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Research Article

CUT DIAMETER OF CYCLONE SEPARATORS: PART I. MULTIPLE NONLINEAR REGRESSION

Selami DEMİR*[1], Aykut KARADENİZ[2], Kadir ULUTAŞ[3]

[1]*Yildiz Technical University, Environmental Eng. Department, İSTANBUL;* ORCID: 0000-0002-8672-9817
[2]*Yildiz Technical University, Environmental Eng. Department, İSTANBUL;* ORCID: 0000-0002-9754-9088
[3]*Eflani Vocational School, Karabuk University, KARABUK;* ORCID: 0000-0002-2931-3559

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ABSTRACT

Pressure drop and particle collection efficiency are the two operating parameters for assessing the performance of cyclone separators. Although a great number of practical models exists for predicting the cyclone pressure drop in the design phase, models for estimating particle collection efficiency is very limited. In this study, an improved mathematical model for calculating cut diameter in cyclone separators, which is a measure of particle collection efficiency, was developed based on Lapple's formula. Modified Lapple's formula represents the cut diameters with R^2 = 0.9969 and relative mean square error (RMSE) of 2.533*10^{-9}. Also, a new empirical regression model was proposed (R^2 = 0.9619). The average errors of both models were very close to zero. Performance tests indicated that both models can be used confidently to predict cut diameter in cyclone separators.
Keywords: Cut diameter, cyclone separators, multiple nonlinear regression, particle collection efficiency.

1. INTRODUCTION

Pressure drop and particle collection efficiency are the two main operating parameters that determine the overall performance of a cyclone separator [1]. Therefore, simple models and rule of thumb approximations are required for especially design purposes. Field engineers, most of the time, need quick estimations of pressure drop and collection efficiency [1].

Of the two main operating parameters, pressure drop in cyclone separators are relatively easy. A great number of practical models have been developed over the years that accurately predict the pressure drop for a given cyclone geometry including [1–8]. A detailed discussion of these models can be found in Demir [1].

Two main options for estimating particle collection efficiency in cyclone separators are Computational Fluid Dynamic (CFD) simulations with Discrete Phase Modeling (DPM) and empirical model given by Theodore and De Paola [9]. Of these, CFD modeling involves simulations for pressure drop as well as particle trajectories. Although extremely accurate and precise for cyclone simulations [10–19], CFD modeling is time-consuming and is usually suitable for scientific purposes. For practical calculations, on the other hand, the empirical model given by

* Corresponding Author: e-mail: seldemir@yildiz.edu.tr, tel: (212) 383 53 72

Theodore and De Paola [9], which is based on experimental data provided by Lapple [20], has long been used confidently to estimate particle collection efficiency i n a cyclone separator as

$$\eta = \frac{1}{1+\left(\frac{d_c}{d_p}\right)^2} \tag{1}$$

where η is particle collection efficiency, d_c is cut diameter (m), and d_p is particle's diameter (m). Lapple [20] defined the cut diameter as the diameter of particle which is captured with 50% efficiency in the cyclone and formulated as

$$d_c = \sqrt{\frac{9\mu_g b}{2\pi N_e V_i(\rho_p - \rho_g)}} \tag{2}$$

where b is the inlet width of the cyclone (m), V_i is the inlet velocity of the gas (m/s), N_e is the number of effective turns of particles within the cyclone, ρ_p is the density of particles (kg/m^3), ρ_g is the density of the gas (kg/m^3), and μ_g is the dynamic viscosity of the gas (kg/m.s).

Lapple's cut diameter formula is defined by analogy to discrete settling of particles in a plug-flow type settling chamber, where b represents the chamber's height and the term πN_e corresponds to the length of the chamber. In this form, the formulation does not account for particle breakup and agglomeration in cyclone separators. Although the empirical model (Eqn. 1) gives good predictions of particle collection efficiency, the cut diameter formulation (Eqn. 2) may be improved to account for any kind of particle-particle and particle-gas interactions within the cyclone.

