Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2021.00018

Technical Note

Global stability of Susceptible Diabetes Complication (SDC) model in discrete time

Şeyma ŞİŞMAN[1], Mehmet MERDAN[2,*]

[1]*Gumushane University, Mathematical Engineering Department, Gumushane, Turkey*
[2]*Gumushane University, Mathematical Engineering Department, Gumushane, Turkey*

ARTICLE INFO

Article history
Received: 29 September 2020
Accepted: 14 January 2021

Key words:
Susceptible Diabetes Complication (SDC) model; Global stability; Equilibrium points

ABSTRACT

In this study, the mathematical model (DC) of diabetes disease is discussed. This model divides people into (D) uncomplicated and (C) complex diabetics two. In addition, diabetes is a disease known to be caused by genetic and environmental factors, and this factor is one of the main causes of genetic disorder at birth. Considering these two factors, the diabetes complication (SDC) model, which is sensitive from the diabetes complication (DC) model, is being developed. In this model, the responsive diabetes complication (SDC) model of a nonlinear system of differential equations is transformed into a discrete-time system of equations. The positivity and limitation of Model solutions were examined R_0 the basic increment number is calculated. If $R_0 < 1$, it has a global asymptotically stable balance for the situation where there is no genetic disorder at birth, and for $R_0 < 1$, the system has an unstable balance. In addition, random behavior of the discrete model was examined for different probability distributions.

Cite this article as: Şeyma Ş, Mehmet M. Global stability of Susceptible Diabetes Complication (SDC) modelin discrete time. Sigma J Eng Nat Sci 2021;39(3):290–312.

INTRODUCTION

Epidemiology has been gaining more and more attention over the past few years for diseases that have spread to a living organism. Mathematical modeling is used to study the epidemology of a disease. With the development of science, mathematical modeling is used to study not only the spread of infectious diseases, but also non-communicable diseases. Analysis of these disease models with discrete-time equation systems is also obtained. Diabetes is a disease commonly referred to as diabetes, which is usually caused by a combination of hereditary and environmental factors, and the blood glucose level rises excessively. The most important of the hormones that play a role in the regulation of sugar metabolism is the insulin hormone secreted from the beta cell of the pancreas. Insulin enables the sugar to enter the cell and to be stored as glycogen in the cell. People with diabetes cannot use glucose, which passes from the food they eat to the blood, and blood sugar levels rise, causing damage to many tissues and organs. There are two types of diabetes: type 1 diabetes, body cells

***Corresponding author.**
*E-mail address: mehmetmerdan@gmail.com

This paper was recommended for publication in revised form by Regional Editor Aydın Seçer

cannot absorb and process glucose without insulin so blood sugar levels increase, in type 2 diabetes, it occurs because the body cannot produce enough insulin. With the introduction of insulin in 1921, all types of diabetes are treated but there is no definitive cure. The most basic treatments of type 1 diabetes are injecting insulin syringes or pens, while in type 2 diabetes, diet and sugar-lowering drugs are used. Treatment methods used in diabetes lead to many complications. In 2004, Boutoyeb and colleagues introduced the (DC) diabetes complication model to find diabetes without complications (D) and diabetes with complications (C), and the following is the continuous time model to be studied in this study.

$$\begin{aligned}\frac{dD}{dt} &= I-(\lambda+\mu)D+\gamma C\\ \frac{dC}{dt} &= \lambda D-(\gamma+\delta+\nu+\mu)C\end{aligned} \tag{1}$$

Here I, λ, γ, δ, ν, μ > 0. Then, unlike model (1), it determines that the number of incidences is not constant and the number of events taking into account the genetic and environmental factors. With this difference in mind, the (1) model is transformed into a responsive diabetes complication (SDC) model and the SDC is expressed below as a continuous time model [4].

$$\begin{aligned}\frac{dS}{dt} &= \alpha S+\alpha(1-\rho)(D+C)-\frac{\beta SD}{N}-\mu S\\ \frac{dD}{dt} &= \frac{\beta SD}{N}+\alpha\rho(D+C)-(\lambda+\mu)D+\gamma C\\ \frac{dC}{dt} &= \lambda D-(\gamma+\delta+\mu)C\end{aligned} \tag{2}$$

Here, (2) model with advanced difference method

$$\begin{aligned}\frac{S(n+1)-S(n)}{h} &= \alpha S(n)+\alpha(1-\rho)\big(D(n)+C(n)\big)\\ &\quad -\frac{\beta S(n)D(n)}{N}-\mu S(n)\\ \frac{D(n+1)-D(n)}{h} &= \frac{\beta S(n)D(n)}{N}+\alpha\rho\big(D(n)+C(n)\big)\\ &\quad -(\lambda+\mu)D(n)+\gamma C(n)\\ \frac{C(n+1)-C(n)}{h} &= \lambda D(n)-(\gamma+\delta+\mu)C(n)\end{aligned} \tag{3}$$

$$\begin{aligned}S(n+1) &= S(n)+h\Bigg(\alpha S(n)+\alpha(1-\rho)\big(D(n)+C(n)\big)\\ &\quad -\frac{\beta S(n)D(n)}{N}-\mu S(n)\Bigg)\\ D(n+1) &= D(n)+h\Bigg(\frac{\beta S(n)D(n)}{N}+\alpha\rho\big(D(n)+C(n)\big)\\ &\quad -(\lambda+\mu)D(n)+\gamma C(n)\Bigg)\\ C(n+1) &= C(n)+h\big(\lambda D(n)-(\gamma+\delta+\mu)C(n)\big)\end{aligned} \tag{4}$$

It is being transformed into a discrete-time system of equations. Where $S(0) > 0$, $D(0) > 0$, $C(0) > 0$, and $h = 0.01$. The parameters α, β, γ, δ, λ, μ, ρ > 0 and $0 \le \rho \le 1$, respectively, are birth rate, interaction rate, recovery rate of complications, complication-related mortality rate, occurrence rate of complications, and rate of genetic disorder at birth [4–6].

Model (3) is obtained by a sensitive diabetes complication analysis. Let's add $N(n)$ carrier complications by adding up all equations of this model:

$$\begin{aligned}N(n+1) &= N(n)+\alpha N(n)-\mu N(n)-\delta C(n)\\ &\le (1+\alpha-\mu)N(n)\end{aligned} \tag{5}$$

In this equation, $\breve{N}(n+1) \le N_0\,(1+\alpha-\mu)^n$ global asymptotic stable $\lim_{n\to\infty}\breve{N}(n) = 0$ has a single balance. The disease equilibrium point of the model (3) is indicated by E^0, and as a single equilibrium is found as $E^0(0,0,0)$ [1–3]. Recently, various studies have been done on random differential equation and difference equation [13–16].

DISCRETE TIME PROBABILITY DISTRIBUTIONS

In this section, definitions related to some probability concepts used are given.

Discrete Uniform Distribution

Definition. Let k be a positive bit integer. A random variable X with probability function

$$P(x,k) = \begin{cases} 1/k, & x = 1,2,3,\ldots,k\\ 0, & other\end{cases}$$

is called a discrete uniform chance variable [12]

Table 1. Parameter values of the (2) model

Parameters	Descriptions	Values
γ	Recovery rate of complications	0.37141
α	Birthrate	0.01623
δ	Complication-related mortality	0.0068
λ	Rate of occurrence of complications	0.67758
μ	Death rate	0.00764
ρ	Genetic disorder in childbirth	0.077
β	Interaction rate	0.16263

Theorem. If X has a discrete uniform distribution, then

a. $E(X)=\frac{k+1}{2}$,

b. $V(X)=\frac{k^2-1}{12}$,

c. $M_x(t)=\frac{1}{k}\sum_{x=1}^{k}e^{tx}$,

Binomial Distribution

Definition. Let the total number of those who succeeded in independent Bernoulli trials be the random variable X. For a single experiment, the probability of success is denoted by p, and the probability of failure is $(1 - p)$. The binomial random variable X has the following probability function

$$f\left(x;n,p\right)=\binom{n}{x}p^x(1-p)^{n-x};x=0,1,2,\ldots,n.$$

Calculation of consecutive binomial probabilities,

$$f(x+1;n,p)=\frac{(n-x)p}{(x+1)(1-p)}f(x;n,p);x=0,1,\ldots,n-1.$$

Theorem. If X has a binomial distribution,

a. $E(X) = np$,

b. $V(X) = np(1–p)$,

c. $M_x(t) = [e^tp + (1 - p)]^n$.

Geometric Distribution

Definition. The number of experiments done to obtain the first desired result (success or unsuccessful) in a Bernoulli experiment repeated n times in succession is called a geometric random variable X. The distribution of this variable is called the geometric distribution and the probability function of the geometric random variable X, with probability of unsuccessfulness $q = 1 – p$ and probability of success p in a single experiment [12]

$$f(x) = P(X = x) = q^{x-1}p; x = 1, 2, 3,\ldots$$

Theorem. If X has a geometric distribution,

a. $E(X)=\frac{1}{p}$,

b. $V(X)=\frac{(1-p)}{p^2}$,

c. $M_x(t)=pe^t\frac{1}{1-\left[e^t(1-p)\right]}$.

Poisson Distribution

Definition. $f(x)=P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!};x=0,1,2,\ldots,\lambda>0.$

The Taylor expansion of the function e^y and the probability function gives $\left(e^y=\sum_{i=0}^{\infty}\frac{y^i}{i!}\right)$:

$$\sum_{x=0}^{\infty}f\left(X=x;\lambda\right)=e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}=e^{-\lambda}e^{\lambda}=1.$$

Theorem. If X has a Poisson distribution,

a. $E(X) = \lambda$,

b. $V(X) = \lambda$,

c. $M_x = e^{\lambda(e^t-1)}$.

