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Research Article

A NEW SOLUTION CONCEPT FOR SOLVING MULTIOBJECTIVE FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT

In this work, we have proposed a solution to multiobjective fractional programming problems (MFPPs) by using the first-order Multiplicative Taylor expansion of these objective functions at optimal points of each fractional objective functions in feasible region. MFPP reduces to an equivalent Multiobjective Linear Programming Problem (MLPP). The resulting MLPP is solved assuming that weights of these linear objective functions are equal and considering the sum of the these linear objective functions. Thus, the problem is reduced to a single objective. The proposed solution to MFPP always yields efficient solution. Therefore, the complexity in solving MFPP has reduced easy computational and to show the efficiency of the Multiplicative Taylor series method, we applied the method to some problems.

Keywords: Fractional programming, multiobjective fractional programming, multiplicative derivation.

1. INTRODUCTION

The multiobjective fractional programming problems (MOFPPs) have received of much interest in recent past. This problems are applied to different disciplines such as engineering, business, finance, economics, etc. MOFPPs are generally used for modeling real life problems with one or more objectives such as profit/cost, inventory/sales, actual cost/ standart cost, output/employee etc. (see, for example, [1-8]).

Multiobjective Fractional Programming Problems (MFPPs) pose some computational difficulties, so they are converted into single objective programming problems and then solved using the method of Bitran and Novaes [9] or Charnes and Cooper [10].

Michael Grossman and Robert Katz gave definitions of a new kind of derivative and integral, moving the roles of subtraction and addition to division and multiplication, and thus established a new calculus, called multiplicative calculus [11].

In this paper, we proposed a solution to Multiobjective Fractional Programming Problems using the first order Multiplicative Taylor polynomial series method at optimal point of each fractional objective function in feasible region.

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2. PRELIMINARIES

Definition: If the nu merator and denominator in the objective function as well as the constraints are linear, we have a linear fractional programming problem (LFPP) as follows:

$$\begin{aligned} &Optimize\ \frac{cx+\alpha}{dx+\beta}, \\ &s.t.:\ x \in S = \left\{ x \mid Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b,\ x \geq 0 \right\} \end{aligned} \tag{1}$$

where A is a real $m \times n$ matrix, $b \in R^m$, $x \in R^n$ and S is a nonempty and bounded set. For some values of x, $dx+\beta$ may be equal to zero. To avoid such cases, is generally set to be greater than zero.

Charnes and Cooper [10] showed that if the denominator is constant in sign on the feasible region, the LFPP can be optimized by solving a linear programming problem. However, in many applications, there are two or more conflicting objective functions which are relevant, and some compromise must be bought between them. Such types of problems are inherently multiobjective linear fractional programming problems and can be written as:

$$Optimize\ Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k},\ k = 1,\dots,K \tag{2}$$

$$s.t.:\ x \in S = \left\{ x \mid Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b,\ x \geq 0 \right\}$$

where S, A, b and x are as defined in problem (1), and $\forall x \in S,\ d_k x + \beta_k > 0$ (k=1,...,K).

Definition 1 Let f be a positive function on A ⊆ R, and it is derivative at x exists. The multiplicative derivation of f is given by

$$f^{\circledast}(x) = \lim_{h \to 0} \left(\frac{f(x+h)}{f(x)} \right)^{\frac{1}{h}} = e^{(\ln \circ f)'(x)} \tag{3}$$

where $(\ln \circ f)(x) = \ln f(x)$. If, additionaly, f is a smooth function. then the higher order multiplicative derivation is given by the following formula [12]

$$f^{\circledast(n)}(x) = e^{(\ln \circ f)^{(n)}(x)}, n = 0,1,\dots \tag{4}$$

The multiplicative derivation has the following rules:

$$\begin{aligned} &(a)(cf)^{\circledast}(x) = f^{\circledast}(x) \\ &(b)(fg)^{\circledast}(x) = f^{\circledast}(x).g^{\circledast}(x) \\ &(c)(f/g)^{\circledast}(x) = f^{\circledast}(x)/g^{\circledast}(x) \\ &(d)(f^h)^{\circledast}(x) = f^{\circledast}(x)^{h(x)}.f(x)^{h'(x)} \\ &(e)(f \circ h)^{\circledast}(x) = f^{\circledast}(h(x))^{h'(x)} \end{aligned} \tag{5}$$

(f)$(f+g)^{\circledast}(x) = f^{\circledast}(x)^{\frac{f(x)}{f(x)+g(x)}}. g^{\circledast}(x)^{\frac{g(x)}{f(x)+g(x)}}.$

where c is a positive c onstant, f and g are $\circledast$ differentiable, h is differentiable and $f \circ h$ in part (e) is defined[12].

