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Review Article

UNI-SOFT IDEALS IN CORESIDUATED LATTICES

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ABSTRACT

The notion of uni-soft ideal is introduced, and related properties are investigated. Characterizations of uni-soft ideal are discussed. The transfer of uni-soft ideal is considered, and relations between uni-soft ideal and its transfer are investigated. Given a soft set, conditions for the transfer of soft set to be a uni-soft ideal.
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1. INTRODUCTION

As a generalization of fuzzy set theory, Molodtsov [2] introduced the soft set theory in 1999 to deal with uncertainty in a parametric manner. Co-residuated lattice is discussed in [3] and [4] by Zheng et al.

In this paper, we introduce the notion of uni-soft ideal in coresiduated lattices, and investigate related properties. We discuss characterizations of uni-soft ideal. We also consider the transfer of uni-soft ideals, and discuss relations between uni-soft ideal and its transfer. We provide conditions for the transfer of a soft set to be a uni-soft ideal.

2. PRELIMINARIES

We display basic notions of coresiduated lattices. We refer to the papers [3] and [4] for more details.

A structure $(\mathcal{L}; \vee, \wedge, \oplus, \ominus, 0, 1)$ is called a coresiduated lattice if the following conditions are valid.

(1) $(\mathcal{L}, \vee, \wedge)$ is a bounded lattice with the smallest element 0 and the greatest element 1.

(2) $(\oplus, \ominus)$ is a coadjoint pair on $\mathcal{L}$.

(3) $(\mathcal{L}, \oplus, 0)$ is a commutative monoid.

In a coresiduated lattice $\mathcal{L}$, the following are valid.

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$$x \ominus 0 = x, \tag{2.1}$$

$$x \le y \iff x \ominus y = 0, \tag{2.2}$$

$$x \oplus y = 0 \iff x = y = 0, \tag{2.3}$$

$$(x \ominus y) \ominus z = (x \ominus z) \ominus y = x \ominus (y \oplus z), \tag{2.4}$$

$$(x \oplus y) \ominus y \le x \le (x \ominus y) \oplus y. \tag{2.5}$$

A subset I of a coresiduated lattice $\mathcal{L}$ is called an *ideal* of $\mathcal{L}$ if it sati sfies:

(I1) $0 \in I$,

(I2) $(\forall x, y \in \mathcal{L})\,(x \in I,\ y \le x \ \Rightarrow\ y \in I)$,

(I3) $(\forall x, y \in \mathcal{L})\,(x \in I,\ y \in I \ \Rightarrow\ x \oplus y \in I)$.

A subset I of a coresiduated lattice $\mathcal{L}$ is an ideal of $\mathcal{L}$ if and only if the following assertions are valid.

$$0 \in I \tag{2.6}$$

$$(\forall x, y \in \mathcal{L})\,(x \in I,\ y \ominus x \in I \ \Rightarrow\ y \in I)\,. \tag{2.7}$$

Let U be an initial universe set and E be a set of parameters. Let 2^U denotes the power set of U and $A \subseteq E$.

A soft set (see [1, 2]) (f, A) over U is defined to be the set of ordered pairs

$$(\tilde{f}, A) := \left\{(x, \tilde{f}_A(x)) : x \in E,\ \tilde{f}_A(x) \in 2^U\right\},$$

where $f_A : E \to 2^U$ such that $f(x) = \varnothing$ if $x = \notin A$: The soft set (f, A) is simply denoted by f_A.

For a soft set f_A over U and a subset τ of U, the τ-exclusive set of f_A, denoted by $e(f_A; \tau)$; is defined to be the set

$$e\left(\tilde{f}_A; \tau\right) := \left\{x \in A \mid \tilde{f}_A(x) \subseteq \tau\right\}.$$

3. UNI-SOFT IDEALS

In what follows, we take a coresiduated lattice $\mathcal{L}$ as a set of parameters.

