Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

NUMERICAL INVESTIGATION OF WAVES GENERATED BY OSCILLATING SIDEWALL OF A TANK: A LATTICE-BOLTZMANN STUDY

Hamed Akhtari SHISHAVAN*[1], Iraj MIRZAEE[2], Nader POURMAHMOUD[3]

[1]*Department of Mechanical Engineering, Urmia University, Urmia, IRAN;* ORCID: 0000-0003-0805-6490
[2]*Department of Mechanical Engineering, Urmia University, Urmia, IRAN;* ORCID: 0000-0002-3523-5251
[3]*Department of Mechanical Engineering, Urmia University, Urmia, IRAN;* ORCID: 0000-0002-9974-6149

Received: 05.12.2019 Revised: 02.01.2020 Accepted: 25.01.2020

ABSTRACT

Tsunamis happen every year in the seas and the oceans throughout the world. These tidal waves propagate at high speeds and in various directions and, if they reach the shoreline, tremendous damage can occur to these areas as well as their structures and facilities. Therefore, understanding this complex phenomenon and predicting its behavior can mitigate such damages. In the present study, the Lattice-Boltzmann method (LBM) is utilized to simulate the phenomenon of wave formation in a tank. Considering seawater as viscous flow, the governing Navier-Stokes equations for shallow water along with the LBM is used to simulate water level. Wave generation is also created by simulating a tank that flushes its left wall and does it once again. The obtained results show small waves at early times which become more intense over time. Besides, the pressure at the end of the tank is at maximum and it consequently decreases as moves upwards. This reveals that hydrostatic pressure variations are due to fluid's height. In addition, the effect of shaking sidewall angles on the waves is investigated. The simulation results demonstrate a significant rising trend in wave height attributable to angularization of the sidewall. Moreover, by 30^o oblique, wave production amplifies but no change is observed in the wave height at distant points from the slope.
Keywords: Lattice-Boltzmann Method, tank wave, sidewall angle, oscillating motion.

1. INTRODUCTION

Irregular waves formed in the seas give rise to a large number of regular waves with various alternation times, directions, amplitudes, and phases. Irregular leveling of the water surface due to these waves is also the result of a number of regular waveforms having constant amplitudes and phases whose choice is perfumed randomly. In order to design marine structures and to calculate forces generated by wave fields on these structures, characteristics of waves should be delineated [1]. Waves must be similarly recorded under different conditions to take into account their characteristics in the area in better conditions. For this purpose, floating booms on water are typically employed. The data recorded in this way can accordingly lead to a profile of changes in water level [2].

In this respect, Zheng et al. [3] presented a new method for nonlinear simulation of sea waves. In their method, a nonlinear term including the effect of history of waves on formation of each

* Corresponding Author: e-mail: aktari1362hamed@gmail.com, tel: +989141218213

wave had been added to a linear term of the wave. It should be also noted that the obtained wave profile did not have normal distribution characteristics in the nonlinear wave simulation method provided. In fact, in this study, the nonlinear simulation method had presented a more realistic waveform compared with linear simulation ones. Similarly, Valipour et al. [4] calculated fatigue rate of offshore pipelines on open span with a new attitude. In their work, they proposed a new method for nonlinear wave simulation. In this method, they tried to distribute wave profiles as a normal distribution or the Gaussian distribution via applying corrections to nonlinear equations suggested by Zheng et al.

Besides, the use of the boundary element method (BEM) is of utmost importance in order to investigate wave profile at free surface and solve related equations. Utilization of the Stochastic Eulerian Lagrangian Method (SELM) can also help in eliminating numerical instabilities in this method. Longuet-Higgins and Cokelet [5] were among the first researchers examining nonlinear surface waves using the BEM. In their work, they first employed the SELM to improve free surface. As well, Lin [6] investigated waveforms generated both analytically and by means of the BEM and compared his findings with experimental results. Moreover, Clelebi et al. [7] simulated a linear and nonlinear wave in a three-dimensional (3D) wave pool using the BEM and the SELM. They additionally utilized smoothing and re-meshing techniques to remove numerical instabilities in the Lagrangian part. Grilli et al. [8] further studied the phenomenon of wave fracture numerically using the BEM and the SELM whose range of solution was based on a 3D wave and optionally in the amplitude domain. Simulating nonlinear waves, Dommermuth and Yue [9] presented a new method for improving free surface and excluding numerical instabilities.