BASIC R_0 INCREMENT NUMBER

Using the Matrix method, we get the basic increment number of the model. Consider the equilibrium point E^0 (0,0,0). If $x = (S,D)^T$, the model can be rewritten as follows:

$$x' = F(x) – V(x)$$

and

$$F\left(x\right)=\begin{bmatrix}-\alpha\rho\left(D\left(n\right)+C\left(n\right)\right)\\ \alpha\rho\left(D\left(n\right)+C\left(n\right)\right)\end{bmatrix},$$

$$V\left(x\right)=\begin{bmatrix}-S\left(n\right)-\alpha S\left(n\right)-\alpha\left(D\left(n\right)+C\left(n\right)\right)+\frac{\beta S\left(n\right)D\left(n\right)}{N}\\ +\mu S\left(n\right)\\ -D\left(n\right)+\left(\lambda+\mu\right)D\left(n\right)-\gamma C\left(n\right)-\frac{\beta S\left(n\right)D\left(n\right)}{N}\end{bmatrix}$$

Jacobian matrices of $F(x)$ and $V(x)$ in E^0

$$DF\left(E^0\right)=\begin{bmatrix}0 & -\alpha\rho\\ 0 & \alpha\rho\end{bmatrix},$$

$$DV\left(E^0\right)=\begin{bmatrix}-1-\alpha+\mu+\frac{\beta D}{n} & -\alpha+\frac{\beta S}{N}\\ \frac{\beta D}{N} & -1+\lambda+\mu\end{bmatrix}$$

for

$$F=\begin{bmatrix}0 & -\alpha\rho\\ 0 & \alpha\rho\end{bmatrix},V=\begin{bmatrix}-1-\alpha+\mu & -\alpha\\ 0 & -1+\lambda+\mu\end{bmatrix}$$

G = FV^{-1} is found as $R_0 = \frac{\alpha\rho}{\mu+\lambda-1}$, which is the basic increase number given by the radius of the new generation matrix. [1]

EXTINCTION AND PERSISTENCE OF THE DISEASE

This section focuses on disease-free equilibrium stability and the absence and persistence of disease determined by the presence of endemic equilibrium of the model. E^0 (0,0,0) indicates an equilibrium. *S, D, C* components are zero, so disease-free balance is called. The stability of the disease-free equilibrium E^0 is given in the following theorem.

Theorem 1 (Jury Theorem). For this criterion for | θ | < 1 values

$$\theta^3 + a_3\theta^2 + a_2\theta + a_1 = 0$$

the roots of the cubic equation can be shown by the following conditions [9].

$$1+a_3+a_2+a_1>0,\ \ 1-a_3+a_2-a_1>0,$$
$$3+a_3-a_2-3a_1>0,\ \ 1+a_3a_1-a_2-a_1^2>0,$$

Theorem 2. For the equilibrium point E^0 of the model

i. $R_0 < 1$ is global asymptotic stable
ii. $R_0 > 1$ for unstable.

Proof. (i) Characteristic equation by giving the *H* matrix in E^0 by the Jacobian matrix in (5)

$$\theta^3 + a_3\theta^2 + a_2\theta + a_1 = 0$$

for

$$a_3 = A - C + E - 3 < 0,$$
$$a_2 = 3 + 2C - 2E - 2A - AC + AE - CE - \lambda D > 0$$
$$a_1 = -1 + E - C + CE + \lambda D + A - AE + AC - ACE - AD\lambda > 0$$

If the Jury Theorem is applied to this cubic equation,

$$1+a_3+a_2+a_1 = 1+A-C+E-3+3 + 2C-2E-2A-AC+AE-CE-\lambda D-1+E-C + CE+\lambda D+A-AE+AC-ACE-AD\lambda>0$$
$$3+a_3-a_2-3a_1 = 3+A-C+E-3-(3+2C-2E-2A-AC+AE-CE-\lambda D) -3(-1+E-C+CE+\lambda D+A-AE+AC-ACE-AD\lambda)>0$$
$$1-a_3+a_2-a_1 = 1-(A-C+E-3)+3+2C-2E-2A-AC+AE-CE-\lambda D -(-1+E-C+CE+\lambda D+A-AE+AC-ACE-AD\lambda)>0$$
$$1+a_3a_1-a_2-a_1^2 = 1+(A-C+E-3)\left(-1+E-C+CE+\lambda D+A-AE+AC-ACE-AD\lambda\right) -(3+2C-2E-2A-AC+AE-CE-\lambda D) -\left(-1+E-C+CE+\lambda D+A-AE+AC-ACE-AD\lambda\right)^2>0$$

Therefore, if the model $R_0 < 1$, it is global asymptotic stable around E^0. [7–9]

Theorem 3. If the disease-free equilibrium point of Model (3) at E^0 is $R_0 < 1$, it is global asymptotically stable and if $R_0 > 1$ is unstable at E^0.

Proof. Model (3) Linearized matrix in E^0 equilibrium,

$$H = \begin{pmatrix} 1+\alpha-\mu & \alpha(1-\rho) & \alpha(1-\rho) \\ 0 & 1+\alpha\rho-\lambda-\mu & \alpha\rho+\gamma \\ 0 & \lambda & 1-\gamma-\delta-\mu \end{pmatrix} \quad (6)$$

Here, let us make it simple by writing instead $A = \alpha - \mu$, $B = \alpha(1-\rho)$, $C = \alpha\rho - \lambda - \mu$, $D = \alpha\rho + \gamma$, $E = \gamma + \delta + \mu$ in the H matrix. Eigenvalues of this matrix obtained are $\theta_1 = 1 - A, \theta_2 = \frac{C}{2} - \frac{E}{2} - \frac{\left(C^2+2CE+E^2+4D\lambda\right)^{\frac{1}{2}}}{2} + 1, \theta_3 = \frac{C}{2} - \frac{E}{2} + \frac{\left(C^2+2CE+E^2+4D\lambda\right)^{\frac{1}{2}}}{2} + 1$ accessible. Therefore, the eigenvalues of $|\theta_i| = 1, 2, 3$ for $R_0 < 1$ are globally asymptoticly stable at E^0 disease-free equilibrium, $R_0 > 1$, E^0 and at is unstable for the disease-free equilibrium point.

When $S(N) \le N(n)$ from this (1) model

$$\begin{aligned} D(n+1) &\le D(n) + \beta D(n) + \alpha\rho(D(n)+C(n)) \\ &\quad -(\lambda+\mu)D(n) + \gamma C(n) \\ &= (1+\beta-\lambda-\mu+R_0(\mu+\lambda-1))D(n) \end{aligned} \quad (7)$$

If the conditions of $\lambda + \mu < 1$ and $R_0 < 1$ become $0 < 1 + \beta - \lambda - \mu + R_0(\mu + \lambda - 1) < 1$. From here (7) repeated inequality use of the equation

$$D(n) = (1+\beta-\lambda-\mu+R_0(\mu+\lambda-1))^n D(0) \quad (8)$$

(8) the equation is $\lim_{n\to\infty} D(n) = 0$.

Since $n \ge N_1$ for any $\varepsilon > 0$ from $\lim_{n\to\infty} D(n) = 0$, $D(n) < \varepsilon$ is a large such that we know that the positive integer is N_1. As a result,

$$C(n+1) = C(n) + \lambda D(n) - (\gamma+\delta+\mu)$$
$$C(n) \le C(n) + \lambda\varepsilon - ((\gamma+\delta+\mu)\,C(n)) \text{ for } n \ge N_1. \quad (9)$$

For the odd balance of $\tilde{C}(n+1) = \tilde{C}(n) + \lambda\varepsilon - ((\gamma+\delta+\mu)\tilde{C}(n)$ in this equation, $\tilde{C}^* = \frac{\lambda\varepsilon}{\gamma+\delta+\mu}$ is

globally asymptotically stable. In comparison principle $C(n) \le \tilde{C}(n) < \frac{2\lambda\varepsilon}{\gamma+\delta+\mu}$ indicates that $N_2 > N_1$ is the integer. For arbitrary ε, this limit $\lim_{n\to\infty} C(n) = 0$ ifade expresses (5) from the equation,

$$N(n) + \alpha N(n) - \mu N(n) - \delta C(n) \le N(n+1) \le (1 + \alpha - \mu)\, N(n) \text{ eğer } n > N_1 \quad (10)$$

On the left side of the inequality of the (10) equation, and from the principle of comparison, we know that for any given $\varepsilon_1 > 0$ for all $n > N_3$, integer. (10) for any $\varepsilon_2 > 0$ according to the comparison principle given on the right side of the inequality of equation, all $n > N_4$. We know that there is an integer $N_4 > N_1$ such that $N(n) \le N_0 (1 + \alpha - \mu)n + \varepsilon_2$.

$N_5 = N_3 + N_4$, inequalities,

$$\frac{\delta\lambda\varepsilon}{(\gamma+\delta+\mu)(\alpha-\mu)} - \varepsilon_1 \le N(n) \le N_0 (1+\alpha-\mu)^n + \varepsilon_2 \text{ for } n > N_5$$

and arbitrary ε is ε_1, ε_2 and $\lim_{n\to\infty} N(n) = \varepsilon_2$, i.e.

$$\lim_{n\to\infty} S(n) = \varepsilon_2,\ \lim_{n\to\infty} D(n) = 0,\ \lim_{n\to\infty} C(n) = 0$$

Meaning that the disease-free equilibrium of (3) is global asymptotically stable since $R_0 < 1$ it is found. $R_0 > 1$ means that the average number of new infections by an infected person is more than one. Its epidemiological interpretation suggests that the disease may be permanent in the population. The theorem below confirms the continuity of the disease in case of $R_0 > 1$.

Theorem 4. If $R_0 > 1$, the disease will remain persistent in the population, that is, the solution of the model with the initial value D(0) > 0(2) has a positive ε value such that $\lim_{n\to\infty} \inf D(n) > \varepsilon$.

Proof. $X = \Omega_1 = \{(S,D,C)\ R_+^3 | S + D + C \le N_0 (1 + \alpha - \mu)\}$, $X_0 = \{(S,D,C)\ X | D > 0,\ C > 0\}$ and $\partial X_0 = \frac{X}{X_0}$. The solution maps of the (2) model for $\Phi: X \to X$, $\Phi_n (x^0) = \phi(n, x^0)\phi(0, x^0) = x^0$ and $x^0 = (S(0), D(0), C(0))$. Where, $M = \{\Phi_0\} = (0,0,0)$ and

$$\mathcal{M}_\partial = \{(S,D,C) \in \partial X_0 |\ \ldots (S\ D0\ C) \in \partial X_0\ \ \forall n \ge$$

This $\{(S,0,0)\partial X_0 | S \ge 0\} M_\partial$ is open and $M_\partial = \{(S,D,C)\ \partial X_0 | D = 0\}$. Also, for Φ_0, Φ is a fixed point in M_∂. Equation,

$$S_1(n+1) = (1 + \alpha - \mu)\, S_1(n)$$

It is the global attractor for the balance $S_* = 0$. Using Lemma 5.9 [10], we know that no subset of M forms a cycle in ∂X_0. $\Phi_n (M_\partial) M_\partial$ state $\Phi_n ((S(0), 0, C(0))) = (S(n), 0, C(n))$ let's imply. If $x^0 = (S(0), 0, C(0)) M_\partial$, $\lim_{n\to\infty} S(n) = 0$, $\lim_{n\to\infty} C(n) = (1 - \gamma - \delta - \mu)^n\, C(0) = 0$ ve $\Omega(M_\partial) = \Phi^0$.