Theorem 2 (Multiplicative Taylor's Theorem for One Variable)[12]. Let A be an open interval and let

$\mathrm{f}: \mathrm{A} \rightarrow \mathrm{R}$ be $\mathrm{n}+1$ times $\circledast$ differentiable on A. Then for any $\mathrm{x}, \mathrm{x}+\mathrm{h} \in \mathrm{A}$, there exists a number $\theta \in (0,1)$ such that

$$f(x+h) =_{m=0}^{n} \left(f^{\circledast(m)}(x)\right)^{\frac{h^m}{m!}}. \left(\left(f^{\circledast(n+1)}(x+\theta h)\right)^{\frac{h^{n+1}}{(n+1)!}}\right). \tag{6}$$

The above result can be extended to functions of several variables as well. For simplicity, consider the function $f(x,y)$ of two variables defined on some open subset of $R^2 (= R \times R)$. We can define partial $\circledast$ derivative of f in x, considering y as fixed, which is denoted by $f_x^{\circledast}$ or $\partial^{\circledast} f / \partial y$. One can go on and define higher-order partial $\circledast$ derivatives of f for which the respective $\circledast$ notations are used.

Two results, generalizing the property (e) of $\circledast$ differentiation and Multiplicative Taylor's Theorem for One Variable, are as follows. They can also be proved by application of the respective results of Newtonian calculus to the function $\ln \circ f$.

Theorem 3 *(Multiplicative Chain Rule).* Let f be a function of two variables y and z with continuous partial $\circledast$ derivatives. If y and z are differentiable functions on (a, b) such that $\mathrm{f}(\mathrm{y}(\mathrm{x}), \mathrm{z}(\mathrm{x}))$ is defined for every $\mathrm{x} \in (\mathrm{a}, \mathrm{b})$, then

$$\frac{d^{\circledast} f(y(x), z(x))}{dx} = f_y^{\circledast}(y(x), z(x))^{y'(x)}. f_z^{\circledast}(y(x), z(x))^{z'(x)}. \tag{7}$$

Theorem 4 *(Multiplicative Taylor's Theorem for Two Variables)*[13]. Let A be an open subset of R^2. Assume that the function $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{R}$ has all partial $\circledast$ derivatives of order $\mathrm{n}+1$ on A. Then for every $(\mathrm{x}, \mathrm{y}), (\mathrm{x}+\mathrm{h}, \mathrm{y}+\mathrm{k}) \in \mathrm{A}$ so that the line segment connecting these two points belongs to A, there exits a number $\theta \in (0,1)$, such that

$$f(x+h, y+k) =_{m=0}^{n}{}_{i=0}^{m} \left(f_{x^i y^{m-i}}^{\circledast(m)}(x,y)\right)^{\frac{h^i k^{m-i}}{i!(m-i)!}}{}^{n+1}_{i=0} \left(\left(f_{x^i y^{n+1-i}}^{\circledast(n+1)}(x+\theta h, y+\theta k)\right)^{\frac{h^i k^{n+1-i}}{i!(n+1-i)!}}\right). \tag{8}$$

3. MULTIPLICATIVE TAYLOR METHOD

In this section, we consider the Multiobjective Fractional Programming Problem (MFPP). If $Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i}$ then,

$$\begin{aligned} &Max\ \ Z(x) = (Z_1(x), Z_2(x), \ldots, Z_k(x)), \\ &s.t. \quad x \in X = \{x \in R^n, Ax \le b, x \ge 0\} \\ &A \in R^{m \times n},\ b, c_i, d_i \in R^n, \alpha_i, \beta_i \in R \end{aligned} \tag{9}$$

We will transform the model (9) to a new model obtained by the following three steps:

Step 1 : Determine $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*)$ which is the value(s) that is used to maximize the ith objective function $Z_i(x)$ $(i = 1, \ldots k)$ where n is the number of the variable.

Step 2: Transform each objective functions by using first-order Multiplicative Taylor polynomial series as follows:

$$Z_i(x) \cong L_{i1}(x) = Z_i(x_i^*) \cdot \left[(x_1 - x_{i1}^*)\frac{d^{\circledast} Z_i(x_i^*)}{dx_1} \cdot \ldots \cdot (x_n - x_{in}^*)\frac{d^{\circledast} Z_i(x_i^*)}{dx_n})\right] \cdot O(h^2) \tag{10}$$

Step 3: Find $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ by solving the reduced problem to a single objective.