Clearly, use of the computational fluid dynamics (CFD) technique reduces complexities and costs of empirical works [10, 14]. The LBM is thus a practical simulation technique for solving flow problems numerically. This method is also feasible as a simulation technique for systems such as suspended solid particles or a polymeric liquid. Unlike conventional numerical schemes based on discretization of macroscopic continuum equations, the LBM is founded on microscopic models and mesoscopic kinetic equations [15]. One of the most important benefits of the LBM is the explicit form of governing equations and easy solution of parallel ones as well as employment of boundary conditions on curved boundaries. The applications of the LBM are in the fields of incompressible flow simulation in complicated geometries like blood flow in vessels, multiphase flows, free convection problems, moving boundaries, chemical reactions, porous media flows, suspended particles, magnetohydrodynamic (MHD) flows, non-Newtonian fluid flows, Large Eddy simulations (LESs), turbulence flows in aerodynamics, as well as other applications [16]. It was also developed into an efficient method to deal with problems including flow-solid interactions [17]. Accordingly, applications in a multitude of fields such as drug delivery in medicine or solid-liquid separation in process engineering rely on physical laws of particulate flows. A raw approximation often suffices for the design of facilities; however, a more in-depth understanding of the dynamics and the impact of the acting forces is crucial in the improvement process [18].

Therefore, they provide a reasonable base for simulations of a large number of particles, which are computationally expensive. Thanks to advances in computing architecture and algorithms over recent years, it is possible to simulate a large number of single particles [19]. Such ELMs have become a feasible approach since they can be based on simple differential equations; therefore, they allow fast computations for a reduced complexity. In such discrete element methods (DEMs), particles can be approximated as spheres in a first step, yielding good accuracy for numerous applications [20].

In the present study, wave generation is created through simulating a tank whose bed oscillates.

In the following, the effect of shaking sidewall angles on the waves is investigated. The simulation results also show a significant rising trend in wave height attributable to the angularization of the oscillating sidewall by the LBM.

2. NUMERI CAL SIMULATION

In this study, numerical simulation of the water-wave phenomenon has been carried out on the surface using the LBM. The purpose of this section is to solve a laminar two-dimensional (2D) fluid flow in a tank. The oscillating motion of the wall can thus create waves inside the tank, especially when it is filled with a fluid and its surface is open to the air. Smooth waves can be thus produced by a set of suitable frequencies and amplitudes. Besides, the bed wall of the tank moves sinusoidally.

A rectangular tank with a length of *L* and a height of *H* is investigated, as shown in Figure 1. The left wall is also assumed to have a motion with sinusoidal changes. The upper wall is in the atmosphere; therefore, it remains at atmospheric pressure. The flow is additionally assumed laminar. The number of grids on the sidewalls is 60 and they are 230 on the lower and upper walls of the tank. In this shallow water depth, it is expected that the free surface has horizontal dominating velocity components, and it is thus suitable for the present model formulation. Moreover, it is emphasized that a traveling wave or a bore may be generated in shallow water depth.

Herein, the free-surface behavior in relation to the waves generated by the oscillating walls based on excitation in a rectangular tank is discussed. Although the bore formation is confined in tanks, the physics at the location of the bore is similar to the ones observed on shorelines. Moreover, fully-developed tank bores resemble the tongue of tsunami waves. However, the present numerical study is limited to weak bores due to the non-overturning numerical formulation.

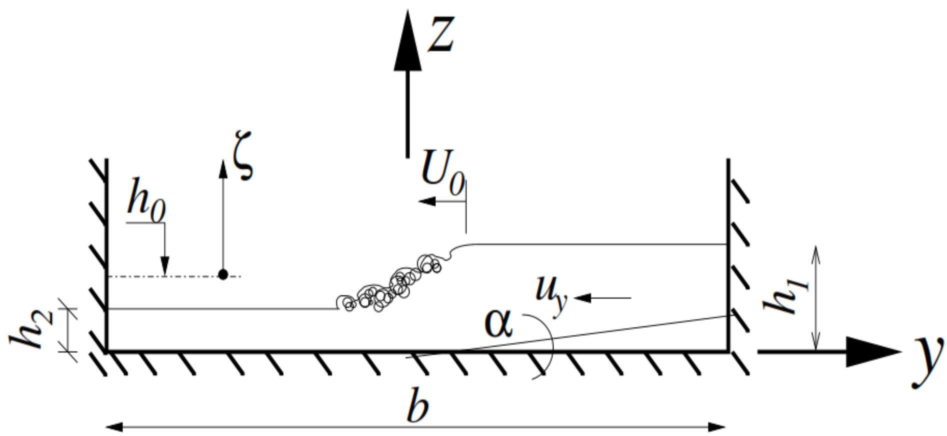


Figure 1. Tank geometry

3. BOUNDARY CONDITIONS

In this model, the right sidewall and the bottom wall are stationary, and the left wall is fluctuated vertically as shown in Figure 2. The top of the free-surface tanks is also considered as a pressure outlet.

