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Research Article

BUCKLING OF RECTANGULAR FSDT PLATES RESTING ON ORTHOTROPIC FOUNDATION BY MIXED FEM

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ABSTRACT

This study presents a mixed type finite element procedure for the linear buckling analysis of moderately thick plates lying on orthotropic elastic foundation. Kinematical expressions are due to the Mindlin plate theory and von Kármán strains. The force intensity exerted by orthotropic foundation on the plate is reflected according to the Pasternak model. Material directions of the foundation coincides with the global axes of the plate. The first variation of the systems nonlinear functional is obtained by following the Hellinger-Reissner principle. This expression is linearized according to the incremental formulation, thus the system and geometric matrices of the problem are obtained. Finite element equations are constructed by discretizing the plate domain with four noded isoparametric quadrilateral elements. After a static condensation procedure, force and couple type field variables are removed from the equations in order to reduce the problem into the solution of a standard Eigen-value system. Firstly, a convergence and comparison study is presented to verify the formulation and numerical procedure. The effects of foundation and plate parameters on the critical buckling loads are investigated.
Keywords: Mindlin plate, orthotropic pasternak foundation, Hellinger-Reissner, mixed finite elements, linear buckling.

1. INTRODUCTION

Structural elements encounter different loading cases under their service conditions. It is especially important for thin-walled structures (e.g. columns, plates, shells) to be analyzed by means of buckling states in order to build a proper design. Determination of critical load of linear elastic buckling is therefore a crucial step for structural systems. Limited to some special cases, elastic buckling loads of structures can be determined analytically [1]. It is well known that, mechanical responses of structures are dramatically affected from the interaction with an elastic medium [2]. This complicated state can be simulated using simple mechanical models e.g. elastic foundation assumptions. The literature on buckling analysis of plates resting on elastic foundation involves various plate theories (Classical Plate Theory (CPT), First Order Shear Deformation Theory (FSDT) etc.) and solution methods. Xiang et al. [3] presented an analytical solution for the free vibration analysis of Mindlin plates resting on Pasternak foundation under the action of in-plane loading. As a special case of the problem they also presented expressions for buckling

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analysis. Xiang et a l. [4], expanded the problem in [3] to the analysis of simply supported symmetrically laminated cross-plies. Lam et al. [5] presented exact solutions for static, free vibration and buckling analyses of Levy plates on Pasternak foundation. In order to solve the problem presented in [3] for different boundary conditions and nonuniform loading case Akhavan et al. [6] suggested an exact solution procedure. Matsunaga [7] employed power series to obtain free vibration and buckling characteristics of higher order shear deformable plates interacting with two parameter foundation. Park and Choi [8] adopted simplified FSDT and presented an analytical solution procedure for bending, free vibration and buckling analyses of plates resting on Pasternak foundation. Doğruoğlu and Omurtag [9] performed the buckling analysis of laminated composite thin plates resting on Pasternak foundation by introducing a mixed finite element method (MFEM). Setoodeh and Karami [10] employed a finite element procedure based on 3D elasticity theory to investigate static, free vibration and buckling behavior of plates with distributed and point type elastic supports. Recently, Kutlu and Omurtag [11] presented a mixed finite element solution of the linear buckling problem of moderately thick plates resting on isotropic Pasternak foundation. This study extends the study of the authors [11] by presenting a mixed finite element solution procedure for the linear buckling analysis of isotropic and homogeneous FSDT plates resting on orthotropic foundation. The field equations of the problem are based on the von Kármán deformation field and Mindlin plate assumptions. Different shear foundation parameters are taken into account in global coordinate directions to reflect a orthotropic version of Pasternak foundation model. Hellinger-Reissner variational principle is used to obtain the first variation of the defined problem. Nonlinear expression is linearized according to incremental formulation, hence the system and geometric matrices are determined. Plate domain is discretized by four noded quadrilateral elements where field variables are interpolated by bilinear shape functions. Integrals are calculated numerically according to 2×2 Gauss scheme. Generated system and geometric matrices are condensed statically in order to produce a standard eigen-value system. Firstly, the proposed formulation and numerical scheme is verified through some convergence studies. The effect of foundation and plate parameters on the buckling load is examined by introducing some original results.