Thus, MFPP (9) reduces the following MLPP (11) :

$$\max\{L_1(x), L_2(x), .., L_k(x)\} \quad (11)$$
$$x \in X = \{x \in R^n, Ax \le b, x \ge 0\}$$

if we assume that the weights of objective functions in problem (11) are equal, then the problem (11) is transformed to the following linear programming problem:

$$\max\{L_1(x) + L_2(x) + \ldots + L_k(x)\} \quad (12)$$
$$x \in X = \{x \in R^n, Ax \le b, x \ge 0\}$$

In problem (12), set X is non-empty convex set having feasible points. The optimal solution of the problem (12) gives the efficient solution of the MFPP (9).

4. NUMERICAL EXAMPLES

Example 1 We consider the example studied by [4]

$$Maximize Z_1(x) = \frac{-3x_1+2x_2}{x_1+x_2+3}$$

and

$$Maximize Z_2(x) = \frac{7x_1+x_2}{5x_1+2x_2+1} \quad (13)$$
$$Subject to x_1 - x_2 \ge 1$$
$$2x_1 + 3x_2 \le 15$$
$$x_1 \ge 3$$
$$x_1, x_2 \ge 0$$

It is observed that $Z_1 < 0$, $Z_2 \ge 0$, for each x in the feasible region. If the problem is solved for each of objectives one by one $Z_1^*(3.6,2.6) = -14/23$, and $Z_2^*(7.5,0) = 15/11$, By expanding the first-order Multiplicative Taylor polynomial series for objective functions $Z_1(x)$ and $Z_2(x)$ about points (3.6,2.6) and (7.5,0) in feasible region, respectively are obtained from

$$Z_1(x) = Z_1(3.6,2.6).\left[(x_1 - 3.6)^{\frac{d^{\odot}Z_1(3.6,2.6)}{dx_1}}.(x_2 - 2.6)^{\frac{d^{\odot}Z_1(3.6,2.6)}{dx_2}}\right] \quad (14)$$
$$= -0.60869565.e^{0.427(x_1-3.6)-0.4658385(x_2-2.6)}$$

$$Z_2(x) = Z_2(7.5,0).\left[(x_1 - 7.5)^{\frac{d^{\odot}Z_2(7.5,0)}{dx_1}}.(x_2 - 0)^{\frac{d^{\odot}Z_2(7.5,0)}{dx_2}}\right] \quad (15)$$
$$= 1.363636.e^{0.0034632(x_1-7.5)-0.0329x_2}$$

we get

$$L(x) = Z_1(x) + Z_2(x)$$
$$= -0.60869565.e^{0.427(x_1-3.6)-0.4658385(x_2-2.6)}$$
$$+ 1.363636.e^{0.0034632(x_1-7.5)-0.0329x_2}$$

Thus, the final form of the MLFP problem is obtained as follows:

$$Maximize L(x)$$
$$s.t. x_1 - x_2 \ge 1$$
$$2x_1 + 3x_2 \le 15$$
$$x_1 \ge 3$$
$$x_1, x_2 \ge 0$$

Optimal solution of the problem (13) is at point (3,2) and maximum value 0.634. The point (3,2) is efficient solution of the given problem in the feasible region. The solution for the problem (13) is obtained as:

$$x_1 = 3, x_2 = 2, Z_1(x) = -5/8, Z_2(x) = 23/20$$

Comparing our results and Guzel's results [4] then it can be seen that the results are same.

Example 2 We consider the example studied by [13]

$$Maximize Z_1(x) = \frac{-3x_1+2x_2}{x_1+x_2+3}$$
$$Maximize Z_2(x) = \frac{7x_1+2x_2}{5x_1+2x_2+1}$$

and

$$Maximize Z_3(x) = \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2}$$
$$Subject to x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 \leq 15$$
$$x_1 \geq 3$$
$$x_1 + 9x_2 \geq 9$$
$$x_1, x_2 \geq 0$$

It is observed that $Z_1 < 0$, $Z_2 \geq 0$, $Z_3 \geq 0$, for each x in the feasible region. If the problem is solved for each of objectives one by one $Z_1^*(3.6,2.6) = -0.608695652$, $Z_2^*(7.2,0.2) = 1.35828877$ and $Z_3^*(3.6,2.6) = 0.8235294$. By expanding the first-order Multiplicative Taylor polynomial series for objective functions $Z_1(x)$, $Z_2(x)$ and $Z_3(x)$ about points (3.6,2.6), (7.2,0.2) and (3.6,2.6) in feasible region, respectively are obtained from

$$Z_1(x) = Z_1(3.6,2.6).\left[(x_1 - 3.6)^{\frac{d^{\odot}Z_1(3.6,2.6)}{dx_1}}.(x_2 - 2.6)^{\frac{d^{\odot}Z_1(3.6,2.6)}{dx_2}}\right] \tag{16}$$
$$= -0.60869565.e^{0.427(x_1-3.6)-0.4658(x_2-2.6)}$$