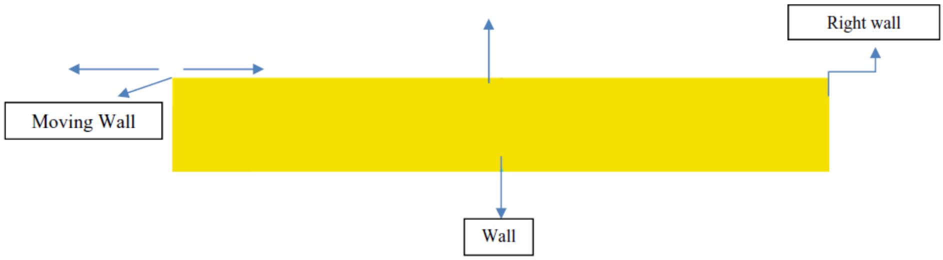


Figure 2. Simulated tank boundary conditions

4. GOVERNING EQUATIONS

The LBM is a useful simulation technique for elucidating flow problems in a numerical manner. This method is also feasible as a simulation technique for systems such as suspended solid particles or a polymeric liquid. The LBM was first developed by Mcnamara and Zanetti [21] in 1998 to explain the problems with lattice gas automata method. Unlike conventional numerical schemes based on the discretization of macroscopic continuum equations, the LBM is founded on microscopic models and mesoscopic kinetic equations. The LBM similarly recovers the N-S using the Chapman-Enskog expansion. Accordingly, the forced horizontal acceleration of the tank in the y-direction is:

$$Z'' = a_h \omega_h^2 \cos(\omega_h t) \tag{1}$$

where a_h and ω_h represent forcing amplitude and frequency, respectively. The water wave equation in the rotating flows is also discretized and approximated by an LB single-time relaxation model (LBGK) to simulate free-surface flows in shallow water. Besides, the flow field is signified by the particles that follow the lattice points, wherein the streams collide when meeting. With reference to the Boltzmann equation of classical kinetic theory, distribution of the fluid molecules is represented by the particle distribution function $f_i(x, c, t)$ where i denotes propagation direction. The function also defines the mass density ρ of the lattice particles (herein equivalent to a water column h) located at the lattice point (x) at time t which moves with velocity c_i. At the lattice point, the particles have different velocities following a Maxwell-Boltzmann distribution of equilibrium. The single-phase one-dimensional (1D) LBGK formulation for shallow liquid flows is thus introduced as:

$$\frac{\partial f_i}{\partial t} + c\frac{\partial f_i}{\partial t} = -\frac{(f_i - f_i^{eq})}{\tau} + \frac{1}{N_i c^2} c_i F_i \tag{2}$$

where, N is a constant depending on lattice geometry and is defined as N_i = 9 for the D_2Q_9 velocity set (2D). The first term of Equation (2) represents the effect of the local change of the fluid motion in time and the second term describes the convection. It should be noted that the convection operator of the LBM is linear. This property originates from kinetic theory. The first term (1) on the right-hand-side (RHS) is the non-equilibrium distribution function which designates the effect of the collisions and the second term on the RHS characterizes the force term defined as $F_i = -gh\frac{\partial h_b}{\partial x}$ (where h_b is the bed slope). The time-scale parameter t also describes collisional relaxation to local equilibrium and $1/\tau$ refers to collision frequency. In the present formulation, it is limited to a single value (i.e. BGK approximation) and is defined as $\tau = 3\,\nu/c^2 + \Delta t/2$ where $\nu = 10^{-6}$.

m^2/s is the kinematic viscosity of water and $c = \Delta x/\Delta t$. The time step is Δt and the grid spacing is Δx. $\tau > \Delta t/2$. If $\tau > 1$ is selected to ensure numerical stability although the results may be unreliable. Therefore, it is better to select $0.5 < \tau \leq 1$. In the present work, $\tau = 1$ has been taken. It should be noted that free-surface nonlinearities are captured via the first term (2) on the RHS and assuming a constant value of τ is a crude approximation. Equation (2) may be viewed as a special finite difference discretization of the single-time relaxation approximation of the Boltzmann equation for discrete velocities. One approach to solve the discrete Boltzmann equations is to use a first-order Euler time difference scheme and a first-order upwind space discretization for the convection term with the uniform lattice spacing Δx. One can then obtain the following algebraic relations:

$$f_i(\vec{x} + \Delta\vec{x}, t + \Delta t) - f_i(\vec{x}, t) = -\frac{\Delta t \times (f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t))}{\tau} - \frac{\Delta t}{N_i c^2} c_i g h \frac{\partial h_b}{\partial x} \quad (3)$$

Equation (3) is commonly referred to as the Lattice-BGK equation or Lattice-Boltzmann equation (LBE). It is observed that Equation (3) is the result of replacing time derivatives of Equation (2) by a first-order time difference and a first-order upwind discretization for the convective term. Furthermore, it is reiterated and observed that Equation (2) represents a finite difference equation. Therefore, if the LBE is viewed as a finite difference model for solving the discrete velocity Boltzmann equations, it becomes necessary to address numerical instability and accuracy like other numerical methods. Importantly to note is that the equation, at first glance, is first-order accurate in time and space.