2. FIELD EQUATIONS AND FORMULATION

Under the assumptions of Mindlin plate theory, mid-plane deformations (ε and γ) and curvatures (κ) of a plate involving von Kármán terms can be described in terms of the displacement field as follows [12]:

$$\left.\begin{aligned} &\varepsilon_{xx}=u_{,x}+\tfrac{1}{2}\left(w_{,x}\right)^2 \;;\; \varepsilon_{yy}=v_{,y}+\tfrac{1}{2}\left(w_{,y}\right)^2 \;;\; \gamma_{xy}=u_{,y}+v_{,x}+w_{,x}w_{,y} \;;\; \gamma_{xz}=w_{,x}+\varphi_x \\ &\kappa_{xx}=\varphi_{x,x} \;;\; \kappa_{yy}=\varphi_{y,y} \;;\; \kappa_{xy}=\varphi_{x,y}+\varphi_{y,x} \;;\; \gamma_{yz}=w_{,y}+\varphi_y \end{aligned}\right\} \tag{1}$$

Here, u,v,w are mid-plane displacements of the plate in global x,y,z directions and φ_x and φ_y are rotation of plate sections about y and x axes respectively. In subscripts, the terms inserted after a comma refers to a partial derivative of the variable with respect to that term. Equilibrium equations of a large deflecting Mindlin plate interacting with orthotropic foundation can be expressed as follows [13].

Here k is the Winkler parameter and G_{fx} and G_{fy} are shear foundation parameters in global x and y directions, respectively. N_{xx},N_{yy},N_{xy} are in-plane forces, M_{xx},M_{yy},M_{xy} are moments and Q_{xz},Q_{yz} are transverse shear forces. q_x,q_y,q_z refer to the intensity of external loads acting in x,y,z directions, respectively (Fig. 1). The Hellinger-Reissner variational principle describes the first variation of a system as follows [14]:

$$\left.\begin{aligned} q_x + N_{xx,x} + N_{xy,y} &= 0 \\ q_y + N_{xy,x} + N_{yy,y} &= 0 \\ q_z + \left(N_{xx}w_{,x} + N_{xy}w_{,y}\right)_{,x} + \left(N_{xy}w_{,x} + N_{yy}w_{,y}\right)_{,y} + Q_{xz,x} + Q_{yz,y} - kw + G_{fx}w_{,xx} + G_{fy}w_{,yy} &= 0 \\ M_{xx,x} + M_{xy,y} - Q_{xz} &= 0 \\ M_{yy,y} + M_{xy,x} - Q_{yz} &= 0 \end{aligned}\right\} \tag{2}$$

$$\delta\Pi_{HR} = \int_V \left(\boldsymbol{\varepsilon}^{\mathbf{u}} - \boldsymbol{\varepsilon}^{\sigma}\right)^T \delta\boldsymbol{\sigma}^{\sigma}\mathrm{d}V + \int_V \left(\left(\boldsymbol{\sigma}^{\sigma}\right)^T \delta\boldsymbol{\varepsilon}^{\mathbf{u}} - \mathbf{q}^T\delta\mathbf{u}\right)\mathrm{d}V - \int_\Gamma \hat{\mathbf{t}}^T\delta\mathbf{u}\mathrm{d}\Gamma = 0 \tag{3}$$

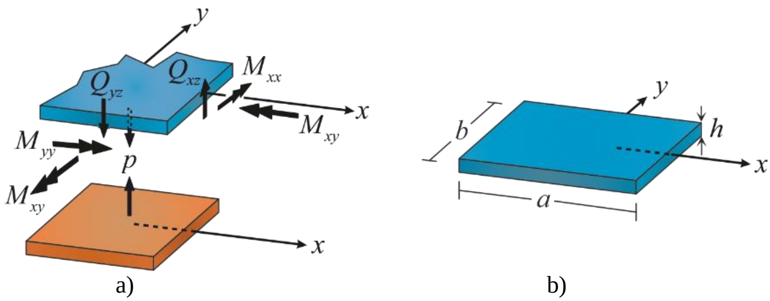


Figure 1. a) Directions of the stress res ultants of the plate interacting with foundation b) Plate dimensions