The purpose of this study is to improve Lapple's formula and to develop an empirical relationship for estimating cut diameter of cyclone separators. For this purpose, experimental cut diameters at various inlet widths and inlet velocities in a Stairmand high-efficiency type cyclone were obtained. CFD model was calibrated using the experimental pressure drops and particle collection efficiencies. The calibrated model was used to estimate cut diameters. These cut diameters were used as the dependent variable in multiple nonlinear regression analyses. The resulting equations (regression constants) represent synergistic effects of both geometry and the particle-particle as well as particle-gas interactions in the cyclone separators.

2. MATERIALS AND METHODS

2.1. Experimental Setup

The details of the experimental setup (Fig. 1) are given in Karadeniz [21]. The lab-scale cyclone system consists of a particle dosing equipment, a differential pressure transmitter, a cyclone separator, and an air fan.

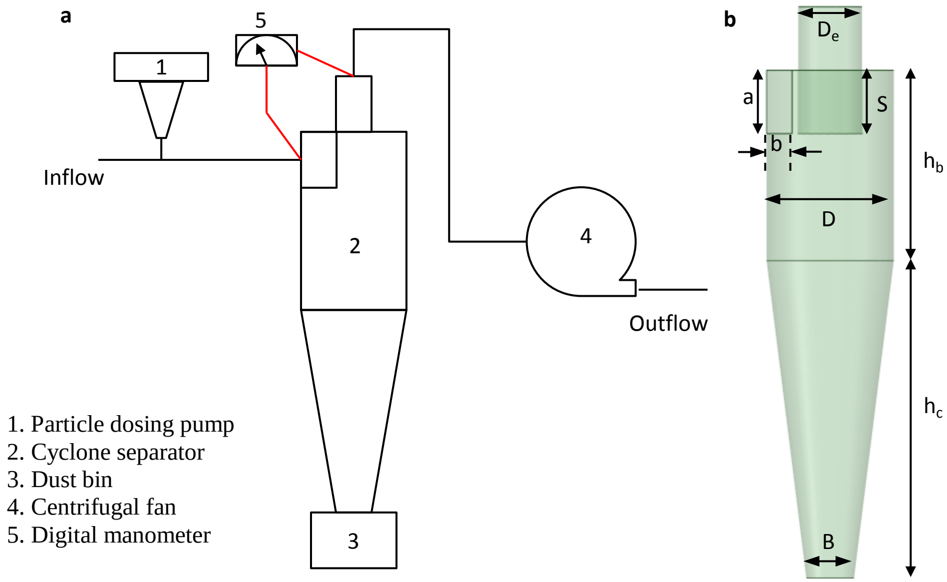

Figure 1. a. Experimental setup, **b.** Solid model of the cyclone

Three cyclone separators (Fig. 1.b) were used for collec ting experimental data. The geometry of the cyclones, which were variations of Stairmand high-efficiency type, is given in Table 1. The three cyclones were different only in inlet widths, which were 0.20*D*, 0.25*D*, and 0.30*D*. The pressure drops in the cyclones at 10, 13.5, and 17 m/s inlet velocities were obtained from Karadeniz [21]. In the scope of an ongoing research project, the collection efficiencies at these inlet velocities were measured. A total of nine data points that consists of pressure drop-collection efficiency couples were obtained. These data points were used for the sole purpose of calibrating CFD model and cut diameters calculated by calibrated CFD model were reported here. For CFD modeling Ansys Fluent v15.0 was used. Pressure drops at three different inlet velocities and three different inlet widths were used for calibrating the gaseous phase. After a calibrated model is obtained, the next step was to run Discrete Phase Modeling (DPM) for particle tracking. The model was calibrated against the experimental cut diameters. In the final step, the calibrated model was run with various inlet widths (0.2*D*, 0.25*D*, and 0.30*D*), inlet velocities (10, 15, 20, and 25 m/s), and particle densities (500, 1000, 1500, 2000, and 2500 kg/m^3) to obtain cut diameters at the specified values of independent variables (inlet width, inlet velocity, and particle density).