$0 \le C(n) \le D(n) \le N(n)$ and $N(n+1) = N(n) + \alpha N(n) - \mu N(n) - \delta C(n)$ due to $N(n+1) \ge N(n) + (\alpha - \mu - \delta) N(n)$ and $N(n+1) \le N(n) + \alpha N(n) - \mu N(n)$. This difference equation $N_1 (n+1) = (1 + \alpha - \mu - \delta)\, N_1(n)$ single balance $N_1^* = (\alpha - \mu - \delta)^n\, N(0)$ and $N(n+1) = N(n) + \alpha N(n) - \mu N(n)$ is the only equation of the equation of $N_2^* = (1 + \alpha - \mu)^n\, N_0$ and is global asymptotic stable. Therefore, for any ε>0, All $n \ge N_1$, $(\alpha - \mu - \delta)^n\, N(0) - \varepsilon \le N(n) \le (1 + \alpha - \mu)^n\, N(0) + \varepsilon$.

If $R_0 > 1$ then we can prove that σ is a small positive number such that

$$\lim_{n\to\infty} \sup d(\Phi_n(S^0, D^0, C^0), \Phi_0) \ge \sigma \quad \text{for } (S^0, D^0, C^0) \in X_0 \quad (11)$$

If the result in (11) is not valid, then any $(S_\partial^0, D_\partial^0, C_\partial^0) X_0$ is a positive number and there is a dot a large $N_2 > N_1$,

$$d(\Phi_n (S^0, D^0, C^0), \Phi_0) < \sigma \text{ için } n > N_2 \quad (12)$$

Inequality in (12),

$$D(n) \le \sigma \text{ and } S(n) > -\sigma \text{ if } n > N_2 \quad (13)$$

Since $n > N_2$, the equations in (3)

$$N(n+1) \le N(n)(1+\alpha-\mu),$$
$$D(n+1) > D(n) + \frac{\beta D(n)(-\sigma)}{N(n)} + \alpha\rho(D(n) + C(n)) - (\lambda+\mu)D(n) + \gamma C(n) \quad (14)$$

From the first inequality in(14), we know that $N(n) \le (1 + \alpha - \mu)^n\, N(0)$ is a $n > N_3$ number that will hold for all $N_3 > N_2$. Since $n > N_3$, we change $N(n) \le (1 + \alpha - \mu)^n\, N(0)$ to the second inequality of (14) to obtain the inequality of.

$$D(n+1) > D(n) + \frac{\beta D(n)(-\sigma)}{(1+\alpha-\mu)^n N_0} + R_0(\lambda+\mu-1)(D(n)) - (\lambda+\mu)D(n) \quad (15)$$

by selecting small enough, the state is expressed as

$$R_0(\lambda+\mu-1) > 0, \text{ and}$$
$$1 + \frac{\beta D(n)(-\sigma)}{(1+\alpha-\mu)^n N_0} + R_0(\lambda+\mu-1) - (\lambda+\mu) \quad (16)$$

From inequalities in (15) and (16), this limit $\lim_{n\to\infty} D(n) = \infty$. Limit limit $\lim_{n\to\infty} D(n) = \infty$ (10) in $D(n)$ contradicts with the inequality of $D(n) < \sigma$. The contradiction comes from the conjecture given in (12), so the result in (11) is true. Then, $W^s(\Phi_0) \cap X_0 = \varnothing$ and Φ_0, is isolated by X. It is equally permanent with respect to $(X_0, \partial X_0)$ in theorem 3.

Also in theorem 4, it implies that the solutions of the (3) model are permanent in the same way as $(X_0, \partial X_0)$ when $R_0 > 1$, so that there is a $\varepsilon>0$ similar to this boundary entry, and $\lim_{n\to\infty}$ in $fD(n) > \varepsilon > 0$.[8–10]■

NUMERICAL EXAMPLES

In this section, after giving information about SDC model, random models will be established and examined [11–12].

DISCRETE TIME PROBABILITY DISTRIBUTION

Uniform Distribution

$$S(n+1) = S(n) + h$$
$$\left(\alpha S(n) + \alpha(1-\rho)(D(n)+C(n)) - \frac{\beta S(n)D(n)}{N} - \mu S(n)\right)$$
$$D(n+1) = D(n) + h$$
$$\left(\frac{\beta S(n)D(n)}{N} + \alpha\rho(D(n)+C(n)) - (\lambda+\mu)D(n) + \gamma C(n)\right)$$
$$C(n+1) = C(n) + h(\lambda D(n) - (\gamma+\delta+\mu)C(n))$$

In the random SDC difference equation defined as if α, β, γ, δ, λ, μ, ρ is a random variable with a parameterized uniform distribution and $K = 10$, then the probability characteristics obtained from 10^5 simulations are given below.

Within the SDC model process (n∈ [0,10]), variability is observed to increase. The end values are shown in the Table (Table 1.1 and figure 1.1).

It appears that the expected diabetes reached its highest level at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(S(10)) = 290$ was obtained for the expected value at the end of the process $n = 10$.

Similarly, variance change (n ∈ [0,10]) appears to increase for the SDC model. Extreme values are seen in the table (Table 1.2 and Figure 1.1).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(S(10)) = 0.006742$ was obtained for variance, $(n = 10)$.

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 1.1). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are

Table 1.1. Expected value of random $S(n)$ number of susceptible individuals, end values and times

Variable	Minimum	Time	Maximum	Time
$E(S(n))$	289.8	0	290	10

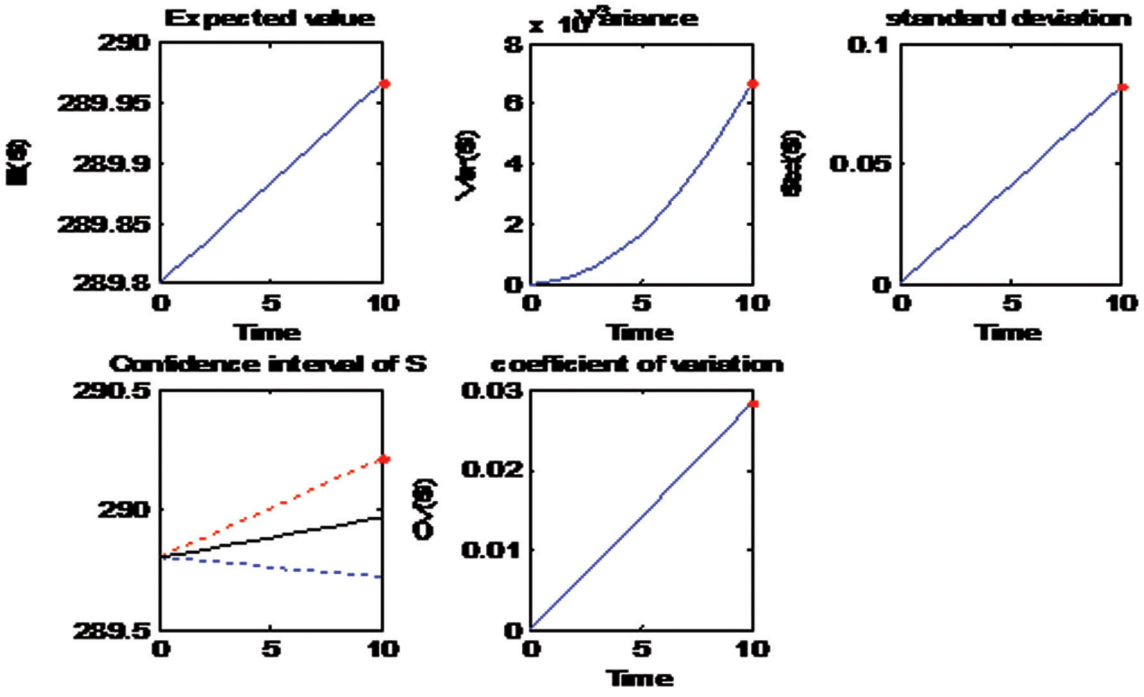

Figure 1.1. random behavior of $S(n)$ number of susceptible individuals.

expected to behave similarly. Extreme values for standard deviations are shown below (Table 1.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(S(10))$ = 0.08211 was obtained for variance (n = 10) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables S(n) in the random model (3) were also calculated as follows (Figure 1.1).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(S(n))/E(S(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $S(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 1.4).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of S(n) is constantly increasing and reaches %0.0002832 at n=10 Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 1.1). The confidence intervals given in the figure are calculated as $Cl = (E(S(n)) - 3.\ std(S(n)), E(S(n)) + 3.\ std(S(n)))$, and three gives the range of variation within the standard deviation. For uniform distribution, this range includes about 99% of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 1.5).

Table 1.2. Extreme values and times of variance of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Var(S(n))$	0	0	0.006742	10

Table 1.3. Extreme values and times of standard deviation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Std(S(n))$	0	0	0.08211	10

Table 1.4. Extreme values and times of the coefficient of variation of random susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CV(S(n))$	0	0	0.02832	10

Table 1.5. End values and times in confidence interval of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CI(S(n))$	0	0	0.02832	10

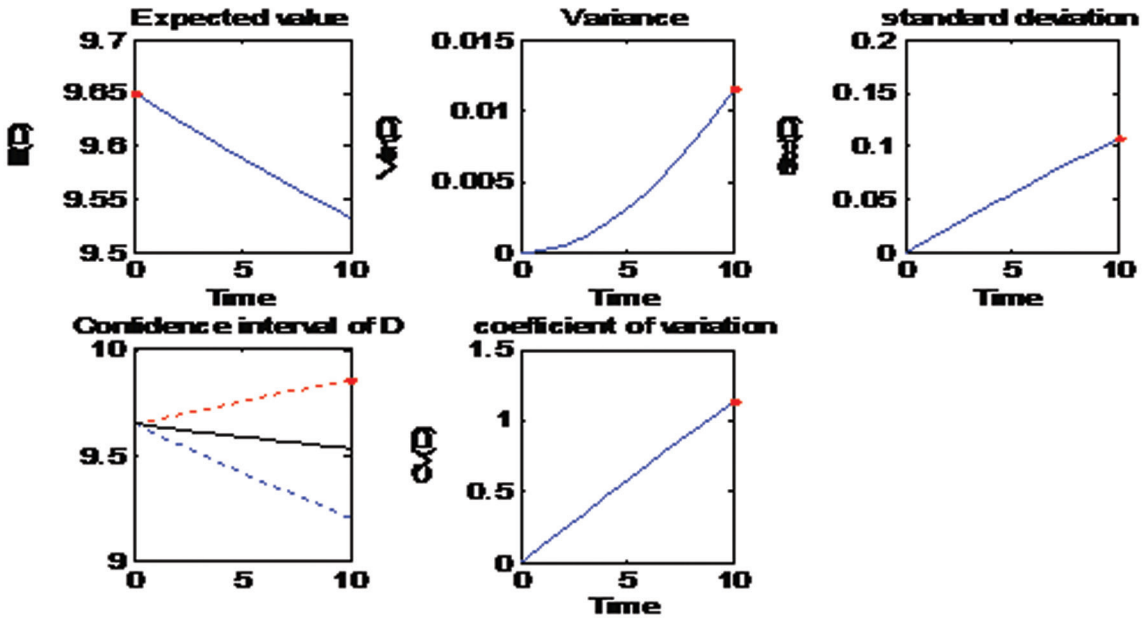

Figure 1.2. $D(n)$ uncomplicated random behaviors.