According to Eq. (3), both sets of constitutive equations and equilibrium equations are satisfied in a weak form. Inserting field Eqs. (1) and (2) in Eq. (3) yields the first variation of the system:

$$\begin{aligned} \delta\Pi_{HR} &= \int_\Omega \left[u_{,x} + \tfrac{1}{2}\left(w_{,x}\right)^2 - \frac{1}{Eh}\left(N_{xx} - \upsilon N_{yy}\right)\right]\delta N_{xx}\mathrm{d}\Omega \\ &+ \int_\Omega \left[v_{,y} + \tfrac{1}{2}\left(w_{,y}\right)^2 - \frac{1}{Eh}\left(N_{yy} - \upsilon N_{xx}\right)\right]\delta N_{yy}\mathrm{d}\Omega + \int_\Omega \left[u_{,y} + v_{,x} + w_{,x}w_{,y} - \frac{N_{xy}}{Gh}\right]\delta N_{xy}\mathrm{d}\Omega \\ &+ \int_\Omega \left[\varphi_{x,x} - \frac{12}{Eh^3}\left(M_{xx} - M_{yy}\upsilon\right)\right]\delta M_{xx}\mathrm{d}\Omega + \int_\Omega \left[\varphi_{y,y} - \frac{12}{Eh^3}\left(M_{yy} - M_{xx}\upsilon\right)\right]\delta M_{yy}\mathrm{d}\Omega \\ &+ \int_\Omega \left[\varphi_{x,y} + \varphi_{y,x} - \frac{12M_{xy}}{Gh^3}\right]\delta M_{xy}\mathrm{d}\Omega + \int_\Omega \left[w_{,x} + \varphi_x - \frac{6Q_{xz}}{5Gh}\right]\delta Q_{xz}\mathrm{d}\Omega \\ &+ \int_\Omega \left[w_{,y} + \varphi_y - 6Q_{yz}/(5Gh)\right]\delta Q_{yz}\mathrm{d}\Omega \\ &+ \int_\Omega \left[N_{xx}\delta u_{,x} + N_{xy}\delta u_{,y}\right]\mathrm{d}\Omega + \int_\Omega \left[N_{xy}\delta v_{,x} + N_{yy}\delta v_{,y}\right]\mathrm{d}\Omega \\ &+ \int_\Omega \left[\left(N_{xx}w_{,x} + N_{xy}w_{,y} + Q_{xz}\right)\delta w_{,x} + \left(N_{xy}w_{,x} + N_{yy}w_{,y} + Q_{yz}\right)\delta w_{,y}\right]\mathrm{d}\Omega \\ &+ \int_\Omega \left[kw\delta w + G_{fx}w_{,x}\delta w_{,x} + G_{fy}w_{,y}\delta w_{,y}\right]\mathrm{d}\Omega + \int_\Omega \left[M_{xx}\delta\varphi_{x,x} + M_{xy}\delta\varphi_{x,y} + Q_{xz}\delta\varphi_x\right]\mathrm{d}\Omega \\ &+ \int_\Omega \left[M_{xy}\delta\varphi_{y,x} + M_{yy}\delta\varphi_{y,y} + Q_{yz}\delta\varphi_y\right]\mathrm{d}\Omega - \int_\Omega \delta w q_z\mathrm{d}\Omega - \int_\Gamma \hat{t}_j\delta u_j\mathrm{d}\Gamma = 0 \end{aligned} \tag{4}$$

Here, E, G and h are Young modulus, shear modulus and thickness of the plate respectively. Ω reflects domain of plate's mid-plane and Γ denotes plate boundary where $\hat{\mathbf{t}}$ is traction at the boundary. The first variation with given in Eq. (4) involves both displacement and stress resultant type field variables with nonlinear terms. Applying the incremental formulation [15] yields the following scheme for an iterative $(i = \text{iteration step})$ solution of static problem:

$$\left(\overset{++}{\mathbf{K}} + \overset{++}{\mathbf{K}}^{nl(i-1)}\right)\overset{+}{\mathbf{X}}^{(i)} = \overset{+}{\mathbf{F}} - \overset{+}{\mathbf{K}}^{(i-1)} \quad ; \quad \bar{\mathbf{X}}^{(i)} = \mathbf{X}^{(i-1)} + \overset{+}{\mathbf{X}}^{(i)} \tag{5}$$