Table 1. Dimensions of cyclones used

Dimension	Symbol	Value	Ratio to body diameter
Body diameter	D	290 mm	1.000
Inlet height	a	145 mm	0.500
Inlet width	b	58 mm	0.200
		72.5 mm	0.250
		87 mm	0.300
Body height	h_b	435 mm	1.500
Cone height	h_c	725 mm	2.500
Cone-tip diameter	B	10.88 mm	0.375
Vortex finder height	S	145 mm	0.500
Vortex finder diameter	D_e	145 mm	0.500

2.2. Cut Diameter

Lapple's cut diameter model (Eqn. 1) was formulated by analogy to discrete settling of particles in a plug-flow type settling chamber, where b corresponds to the height of the settling chamber and the term πN_e is a measure of the settling chamber's length. Although widely used with confidence, the formulation does not account for the effects of complex flow in cyclone separators (particle-gas interactions) and particle-particle interactions on the particle trajectories. In cyclone separators, particles may agglomerate to form larger ones, or breakup into smaller particles as a result of high turbulence. Also, smaller particles may be swept towards the cyclone walls by larger ones when the inlet particle concentration is high. Simulating the effects of these mechanisms on the particle collection efficiency by CFD tools, for instance, would not be feasible in terms of computational power and time of simulation. On the other hand, empirical models could easily represent the effects of all mechanisms on particle trajectories. Also, an empirical model for cyclone cut diameter would be very useful for field engineers.

In this study, Lapple's formula for cut diameter (Eqn. 1) was modified to obtain better predictions of cut diameter as

$$d_c = \sqrt{\frac{9\mu_g}{2\pi N_e}}\sqrt{\frac{b^{a_1}}{{V_i}^{a_2}{\rho_p}^{a_3}}} \tag{3}$$

where a_1, a_2, and a_3 are regression constants. This formula is referred to as modified Lapple's. Note that the term ρ_g is dropped from the equation for simplification purposes. Since the density of gas is usually very small compared to density of particles, this simplification does not cause a significant error.

In the next step, a simpler empirical model was proposed as

$$d_c = a_1 K_b + \frac{a_2}{V_i} + \frac{a_3}{\rho_p} + a_4 \tag{4}$$

where a_1, a_2, a_3, and a_4 are regression constants, d_c is in micrometers (μm), V_i is in meters per second (m/s), ρ_p is in grams per cubic centimeter (g/cm^3), and K_b is defined as the ratio of inlet width to the body diameter (dimensionless). This new formula was referred to as YTUCut.

2.3. Multiple Nonlinear Regression

A multiple nonlinear regression (MNLR) tool by Demir et al [22] was used for improving Lapple's formula as well as for developing an empirical relationship. The tool uses an MS Excel VBA code with a user-friendly interface on MS Excel sheets. The nonlinear regression algorithm is the Gauss-Newton method, which is based on first-order Taylor expansions of the given regression formula with respect to each regression constant. The tool involves an iterative approach by updating estimated regression constants in each iteration, attempting to minimize the mean square error calculated between experimental and simulated dependent variables. It allows typing any kind of mathematical equation and regression constants and can be used for estimating regression-based parameters in any field of environmental engineering from air pollution [22] to anaerobic processes [23].

3. RESULTS AND DISCUSSIONS

Computational Fluid Dynamics (CFD) simulations were performed. CFD model was calibrated using experimental pressure drops for 0.058 m, 0.0725 m, and 0.087 m inlet widths at 10, 13.5, and 17 m/s inlet velocities. For calibrating the DPM trajectories, experimental collection efficiencies, which were previously obtained in the scope of an ongoing project, were used, CFD results are reported here.