At the end of the process, three standard deviation intervals for S(n) variables are obtained as follows: $CI(S(10)) \in (289.8, 290.2)$

Model (3) states that the expectation for this value is $CI(S(10)) = 290.2$, that is, approximately %2.902, and the expected approximate diabetes ratio is in the range of %99 probability (289.8, 290.2) at time $n = 10$.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 1.6 and Figure 1.2).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(D(0)) = 9.65$ was obtained for the expected value at the end of the process ($n = 0$).

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 1.7 and Figure 1.2).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(D(10)) = 0.01166$ was obtained for variance, ($n = 10$).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 1.2). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 1.8).

Table 1.6. Random $D(n)$ uncomplicated expected value end values and times

Variable	Minimum	Time	Maximum	Time
$E(D(n))$	9.54	10	9.65	0

Table 1.7. Extreme values and times of random $D(n)$ uncomplicated variance

Variable	Minimum	Time	Maximum	Time
$Var(D(n))$	0	0	0.01166	10

Table 1.8. End values and times of random D(n) uncomplicated standard deviation

Variable	Minimum	Time	Maximum	Time
$Std(D(n))$	0	0	0.0108	10

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(D(10)) = 0.0108$ was obtained for variance ($n = 10$) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $D(n)$ in the random model (3) were also calculated as follows (Figure 1.2).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(D(n))/E(D(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $D(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 1.9).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $D(n)$ is constantly increasing and reaches %0.01133 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 1.2). The confidence intervals given in the figure are calculated as $CI = (E(D(n)) - 3.\ std(D(n)), E(D(n)) + 3.\ std(D(n)))$, and three gives the range of variation within the standard deviation. For uniform distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 1.10).

At the end of the process, three standard deviation intervals for $D(n)$ variables are obtained as follows: $CI(D(0)) \in (9.209,9.65)$

Model (3) states that the expectation for this value is $CI(D(0)) = 9.65$, that is, approximately %0.0965, and the expected approximate diabetes ratio is in the range of %99 probability (9.209,9.65) at time $n = 0$.

Table 1.9. Extreme values and times of the coefficient of variation of random $D(n)$ uncomplicated variation coefficient

Variable	Minimum	Time	Maximum	Time
$CV(D(n))$	0	0	1.133	10

Table 1.10. End values and times in random $D(n)$ uncomplicated confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(D(n))$	9.209	10	9.65	10

It is observed that the variability increases in the SDC model process ($n \in [0.10]$). Extreme values are seen in the table (Table 1.11 and Figure 1.3).

It appears that the expected diabetes reached its highest level at the time of n=10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(C(10)) = 11.32$ was obtained for the expected value at the end of the process $n = 10$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 1.12 and Figure 1.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(C(10)) = 0.1049$ was obtained for variance ($n = 10$)at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $C(n)$ in the random model (3) were also calculated as follows (Figure 1.3).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(C(n))/E(C(n))$ and random α, β, γ, δ, λ, μ, ρ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $C(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 1.14).

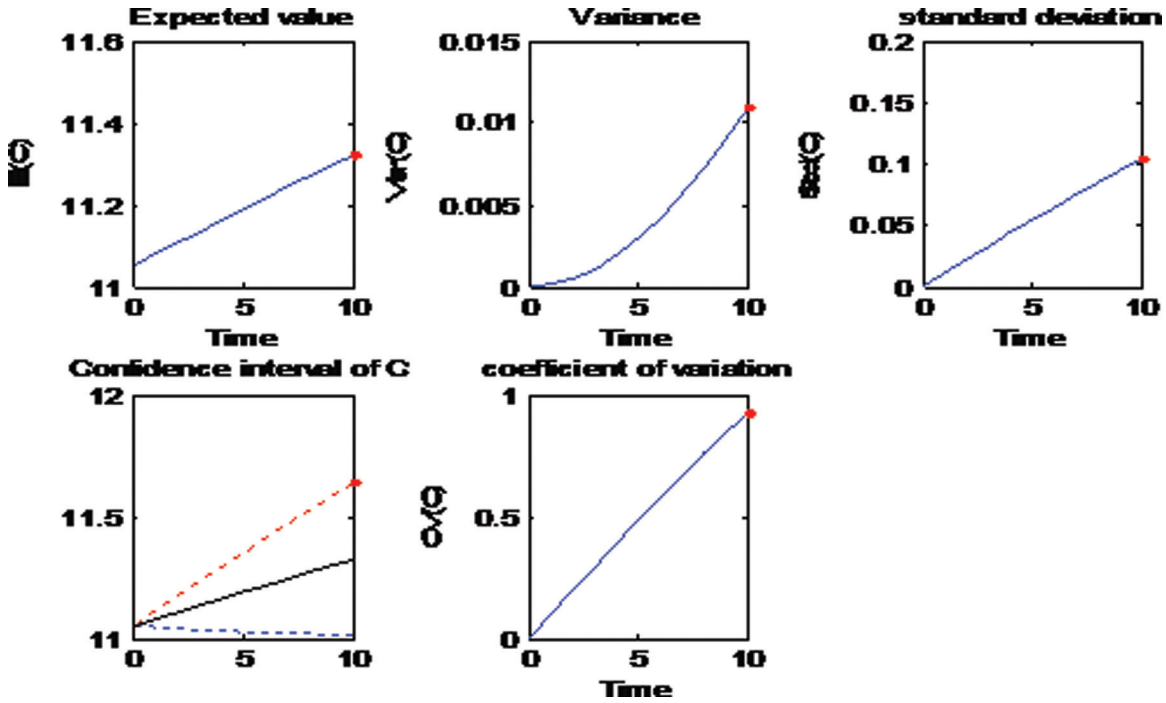

Figure 1.3. $C(n)$ random behavior of complication rate.

Table 1.11. Expected value of random $C(n)$ complication rate, extreme values and times

Variable	Minimum	Time	Maximum	Time
$E(C(n))$	11.05	0	11.32	10

Table 1.12. Extreme values and times of variance of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Var(C(n))$	0	0	0.01101	10

Table 1.13. Extreme values and times of standard deviation of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Std(C(n))$	0	0	0.1049	10

Table 1.14. Extreme values and times of variation coefficient of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$CV(C(n))$	0	0	0.940661	10

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of C(n) is constantly increasing and reaches %0.009406 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 1). The confidence intervals given in the figure are calculated as $GA = (E(C(n))–3.std(C(n)), E(C(n)) + 3.std(C(n)))$, and three gives the range of variation within the standard deviation. For uniform distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 1.15).

At the end of the process, three standard deviation intervals for $C(n)$ variables are obtained as follows: $CI(C(10)) \in (11.05,11.64)$

Model (3) states that the expectation for this value is $(C(10)) = 11.64$, that is, approximately %0.1164, and the expected approximate diabetes ratio is in the range of %99 probability ((11.05,11.64)) at time n =10.

Binomial Distribution

In the random SDC difference equation defined as (3) if $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ is a random variable with a parameterized Binomial distribution and $K = 10$, then the probability characteristics obtained from 10^5 simulations are given below.

It is seen that the variability decreases in the SDC model process ($n \in [0.10]$). Extreme values are seen in the table (Table 2.1 and Figure 2.1).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $E(S(0)) = 289.8$ was obtained for the expected value at the end of the process $n = 0$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 2.2 and Figure 2.1).

It is observed that the diabetes has reached its highest level of deviation from the average at t he time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In

Table 1.15. End values and times of random $C(n)$ complication rate in confidence interval

Variable	Minimum	Time	Maximum	Time
$Std(D(n))$	11.05	10	11.64	10

Table 2.1. Expected value of random number of $S(n)$ susceptible individuals, end values and times

Variable	Minimum	Time	Maximum	Time
$E(S(n))$	280.1	10	289.8	0

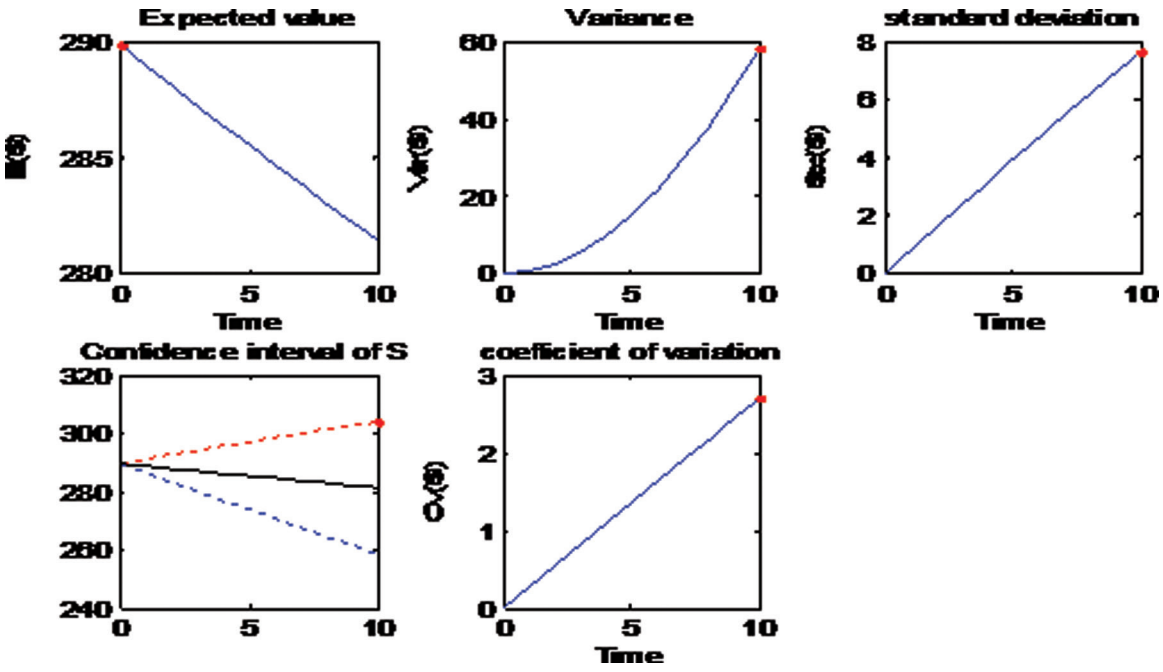

Figure 2.1. Random behavior of $S(n)$ number of susceptible individuals.

addition, at the end of the process, $Var(S(10)) = 57.96$ was obtained for variance, $(n = 10)$.

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 2.1). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 2.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(S(10)) = 7.613$ was obtained for variance $(n = 10)$ at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $S(n)$ in the random model (3) were also calculated as follows (Figure 2.1).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(S(n))/E(S(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of S(n) variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 2.4).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $S(n)$ is constantly increasing and reaches %0.0270403 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 2.1). The confidence intervals given in the figure are calculated as $CI = (E(S(n)) - 3.std(S(n)), E(S(n)) + 3.std(S(n)))$, and three gives the range of variation within the standard deviation. For binomial distribution, this range includes about 99% of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 2.5).