Definition 3.1. A soft set $f_{\mathcal{L}}$ over U is called a *uni-soft ideal* of $\mathcal{L}$ if it satisfies:

$$(\forall x \in L)\left(\tilde{f}_{\mathcal{L}}(0) \subseteq \tilde{f}_{\mathcal{L}}(x)\right), \tag{3.1}$$

$$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}(y) \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y \ominus x)\right). \tag{3.2}$$

Proposition 3.2. If $\sim f_{\mathcal{L}}$ *is a uni-soft ideal* of $\mathcal{L}$, then

$$(\forall x, y \in \mathcal{L})\left(x \le y \ \Rightarrow\ \tilde{f}_{\mathcal{L}}(x) \subseteq \tilde{f}_{\mathcal{L}}(y)\right). \tag{3.3}$$

Proof. For any x, y $\in \mathcal{L}$, if x $\le$ y then x $\ominus$ y = 0. It follows from (3.1) and (3.2) that

$$\tilde{f}_{\mathcal{L}}(x) \subseteq \tilde{f}_{\mathcal{L}}(y) \cup \tilde{f}_{\mathcal{L}}(x \ominus y) = \tilde{f}_{\mathcal{L}}(y) \cup \tilde{f}_{\mathcal{L}}(0) = \tilde{f}_{\mathcal{L}}(y).$$

This completes the proof.

Theorem 3.3. *A soft set $f_{\mathcal{L}}$ over U is a uni-soft ideal of $\mathcal{L}$ if and only if*

$$(\forall x, y, z \in \mathcal{L}) \left(z \leq x \oplus y \;\Rightarrow\; \tilde{f}_{\mathcal{L}}(z) \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y) \right). \tag{3.4}$$

*Proof.*_Assume that $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$. Let $x, y, z \in \mathcal{L}$ be such that $z \leq x \oplus y$.

Then $z \ominus y \leq x$, and so $f_{\mathcal{L}}(z \ominus y) \mathcal{L} \sim f_L(x)$ by (3.3). It follows from (3.2) that

$$\tilde{f}_{\mathcal{L}}(z) \subseteq \tilde{f}_{\mathcal{L}}(y) \cup \tilde{f}_{\mathcal{L}}(z \ominus y) \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y).$$

Thus (3.4) is valid.

Conversely suppose that $f_{\mathcal{L}}$ satis_es the condition (3.4). Since $0 \leq x \oplus x$ for all $x \in \mathcal{L}$, we have $f_{\mathcal{L}}(0) \subseteq f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(x) = f_{\mathcal{L}}(x)$ by (3.4). Note that $y \leq (y \ominus x) \oplus x$ for all $x, y \in \mathcal{L}$ by (2.5). Hence $f_{\mathcal{L}}(y) \subseteq f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y \ominus x)$ for all $x, y \in \mathcal{L}$. Therefore $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$. □

Corollary 3.4. For any soft set $f_{\mathcal{L}}$ over U, the following are equivalent.

(1) *$f_{\mathcal{L}}$ is a uni-soft ideal of L.*
(2) *$f_{\mathcal{L}}$ satis_es the condition (3.3).*
(3) *$f_{\mathcal{L}}$ satis_es the condition (3.4).*
(4) $(\forall x, y, z \in \mathcal{L}) \left(z \ominus y \leq x \;\Rightarrow\; \tilde{f}_{\mathcal{L}}(z) \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y) \right).$

Proof. It is straightforward by Theorem 3.3.

Theorem 3.5. *A soft set $f_{\mathcal{L}}$ over U is a uni-soft ideal of $\mathcal{L}$ if and only if the following condition is true.*

$$y \leq x_1 \oplus x_2 \oplus \cdots \oplus x_n \;\Rightarrow\; \tilde{f}_{\mathcal{L}}(y) \subseteq \bigcup_{k=1}^{n} \tilde{f}_{\mathcal{L}}(x_k) \tag{3.5}$$

for all $y, x_1, x_2, \ldots, x_n \in \mathcal{L}$.

Proof. Assume that $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$. If $n = 2$, then it is clear by Theorem 3.3. Suppose the condition (3.5) is valid for $n = k$, that is,

$$y \leq x_1 \oplus x_2 \oplus \cdots \oplus x_k \;\Rightarrow\; \tilde{f}_{\mathcal{L}}(y) \subseteq \bigcup_{i=1}^{k} \tilde{f}_{\mathcal{L}}(x_i) \tag{3.6}$$

for all $y, x_1, x_2, \ldots, x_k \in \mathcal{L}$. Let $y, x_1, x_2, \ldots, x_k, x_{k+1} \in \mathcal{L}$ be such that

$$y \leq x_1 \oplus x_2 \oplus \cdots \oplus x_k \oplus x_{k+1}.$$

Then $y \oplus x_{k+1} \leq x_1 \oplus x_2 \oplus \ldots \oplus x_k$, and so

$$\tilde{f}_{\mathcal{L}}(y \ominus x_{k+1}) \subseteq \bigcup_{i=1}^{k} \tilde{f}_{\mathcal{L}}(x_i) \tag{3.7}$$