The CFD calibration were accomplished using experimental pressure drops and collection efficiencies with an average discrepancy of 1.2% for pressure drops and 3.2% for cut diameters. Then, the calibrated CFD model was run for calculating cut diameters of the cyclone for various inlet widths (b=0.20D=0.058 m; b=0.25D=0.0725 m; b=0.30D=0.087 m), inlet velocities (V_i = 10, 15, 20, 25 m/s), and particle densities (ρ_p = 500, 1000, 1500, 2000, 2500 kg/m^3). CFD-estimated cut diameters (will be referred to as simulated cut diameters for this study) are shown in Fig. 2. As expected, the cut diameters decreased with increasing inlet velocities between 10 and 25 m/s, indicating a gradual increase in particle collection efficiency. A rule of thumb pertaining to the relationship between the inlet velocity and the cut diameter is that the cut diameter decreases by 40% when the inlet velocity increases by approximately 67%. The effect of particle density on cut diameter was similar to that of inlet velocity. The cut diameter decreased by approximately 80% when the particle density increased by four times. The effect of inlet width was somewhat different than that of inlet velocity and particle density. The cut diameters increased by increasing inlet width. All of the findings conform with the Lapple's formula (Eqn. 1).

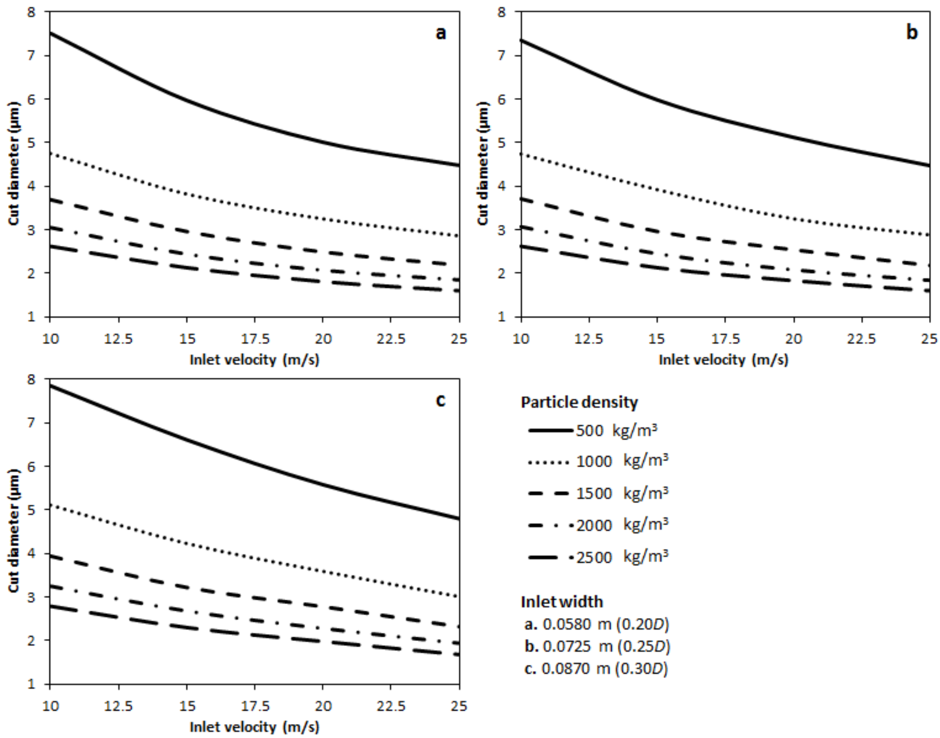

Figure 2. Simulated cut diameters

MNLR 2.0 tool [22] was run with simulated cut diameters (a total of 60 data points). Eqn. 3 was used as the regression equation. The statistics are shown in Table 2. The coefficient of determination (R^2) was calculated as 0.9964 with adjusted R^2 of 0.9962. Relative mean square error was calculated as the ratio of the squared error between simulated and regression cut diameters to the simulated cut diameters, which was $2.533 * 10^{-9}$. All statistics pointed out an extremely good agreement between simulated and regression cut diameters showing that the modified Lapple's model can be confidently used to estimate cut diameter of cyclones. The exponents in the modified Lapple's model were calculated as 0.319 ± 0.052 for inlet width (*b*), 1.095 ± 0.034 for inlet velocity (V_i) and 1.276 ± 0.018 for particle density (ρ_p) at confidence level of 95%.