Here $\overset{++}{\mathbf{K}}$ collects the linear part of system matrix while $\overset{++}{\mathbf{K}}^{nl(i-1)}$ involves nonlinear terms. $\overset{+}{\mathbf{F}}$ is the external load vector and $\overset{+}{\mathbf{K}}^{(i-1)}$ is the state vector. $\mathbf{X}$ is the vector of field variables, namely displacements, forces and force couples. In order to revolve into a buckling analysis, in-plane force components will be considered constant which yields their variations become zero $\delta N_{xx} = \delta N_{yy} = \delta N_{xy} = 0$. Finally, neglecting the external loads on the system Eq. (5) becomes an eigen-values system:

$$\left(\begin{bmatrix} [\mathrm{k}_{ss}] & [\mathrm{k}_{su}] \\ [\mathrm{k}_{us}] & [\mathrm{k}_{uu}] \end{bmatrix} - \lambda_b \begin{bmatrix} [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{K}_g] \end{bmatrix}\right)\begin{Bmatrix} \{\mathbf{M}\}^T \\ \{\mathbf{U}\}^T \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{0}\} \\ \{\mathbf{0}\} \end{Bmatrix} \tag{6}$$

Here, λ_b corresponds to the critical value of the in-plane loading regarding the buckling load. $\mathbf{K}_g$ is the geometric matrix of the system. After eliminating the stress resultant type field variables from Eq. (6), a system in terms of displacements can be obtained. After static condensation the condensed system matrix is obtained as follows:

$$\left[\mathbf{K}^*\right] = \left[\mathrm{k}_{uu}\right] - \left[\mathrm{k}_{su}\right]^{\mathrm{T}}\left[\mathrm{k}_{ss}\right]^{-1}\left[\mathrm{k}_{su}\right] \tag{7}$$

Hence, the following form of the eigen-value problem can be expressed:

$$\left(\left[\mathbf{K}^*\right] - \lambda_b\left[\mathbf{K}_g\right]\right)\left\{\mathbf{U}^T\right\} = \{0\} \tag{8}$$

$\mathbf{U}$ reflect the mode vector (eigen-vector) corresponding to a buckling load of the system obtained from Eq. (8) as an eigen-value.

3. NUMERICAL RESULTS

In order to verify the proposed procedure and numerical solution, some comparison and convergence analyses are performed for square plates. Then the effect of orthotropic foundation parameters on the buckling results of rectangular plates of different parameters will be discussed. First, a very thin plate is analyzed without a contact with foundation and then thin plate and moderately thick plate cases are considered with foundation interaction. Dimensions a, b denotes side length of rectangular plate along x, y directions, respectively (Fig. 1b). In terms of plate's bending rigidity $D = Eh^3/12(1-\upsilon^2)$ and side length, foundation parameters and buckling load are presented in nondimensional form as $\tilde{k} = kb^4/D$, $\tilde{G}_f = G_f b^2/D$, and $\tilde{N}_b = N_b b^2/D$. Poisson's ratio of the plate is kept constant $\upsilon = 0.3$ in calculations.

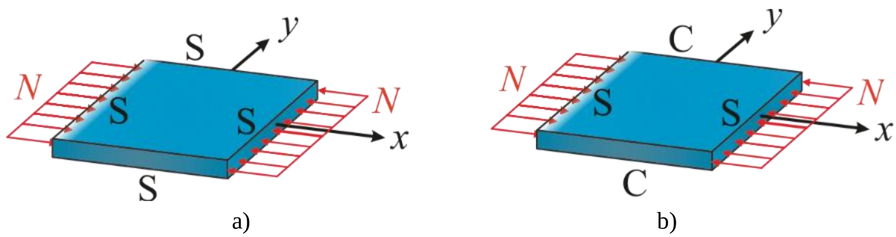


Figure 2. Uniaxially loaded plate a) SSSS boundary conditions b) SCSC boundary conditions

3.1. Convergence and Comparison Example 1: Very Thin Plate (No Foundation)

The proposed mixed finite element formulation overcomes the shear locking problem which is encountered in shear deformable plate solutions. In order to reflect this property of the formulation, a very thin square plate ($b/h = 1000$) is handled. Four edges of the plate are considered to be simply supported. A range from 4 to 400 elements are employed in domain discretization and convergence is observed for increasing number of elements. Critical value of the uniformly acting unidirectional in-plane load (Fig. 2a) is presented in Fig. 3 and compared with the result given in [8] (Table 4). Observing Fig. 3 yields that the mixed finite element solution for very thin plate case converges consistently and is in very good agreement with literature. For instance, 20×20 mesh configuration has a difference of 0.2% with CPT result. This example reveals that the formulation does not suffer shear locking problem and gives good results even for very thin plate case.