by (3.6). It follows from (3.2) that

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(y) &\subseteq \tilde{f}_{\mathcal{L}}(y \ominus x_{k+1}) \cup \tilde{f}_{\mathcal{L}}(x_{k+1}) \\ &\subseteq \left(\bigcup_{i=1}^{k} \tilde{f}_{\mathcal{L}}(x_i)\right) \cup \tilde{f}_{\mathcal{L}}(x_{k+1}) \\ &= \bigcup_{i=1}^{k+1} \tilde{f}_{\mathcal{L}}(x_i).\end{aligned}$$

Therefore (3.5) is valid.

Conversely suppose that (3.5) holds. If $n = 2$, then it is true by Theorem 3.3. Suppose $n > 2$. If we take $y = 0$ and $xi = x$ for $i = 1, 2, \dots, n$ in (3.5), then $\sim f_{\mathcal{L}}(0) \subseteq \sim f_{\mathcal{L}}(x)$. Hence the condition (3.4) is induced by taking $y = z$, $x_1 = x$, $x_2 = y$ and $x_i = 0$ for $i = 3, 4, \dots, n$ in (3.5). Therefore $\sim f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ by Theorem 3.3.

Theorem 3.6. *A soft set $f_{\mathcal{L}}$ over U is a uni-soft ideal of $\mathcal{L}$ if and only if $f_{\mathcal{L}}$ satisfies (3.3) and*

$$(\forall x, y \in \mathcal{L}) \left(\tilde{f}_{\mathcal{L}}(x \oplus y) = \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y)\right). \tag{3.8}$$

Proof. Let $f_{\mathcal{L}}$ be a uni-soft ideal of $\mathcal{L}$. Since $x \le x \oplus y$ and $y \le x \oplus y$, we have $f_{\mathcal{L}}(x) \subseteq f_{\mathcal{L}}(x \oplus y)$ and $f_{\mathcal{L}}(y) \subseteq f_{\mathcal{L}}(x \oplus y)$ by Proposition 3.2. Hence $f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y) \subseteq f_{\mathcal{L}}(x \oplus y)$.

Since $x \oplus y \le x \oplus y$, we get $f_{\mathcal{L}}(x \oplus y) \subseteq f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y)$ by Theorem 3.3. Therefore $f_{\mathcal{L}}(x \oplus y) = f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y)$ for all $x, y \in \mathcal{L}$.

Conversely, suppose that $f_{\mathcal{L}}$ satisfies two conditions (3.3) and (3.8). Let $x, y, z \in \mathcal{L}$ be such that $z \le x \oplus y$. Then $f_{\mathcal{L}}(z) \subseteq f_{\mathcal{L}}(x \oplus y) = f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y)$, and thus $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ by Theorem 3.3.

Theorem 3.7. *A soft set $f_{\mathcal{L}}$ over U is a uni-soft ideal of $\mathcal{L}$ if and only if the τ-exclusive set*

$$e(\tilde{f}_{\mathcal{L}}; \tau) := \left\{x \in \mathcal{L} \mid \tilde{f}_{\mathcal{L}}(x) \subseteq \tau\right\}$$

is an ideal of $\mathcal{L}$ for all $\tau \in 2^U$ with $e(f\mathcal{L}; \tau) \neq \emptyset$

Proof. Suppose that $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ and let $\tau\, 2^U$ be such that $e(f_{\mathcal{L}}; \tau) \neq \emptyset$. Then $f_{\mathcal{L}}(x0) \subseteq \tau$ for some $x_0 \in \mathcal{L}$. It follows from (3.1) that $\sim f_{\mathcal{L}}(0) \subseteq \sim f_{\mathcal{L}}(x_0) \subseteq \tau$ and so that $0 \in e(f_{\mathcal{L}}; \tau)$. Let $x, y \in \mathcal{L}$ be such that $x \in e(f_{\mathcal{L}}; \tau)$ and $y \ominus x \in e(f_{\mathcal{L}}; \tau)$. Then $f_{\mathcal{L}}(x) \subseteq \tau$ and $f_{\mathcal{L}}(y \ominus x) \subseteq \tau$, which imply from (3.2) that $f_{\mathcal{L}}(y) \subseteq f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y \ominus x) \subseteq \tau$. Hence $y \in e(f_{\mathcal{L}}; \tau)$, and therefore $e(f_{\mathcal{L}}; \tau)$ is an ideal of $\mathcal{L}$.