At the end of the process, three standard deviation intervals for $S(n)$ variables are obtained as follows: $CI(S(10)) \in (258.5,304.1)$

Model (3) states that the expectation for this value is $CI(S(10)) = 304.189$, that is, approximately %3.04189, and the expected approximate diabetes ratio is in the range of %99 probability (258.5, 304.1) at time $n = 10$.

It is seen that the variability decreases in the SDC model process $(n \in [0,10])$. Extreme values are seen in the table (Table 2.6 and Figure 2.2).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(D(0)) = 9.65$ was obtained for the expected value at the end of the process $(n = 0)$.

Similarly, variance change $(n \in [0,10])$ appears to increase for the SDC model. Extreme values are seen in the table (Table 2.7 and Figure 2.2).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(D(10)) = 0.0638595$ was obtained for variance, $(n = 10)$.

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 2.2). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 2.8).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In

Table 2.2. Extreme values and times of variance of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Var(S(n))$	0	0	57.96	10

Table 2.3. Extreme values and times of standard deviation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Std(S(n))$	0	0	7.613	10

Table 2.4. Extreme values and times of the coefficient of variation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CV(S(n))$	0	0	2.70403	10

Table 2.5. End values and times in confidence interval of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CI(S(n))$	258.5	10	304.189	10

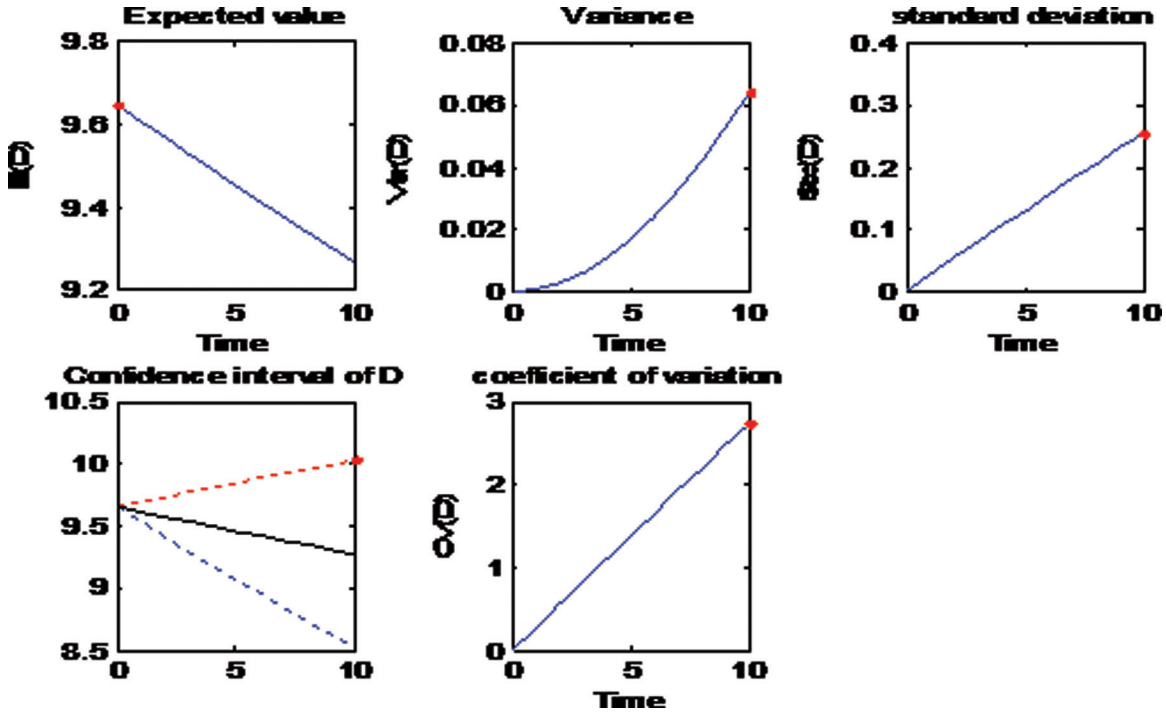

Figure 2.2. $D(n)$ uncomplicated random behaviors.

Table 2.6. Random $D(n)$ uncomplicated expected value end values and times

Variable	Minimum	Time	Maximum	Time
$E(D(n))$	9.3	0	9.65	0

Table 2.7. Extreme values and times of random $D(n)$ uncomplicated variance

Variable	Minimum	Time	Maximum	Time
$Var(D(n))$	0	0	0.0638595	10

Table 2.8. End values and times of random $D(n)$ uncomplicated standard deviation

Variable	Minimum	Time	Maximum	Time
$Std(D(n))$	0	0	0.0252704	10

Table 2.9. Extreme values and times of the coefficient of variation of random uncomplicated variation coefficient

Variable	Minimum	Time	Maximum	Time
$CV(D(n))$	0	0	2.72691	10

addition, $Std(D(10)) = 0.0252704$ was obtained for variance ($n = 10$) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $D(n)$ in the random model (3) were also calculated as follows (Figure 2.2).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(D(n))/E(D(n))$ and random α, β, γ, δ, λ, μ, ρ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $D(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 2.9).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $D(n)$ is constantly increasing and reaches %0.027691 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 2.2). The confidence intervals given in the figure are calculated as $CI = (E(D(n)) - 3.std(D(n)), E(D(n)) + 3.std(D(n)))$, and three gives the range of variation within the standard deviation. For binomial distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 2.10).

At the end of the process, three standard deviation intervals for $D(n)$ variables are obtained as follows: $CI(D(10)) \in (8.51,10.03)$

Model (3) states that the expectation for this value is $CI(D(10)) = 10.03$, that is, approximately %0.1003, and the expected approximate diabetes ratio is in the range of %99 probability (8.51, 10.03) at time $n = 10$.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 2.11 and Figure 2.3).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(C(0)) = 11.05$ was obtained for the expected value at the end of the process $n = 0$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 2.12 and Figure 2.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(C(10)) = 0.088476$ was obtained for variance, ($n = 10$).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 2.3). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 2.13).

Table 2.10. End values and times in random $D(n)$ uncomplicated confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(D(n))$	8.51	10	10.03	10

Table 2.11. Expected value of random $C(n)$ complication rate, extreme values and times

Variable	Minimum	Time	Maximum	Time
$E(C(n))$	10.95	10	11.05	0

Table 2.12. Extreme values and times of variance of random complication rate

Variable	Minimum	Time	Maximum	Time
$E(C(n))$	10.95	10	11.05	0

Table 2.13. Extreme values and times of standard deviation of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Std(C(n))$	0	0	0.297449	10

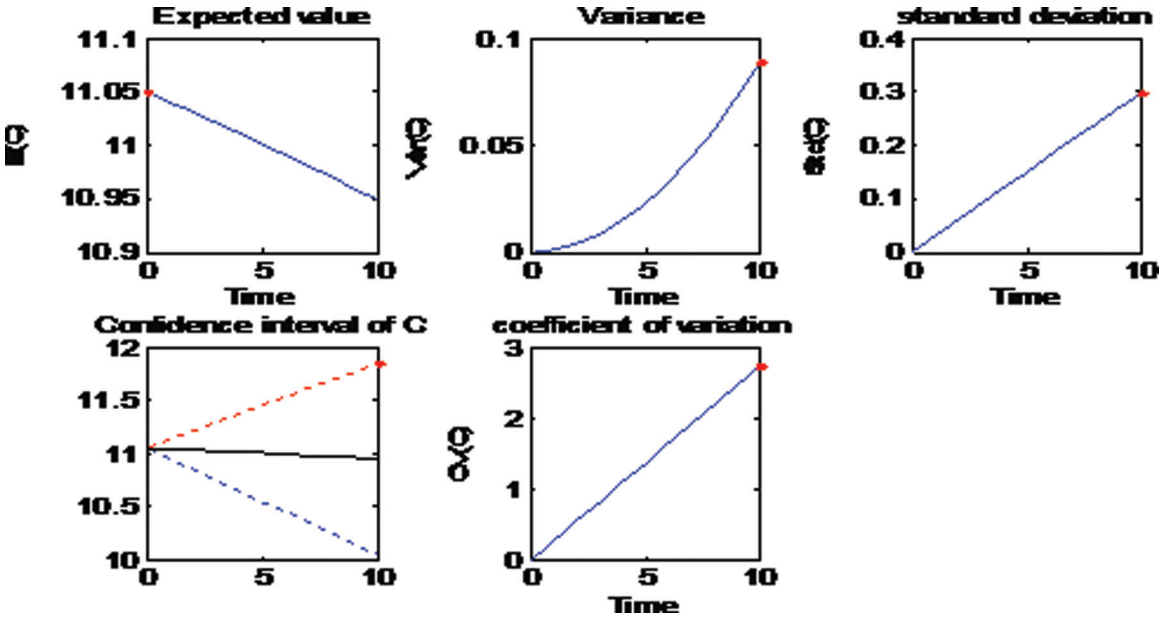

Figure 2.3. $C(n)$ random behavior of complication rate.

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(C(10))$ = 0.297449 was obtained for variance (n = 10)at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $C(n)$ in the random model (3) were also calculated as follows (Figure 2.3).

Coefficient of Variation (CV) is calculated by definition as 100 × $std(C(n))/E(C(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $C(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 2.14).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $C(n)$ is constantly increasing and reaches %0.02716 at n = 10. Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 2.3). The confidence intervals given in the figure are calculated as $Cl = (E(C(n)) - 3.std(C(n)), E(C(n)) + 3.\text{std}(C(n)))$, and three gives the range of variation within the standard deviation. For binomial distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 2.15).

At the end of the process, three standard deviation intervals for $C(n)$ variables are obtained as follows: $CI(C(10)) \in$ (10.06,11.84)

Model (3) states that the expectation for this value is $CI(C(10))$ = 11.8407, that is, approximately %0.118407, and the expected approximate diabetes ratio is in the range of %99 probability ((10.06,11.84)) at time n = 10.

GEOMETRIC DISTRIBUTION

In the random SDC difference equation defined as (3) if $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ is a random variable with a parameterized geometric distribution and K = 10, then the probability characteristics obtained from 10^5 simulations are given below.

