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Research Article

STRUCTURED SINGULAR VALUES FOR ANTI-ALIASING FILTER

Mutti-Ur REHMAN*[1]

[1]*Department of Mathematics, Sukkur IBA University, Sukkur 65200, PAKISTAN;*
ORCID: 0000-0002-5828-3133

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ABSTRACT

We present the numerical computation of bounds of a well- known mathematical quantity known as Sructured Singular Value (SSV) for anti-aliasing filter. The computation of SSV provides the bounds which estimates the behavior of linear input-output system in control. The proposed methodology is based on two level algorithm, that is, inner-outer algorithm. The numerical experimentation shows the com- parison of obtained results for SSV lower bounds when compare with MATLAB routine mussv.
Keywords: Eigenvalues, singular values, structured singular values, low-rank ODEs.

1. INTRODUCTION

The Structured Singular Value known as μ-value was first introduced by J. C. Doyle around 1980's [1]. The μ-value can be used for the analyses and synthesis of robustness and performance of the linear systems in control and system theory subject to presence of structured uncertainty. The interesting applications of μ-values can be found in [2, 3].

The computation of an exact value of μ-value is NP-hard problem [4, 5]. Much research has been carried out while providing numerical methods in order to approximate the bounds of μ-values, that is, the approximation of lower and upper bounds.

The power method developed by [3] approximates the lower bounds of μ-values. The power method realize only on matrix-vector products and work well for the case when pure complex uncertainties are under consideration. The power method was extended to μ-value problem when mixed real and complex uncertainties were under consideration by [6, 7]. The power method was extended to skew μ-value problems by [8, 9]. The power method may fail to converge when purely real uncertainties are under consideration [10].

The computation of μ-values for pure real uncertainties involves fundamental difficulties because of the fact that real μ-values could be a discontinuous function of the problem data [11].

This paper describes a low rank Ordinary Differential Equations ODE's based technique in order to approximate the lower bounds of μ-values. The low rank ODE's technique is based upon inner algorithm and outer algorithm. In an inner-algorithm, the main goal is to construct and then solve a system of ODE's. While discussing with outer-algorithm, we vary perturbation level $E >$ 0 by means of fast Newton's iterations.

* Corresponding Author: e-mail: mutti.rehman@iba-suk.edu.pk, tel: 0092-3040140607

We compare the results obtained for the approximation of μ-values lower bounds with one computed with the help of Gain Based Algorithm GBA [12] while considering both pure real and mixed real/complex uncertainties. The basic idea of GBA [12] involves the worst-case gain problem for the computation of real blocks of underlying uncertainties. The computation of complex blocks for the uncertainties involves the power method.

Computing μ-values by means of a related worst-case gain problem can be applied to the problem which exists in control systems. Furthermore, this idea can be generalized in order to deal with analysis of robustness problems which taking uncertainty norms, that is, matrix 2-norm for ellipsoidal uncertainty [13]. This idea can also be applied to the analysis of non-linear lumped and distributed parameter systems [14]. The idea of GBA [12] is also useful for the analysis of finite-time control in batch and semi-batch process.

2. NOTATIONS

The notation $\mathrm{C}^{m,n}$ and $\mathrm{R}^{m,n}$ denotes complex and real matrices respectively. The block diagonal uncertainties are denoted by B and B^* when mixed real and complex uncertainties and pure complex uncertainties are under consideration. The matrix valued function $\Delta \in \mathrm{B}$ or $\Delta \in \mathrm{B}^*$ denotes the uncertainties belonging to block diagonal matrices. The 2-norm of Δ, that is, $\|\Delta\|_2$ denotes the largest singular value of the matrix valued function $\Delta \in \mathrm{B}$ or $\Delta \in \mathrm{B}^*$. The notation for structured singular value μ is standard. Δ_R and Δ_C in B denotes the number of repeated real blocks and the number of pure complex blocks, that is,

$$\Delta_R := \{diag(\delta_1 I_1, \delta_2 I_2, \ldots, \delta_S I_S) : \delta_i \in \mathrm{R}, \forall i = 1 : s\};$$

and

$$\Delta_C := \{diag(\delta_1 I_{S+1}, \delta_2 I_{S+2}, \ldots, \delta_F I_{S+F}\,; \Delta_1, \Delta_2, \ldots, \Delta_F) : \delta_j \in \mathrm{C}, \Delta_j \in \mathrm{C}^{m_j, m_j}\ \forall j = 1 : F\}.$$

