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Research Article

DATA ENVELOPMENT ANALYSIS BASED METAMODELING FOR MULTI OBJECTIVE SIMULATION OPTIMIZATION IN A MANUFACTURING LINE

Onder BELGIN*[1]

[1]*Republic of Turkey Ministry of Industry and Technology, ANKARA;* ORCID: 0000-0001-6702-2608

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ABSTRACT

To adapt changing market conditions, firms must make quick decisions and response them as fast as possible. Simulation is a powerful tool to analyze the effects of changes in an industrial or service system on a virtual environment and usage of simulation models have become widespread with the developments in computers. Simulation isn't adequate to optimize the system parameters and additional methods are needed to integrate with simulation for optimization. In this study, a multi-objective optimization of a production system is considered. In this system, management aims to decide the optimal combination of workers in considered workstations. To cope with the problem a Data Envelopment Analysis (DEA) based metamodel is obtained and this metamodel is used as the objective function of the mathematical model with relevant constraints. In metamodeling stage two level factorial design is used.
Keywords: Simulation optimization, multi-objective simulation optimization, data envelopment analysis, metamodeling.

1. INTRODUCTION

Analytically modelling real world problems is very complex and needs many assumptions. It is usually difficult to obtain an accurate and reliable analytical model that reflects the behavior of the system. Simulation models are more capable of reflecting the behavior of the system and evaluating performance of the alternatives. Simulation has been applied to various sectors, such as manufacturing, services, defence, healthcare, and public services widely owing to invention and evolution of the computer, which has supported the uptake of practical simulation tools and techniques [1].

Simulation is not sufficient to optimize the system parameters, so an approach of integrating both simulation and optimization is needed. Simulation optimization (SO) requires the evaluation of a simulation model in the form of responses to a "What if" question and this question demand answers on certain performance measures for a given set of values for the decision variables of the system [2]. Simulation optimization aims to find the best values for simulation model input parameters in the search of one or more desired outputs [3]. There are many approaches to carry out the integration of simulation model and optimization techniques such as response surface methods, gradient search methods, stochastic approximation methods and heuristic search

* Corresponding Author: e-mail: onderbelgin@gmail.com, tel: (553) 667 16 02

methods [4]. Simulation optimization is generally a slow process that takes a large amount of time and the accomplishment of innumerous experiments [5].

A general form of the simulation optimization problem minimizes the expected value of the objective function with respect to its constraint set as in Equation 1.

$$\min_{\theta\in\Theta} J(\theta), \tag{1}$$

where $J(\theta) = E[L(\theta,\varepsilon)]$ is the performance measure of the problem, $L(\theta,\varepsilon)$ is the sample performance, ε represents the stochastic effects in the system, θ is a vector of controllable factors and Θ is the set containing all the feasible θ [6].

Many real-world problems have multiple conflicting objectives and these need to be optimized simultaneously. When the simulation optimization is considered, obtaining the compromise solution set becomes very complex and time-consuming due to the stochastic nature of the simulation.

In this paper, I propose a framework for multi-objective simulation optimization that combines meta modelling with data envelopment analysis (DEA) to determine the optimum levels of the factors for a manufacturing line. In literature there are some studies combining simulation optimization and DEA. Azadeh et. al [7], solved workshop facility layout design problem with ambiguous environmental and health indicators using an integrated algorithm based on fuzzy simulation, fuzzy linear programming and fuzzy DEA. Authors use fuzzy simulation to model different layout alternatives and fuzzy DEA (FDEA) to rank the alternatives to find the optimal layout design alternatives. Miranda et. al [8], proposed a method to identify the best ranges for each integer decision variable, in which providing a reduction in computational cost without loss of the quality in the response combining discrete event simulation, and DEA. Orthogonal arrays are used to obtain the input scenarios and super-efficiency analysis is applied in DEA stage. Shadkam and Bijari [9], solved supplier selection and order quantity allocation to each supplier problem using Cuckoo optimization algorithm, discrete event simulation, supply chain model and generalized DEA. Zarrin and Azadeh [10], simulated a manufacturing organization with maintenance strategy to evaluate the impacts of resilience engineering principles on lean practices. The researchers calculated considered outputs by simulation and analyzed the optimal values of scenarios' efficiencies using DEA. The results of the DEA were validated by using principal component analysis. Azadeh et. al [11], used simulation optimization to assess the appropriate scenarios approved by experts in an emergency department in Iran. Scenarios were examined and evaluated by stochastic DEA. Azadeh and Moradi [12] used fuzzy simulation to model different layout alternatives with safety and ergonomics factors using FDEA and fuzzy analytic hierarchy process (FAHP) In the study, feasible layout alternatives are generated using fuzzy simulation, FAHP for weighting non-crisp ergonomics and safety factors and finally, FDEA to obtain the optimum layout alternatives. Miranda et. al [5], aimed to define optimum variation intervals for each decision variable combining DEA with fuzzy logic. In the study, Taguchi's orthogonal arrays were utilized to generate the decision making units, and the output variables were obtained by the simulation model. Lin et. al [13], proposed a framework combining genetic algorithm (GA) and DEA for multi-objective simulation optimization. In this framework, a design point's relative efficiency score was obtained by DEA and this score was used as its fitness value in the selection operation of GA. Villarreal-Marroquin et. al [14], compared two metamodel based methodologies for multi-criteria simulation optimization for a molding process. The first methodology with linear regression metamodels and DEA, while the second one with a Gaussian process metamodel and calculates an expected improvement to determine the new input runs sequentially.