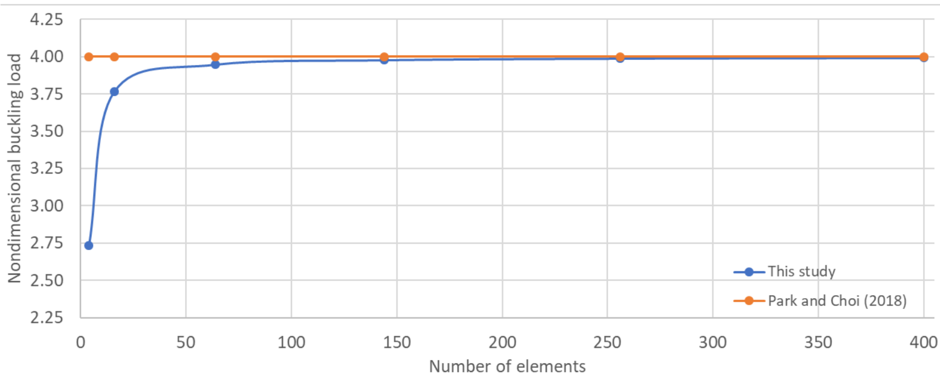
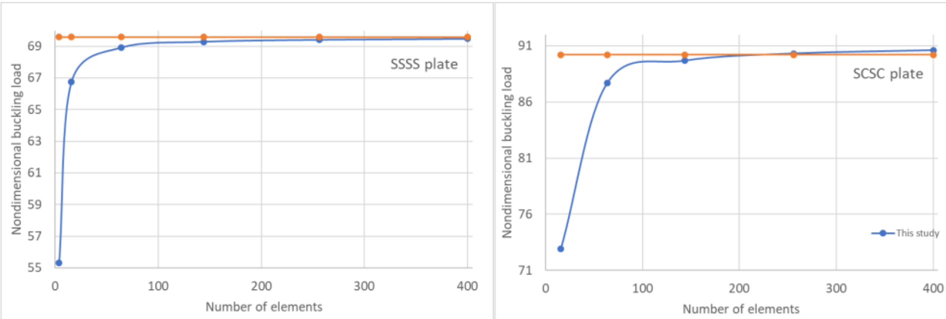


Figure 3. Critical buckling load $\tilde{N}_b / \pi^2$ of simply supported very thin square plate with respect to number of elements

3.2. Convergence and Comparison Example 2: Thin Plate on Isotropic Elastic Foundation

In this example, buckling loads are calculated for thin ($b/h = 100$) square plates resting on Pasternak foundation and under the action of a uniaxial loading along x direction. Nondimensional foundation parameters are selected as $\tilde{k} = 100$ and $\tilde{G}_{fx} = \tilde{G}_{fy} = 10$ and results are compared with the exact solutions presented in [6]. Nondimensional buckling loads of SSSS (Fig. 2a) and SCSC (Fig. 2b) supported plates are presented in Fig. 4. Denoting number of half waves in (x, y) directions as (m,n); buckling loads corresponding to (m,n)=(1,1) mode and (m,n)=(2,1) mode are calculated for SSSS and SCSC plate, respectively. According to Fig. 4, a consistent convergence behavior and good agreement with exact solution is obtained for both support configurations. However, it is observed that results of SSSS plate converges faster compared to CCCC plate.



a) b)

Figure 4. Nondimensional buckling load $\tilde{N}_b$ of thin square plate resting on Pasternak foundation a) SSSS Plate b) SCSC Plate

3.3. Convergence and Comparison Example 3: Moderately Thick Square Plate on Elastic Foundation

This example updates the parameters of the Example 2 by changing the thickness to width ratio to $b/h = 10$. This moderately thick plate is investigated under the SSSS boundary condition. The critical buckling load of the square plate is compared with the results presented in [6] and given in Fig. 5. A consistent convergence behavior is also observed in this example. However, compared to thin plate case, a minor difference with the results of [6] is noticed.