Conversely assume that $e(f_{\mathcal{L}}; \tau)$ is an ideal of $\mathcal{L}$ for all $\tau \in 2^U$ with $e(f_{\mathcal{L}}; \tau) \neq \emptyset$. For any $x, y \in \mathcal{L}$, let $f_{\mathcal{L}}(x) = \tau_0$. Then $e(f_{\mathcal{L}}; \tau_0) \neq \emptyset$, and so $0 \in e(f_{\mathcal{L}}; \tau_0)$. Hence $f_{\mathcal{L}}(0) \subseteq \tau 0 = f_{\mathcal{L}}(x)$, and thus (3.1) is valid. If we put $\tau_1 = f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y \ominus x)$, then $f_{\mathcal{L}}(x) \subseteq \tau_1$ and $f_{\mathcal{L}}(y \ominus x) \subseteq \tau_1$, that is, $x \in e(f_{\mathcal{L}}; \tau 1)$ and $y \ominus x \in e(f_{\mathcal{L}}; \tau 1)$. Since $e(f_{\mathcal{L}}; \tau_1)$ is an ideal of $\mathcal{L}$, it follows that $y \in e(f_{\mathcal{L}}; \tau_1)$. Hence $f_{\mathcal{L}}(y) \subseteq \tau_1 = f_{\mathcal{L}}(x) \cup f_{\mathcal{L}}(y \ominus x)$. Therefore $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$.

For any soft set $f_{\mathcal{L}}$ over U and $a \in \mathcal{L}$, consider the set

$$\mathcal{L}_a(\tilde{f}_{\mathcal{L}}) := \left\{x \in \mathcal{L} \mid \tilde{f}_{\mathcal{L}}(x) \subseteq \tilde{f}_{\mathcal{L}}(a)\right\}.$$

Obviously, $a \in \mathcal{L}_a(f_{\mathcal{L}})$. Using Theorem 3.7, we have the following corollary.

Corollary 3.8. *If $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$, then the set $\mathcal{L}_a(f_{\mathcal{L}})$ is an ideal of $\mathcal{L}$ for all a 2 $\mathcal{L}$.*

Using (3.1) and Corollary 3.8, we have the following corollary.

Corollary 3.9. *If $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$, then the set*

$$\mathcal{L}^*(\tilde{f}_{\mathcal{L}}) := \left\{ x \in \mathcal{L} \mid \tilde{f}_{\mathcal{L}}(x) = \tilde{f}_{\mathcal{L}}(0) \right\}.$$

is an ideal of $\mathcal{L}$.

Theorem 3.10. *Given a soft set $f_{\mathcal{L}}$ over U and $a \in \mathcal{L}$, if $f_{\mathcal{L}}$ satisfies (3.3) and*

$$(\forall x, y \in \mathcal{L}) \left(x, y \in \mathcal{L}_a(\tilde{f}_{\mathcal{L}}) \;\Rightarrow\; x \oplus y \in \mathcal{L}_a(\tilde{f}_{\mathcal{L}}) \right), \tag{3.9}$$

then $\mathcal{L}_a(f_{\mathcal{L}})$ is an ideal of $\mathcal{L}$.

Proof. Clearly, $0 \in \mathcal{L}_a(f_{\mathcal{L}})$. Let $x, y \in \mathcal{L}$ be such that $y \le x$ and $x \in \mathcal{L}_a(f_{\mathcal{L}})$. Then $f_{\mathcal{L}}(y) \subseteq f_{\mathcal{L}}(x) \subseteq f_{\mathcal{L}}(a)$, that is, $y \in \mathcal{L}_a(f_{\mathcal{L}})$. The condition (3.9) implies that $x \oplus y \in \mathcal{L}_a(f_{\mathcal{L}})$ for all $x, y \in \mathcal{L}_a(f_{\mathcal{L}})$. Therefore $\mathcal{L}_a(f_{\mathcal{L}})$ is an ideal of $\mathcal{L}$.