The moments of the equilibrium distribution functions also signify conservation of mass, momentum, and static/dynamic pressures, and are expressed as follows:

$$h = \sum_{i=0}^{2} f_i^{eq} \quad ; \qquad \rho u = \sum_{i=0}^{2} f_i^{eq} c_i \quad ; \qquad \frac{1}{2} g h^2 + h u^2 = \sum_{i=0}^{2} f_i^{eq} c_i c_i \quad (4)$$

5. RESULTS

5.1. Validation

Wave height is considered as one of the most important features. In a local area, orientation, tidal surface, and magnitude of the waves can be very different; depending on the topography of the sea floor.

In Figure 3, variations in the dimensionless wave height are shown in this model in relation to the dimensionless depth. As observed, there is no change in the wave height at the distant points from the slope. After the arrival of the wave to the sloping area, there is a slight change in height, which can be seen with the advancement of the wave towards the coast and, consequently, the reduction in the depth of water increases in the wave height. To validate the model, the results of the wave height change are compared with the Green’s function equation [22], which is the theory of shallow water wave, expressed as follows [22]:

$$\frac{H}{H_0} = \left(\frac{h_0}{h}\right)^{1/4} \quad (5)$$

In this case, H is wave height, h refers to depth of water, and sub-zero represents values at reference point (deep water). Figure 3 compares the results of variations in the wave height between the present LBM and the shallow water theory [22]. Based on this comparison, there is an error of about 3% between the results of the present model and the Green’s function equation. Therefore, a good correlation is observed between the model and the theory results.

In this study, a numerical model is developed for wave motion analysis based on the numerical solution of the Navier-Stokes equations for viscous fluid and suitable boundary conditions. The analytical equation i.e. the Green analytical model is applicable to very simple situations including the slope of the floor and the fixed wall. Therefore, the present innovation of the manuscript is to investigate the changes in the complex wave, including non-sequential and oscillating ones.

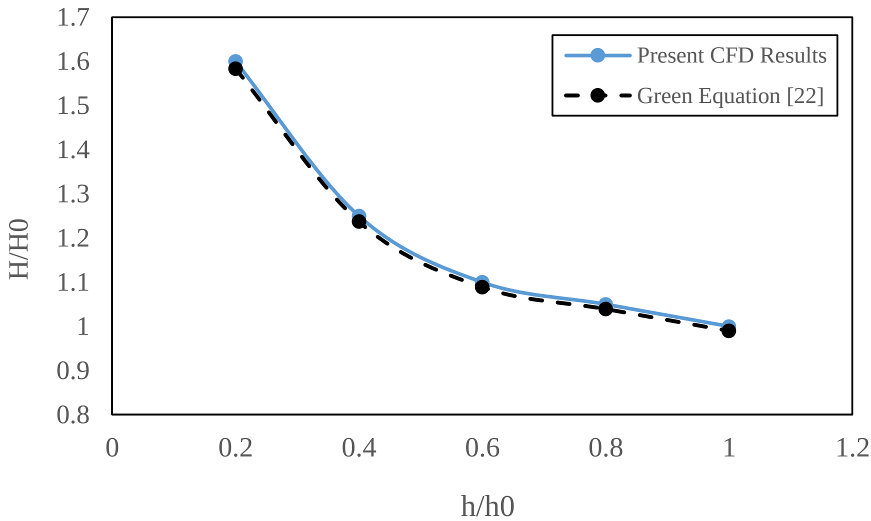
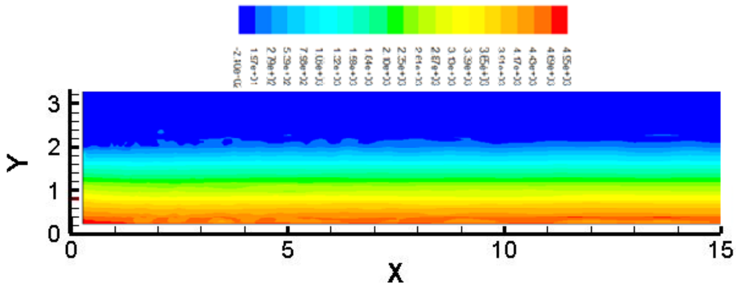


