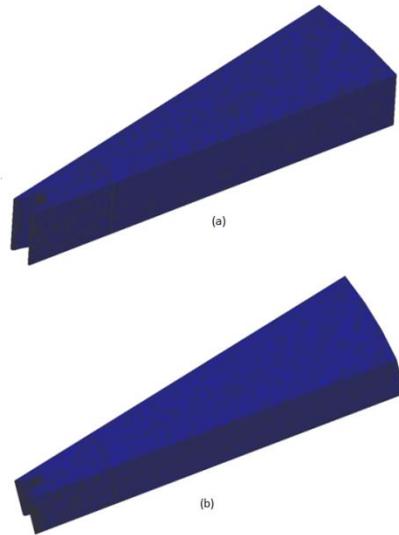
## 4. FINITE ELEMENT MODELING

For the symmetric and asymmetric gear dies, 3D single tooth models were created in SolidWorks and exported to Simufact Forming FE package. The generated mesh type and the number of elements were tetrahedral and 536100, respectively. Symmetry plane boundary condition applied to the both sides of the models. A uniform friction (0.2) and interference  $z=0.29$  mm were applied between the gear shaped die inserts and the outer rings. The 3D FE models of the symmetric and asymmetric gear dies are shown Figure 4.

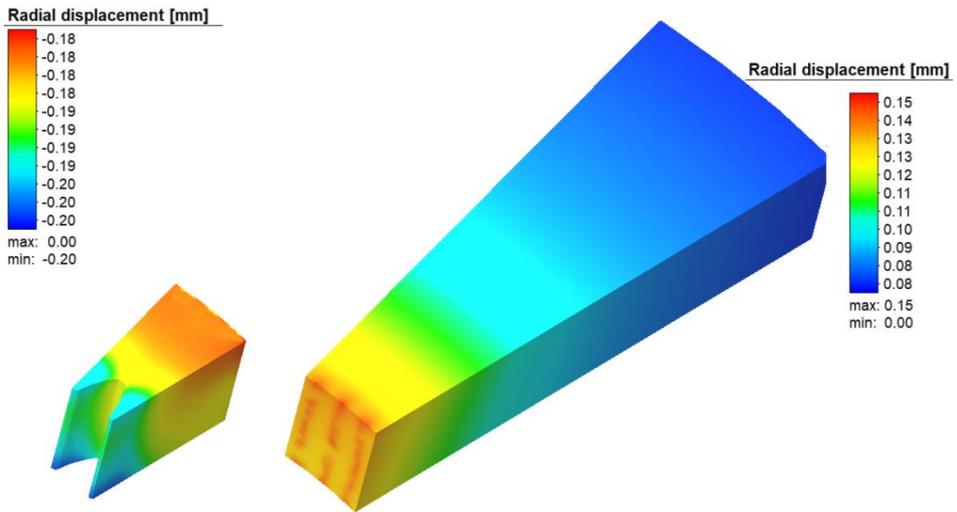
## 5. RESULTS AND DISCUSSION

### 5.1. The Symmetric Gear Die

For the simple cylinders, the radial displacement (contraction) of the inner die bore radius is calculated as  $-0.1641$  mm by using equation 4. The results of finite element analyses for the symmetrical gear die model are given in Figure 5 and Figure 6. The dimensional change of the gear tooth profile of the symmetrical gear die is not uniform and changing from tip ( $-0.186$  mm) to root ( $-0.194$ ) of the profile as shown in Figure 7a. The finite element results and the 3D CMM measurements (see Figure 6) are in well agreement. The slight differences between, CMM and FEM are coming from the nature of the measurement and truncated values during re-meshing. These results show that the radial displacement of the symmetric gear profile due to shrink fit is higher than the calculated one by using Lamé Equation (Eq'n 4).