4. TRANSFER OF UNI-SOFT IDEALS

For any soft set $f_{\mathcal{L}}$ over U, we consider the set

$$\Omega_{\tilde{f}_{\mathcal{L}}} := U \setminus \bigcup_{x \in \mathcal{L}} \tilde{f}_{\mathcal{L}}(x). \tag{4.1}$$

It is clear that $\Omega_{f_{\mathcal{L}}}$ and $f_{\mathcal{L}}(x)$ are disjoint for all $x \in \mathcal{L}$.

Definition 4.1. For a soft set $f_{\mathcal{L}}$ over U and $\varepsilon \subseteq f_{\mathcal{L}}$, a soft set $f^{\varepsilon}_{\mathcal{L}}$ over U is called the ε-*soft transfer* of $f_{\mathcal{L}}$ where

$$\tilde{f}^{\varepsilon}_{\mathcal{L}} : \mathcal{L} \to 2^U, \; x \mapsto \tilde{f}_{\mathcal{L}}(x) \cup \varepsilon.$$

Theorem 4.2. *If $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U, then the ε-soft transfer $f^{\varepsilon}_{\mathcal{L}}$ of $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U for all $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$.*

Proof. Using (3.1) and (3.2), we have

$$\tilde{f}^{\varepsilon}_{\mathcal{L}}(0) = \tilde{f}_{\mathcal{L}}(0) \cup \varepsilon \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \varepsilon = \tilde{f}^{\varepsilon}_{\mathcal{L}}(x)$$

and

$$\begin{aligned} \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) &= \tilde{f}_{\mathcal{L}}(x) \cup \varepsilon \\ &\subseteq (\tilde{f}_{\mathcal{L}}(x \ominus y) \cup \tilde{f}_{\mathcal{L}}(y)) \cup \varepsilon \\ &= (\tilde{f}_{\mathcal{L}}(x \ominus y) \cup \varepsilon) \cup (\tilde{f}_{\mathcal{L}}(y) \cup \varepsilon) \\ &= \tilde{f}^{\varepsilon}_{\mathcal{L}}(x \ominus y) \cup \tilde{f}^{\varepsilon}_{\mathcal{L}}(y) \end{aligned}$$

for all $x, y \in \mathcal{L}$ and $\varepsilon \subseteq \Omega_{\sim f_{\mathcal{L}}}$. Therefore $f^{\varepsilon}_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U.

Corollary 4.3. *Let $f_{\mathcal{L}}$ be a soft set over U and $\varepsilon \subseteq \Omega f_{\mathcal{L}}$. If $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U, then the ε-soft transfer of $\sim f_{\mathcal{L}}$ satisfies:*

$$(\forall x, y \in \mathcal{L}) \left(x \le y \;\Rightarrow\; \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) \subseteq \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) \right), \tag{4.2}$$

$$(\forall x, y, z \in \mathcal{L}) \left(z \le x \ominus y \;\Rightarrow\; \tilde{f}^{\varepsilon}_{\mathcal{L}}(z) \subseteq \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) \cup \tilde{f}^{\varepsilon}_{\mathcal{L}}(y) \right). \tag{4.3}$$

We consider the converse of Theorem 4.2.

Theorem 4.4. *Let $f_{\mathcal{L}}$ be a soft set over U. If there exists a subset ε of $\Omega_{f_{\mathcal{L}}}$ such that the ε-soft transfer of $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U, then $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U.*

Proof. Assume that the ε-soft transfer $f^{\varepsilon}_{\mathcal{L}}$ of $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U for some $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$. Then

$$\tilde{f}_{\mathcal{L}}(0) \cup \varepsilon = \tilde{f}^{\varepsilon}_{\mathcal{L}}(0) \subseteq \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) = \tilde{f}_{\mathcal{L}}(x) \cup \varepsilon$$

and

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(x) \cup \varepsilon = \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) &\subseteq \tilde{f}^{\varepsilon}_{\mathcal{L}}(x \ominus y) \cup \tilde{f}^{\varepsilon}_{\mathcal{L}}(y) \\ &= (\tilde{f}_{\mathcal{L}}(x \ominus y) \cup \varepsilon) \cup (\tilde{f}_{\mathcal{L}}(y) \cup \varepsilon) \\ &= (\tilde{f}_{\mathcal{L}}(x \ominus y) \cup \tilde{f}_{\mathcal{L}}(y)) \cup \varepsilon\end{aligned}$$

for all $x, y \in \mathcal{L}$. Since $f_{\mathcal{L}}(x)$ and ε are disjoint for all $x \in \mathcal{L}$, it follows that $f_{\mathcal{L}}(0) \subseteq f_{\mathcal{L}}(x)$ and $f_{\mathcal{L}}(x) \subseteq f_{\mathcal{L}}(x \ominus y) \cup f_{\mathcal{L}}(y)$ for all $x, y \in \mathcal{L}$. Therefore $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U.