$$Z_2(x) = Z_2(7.2,0.2).\left[(x_1 - 7.2)^{\frac{d^{\odot}Z_2(7.2,0.2)}{dx_1}}.(x_2 - 0.2)^{\frac{d^{\odot}Z_2(7.2,0.2)}{dx_2}}\right] \tag{17}$$
$$= 1.35828877.e^{0.0041(x_1-7.2)-0.0141(x_2-0.2)}$$

$$Z_3(x) = Z_3(3.6,2.6).\left[(x_1 - 3.6)^{\frac{d^{\odot}Z_3(3.6,2.6)}{dx_1}}.(x_2 - 2.6)^{\frac{d^{\odot}Z_3(3.6,2.6)}{dx_2}}\right] \tag{18}$$
$$= 0.8235294.e^{-0.0462(x_1-3.6)+0.1092(x_2-2.6)}$$

we get

$$L(x) = Z_1(x) + Z_2(x) + Z_3(x)$$

Thus, the final form of the MLFP problem is obtained as follows:

$$Maximize L(x)$$
$$Subject to x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 \leq 15$$
$$x_1 \geq 3$$
$$x_1 + 9x_2 \geq 9$$
$$x_1, x_2 \geq 0$$

The problem is solved and the solution of the problem above is as follows:

$$x_1 = 3.6, x_2 = 2.6, Z_1(x) = -0.60869565, Z_2(x) = 1.29385437, Z_3(x) = 0.8235294$$

and maximum value is 1.508688.

Comparing our results and Gupta's results ($x_1 = 3$, $x_2 = 2$, $Z_1(x) = -0.625$, $Z_2(x) = 1.15$, $Z_3(x) = 0.7857$) then it can be seen that our the results are very effective.

5. CONCLUSIONS

I has been made in this paper to explore a procedure to solve Multiobjective Fractional Programming Problems (MFPPs) based on first-order Multiplicative Taylor series. With the help of first-order Multiplicative Taylor polynomial series at optimal points of each fractional objective function in feasible region. We assumed that the weights of the objective are equal. Then, the proposed solution method was applied to two numerical examples to test the effect of first-order Multiplicative Taylor series method and the results show that the proposed method is more effective when compared to the previous methods. This method is applied to different disciplines such as engineering, business, finance, economics, etc.

REFERENCES

[1] Cevikel, A.C and Ahlatcioglu, M., (2010) Solutions for fuzzy matrix games, *Computers and Mathematics with Applications* 60, 399-410.

[2] Cevikel, A.C and Ahlatcioglu, M., (2018) A Transition from two-person zero-sum games to cooperative games with fuzzy payoffs, *Iranian Journal of Fuzzy Systems* 15, 121-131.

[3] Cevikel, AC and Ozavsar, M., (2019) Q-Taylor method for multiobjective fractional programming problem, *European Journal of Sciences and Technology* 16, 26-31.

[4] Guzel N. and Sivri M., (2005) Taylor series solution of multi objective linear fractional programming problem, *Trakya University Journal Science* 6, 80–87.

[5] Kornbluth J.S.H. and Steuer R.E., (1981) Multiple objective linear fractional programming, *Management Science* 27, 1024–1039.

[6] Nykowski I. and Zolkiski Z., (1985) A compromise procedure for the multiple objective linear fractional programming problem, *European Journal of Operational Research* 19, 91–97.

[7] Saad O., (2007) On stability of proper efficient solutions in multiobjective fractional programming problems under fuzziness, *Mathematical and Computer Modelling* 45, 221–231.

[8] Cevikel, A.C., (2019) Q-Taylor series method for solving fuzzy multiobjective linear fractional programming problem, *European Journal of Science and Technology* 15, 10-17.

[9] Bitran G.R. and Novaes A.G., (1973) Linear programming with a fractional objective function, *Operation Research* 21, 22–29.

[10] Charnes A. and Cooper W.W., (1962) Programming with linear fractional functions, *Naval Research Logistics Quaterly* 9, 181–186.

[11] Grossman M.and Katz R., (1972) Non-Newtonian Calculus, *Lee Press*, Pigeon Cove, MA, USA.

[12] Bashirov A.E., Kurpinar E.M.and Ozyapici A., (2008) Multiplicative calculus and its applications, Journal of Mathematical Analysis and Applications 337, 36-48.

[13] Chakraborty M. and Gupta S., (2002) Fuzzy mathematical programming for multiobjective linear fractional programming problem, *Fuzzy sets and systems* 125 335-342.