On the other hand, it is possible to encounter some studies combining simulation optimization and metamodeling. Mirfenderesgi and Mousavi [15], developed a hybrid optimization-simulation model by linking a stretching particle swarm optimization algorithm and used four metamodel types of artificial neural networks, support vector machines, Kriging and polynomial response functions to optimize water allocation at basin scale. Dengiz et. al [16], used simulation modeling and the regression metamodeling approach – as an objective function – to design and optimize an automotive production system. Pedrielli and Ng [17], proposed a Kriging-Based Trust Region Method for global optimization called G-STAR. Ryu et al [18] proposed a method for approximating the Pareto front of a multi-objective simulation optimization problem using a metamodeling scheme for objective function employing a weighted sum method to convert the MOP into a set of single objective optimization problems. Yang and Tseng [19] used a simulation metamodel for the ink-marking operation using a fractional factorial experimental design and regression analysis. Obtained metamodel solved by a hybrid response surface method and lexicographical goal programming approach. Zakerifar et al. [20] used Kriging metamodeling for multi-objective simulation optimization for an inventory system. Dengiz et al [21], built a multiple regression metamodel as a simulation optimization-based decision support system. Yang et. al [22], solved a multi response simulation problem by using a dual-response system and scatter-search method. Yang and Chou [23] proposed a hybrid Taguchi method and TOPSIS to solve the multi response simulation optimization in an integrated-circuit packaging company. Similarly, Kuo et al. [24] integrates Taguchi method and grey relational analysis and Belgin [25] integrates Taguchi method and grey relational analysis with fuzzy AHP weighting for multi response simulation optimization. Nezhad and Mahlooji [26], presented artificial neural network metamodels for expensive continuous simulation optimization with stochastic constraints. Sreekanth and Datta [27], developed a methodology based on simulation-optimization for the management of production and barrier wells in a coastal aquifer. In the study, a neural network based metamodel and a multi-objective genetic algorithm are used. Shirazi et. al [27], described an intelligent co-simulator for real time production control of a complex flexible manufacturing system using dynamic metamodeling for simulation optimization.

Any study was not encountered in the literature that use DEA and metamodeling for multi-objective simulation optimization. The main contribution of this study is on the integration of DEA and metamodeling approaches for the multi-objective simulation optimization. DEA is employed to unify the more than one objective values into a single value. The results of DEA are used as the response values of the experimental design to obtain the metamodel. After that, metamodel is used as the objective function of a nonlinear mathematical model including relevant constraints and optimum values for the considered variables are obtained solving the mathematical model.

2. PROPOSED METHODOLOGY

In this study, metamodeling and DEA are used together for the multi-objective optimization of the considered production system. Since in metamodeling corresponding model is obtained using only one response value for each experiment, two objectives are combined into a single value using DEA. The efficiency values obtained by DEA are used as response value. In this section firstly, metamodeling approach then details of DEA and finally the structure of the proposed methodology will be mentioned.

2.1. Metamodeling Approach

In simulation optimization, it is required to run simulation models several times and this requires excessive computation. To reduce the time required, usage of metamodeling approach can be appropriate. Metamodels can be described as the *models of the model* briefly. These metamodels are usually deterministic approximating functions for the function that are inexpensive to compute [28].