For any soft set $f_{\mathcal{L}}$ over U, consider a set

$$\mathcal{L}^{\varepsilon}_{\delta} := \{x \in \mathcal{L} \mid \tilde{f}_{\mathcal{L}}(x) \subseteq \delta \setminus \varepsilon\}$$

where $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$ and $\delta \in 2^U$ with $\varepsilon \subseteq \delta$. We say that $\mathcal{L}^{\varepsilon}_{\delta}$ is the *(δ, ε)*-support of $f_{\mathcal{L}}$. Note that

$$(\forall x \in \mathcal{L}) \left(\tilde{f}_{\mathcal{L}}(x) \subseteq \delta \setminus \varepsilon \;\Leftrightarrow\; \tilde{f}_{\mathcal{L}}(x) \cup \varepsilon \subseteq \delta\right)$$

Hence $\mathcal{L}^{\varepsilon}_{\delta} := \{x \in \mathcal{L} \mid f^{\varepsilon}_{\mathcal{L}}(x) \subseteq \delta\}$.

Theorem 4.5. *For any $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$, if a soft set $f_{\mathcal{L}}$ over U is a uni-soft ideal of $\mathcal{L}$ over U, then the (δ, ε)-support of $f_{\mathcal{L}}$ is an ideal of $\mathcal{L}$ for all $\delta \in 2^U$ with $\varepsilon \subseteq \delta$.*

Proof. Assume that $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U. Let $x, y \in \mathcal{L}$. If $x \in \mathcal{L}^{\varepsilon}_{\delta}$, then $f_{\mathcal{L}}(0) \subseteq f_{\mathcal{L}}(x) \subseteq \delta \setminus \varepsilon$ by (3.1), and so $0 \in \mathcal{L}^{\varepsilon}_{\delta}$. Suppose that $x \ominus y \in \mathcal{L}^{\varepsilon}_{\delta}$ and $y \in \mathcal{L}^{\varepsilon}_{\delta}$. Then $f_{\mathcal{L}}(x \ominus y) \subseteq \delta \setminus \varepsilon$ and $f_{\mathcal{L}}(y) \subseteq \delta \setminus \varepsilon$. Using (3.2), we have

$$\tilde{f}_{\mathcal{L}}(x) \subseteq \tilde{f}_{\mathcal{L}}(x \ominus y) \cup \tilde{f}_{\mathcal{L}}(y) \subseteq \delta \setminus \varepsilon$$

and thus $x \in \mathcal{L}^{\varepsilon}_{\delta}$. Therefore the ($\delta$, ε)-support of $f_{\mathcal{L}}$ is an ideal of $\mathcal{L}$.

Using Theorems 4.4 and 4.5, we obtain the following corollary.

Corollary 4.6. *For a soft set $f_{\mathcal{L}}$ over U, if there exists a subset ε of $f_{\mathcal{L}}$ such that the ε-soft transfer of $f_{\mathcal{L}}$ is an ideal of $\mathcal{L}$ over U, then the (δ, ε)-support of $f_{\mathcal{L}}$ is an ideal of $\mathcal{L}$ for all $\delta \in 2^U$ with $\varepsilon \subseteq \delta$.*

Lemma 4.7. *Given $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$ and any $\delta \in 2^U$ with $\varepsilon \subseteq \delta$, let $f_{\mathcal{L}}$ be a soft set over U such that*

$$(\forall x, y \in \mathcal{L})\,(x, y \in \mathcal{L}^{\varepsilon}_{\delta} \;\Rightarrow\; x \ominus y \in \mathcal{L}^{\varepsilon}_{\delta})\,. \tag{4.4}$$

Then the ε-soft transfer $f^{\varepsilon}_{\mathcal{L}}$ of $f_{\mathcal{L}}$ satisfies:

$$(\forall x, y \in \mathcal{L}) \left(\tilde{f}^{\varepsilon}_{\mathcal{L}}(x \ominus y) \subseteq \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) \cup \tilde{f}^{\varepsilon}_{\mathcal{L}}(y)\right). \tag{4.5}$$