Figure 3. Comparison of results of present numerical work and theory [22] for dimensionless variation in wave height vs. depth in the model with a 30^{o} oblique left-side oscillating wall

5.1. The Tank with an Oscillating Sidewall

In this study, at first, a tank with an oscillating (left side) sidewall is simulated and the wave formation is investigated. Figures 4 to 8 show pressure and vorticity (water level) at four times i.e. 0.5, 1.0, 1.5, 2.0, and 4.0 seconds for a simple tank. Accordingly, there are small waves at early times which become more intense over time. The pressure at the end of the tank is at maximum and it consequently decreases as it moves upwards, indicating that the hydrostatic pressure variations are caused by the fluid's height.



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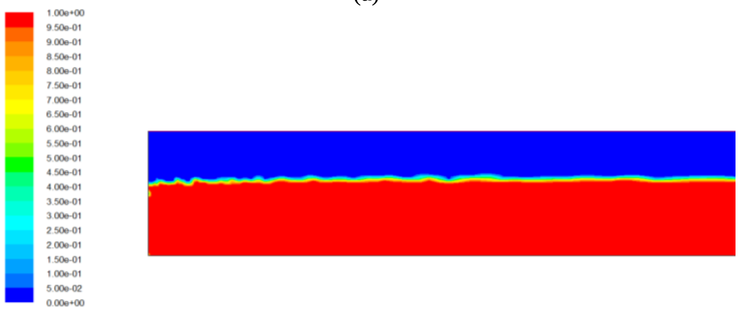
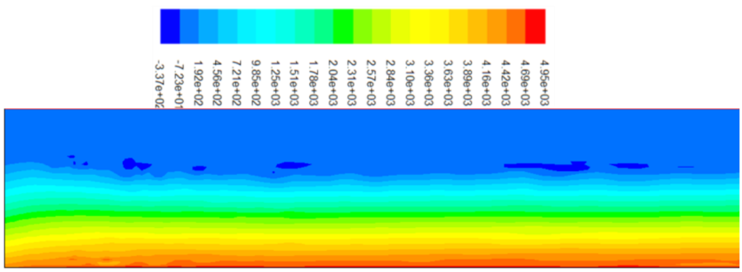


(b)

Figure 4. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a left-side oscillating wall at t=0.5s



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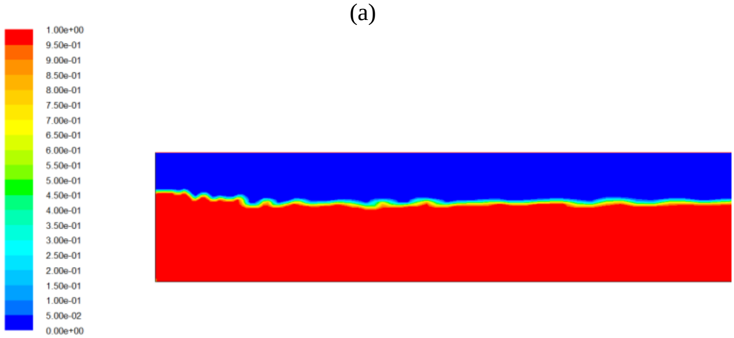
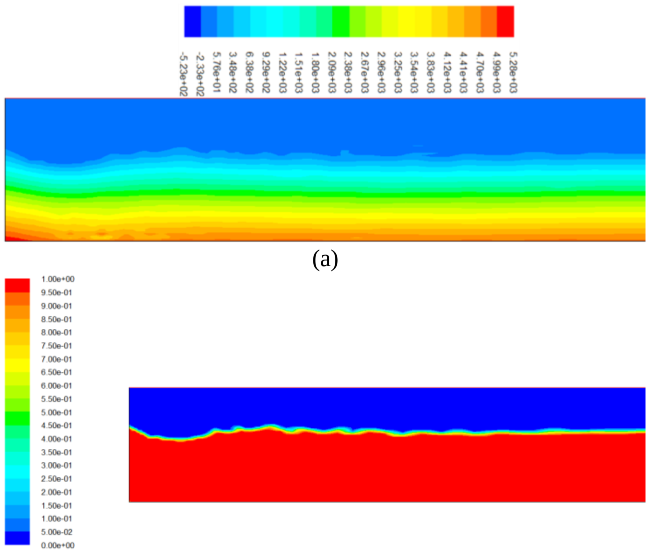