The main steps in metamodeling can be listed as follows [28]:

- the choice of a functional form for the function

- the design of experiments
- fitting the function to the simulation response using the experimental data
- the assessment of the adequacy of the fitted metamodel

In this study two-level (2^k) factorial design is used to obtain the metamodel. In other words, k factors are considered, each at two coded levels as low (-1) and high (+1). This design allows us to obtain a regression model including main effects of the factors and interactions among the factors. The general form of the regression model including three factors are given in Equation 2.

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_i \sum_{\substack{j \\ i<j}} \beta_{ij} x_{ij} + \beta_{123} x_{123} + \varepsilon \quad (2)$$

where Y is response, β_0 is regression intercept, β_i is first order effect of factor i, β_{ij} two factor interaction between factor i and j ($i \neq j$), β_{123} is three-factor interaction between all factors and ε is the fitting error of the regression model.

2.2. Data Envelopment Analysis

DEA is a nonparametric technique based on linear programming and can be used to rank and compare the relative performance of decision making units (DMUs). DEA provides a unified performance efficiency measurement (efficiency score) for each DMU using a set of input and output variables [29]. First paper on DEA was published in 1978 by Charnes, Cooper, and Rhodes [30] and this model is named as CCR model in DEA terminology and it presents a constant return of scale.

The linear form of CCR model is given below.

$$\max E_k = \sum_{r=1}^{s} u_{rk} y_{rk} \qquad k = 1,2,3,\dots,n \quad (3)$$

Subject to

$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0 \qquad j = 1,2,3,\dots,n \quad (4)$$

$$\sum_{i=1}^{m} v_{ik} x_{ik} = 1 \quad (5)$$

$$u_{rk} \geq 0 \qquad r = 1,2,3,\dots,s \quad (6)$$

$$v_{ik} \geq 0 \qquad i = 1,2,3,\dots,m \quad (7)$$

where E_k is the relative efficiency value of each DMUs. m and s are the number of inputs and number of outputs respectively. n denotes for the number of DMUs, x_{ij} is amount of i^{th} input used by the j^{th} DMU, y_{rk} is amount of r^{th} output produced by the j^{th} DMU. The weights of inputs and outputs are u_{rk} and v_{ik} succesively.

Results of a DEA can give more than one efficient DMUs and the decision makers may want to know which DMU is more efficient. For this purpose, super-efficiency DEA model can be employed which eliminates the upper bound on the technical efficiency score and gives additional information regarding the relative performance of the efficient unit [31]. In super-efficiency DEA model an efficient DMU may increase its input vector proportionally while preserving efficiency with a score above one [32]. The mathematical model of super-efficiency DEA model is given below.

$$\begin{aligned} & max\ E_j = \sum_{r=1}^{s} u_r y_{r0} \\ & \sum_{\substack{r=1 \\ j\neq k}}^{s} u_{rk} y_{rj} - \sum_{\substack{i=1 \\ j\neq k}}^{m} v_{ik} x_{ij} \leq 0 \qquad j = 1,2,3,\dots,n \\ & \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & u_{rk} \geq 0 \qquad r = 1,2,3,\dots,s \\ & v_{ik} \geq 0 \qquad i = 1,2,3,\dots,m \end{aligned} \quad (8)$$

The model above gives continuous technical efficiency without upper bound. Main difference in super-efficiency DEA model is exclusion of unit k from the constraint set [31].

Any DEA model has two orientations as input-oriented and output-oriented. Input-oriented models are used to test if a DMU under evaluation can reduce its inputs while keeping the outputs at their current levels, output-oriented models are used to test if a DMU under evaluation can increase its outputs while keeping the inputs at their current levels [33].

2.3. Structure of the Proposed Methodology

The proposed methodology including metamodeling and DEA consists of five main stages as in Figure 1.

The main steps of the methodology are as follows:

1. The considered problem is defined and the factors to be optimized are determined. The lower and upper bounds of the factors are determined.
2. Considering the factors 2^k factorial design is constructed for Objective I and Objective II. For the response value of each design points, simulation model is run. As a result two sets of factorial designs are obtained.
3. After the experiment stage the factorial designs are unified. The inputs values (factor levels) are same in each design but output values (response values) are different. Since every outputs have different measuring units these values are normalized. For unifying stage CCR super-efficiency DEA model is run and the efficiency values are obtained as response values for each design points.
4. The regression metamodel is obtained using factor levels and efficiency values. After the statistical validation tests the most appropriate model is determined.
5. Finally a non-linear mathematical model is constructed subject to certain constraints related to the production line. The objective function of the model is the metamodel which is obtained in previous step.