Proof. Let $x, y \in \mathcal{L}$ be such that $f^{\varepsilon}_{\mathcal{L}}(x) = \delta_x$ and $f^{\varepsilon}_{\mathcal{L}}(y) = \delta_y$. If we take $\delta = \delta_x \cup \delta_y$, then $f_{\varepsilon\mathcal{L}}(x) = \delta_x \subseteq \delta$ and $f_{\varepsilon\mathcal{L}}(y) = \delta_y \subseteq \delta$, that is, $f_{\mathcal{L}}(x) \cup \varepsilon \subseteq \delta$ and $f_{\mathcal{L}}(y) \cup \varepsilon \subseteq \delta$. Since $f_{\mathcal{L}}(x)$ and ε are disjoint for all $x \in \mathcal{L}$, we have $f_{\mathcal{L}}(x) \subseteq \delta \setminus \varepsilon$ and $f_{\mathcal{L}}(y) \subseteq \delta \setminus \varepsilon$, *i.e.*, $x, y \in \mathcal{L}^{\varepsilon}_{\delta}$. Thus $x \ominus y \in \mathcal{L}^{\varepsilon}_{\delta}$ by (4.4). It follows that

$$\tilde{f}^{\varepsilon}_{\mathcal{L}}(x \ominus y) \subseteq \delta = \delta_x \cup \delta_y = \tilde{f}^{\varepsilon}_{\mathcal{L}}(x) \cup \tilde{f}^{\varepsilon}_{\mathcal{L}}(y)$$

which completes the proof.

Theorem 4.8. *Given $\varepsilon \subseteq \Omega_{f_{\mathcal{L}}}$ and any $\delta \in 2^U$ with $\varepsilon \subseteq \delta$, if the (δ, ε)-support of a soft set $f_{\mathcal{L}}$ over U is an ideal of $\mathcal{L}$, then the ε-soft transfer of $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U.*

Proof. Suppose that the (δ, ε)-support $\mathcal{L}^{\varepsilon}_{\delta}$ of a soft set $f_{\mathcal{L}}$ over U is an ideal of $\mathcal{L}$. Let $x, y \in \mathcal{L}^{\varepsilon}_{\delta}$. Then

$$(x \ominus y) \ominus x = (x \ominus x) \ominus y = 0 \ominus y = 0 \in \mathcal{L}^{\varepsilon}_{\delta},$$

and so $x \ominus y \in \mathcal{L}^{\varepsilon}_{\delta}$. It follows from Lemma 4.7 that the ε-soft transfer $f^{\varepsilon}_{\mathcal{L}}$ of $f_{\mathcal{L}}$ satisfies the condition (4.5). Thus $f^{\varepsilon}_{\mathcal{L}}(0) = f^{\varepsilon}_{\mathcal{L}}(x \ominus x) \subseteq f^{\varepsilon}_{\mathcal{L}}(x) \cup f^{\varepsilon}_{\mathcal{L}}(x) = f^{\varepsilon}_{\mathcal{L}}(x)$ for all $x \in \mathcal{L}$. Let $x, y \in \mathcal{L}$ be such that $f^{\varepsilon}_{\mathcal{L}}(x \ominus y) = \delta_{x\ominus y}$ and $f^{\varepsilon}_{\mathcal{L}}(y) = \delta_y$. If we take $\delta := \delta_{x\ominus y} \cup \delta_y$, then $f^{\varepsilon}_{\mathcal{L}}(x \ominus y) = \delta_{x\ominus y} \subseteq \delta$ and $\sim f^{\varepsilon}_{\mathcal{L}}(y) = \delta_y \subseteq \delta$, that is, $x \ominus y \in \mathcal{L}^{\varepsilon}_{\delta}$ and $y \in \mathcal{L}^{\varepsilon}_{\delta}$. Since $\mathcal{L}^{\varepsilon}_{\delta}$ is an ideal of $\mathcal{L}$, we have $x \in \mathcal{L}^{\varepsilon}_{\delta}$. Thus $f^{\varepsilon}_{\mathcal{L}}(x) \subseteq \delta = \delta_{x\ominus y} \cup \delta_y = f^{\varepsilon}_{\mathcal{L}}(x \ominus y) \cup f^{\varepsilon}_{\mathcal{L}}(y)$. Therefore the ε-soft transfer of $f_{\mathcal{L}}$ is a uni-soft ideal of $\mathcal{L}$ over U.

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