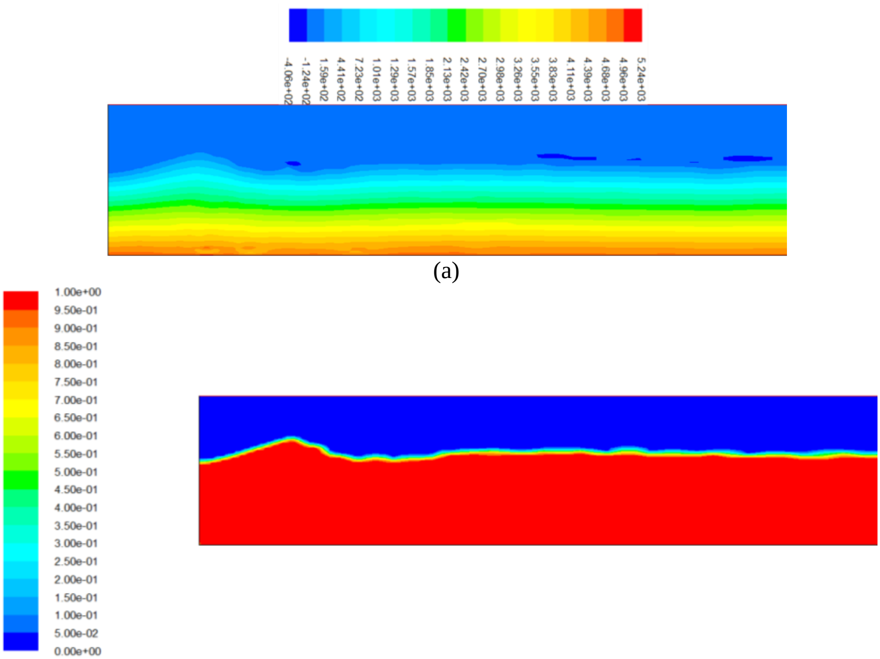
(b)

Figure 5. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a left-side oscillating wall at t=1.0s



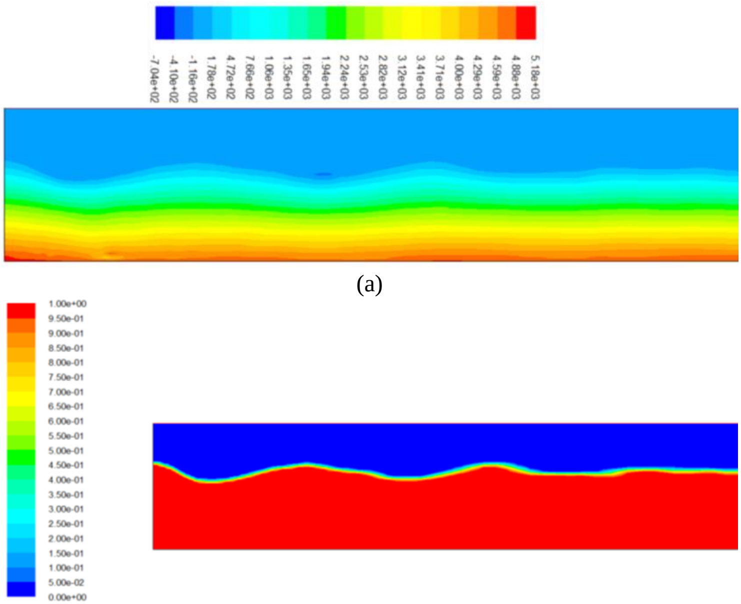
(b)

Figure 6. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a left-side oscillating wall at t=1.5s



(b)

Figure 7. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a left-side oscillating wall at t=2.0s



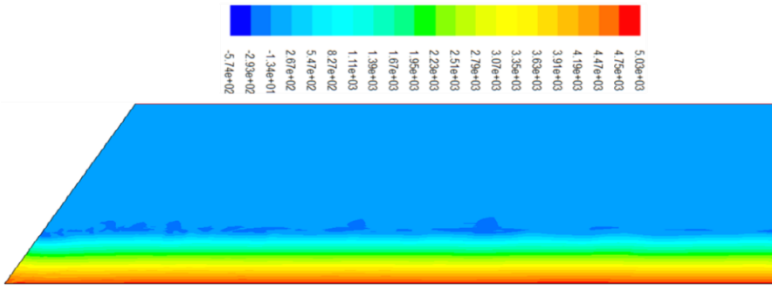
(a)

(b)

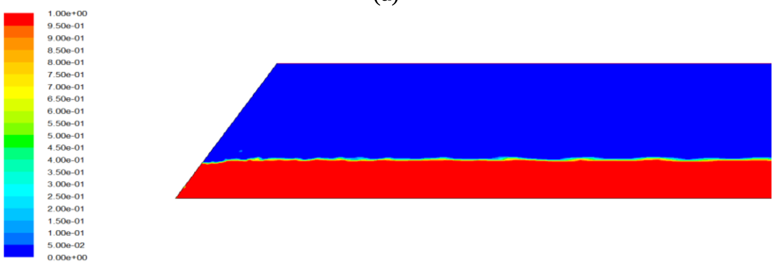
Figure 8. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a left-side oscillating wall at t=4.0s