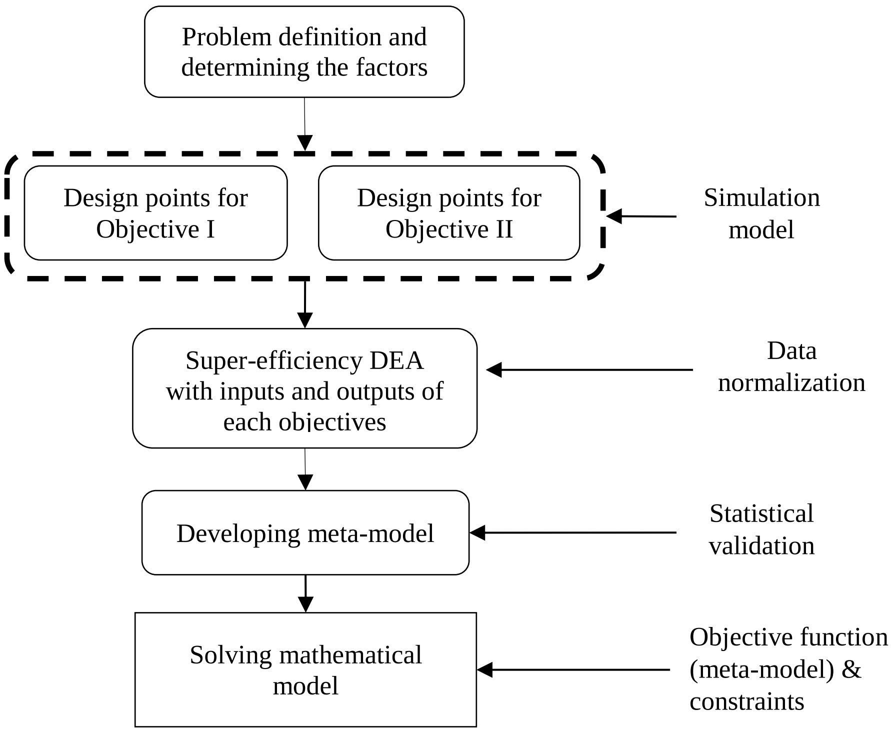

Figure 1. Structure of the proposed methodology

3. CASE STUDY

In this section the case study on determining optimum resource levels in a production line is mentioned. Firstly, the details of the considered production line is given. The working flow of the production line and the current performance of the system are mentioned. After the system description the design of experiment stage is given and then CCR super-efficiency DEA model results is given in the unifying stage of the response values. After that, the metamodel is constructed and the relevant statistical validations are done. Finally, the mathematical model is constructed using metamodel as objective function and relevant constraints are added to the model. At the end, the optimum levels of the resources are obtained.

3.1. Simulation Model of the Production System

The corresponding production system was handled firstly in Belgin (2015). The working flow of the production line is given in Fig. 2 and it has been modeled using Arena® 9.0 simulation software of Rockwell Automation. Water, electric and natural gas meters are produced in this system. These products are manufactured in three separate lines. The characteristics of the production process are based on assembly of the relevant parts. In every line there are control points and if the operator in the production line encounters any problem the meter is sent to repairing station. If the problem is solved, then the meter continues in the production line from the point last sent. In the repairing station the meter may be accepted as scrap with a certain percentage. The meters with no problem are packaged in the packaging station which serves to all production lines. In water meter line there are 12, in electric meter line there are 11 and in natural gas meter line there are 12 processes. One worker is responsible for each process. Water meter and natural gas lines have 6 control points and electric meter line has 5 control points along the assembly line and there are 2 workers in packaging station.

At the current situation, there are 36 workers in all production system totally. The management's priorities are on increasing workforce productivity and decreasing amount of work-in-process. The management aims to obtain optimal number of workers in repairing stations of each line and packaging station. For this reason, firm management encounters a multi-objective optimization problem consisting of two contradicting objectives – increasing workforce productivity and reducing work in process. To solve this problem DEA method and metamodeling are used together. In the current situation workforce productivity level is 1,792 and work in process level is 8,183.