Table 2. Regression statistics for Modified Lapple's

Regression statistics	Value
a_1	0.319 ± 0.052
a_2	1.095 ± 0.034
a_3	1.276 ± 0.018
R^2	0.9969
Adjusted R^2	0.9962
Relative mean square error (RMSE)	$2.533 * 10^{-9}$

The correlation plots for both Lapple's and modified Lapple's formulae are shown in Fig. 3. In the figure, blue dots represent simulated versus Lapple's cut diameters (Fig. 3.a) and regression cut diameters (Fig. 3.b). The red lines indicate one-to-one line (perfect fit). It is clear that Lapple's formulation overestimates the cut diameter slightly, which results in underestimated collection efficiencies. On the other hand, coefficient of determination (R^2) for modified Lapple's formulation was much better.

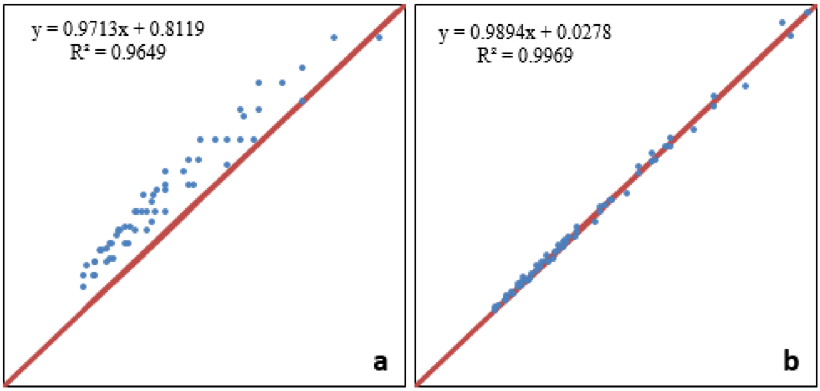

Figure 3. Correlation plots for **a.** Lapple's, **b.** Modified Lapple's formula

An important implication of Lapple's formula is that the inlet width, the inlet velocity, and the particle density has a similar degree of effect on the cut diameter. Results show that these parameters have varying degrees of effects. For instance, Lapple's formula suggests that the cut diameter is directly proportional to the squared root of inlet width. In fact, the cut diameter is a function of inlet width raised to the power of approximately 0.160. Also, a stronger effect on cut diameter by inlet velocity and particle density than what is expected based on Lapple's formula is observed. Modified Lapple's formula takes these effects into account by updating the exponents as 1.095 and 1.276, respectively. The authors confidently suggest, based on regression statistics (Table 2) and correlation plots (Fig. 3), that modified Lapple's formula is better for predicting the cyclone cut diameter.

Although modified Lapple's formula produces very good predictions of cut diameter, it involves a square root and exponents that are not equal to unity. Field engineers could benefit from a simpler regression formula. MNLR 2.0 tool was also run to estimate the regression constants in Eqn. 4. The results are shown in Table 3. Although the coefficient of determination is lower than both the Lapple's and the modified Lapple's formula, YTUCut is satisfactorily accurate (R^2 = 0.9619) and one can make a trade-off between the accuracy and the ease of calculations. The relative mean square error was calculated as 2.300 * 10^{-2}, and the values of regression constants were calculated at a confidence level of 95%. The correlation plot for YTUCut is shown in Fig. 4. Comparing the YTUCut's correlation plot with Lapple's and modified Lapple's correlation plots (Fig. 3), one can conclude that YTUCut, in terms of accuracy, come between Lapple's and modified Lapple's.