Table 2.14. Extreme values and times of variation coefficient of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$CV(C(n))$	0	0	2.71684	10

Table 2.15. End values and times of random $C(n)$ complication rate in confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(C(n))$	10.06	10	11.8407	10

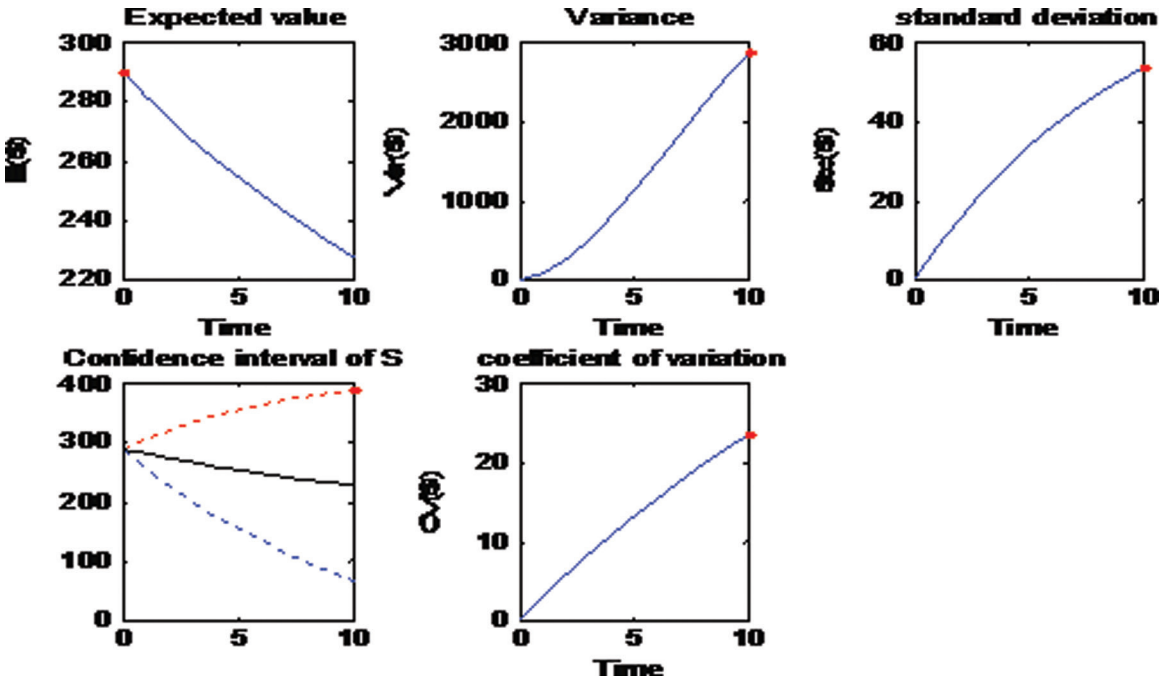

Figure 3.1. random behavior of number of $S(n)$ susceptible individuals.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 3.1 and Figure 3.1).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(S(0)) = 289.8$ was obtained for the expected value at the end of the process $n = 0$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 3.2 and Figure 3.1).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(S(10)) = 2900.9$ was obtained for variance, ($n = 10$).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 3.1). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 3.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(S(10)) = 53.86$ was obtained for variance ($n = 10$) at the end of the process.

Table 3.1. Expected value of random number of $S(n)$ susceptible individuals, end values and times

Variable	Minimum	Time	Maximum	Time
$E(S(n))$	227.3	10	289.8	0

Table 3.2. Extreme values and times of variance of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Var(S(n))$	0	0	2900.9	10

Table 3.3. Extreme values and times of standard deviation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$E(S(n))$	227.3	10	289.8	0

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables S(n) in the random model (3) were also calculated as follows (Figure 3.1).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(S(n))/E(S(n))$ and random α, β, γ, δ, λ ,μ,ρ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $S(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 3.4).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $S(n)$ is constantly increasing and reaches %0.23694 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 3.1). The confidence intervals given in the figure are calculated as $Cl = (E(S(n)) - 3.std(S(n)), E(S(n)) + 3.std(S(n)))$, and three gives the range of variation within the standard deviation. For geometric distribution, this range includes about 99% of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 3.5).

At the end of the process, three standard deviation intervals for $S(n)$ variables are obtained as follows: $CI(S(10)) \in (65.74,388.9)$

Model (3) states that the expectation for this value is $CI(S(10)) = 388.9$, that is, approximately %3.889, and the expected approximate diabetes ratio is in the range of %99 probability (65.74,388.9) at time $n = 10$.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 3.6 and Figure 3.2)

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at

Table 3.4. Extreme values and times of the coefficient of variation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CV(S(n))$	0	0	23.694	10

Table 3.5. End values and times in confidence interval of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CI(S(n))$	65.74	10	388.9	10

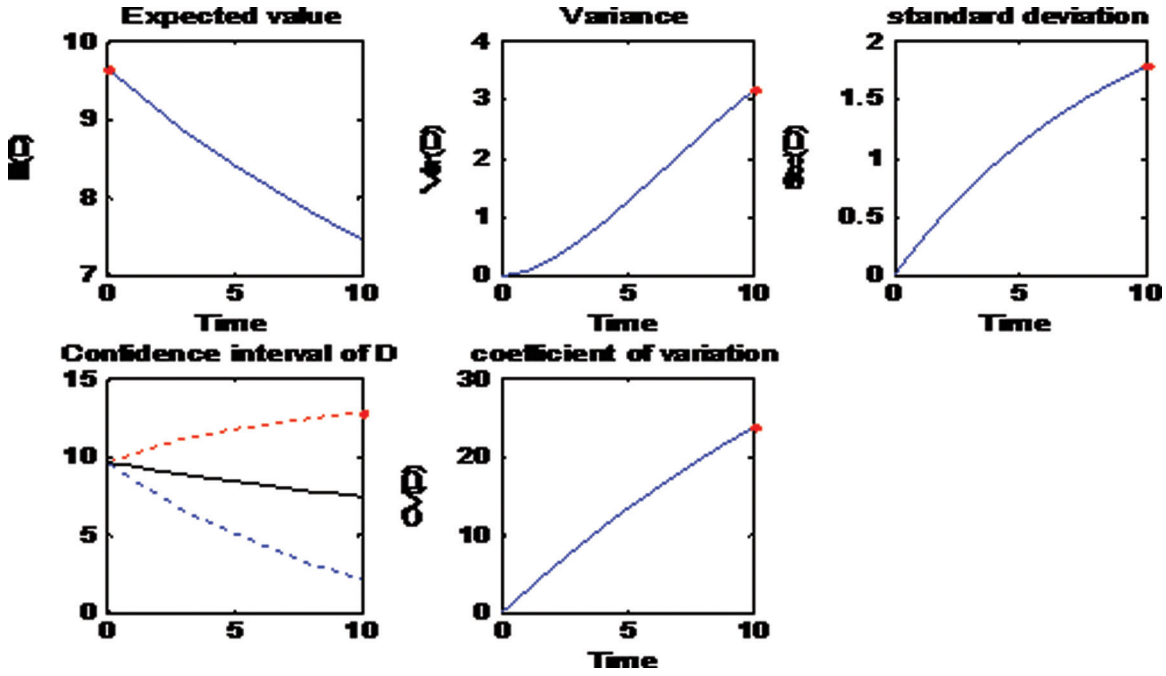

Figure 3.2. $D(n)$ uncomplicated random behaviors.

Table 3.6. Random $D(n)$ uncomplicated expected value end values and times

Variable	Minimum	Time	Maximum	Time
$E(D(n))$	7.45	10	9.65	0

Table 3.7. Extreme values and times of random $D(n)$ uncomplicated variance

Variable	Minimum	Time	Maximum	Time
$Var(D(n))$	0	0	3.208	10

Table 3.8. End values and times of random $D(n)$ uncomplicated standard deviation

Variable	Minimum	Time	Maximum	Time
$Std(D(n))$	7.45	10	9.65	0

Table 3.9. Extreme values and times of the coefficient of variation of random $D(n)$ uncomplicated variation coefficient

Variable	Minimum	Time	Maximum	Time
$CV(D(n))$	0	0	24.0124	10

these moments.In addition, $E(D(0)) = 9.65$ was obtained for the expected value at the end of the process ($n = 0$).

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 3.7 and Figure 3.2).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(D(10)) = 3.208$ was obtained for variance, ($n = 10$).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 3.2). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 3.8).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(D(10)) = 1.79109$ was obtained for variance ($n = 10$) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $D(n)$ in the random model (3) were also calculated as follows (Figure 3.2).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(D(n))/E(D(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $D(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 3.9).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $D(n)$ is constantly increasing and reaches %0.240124 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 3.2). The confidence intervals given in the figure are calculated as $CI = (E(D(n))–3.std(D(n)), E(D(n)) + 3.std(D(n)))$, and three gives the range of variation within the standard deviation. For geometric distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 2.10).

At the end of the process, three standard deviation intervals for $D(n)$ variables are obtained as follows: $CI(D(10)) \in (2.086,12.83)$

Model (3) states that the expectation for this value is $CI(D(10)) = 12.83$, that is, approximately %0.1283, and the expected approximate diabetes ratio is in the range of %99 probability (2.086,12.83) at time $n = 10$.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 3.11 and Figure 3.3).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(C(0)) = 11.05$ was obtained for the expected value at the end of the process $n = 0$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 3.12 and Figure 3.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(C(10)) = 4.516$ was obtained for variance, $(n = 10)$.

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 3.3). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 3.13).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of $n = 10$.

Table 3.10. End values and times in random $D(n)$ uncomplicated confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(D(n))$	2.086	10	12.83	10

Table 3.11. Expected value of random $C(n)$ complication rate, extreme values and times

Variable	Minimum	Time	Maximum	Time
$CI(D(n))$	2.086	10	12.83	10

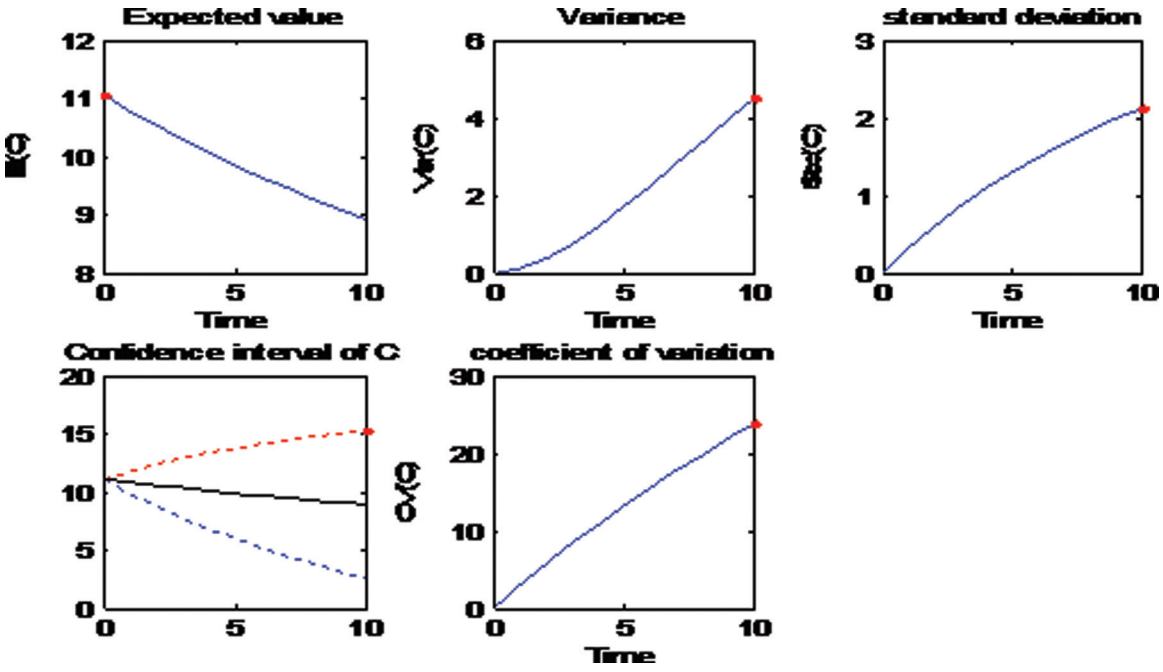

Figure 3.3. $C(n)$ random behavior of complication rate.