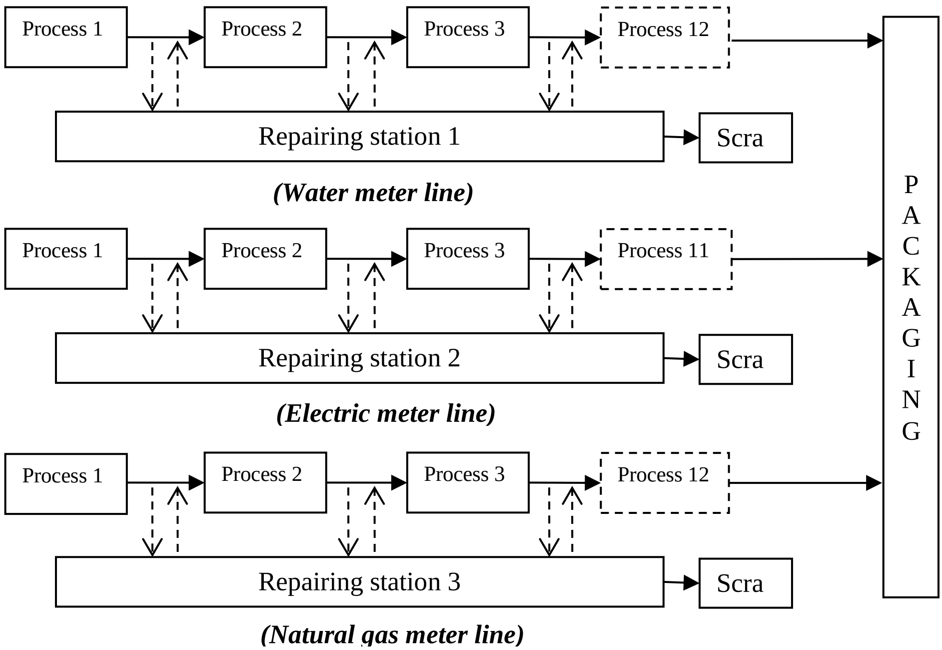

Figure 2. Working flow of production line

3.2. Design of Experiment

To optimize the considered production line with compromise objectives a mathematical description of the system is needed. For this purpose, 2^k factorial design is utilized. Factorial designs are widely used in experiments involving several factors where it is necessary to study the joint effect of the factors on a response [33]. The 2^k means the designs with k factors in which each factor has two levels, i.e. "low" or "high".

In this study 2^4 factorial design is used to develop metamodels. The relevant four factors and their levels are given in Table 1. These factors are number of workers in repairing station 1, number of workers in repairing station 2, number of workers in repairing station 3 and number of workers in packaging station.

Table 1. Relevant factors and their levels

Factors	Lower Bound	Upper Bound
No. of workers in repairing station 1 (A)	1	3
No. of workers in repairing station 2 (B)	1	3
No. of workers in repairing station 3 (C)	1	3
No. of workers in packaging station (D)	2	4

In the simulation optimization manner simulation model is run with the all levels of the factors and the best solution is decided as the optimal solution. This means 3x3x3x3=81 experiments and when all experiments are run 10 times and the factorial design is replicated 5 times 81x10x5=4050 simulation runs are required. By means of factorial design we need $5x2^4$=80 experiments and this means 800 simulation runs totally.

Since two objectives are considered in the problem, the response values for each objective are given separately. These objectives are workforce productivity (PR) and number of work-in-process in the production line (WIP). PR is number of packed goods for each worked hour and WIP is average number of uncompleted goods in the production line. In Table 2 and Table 3 the results of the 2^4 factorial design for PR and WIP are given including 5 replications.