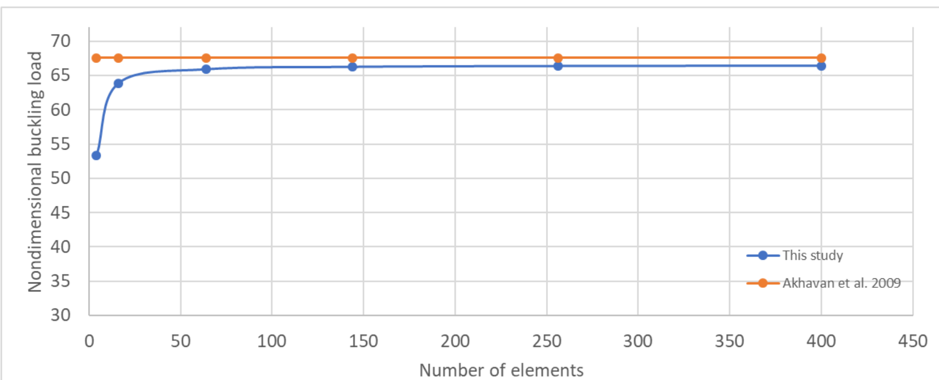


Figure 5. Nondimensional buckling load $\tilde{N}_b$ of moderately thick square plate resting on Pasternak foundation

3.4. Orthotropic Foundation Example: Effect of Shear Foundation Parameter and Aspect Ratio on Buckling Load of Plates with Different Thickness

This example introduces some linear buckling loads of rectangular plates resting on orthotropic Pasternak foundation. For this purpose, the Winkler foundation parameter is kept constant $\tilde{k}=100$ and three different foundation configurations are considered, namely $(\tilde{G}_{fx},\tilde{G}_{fy})$ =(10,10),(10,20),(10,40). Three different aspect ratios are selected as $a/b=$ 1,1.5,2 and two different thickness to width ratios are chosen, namely $h/b=0.01,0.1$. Regarding the presented convergence analyses, three different mesh configurations of 16×16, 24×16, 32×16 elements are employed in three different plate domains of aspect ratios $a/b=$ 1,1.5,2, respectively. Buckling load of SSSS supported and uniaxially loaded plates corresponding to (m,n)□(1,1) mode is presented in Table 1 for different foundation and plate parameters. It is observed from the Table 1 that as shear foundation parameter and plate aspect ratio increases, the buckling load is less effected from the increase of thickness to width ratio. The effect of aspect ratio on the buckling load increases as the ratio of shear foundation parameters ($\tilde{G}_{fy}/\tilde{G}_{fx}$) increases. However, this effect is almost the same for different thickness to width ratios of the plate. Finally, as expected the change in $\tilde{G}_{fy}/\tilde{G}_{fx}$ ratio becomes more influential on the buckling load of the plate as the aspect ratio (a/b) diverges from 1.

Table 1. Buckling load ($\tilde{N}_b$) of uniaxially loaded simply supported rectangular plates for different foundation and plate configurations corresponding to (m,n)=(1,1) mode

		$h/b=0.01$			$h/b=0.1$		
	$(\tilde{G}_{fx},\tilde{G}_{fy})$	(10,10)	(10,20)	(10,40)	(10,10)	(10,20)	(10,40)
a/*b*	1	69.4166	79.4149	99.4116	66.3711	76.2258	95.9353
	1.5	101.5391	124.0766	169.1516	98.6770	120.9726	165.5639
	2	152.1889	192.2812	272.4659	148.7013	168.2994	267.8478

4. CONCLUSION

This study proposes a finite element formulation for the linear buckling analysis of moderately thick plates resting on orthotropic foundation. Shear deformation of the plate is based on Mindlin plate assumptions. Using von Kármán deformations differential equations of the plate regarding large deflection problem are obtained. Pasternak foundation model in orthotropic form is included in the formulation where material directions of the foundation coincide with global axes. First, variation of the problem's functional is obtained by following the Hellinger-Reissner variational principle. Next, this nonlinear equation is linearized according to incremental formulation to yield system and geometric matrices. For the numerical procedures, four noded quadrilateral elements with bilinear shape functions are employed where a 2×2 Gauss scheme is adopted for the calculation of integrals. After constructing the system matrix, it is condensed by eliminating stress resultant type field variables to construct a standard eigen-value problem. The proposed mixed formulation is verified through some convergence and comparison analyses. It is shown that the mixed formulation does not suffer shear locking problem. Some original results are presented to the literature by investigating the effect of orthotropic foundation parameters and aspect ratio of the plate on the buckling behavior of the plate.

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