Table 3.12. Extreme values and times of variance of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Var(C(n))$	0	0	4.516	10

Table 3.13. Extreme values and times of standard deviation of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Std(C(n))$	0	0	2.1251	10

Table 3.14. Extreme values and times of variation coefficient of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$CV(C(n))$	0	0	23.8437	10

Table 3.15. End values and times of random $C(n)$ complication rate in confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(C(n))$	2.537	10	15.2886	10

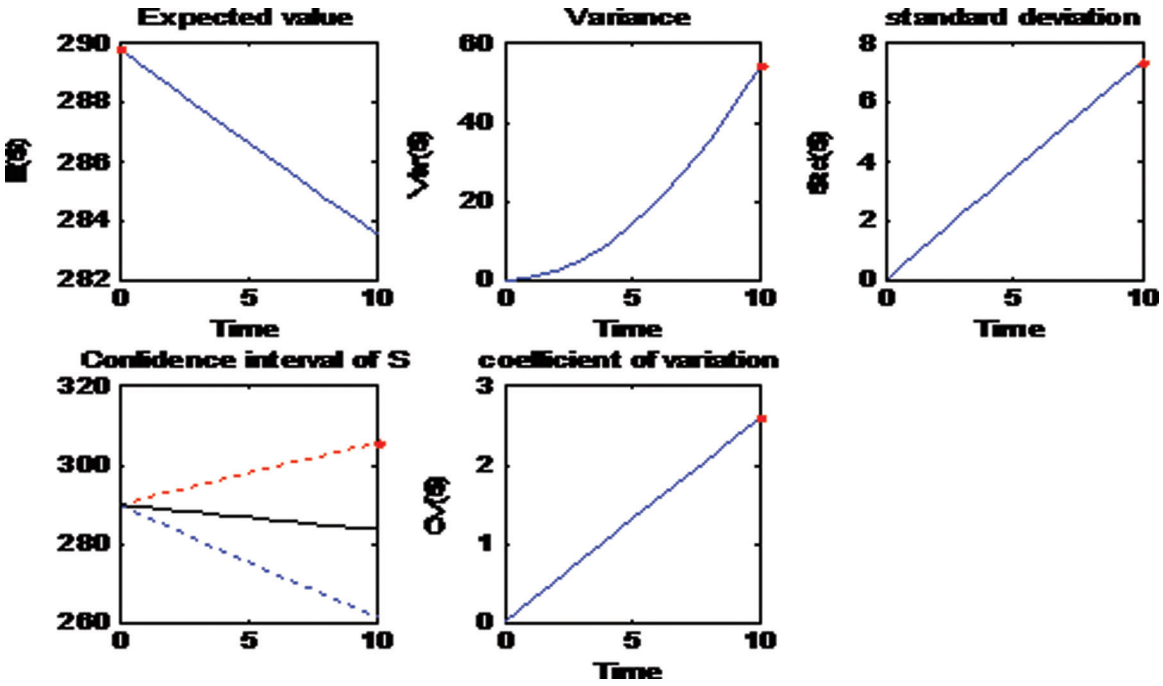

Figure 4.1. random behavior of $S(n)$ number of susceptible individuals.

Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(C(10)) = 2.1251$ was obtained for variance ($n = 10$)at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $C(n)$ in the random model (3) were also calculated as follows (Figure 3.3).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(C(n))/E(C(n))$ and random α, β, γ, δ, λ, μ, ρ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $C(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 3.14)

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $C(n)$ is constantly increasing and reaches %0.238437 at $n = 10$ Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 3.3). The confidence intervals given in the figure are calculated as $Cl = (E(C(n))–3.std(C(n)), E(C(n)) + 3.std(C(n)))$, and three gives the range of variation within the standard deviation. For geometric distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 3.15).

At the end of the process, three standard deviation intervals for $C(n)$ variables are obtained as follows: $CI(C(10)) \in (2.537,15.2886)$

Model (3) states that the expectation for this value is ($C(10)$) = 15.2886, that is, approximately %0.152886, and the expected approximate diabetes ratio is in the range of %99 probability (2.537,15.2886) at time n = 10.

Poisson distribution

In the random SDC difference equation defined as (3) if $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ is a random variable with a parameterized poisson distribution and K = 10, then the probability characteristics obtained from 10^5 simulations are given below.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 4.1 and Figure 4.1).

It appears that the expected diabetes reached its highest level at the time of n = 0. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $E(S(0))$ = 289.8 was obtained for the expected value at the end of the process n = 0.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 4.2 and Figure 4.1).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n=10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(S(10))$ = 54.2702 was obtained for variance, (n = 10).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 4.1). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 4.3).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(S(10))$ = 7.367 was obtained for variance (n = 10) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $S(n)$ in the random model (3) were also calculated as follows (Figure 4.1).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(S(n))/E(S(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $S(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 4.4).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $S(n)$ is constantly increasing and reaches %0.2598 at n = 10 Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 4.1). The confidence intervals given in the figure are calculated as $Cl = (E(S(n))–3.std(S(n)), E(S(n)) + 3.std(S(n)))$, and three gives the range of variation within the standard deviation. For poisson distribution, this range includes about 99% of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 4.5).

At the end of the process, three standard deviation intervals for S(n) variables are obtained as follows: $CI(S(10)) \in$ (261.4,305.6)

Model (3) states that the expectation for this value is $Cl(S(10))$ = 305.6, that is, approximately %3.056, and the

Table 4.1. Expected value of random number of $S(n)$ susceptible individuals, end values and times

Variable	Minimum	Time	Maximum	Time
$E(S(n))$	283.6	10	289.8	0

Table 4.2. Extreme values and times of variance of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Var(S(n))$	0	0	54.2702	10

Table 4.3. Extreme values and times of standard deviation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$Std(S(n))$	0	0	7.367	10

Table 4.4. Extreme values and times of the coefficient of variation of random $S(n)$ susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CV(S(n))$	0	0	2.598	10

Table 4.5. End values and times in confidence interval of random $S(n)$ number of susceptible individuals

Variable	Minimum	Time	Maximum	Time
$CI(S(n))$	261.4	10	305.6	10

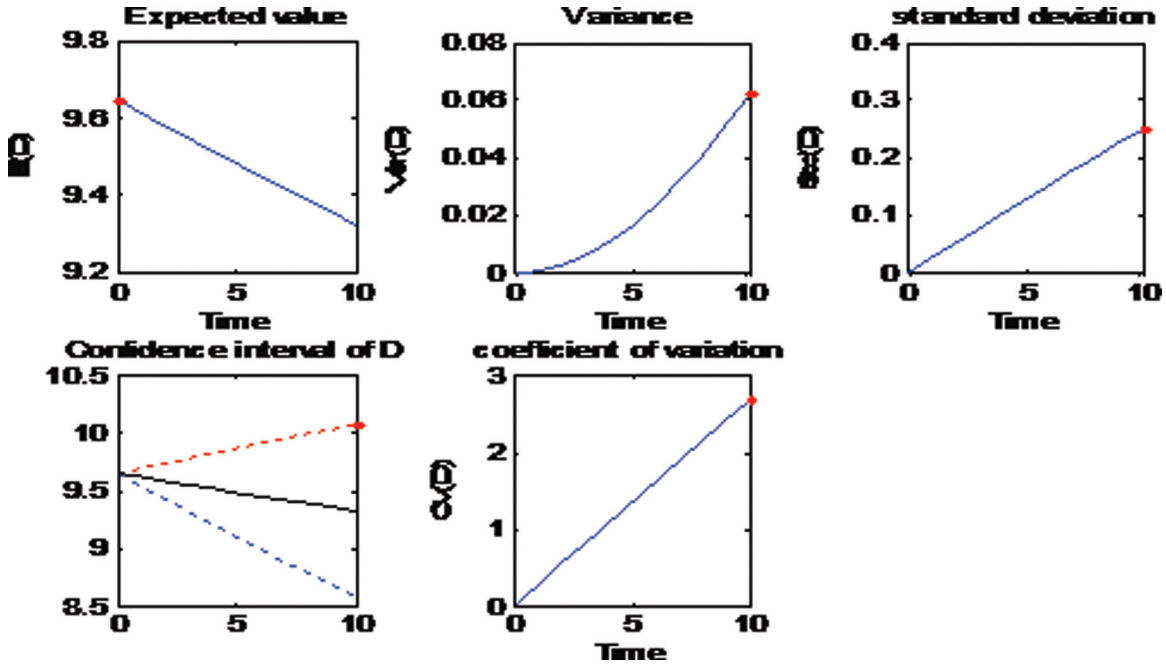

Figure 4.2. $D(n)$ uncomplicated random behaviors.

Table 4.6. Random $D(n)$ uncomplicated expected value end values and times

Variable	Minimum	Time	Maximum	Time
$E(D(n))$	9.326	10	9.65	0

Table 4.7. Extreme values and times of random $D(n)$ uncomplicated variance

Variable	Minimum	Time	Maximum	Time
$Var(D(n))$	0	0	0.0626	10

Table 4.8. End values and times of random $D(n)$ uncomplicated standard deviation

Variable	Minimum	Time	Maximum	Time
$Std(D(n))$	0	0	0.2502	10

Table 4.9. Extreme values and times of the coefficient of variation of random $D(n)$ uncomplicated variation coefficient

Variable	Minimum	Time	Maximum	Time
$CV(D(n))$	0	0	2.683	10

expected approximate diabetes ratio is in the range of %99 probability (261.4,305.7) at time n =10.

It is seen that the variability decreases in the SDC model process ($n \in [0,10]$). Extreme values are seen in the table (Table 4.6 and Figure 4.2).

It appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(D(0)) = 9.65$ was obtained for the expected value at the end of the process ($n = 0$).