Table 2. Results of 2^4 factorial design for PR

No. of Experiment	Factors				PR					
	A	B	C	D	Rep.1	Rep.2	Rep.3	Rep.4	Rep.5	Average
1	1	3	3	4	1,961	1,960	1,959	1,957	1,956	1,959
2	3	1	1	4	1,978	1,940	2,034	2,044	2,055	2,010
3	3	3	1	2	1,976	1,954	1,879	1,869	1,718	1,879
4	3	1	1	2	2,018	1,979	1,950	1,926	1,894	1,953
5	3	1	3	4	1,828	1,860	1,859	1,858	1,851	1,851
6	3	3	3	4	1,854	1,858	1,853	1,860	1,864	1,858
7	1	3	1	2	2,072	2,092	2,106	2,096	2,010	2,075
8	3	3	1	4	1,881	1,885	1,872	1,859	1,873	1,874
9	3	3	3	2	1,909	1,911	1,897	1,890	1,859	1,893
10	1	1	3	4	2,046	2,052	2,070	2,099	2,087	2,071
11	1	1	3	2	2,177	2,247	2,243	2,254	2,234	2,231
12	3	1	3	2	1,898	1,869	1,869	1,896	1,886	1,884
13	1	3	1	4	1,956	1,917	1,893	1,853	1,808	1,885
14	1	1	1	4	2,204	2,164	2,232	2,255	2,304	2,232
15	1	1	1	2	2,002	1,931	1,909	2,025	1,909	1,955
16	1	3	3	2	2,304	2,284	2,296	2,291	2,269	2,289

According to Table 2 Experiment 5 has minimum average PR value with 1,851 and Experiment 16 has maximum average PR value with 2,289. As for Table 3, Experiment 16 has minimum average WIP value with 7,147 and Experiment 12 has maximum average WIP value with 9,449.

Table 3. Results of 2^4 factorial design for WIP

No. of Experiment	Factors				WIP					
	A	B	C	D	Rep.1	Rep.2	Rep.3	Rep.4	Rep.5	Average
1	1	3	3	4	7,242	7,231	7,277	7,291	7,297	7,268
2	3	1	1	4	8,960	8,971	8,954	8,943	8,937	8,953
3	3	3	1	2	8,339	8,356	8,369	8,369	8,419	8,370
4	3	1	1	2	9,166	9,175	9,175	9,172	9,160	9,170
5	3	1	3	4	9,306	9,295	9,313	9,299	9,306	9,304
6	3	3	3	4	8,306	8,327	8,349	8,362	8,376	8,344
7	1	3	1	2	8,530	8,506	8,473	8,443	8,419	8,474
8	3	3	1	4	8,473	8,468	8,484	8,492	8,492	8,482
9	3	3	3	2	8,329	8,332	8,326	8,326	8,349	8,332
10	1	1	3	4	8,100	8,100	8,110	8,110	8,121	8,108
11	1	1	3	2	7,860	7,860	7,872	7,882	7,882	7,871
12	3	1	3	2	9,453	9,456	9,459	9,444	9,435	9,449
13	1	3	1	4	8,790	8,780	8,787	8,818	8,849	8,805
14	1	1	1	4	7,963	7,963	7,948	7,950	7,953	7,955
15	1	1	1	2	7,795	7,792	7,783	7,752	7,715	7,767
16	1	3	3	2	7,161	7,184	7,138	7,129	7,122	7,147

3.3. Unifying Two Objectives Using DEA

As in mentioned before DEA is used to unify two different responses for the metamodel. In this study only the efficiency values are dealt with and for this reason CCR super-efficiency DEA model is used. The CCR model is input-oriented because we have control on input values. Super-efficiency allows efficient DMUs to have efficiency value over one and enables to rank the most efficient DMUs.

Table 4. Super-efficiency values of design points

No. of Experiment	Factors				Super-efficiency values					
	A	B	C	D	Rep.1	Rep.2	Rep.3	Rep.4	Rep.5	Average
1	1	3	3	4	0,301	0,246	0,262	0,254	0,406	0,294
2	3	1	1	4	0,897	0,898	1,059	1,090	1,098	1,008
3	3	3	1	2	0,689	0,643	0,604	0,607	0,636	0,636
4	3	1	1	2	1,711	1,688	1,677	1,689	1,689	1,691
5	3	1	3	4	0,936	0,929	0,937	0,937	0,944	0,937
6	3	3	3	4	0,398	0,404	0,418	0,424	0,441	0,417
7	1	3	1	2	1,521	1,585	1,575	1,533	1,399	1,523
8	3	3	1	4	0,654	0,645	0,661	0,667	0,695	0,664
9	3	3	3	2	0,554	0,558	0,565	0,543	0,606	0,565
10	1	1	3	4	1,050	1,065	1,081	1,088	1,091	1,075
11	1	1	3	2	1,452	1,895	1,689	1,695	1,560	1,658
12	3	1	3	2	1,143	1,141	1,139	1,133	1,135	1,138
13	1	3	1	4	1,190	1,207	1,235	1,285	1,332	1,250
14	1	1	1	4	2,161	1,965	2,300	2,337	2,559	2,264
15	1	1	1	2	0,791	0,679	0,673	0,781	0,684	0,722
16	1	3	3	2	1,364	1,095	1,136	1,092	1,068	1,151