**Figure 4.** 3D FE models of the (a) symmetric (b) asymmetric gear dies



**Figure 5.** FE results of symmetric gear die

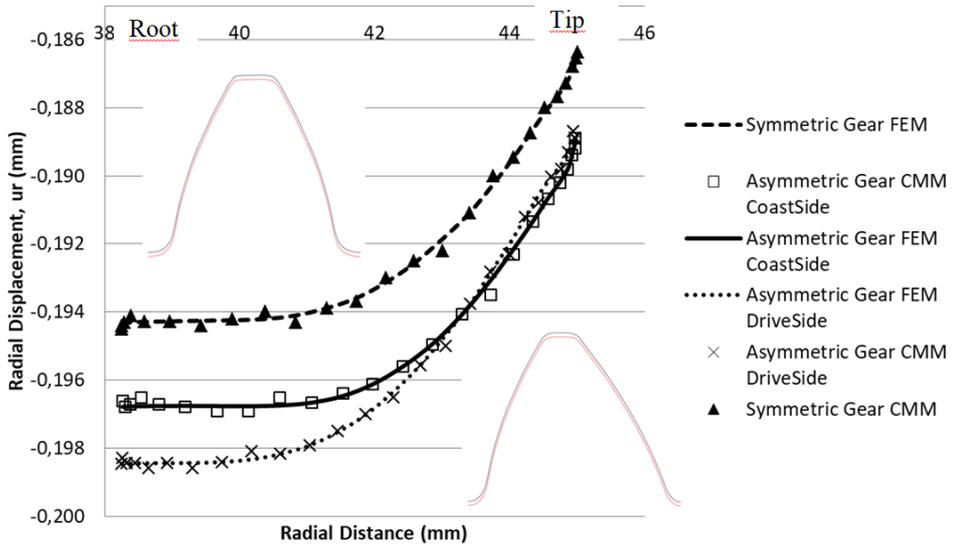
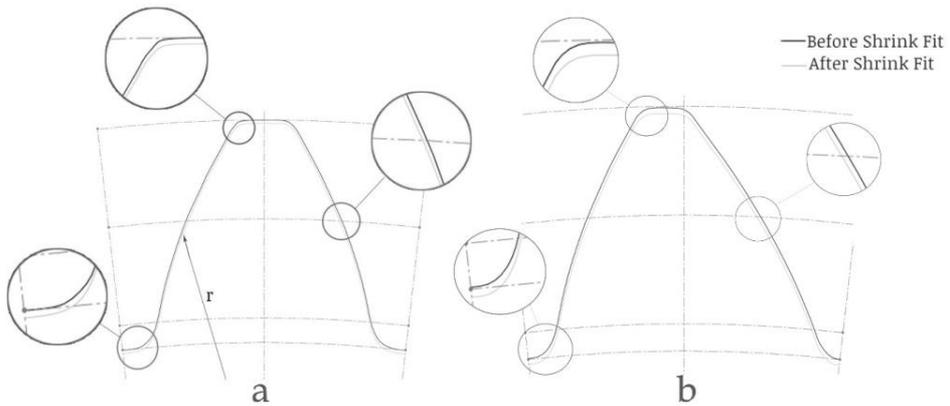


Figure 6. Radial displacement of the gear tooth profile after shrink fit.

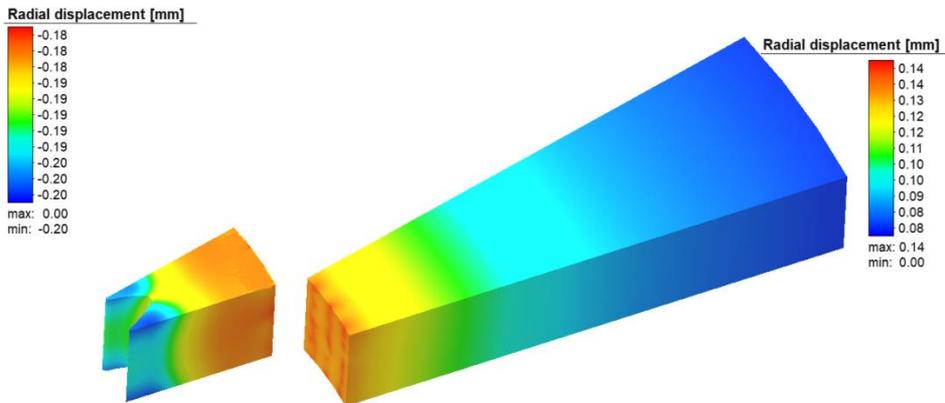
## 5.2. The Asymmetric Gear Die

The finite element results of the radial displacement ( $u_r$ ) of the asymmetric gear profile are given in Figure 8 and Figure 6. The radial displacement of the gear tooth profile of the asymmetrical gear die is not uniform and changing from tip (-0.189 mm) to root (-0.198) of the profile as shown in Figure 7b. The CMM measurements and FE results are in well agreement. The radial displacement of the asymmetric gear die is bigger than the symmetric one.

For the symmetric and asymmetric gear dies, both the experimental (CMM) and the finite element results predict much higher radial displacements than the results of cylindrical die (-0.1641 mm). Therefore, the thick-wall cylindrical approach is inadequate to determine the shape change of the gear tooth profile. In the design of gear forging die, the gear tooth profile have to be modified before cutting according to obtain the standard gear after shrink fitting. This can be done by adding radial displacement due to shrink fit on the radial distance of tooth profile ( $r + u_r$ ).



**Figure 7.** Tooth profiles of (a) symmetric and (b) asymmetric gear die before and after shrink fitting.



**Figure 8.** FE results of the asymmetric gear die

## 6. CONCLUSIONS

The compressive pre-stress due to the shrink fitting causes dimensional variations on the gear profile of the forging die. The amount of radial displacement of the gear tooth profile must be predetermined and the profile has to be modified accordingly to manufacture the forged gear in required accuracy. In this study, the changes of radial dimensions of symmetric and asymmetric precision gear forging dies are evaluated by using FE analyses and experimental study. The design procedure suggested by Eyercioglu [11] is used in determination of sizes of die components and the amount of shrink fit. The followings can be concluded from the study:

- a) The FE results and the experimental (CMM) measurements are in well agreement. Therefore, the FE models are successful to simulate the shrink fit and determine the radial displacements of both symmetric and asymmetric precision gear forging dies.
- b) Both the (CMM) measurements and the finite element results of gear dies predict much higher radial displacements than the results of the cylindrical approach (Lame Equations). Therefore, the thick-wall cylindrical approach is inadequate in determination of the gear forging die tooth profile change.

c) The amount of radial displacements along the tooth profiles of both symmetric and asymmetric gear dies are not uniform and changing from root to tip of the tooth.

d) The amount of radial displacements of the asymmetric gear forging die are higher than the symmetric one.

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