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 4.7 and Figure 4.2).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n=10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(D(10)) = 0.0626$ was obtained for variance, ($n = 10$).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 4.2). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 4.8).

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(D(10)) = 0.2502$ was obtained for variance (n = 10)at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $D(n)$ in the random model (3) were also calculated as follows (Figure 4.2).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(D(n))/E(D(n))$ and random $\alpha, \beta, \gamma, \delta, \lambda, \mu, \rho$ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $D(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 4.9)

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of $D(n)$ is constantly increasing and reaches %0.02683 at $n = 10$. Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 4.2). The confidence intervals given in the figure are calculated as $CI = (E(D(n))–3.\text{std}(D(n)), E(D(n)) + 3.std(D(n)))$, and three gives the range of variation within the standard deviation. For poisson distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 4.10).

At the end of the process, three standard deviation intervals for D(n) variables are obtained as follows: $CI(D(10)) \in (8.574,10.0783)$

Model (3) states that the expectation for this value is $CI(D(10)) = 10.078$, that is, approximately %0.10078, and the expected approximate diabetes ratio is in the range of %99 probability (8.574,10.0783) at time $n = 10$.

It is seen that the variability in the SDC model process ($n \in [0.10]$) is stable and then increases. Extreme values are seen in the table (Table 4.11 and Figure 4.3).

IIt appears that the expected diabetes reached its highest level at the time of $n = 0$. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, $E(C(0)) = 11.05$ was obtained for the expected value at the end of the process $n = 0$.

Similarly, variance change ($n \in [0,10]$) appears to increase for the SDC model. Extreme values are seen in the table (Table 4.12 and Figure 4.3).

Table 4.10. End values and times in random $D(n)$ uncomplicated confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(D(n))$	8.574	10	10.0783	10

Table 4.11. Expected value of random $C(n)$ complication rate, extreme values and times

Variable	Minimum	Time	Maximum	Time
$E(C(n))$	11.04	10	11.05	0

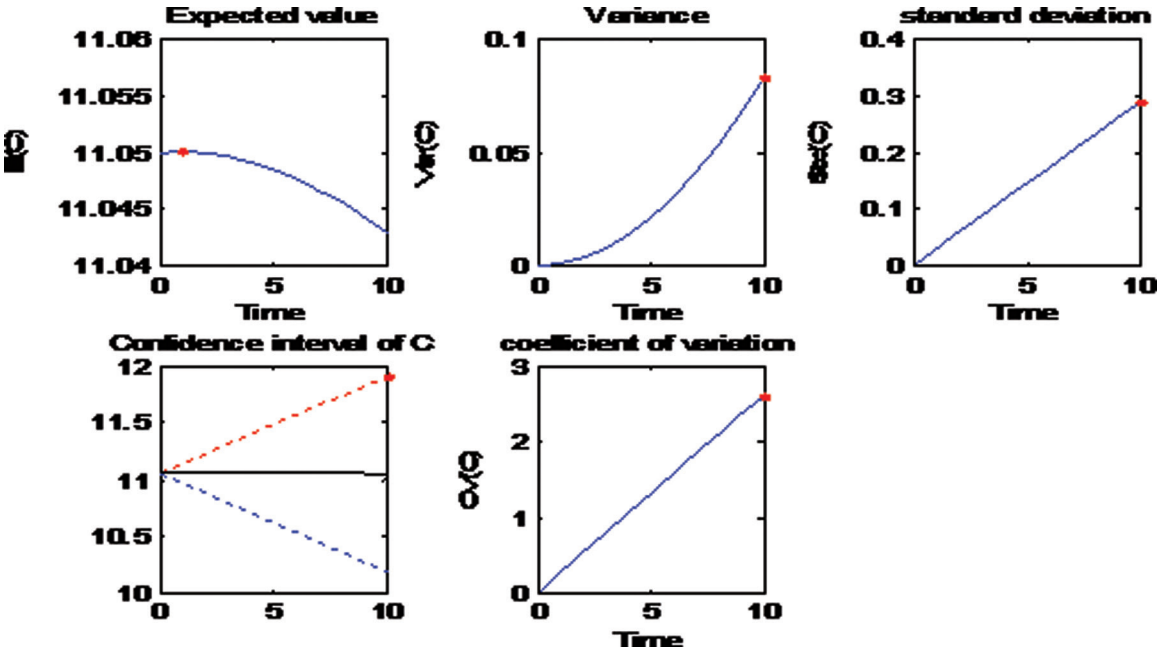

Figure 4.3. $C(n)$ random behavior of complication rate.

Table 4.12. Extreme values and times of variance of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Var(C(n))$	0	0	0.08313	10

Table 4.13. Extreme values and times of standard deviation of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Std(C(n))$	0	0	0.2883	10

Table 4.14. Extreme values and times of variation coefficient of random $C(n)$ complication rate

Variable	Minimum	Time	Maximum	Time
$Std(C(n))$	0	0	0.2883	10

Table 4.15. End values and times of random $C(n)$ complication rate in confidence interval

Variable	Minimum	Time	Maximum	Time
$CI(C(n))$	10.18	10	11.91	10

It is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments.In addition, at the end of the process, $Var(C(10))$ = 0.08313 was obtained for variance, (n = 10).

Similar to the variance, the changes in the standard deviation for the SDC model are shown below (Figure 4.3). By definition, the standard deviation is the square root of the variance, so these two numerical characteristics are expected to behave similarly. Extreme values for standard deviations are shown below (Table 4.13).

IIt is observed that the diabetes has reached its highest level of deviation from the average at the time of n = 10. Therefore, the results obtained from the deterministic model are more likely to be observed differently in an experiment that takes place randomly at these moments. In addition, $Std(C(10))$ = 0.2883 was obtained for variance (n = 10) at the end of the process.

Using the results obtained for the standard deviations and expected values, the variation coefficients for the variables $C(n)$ in the random model (3) were also calculated as follows (Figure 4.3).

Coefficient of Variation (CV) is calculated by definition as $100 \times std(C(n))/E(C(n))$ and random α, β, γ, δ, λ, μ, ρ parameters for the installation of model (3) are defined to have %5 coefficient of variation. However, as a result of examining the model, it is seen that the coefficient of variation of $C(n)$ variables increased to higher rates. The extreme values of the variation coefficients are given in the table below (Table 4.14).

Despite the %5 coefficient of variation in the parameters, it is observed that the variation rate of C(n) is constantly increasing and reaches %0.02611 at n=10 Therefore, it can be interpreted that the variability in random results increases as it progresses.

The results obtained for the expected values of the model (3) are given below (Figure 4.3). The confidence intervals given in the figure are calculated as $CI = (E(C(n))–3.\text{std}(C(n)), E(C(n)) + 3.std(C(n)))$, and three gives the range of variation within the standard deviation. For poisson distribution, this range includes about %99 of the values of the random variable. Therefore, the extreme values obtained for the expected values in these ranges are given below (Table 4.15).

At the end of the process, three standard deviation intervals for $C(n)$ variables are obtained as follows: $CI(C(10)) \in$ (10.18,11.91)

Model (3) states that the expectation for this value is $CI(C(10))$ = 11.91, that is, approximately %0.1191, and the expected approximate diabetes ratio is in the range of %99 probability ((10.18,11.91)) at time n = 10.

CONCLUSION

In this study, the mathematical model is analyzed by converting this system consisting of three differential equations modeling the responsive diabetes complication (SDC) model into discrete time with the advanced difference method. A stability analysis has been performed for this system of equations. The fundamental increase number and global stability for stable states of equilibrium point were studied. The discrete time probability distributions for the random behaviors of the SDC model were studied under random effects with Uniform, Binomial, Geometric and Poisson distributions.The expected value, variance, standard deviation, coefficient of variation and confidence intervals of the obtained solutions were found.The coefficients of variation for the five distributions are compared and for each distribution the parameter is defined to have a coefficient of variation of 5%.Although a 5% deviation rate was used for random parameters, the simulation results showed variability in the sugar ratio. Analysis of the random SDC difference model is provided with the help of graphs and tables.

DATA AVAILABILITY STATEMENT

No new data were created in this study. The published publication includes all graphics collected or developed during the study.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Anaç H, Merdan M, Bekiryazıcı Z. Global stabılıty in a pathogen-specıfic Cd8 immune response predıctıon model. Sigma J Eng Nat Sci 2019;10:325–42.

[2] Lamichhane S, Chen Y. Global asymptotic stability of a compartmantal model for a pandemic. J Egypt Math Soc 2015;23:251–5. [CrossRef]

[3] Side S, Sanusi W, Aidid MK, Sidjara S. Global stability of SIR and SEIR model for tuberculosis disease transmission with Lyapunov function method. Asian Journal of Applied Sciences 2016;9:87–96. [CrossRef]

[4] National Diabetes Statistics Report. Centers for Disease Control and Prevention (CDC), 2015 National Data of Diabetes.

[5] Hill J, Nielsen M, Fox MH. The Permanente Journal 2013;2:67–72.

[6] Widyaningsih P, Affan RC, Saputro DRS. A Mathematical Model for The Epidemiology of Diabetes Mellitus with Lifestyle and Genetic Factors, IOP Conf. Series: Journal of Physics: Conf Series 2018;1028.012110. [CrossRef]

[7] Elaydi S. An Introduction to Difference Equations. BerlinHeidelberg-New York: Springer-Verlag; 2016.

[8] Sandefur J. Discrete Dynamical Systems: Theory and Applications. Oxford: Oxford University Press; 1990.

[9] Samuelson PA. Conditions that a root of a polynomial be less than unity in absolute value. Ann Math Stat 1941;12:360–4. [CrossRef]

[10] Salceanu P, Smith H. Persistence in a discrete-time, stage-structured epidemic model. J Differ Equ Appl 2010;16:73–103. [CrossRef]

[11] Akdeniz F. Olasılık ve İstatistik. Ankara: Akademiyen Kitabevi; 2014

[12] Feller W. An Introduction to Probability Theory and Its Applications. 3rd ed. New York: John Wiley & Sons; 1968.

[13] Merdan M, Bekiryazici Z, Kesemen T, Khaniyev T. Comparison of stochastic and random models for bacterial resistance. Advances in Difference Equations 2017;133:1–19. [CrossRef]

[14] Merdan M, Anac H, Bekiryazici Z, Kesemen T. Solving of some random partial differential equations by using differential transformation method and laplace-padé method. Gumushane Universitesi Fen Bilimleri Enstitusu Dergisi 2019;9:108–18.

[15] Merdan M, Şişman Ş. Analysıs of random discrete time logistic model. Sigma Journal of Engineering and Natural Sciences Sigma Mühendislik ve Fen Bilimleri Dergisi 2020;38:1269–98.

[16] Merdan M, Şişman Ş. Investigation of linear difference equations with random effects. Advances in Difference Equations 2020:579:1–19. [CrossRef]