5.2. The Tank with an Oscillating Sidewall at a 30^{o} Angle

Angularizing the left wall, the effect of the left oblique oscillating wall of the tank on pressure variations and tank water levels was investigated. According to Figures 9 to 13, it is observed that, by 30^{o} oblique, the wave production increases and the pressure contours in the form of lines become almost horizontal. The rotation of the fluid around the left wall from the front to the back, with the production of small vortices, is quite evident, and this vortex continues over time. In addition, the wave indentation before reaching the wall with the sharp corners of the water, as seen in the below figures, is the cause of the tsunami.

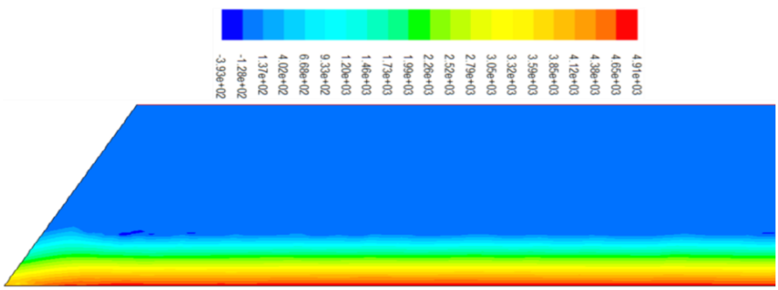


(a)

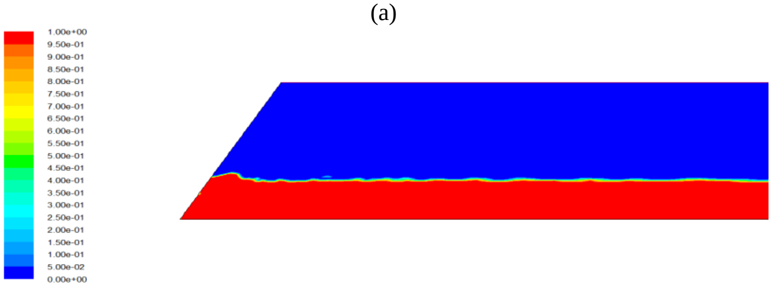


(b)

Figure 9. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a 30^o oblique left-side oscillating wall at t=0.5s

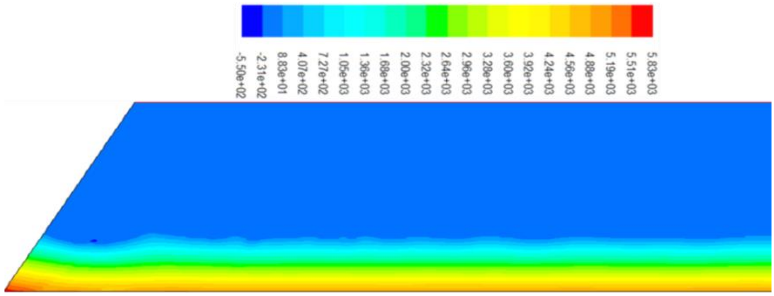


(a)

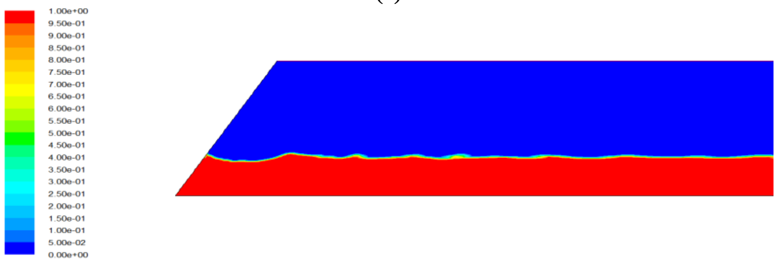


(b)

Figure 10. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a 30^o oblique left-side oscillating wall at t=1.0s

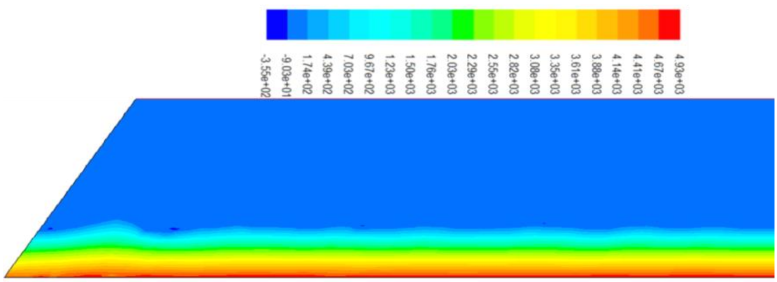


(a)