To reduce the effects of various measuring units of responses on relative efficiency analysis in DEA, experimental data are normalized. For PR variable, it is better to be larger but for WIP value it is better to be smaller. Two types of data normalization are used; smaller-the-better and larger the better. Output values are transformed to $y_{ij}^{'}$ using Eq. 14 and 15.

For smaller the better type,

$$y_{ij}^{'} = \frac{y_{ij} - \max(y_j)}{\max(y_j) - \min(y_j)} \tag{14}$$

For larger the better type,

$$y_{ij}^{'} = \frac{y_{ij} - \min(y_j)}{\max(y_j) - \min(y_j)} \tag{15}$$

The super-efficiency values of each design point obtained using normalized response values are given in Table 4. The values are obtained using EMS (Efficiency Measurement System) [36].

According to Table 4 Experiment 1 has minimum average super-efficiency value with 0,294 and Experiment 14 has maximum average super-efficiency value with 2,264.

3.4. Developing Metamodel

In Table 5 estimated effects and coefficients for unified values of PR and WIP based on super-efficiency DEA values are given. In the first column of the table sources are given. Second column shows degrees of freedom values, third column shows adjusted sum of square values, adjusted mean square values, fifth column shows F-value and last column shows *p* value. The table is obtained using Minitab 17 software.

Table 5. ANOVA table for PR&WIP

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	15	20,926	1,395	184,51	0,000
Linear	4	9,996	2,499	330,51	0,000
A	1	2,592	2,592	342,83	0,000
B	1	4,984	4,984	659,20	0,000
C	1	1,989	1,989	263,06	0,000
D	1	0,431	0,431	56,97	0,000
2-Way Interactions	6	3,193	0,532	70,39	0,000
A*B	1	0,306	0,306	40,48	0,000
A*C	1	0,127	0,127	16,82	0,000
A*D	1	0,217	0,217	28,69	0,000
B*C	1	0,185	0,185	24,40	0,000
B*D	1	0,549	0,549	72,59	0,000
C*D	1	1,810	1,810	239,35	0,000
3-Way Interactions	4	6,225	1,556	205,85	0,000
A*B*C	1	0,596	0,596	78,85	0,000
A*B*D	1	2,545	2,545	336,66	0,000
A*C*D	1	2,840	2,840	375,57	0,000
B*C*D	1	0,244	0,244	32,30	0,000
4-Way Interactions	1	1,511	1,511	199,90	0,000
A*B*C*D	1	1,511	1,511	199,90	0,000
Error	64	0,484	0,008		
Total	79	21,409			

According to the values in Table 5, the main factors A, B, C and D are significant at 0.05 significance level. Furthermore, all two-way interactions (AB, AC, AD, BC, BD and CD) are significant at 0.05 significance level. Also three-way interactions (ABC, ABD and ACD) and four-way interaction (ABCD) are significant at 0.05 significance level. The adjusted R^2 (variation in the dependent variable) is 97.21% and this is a highly enough level.

In Figure 3 main effects plot for PR&WIP is given. According to the figure increase in level of factors cause decrease in mean values of PR&WIP.

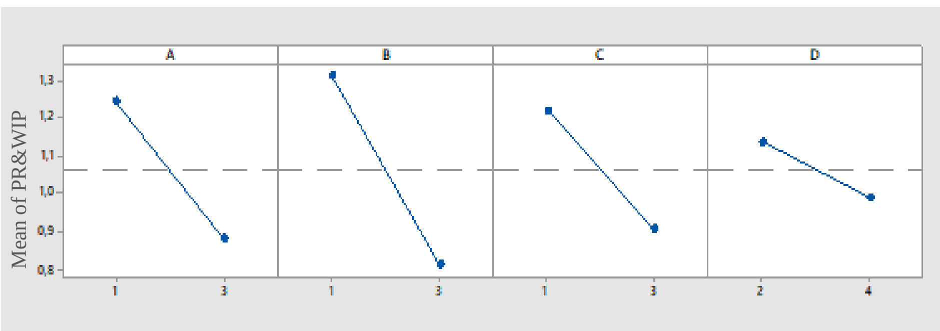

Figure 3. Main effects plot for PR&WIP

After the statistical analysis, the relevant model for PR&WIP in uncoded units is as follows:

$$PR\,\&\,WIP = -8.589 + 4.216A + 3.615B + 3.766C + 3.285D - 1.594AB - 1.523AC - 1.335AD - 1.211BC - 1.100BD - 1.188CD + 0.499ABC + 0.453ABD + 0.463ACD + 0.330BCD - 0.138ABCD \quad (16)$$

3.5. Mathematical Model and Optimum Results

To optimize factor levels mathematical programming is utilized. This model has a nonlinear objective function and other constraints are added subject to the objective function. The mathematical model is given below:

$$Max = -8.589 + 4.216A + 3.615B + 3.766C + 3.285D - 1.594AB - 1.523AC - 1.335AD - 1.211BC - 1.100BD - 1.188CD + 0.499ABC + 0.453ABD + 0.463ACD + 0.330BCD - 0.138ABCD \quad (17)$$

Subject to

$$1 \le A \le 3 \quad (18)$$

$$1 \le B \le 3 \quad (19)$$

$$1 \le C \le 3 \quad (20)$$

$$2 \le D \le 4 \quad (21)$$

$$A + B + C \le 8 \quad (22)$$

$$2A - C = 0 \quad (23)$$

$$A, B, C, D \ge 0, \text{ and integer} \quad (24)$$

In this formulation objective function (17) maximizes the PR&WIP value. Constraints (18) to (21) are lower and upper bounds of number of workers in each station. Constraint (22) gives an upper bound for total workers in all work stations A, B, C and this value is 8. Constraint (23) ensures the number of workers in work station C is doubles the number of workers in work station A. This constraint is needed for the synchronization of the production rate. Finally, Constraint (24) is the integrality constraint.

The mathematical model is solved using LINGO 17.0 and the optimal combination of the workers in the workstations is $A=1$, $B=1$, $C=2$ and $D=4$ respectively. Objective function value is 1,658 for PR&WIP. In addition to this, the simulation model is run again with this factor levels and according to the results of these runs PR value for the new system is 1,862 and WIP value is 7,921. As a result, in the new system an improvement of 3,9% in PR and 3,2% in WIP is obtained. The comparison of the current system and new system is given in Table 6.

Table 6. Comparison of current system and proposed system

Factors	Current System			Proposed System		
	Factor Levels	PR	WIP	Factor Levels	PR	WIP
No. of workers in repairing station 1 (A)	1	1,792	8,183	1	1,862	7,921
No. of workers in repairing station 2 (B)	1			1		
No. of workers in repairing station 3 (C)	1			2		
No. of workers in packaging station (D)	2			4		

4. CONCLUSIONS

In this study, DEA based metamodeling is used to solve a multi-objective simulation optimization problem encountered in a production line. Using proposed methodology in the study number of workers in repairing station 1 (A), number of workers in repairing station 2 (B), number of workers in repairing station 3 (C) and number of workers in packaging station (D) are aimed to be optimized subject to relevant constraints. For metamodeling stage 2^4 factorial design is used and metamodel is obtained for workforce productivity (PR) and work-in-process in production line (WIP) responses. To obtain responses for the factorial design simulation model is used. Since the considered problem has a multi-objective characteristic and metamodel needs only one response to run. PR and WIP objectives are unified into one value using input oriented CCR super-efficiency DEA model. 2^4 factorial design is run using super-efficiency DEA values and a model is obtained which has interactions up to four-ways. After validating statistically the model is used as an objective function subject to relevant constraints for the production system. According to the results of the mathematical formulation the optimum combination of the main variables are found to be $A=1$, $B=1$, $C=2$ and $D=4$. In the proposed system an improvement of 3,9% in PR and 3,2% in WIP is obtained.

By means of this approach firm management can easily decide the optimum combination of workers and the firm management can quickly adapt the corresponding system to new market conditions more economically. For future directions, different DEA models (such as slack-based-measure DEA or fuzzy DEA) can be used for unifying the objectives. Furthermore, multi-criteria decision making methods can be employed for unifying stage.

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