Table 3. Regression statistics for YTUCut

Regression statistics	Value
a_1	2.695 ± 1.974
a_2	28.76 ± 3.539
a_3	2.341 ± 0.139
a_4	-1.237 ± 0.564
R^2	0.9619
Adjusted R^2	0.9598
Relative mean square error (RMSE)	$2.300 * 10^{-2}$

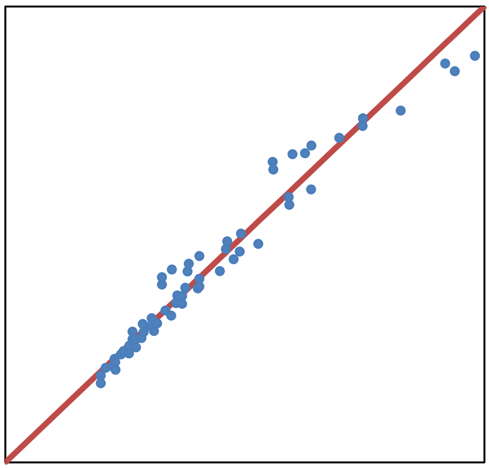

Figure 4. Correlation plot for YTUCut

Calculated errors for all three formulae (Eqn. 2, 3, and 4) are shown in Fig. 5. The Lapple's formula overestimates the cut diameter by approximately 25% on average, while YTUCut formula underestimates by approximately 1.5% on average. On the other hand, modified Lapple's formula has an average margin of error equal to zero. For Lapple's formula, calculated errors are somewhat higher with minimum errors reaching to zero. For modified Lapple's and YTUCut formulae, the calculated errors within the first and the third quartiles are very close to zero. The errors by modified Lapple's formula are almost normally distributed around zero with a very low skewness (-0.11). On the other hand, YTUCut formula, although produces better predictions than Lapple's formula, has the highest skewness (1.09). In contrast to its low skewness (-0.17), Lapple's formula clearly has the highest margin of errors.

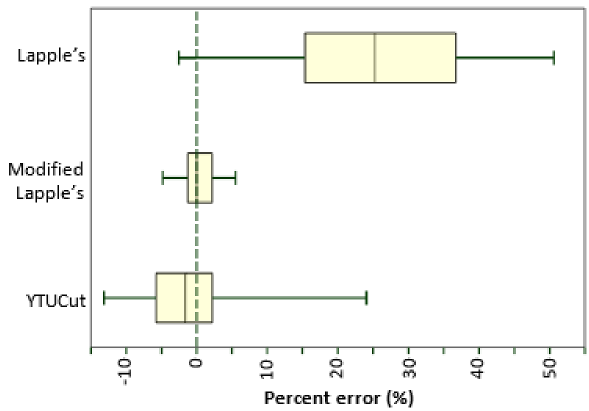

Figure 5. Percent errors of cut diameters estimated by the three formulae

4. CONCLUSIONS

Formulae for predicting the cut diameter of cyclones were investigated in this study. For this purpose, CFD simulations were performed, which were previously calibrated with experimental data. In simulations, cut diameters were calculated for 60 different sets of operating and design parameters (inlet widths of 0.20*D*, 0.25*D*, and 0.30*D*; inlet velocities of 10, 15, 20, and 25 m/s; and particle densities of 500, 1000, 1500, 2000, and 2500 kg/m^3). The calculated cut diameters were used to check Lapple's formula for accuracy and possible improvements. A multiple nonlinear regression tool was used to fit a modified version of Lapple's formula for cut diameter. The modification involved updating the exponents of inlet width (b), inlet velocity (V_i), and particle density (ρ_p). Also a new, empirical model for estimating the cut diameter was proposed. Finally, all the three formulae were evaluated for their performances. The following conclusions can be withdrawn:

- The effects of inlet width, inlet velocity, and particle density on cut diameter are somewhat different than what Lapple's formula implies.
- Lapple's formula for cut diameter of a cyclone neglects particle-particle and particle-gas interactions in cyclone separators. It overestimates cut diameter by around 25%.
- An improved formulation can be used to represent synergic effects of all mechanisms in cyclone separators. Empirical formulations can also be used for accurately predicting cur the diameters.
- Modified Lapple's formula and empirical relationship proposed in this study can be used for predicting cut diameter with small margins of errors.

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