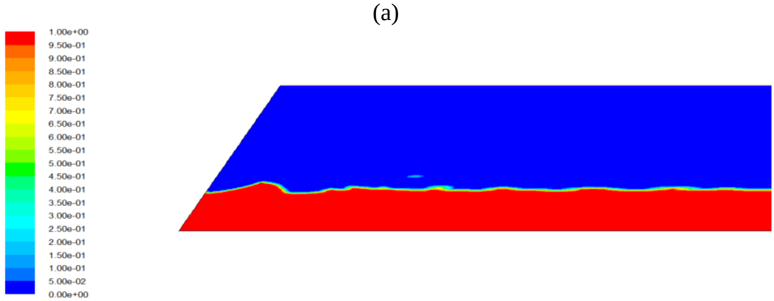


(b)

Figure 11. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a 30^o oblique left-side oscillating wall at t=1.5s



(a)



(b)

Figure 12. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a 30^o oblique left-side oscillating wall at t=2.0s

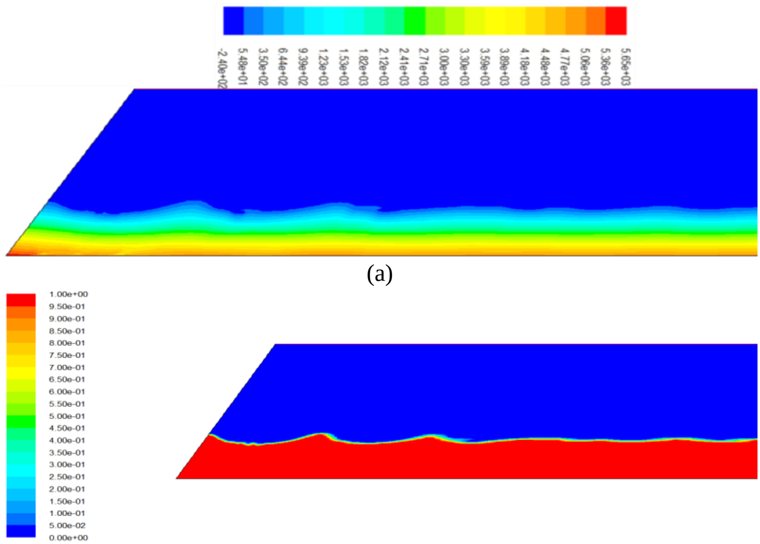


Figure 13. a) Pressure contour b) Vorticity contour (changes in tank water level) for a tank with a 30° oblique left-side oscillating wall at t=4.0s

The wave height for different periodic times is illustrated in Figure 14. As observed, increasing the periodic time can amplify wave height.

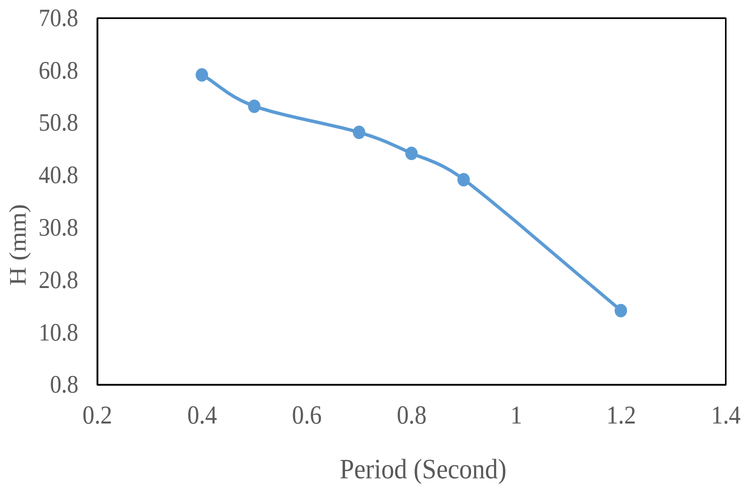


Figure 14. Wave height diagram for different period times

6. CONCLUSION

In this study, wave generation in a simple open tank is modeled using a vertical oscillating sidewall, and then computational results together with fluid flow inside it are presented via the

LMB. The obtained numerical results are also compared with available analytical outputs, suggesting a good agreement. The main findings from this research are as follows:

- There is no change in wave height at distant points from slope.
- There are small waves at early times which become more intense over time.
- By 30^o oblique, wave production increases and pressure contours in the form of lines become almost horizontal.
- Wave height amplifies as periodic time is added.

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