The spectral radius of the matrix M is denoted by $\rho(M)$. The notations $\Lambda_\varepsilon^B(M)$ and $\Sigma_\varepsilon^B(M)$ denotes the structured ε-spectral value sets for the given matrix M with respect to perturbation level $\varepsilon > 0$.

Definition 2.1. [1]. For a given matrix $M \in \mathrm{K}^{m,m}$ where K = C or R and for a given block diagonal uncertainties B, SSV is defined as,

$$\mu_{\mathbb{B}}(M) := \frac{1}{\min\{\|\Delta\|_2 : \Delta \in \mathbb{B},\ \det(I - M\Delta) = 0\}}, \tag{2.1}$$

if there exists $\Delta \in \mathrm{B}$ such that $\det(I - M\Delta) = 0$, otherwise $\mu_{\mathrm{B}}(M) := 0$ for $\Delta \in \mathrm{B}$ such that $\det(I - M\Delta) = 0$.

Definition 2.2. [16]. The structured epsilon spectral value set of given matrix $M \in \mathrm{C}^{n,n}$ w.r.t an admissible perturbation level $\varepsilon > 0$is defined as:

$$\Lambda_\varepsilon^B(M) = \{\lambda \in \Lambda(\varepsilon M\Delta) : \Delta \in \mathrm{B},\ \|\Delta\|_2 \leq 1\}. \tag{2.2}$$

In Equ. (2.2), the quantity $\Lambda(M)$ express the spectrum of a matrix M.

Definition 2.3. [16]. The structured spectral value set of given matrix $M \in \mathrm{C}^{n,n}$ w.r.t an admissible perturbation level $E > 0$ is defined as:

$$\Sigma_\varepsilon^B(M) = \{\xi = 1 - \lambda : \lambda \in \Lambda_\varepsilon^B(M)\}. \tag{2.3}$$

The Equ. (2.1) allows us to reformulate the definition of μ-value as,

Definition 2.4. [16]. The μ-value of a given matrix $M \in K^{m,m}$ where

K = C or R and with respect to a set of block diagonal matrices B is defined as,

$$\mu_{\mathbb{B}}(M) := \frac{1}{\arg\min\{0 \in \Sigma_{\epsilon}^{\mathbb{B}}(M)\}}. \quad (2.4)$$

For pure complex perturbation that is B^* the μ-value is defined as,

$$\mu_{\mathbb{B}^*}(M) = \frac{1}{\arg\min\{\max|\lambda| = 1\}}. \quad (2.5)$$

Here, $\lambda \in \Sigma_{\varepsilon}^{B^*}(M)$.

3. LOW RANK ODE'S BASED TECHNIQUE

In this section, we provide the basic idea of low rank ODE's based technique in order to approximate the lower bound of μ-values. We take the following optimization problem into account

$$\xi(\varepsilon) = \arg\min |\xi|, \quad (3.1)$$

where $\xi \in \Sigma_{\varepsilon}^{B}(M)$, for some fixed perturbation level $\varepsilon > 0$.

As μ-value is the reciprocal of the smallest obtained value for the perturbation level $E > 0$ for which $\xi(\varepsilon) = 0$. This allows us to suggest inner algorithm and outer algorithm. Inner algorithm deals with the solution of the minimization problem as discussed in Equ. (3.1). In the outer-algorithm, the idea is to modify perturbation level $\varepsilon > 0$ by means of Newton's method. This idea leads us in order to compute the extremizers of the matrix valued function $\Delta(\varepsilon)$ with respect to $\varepsilon > 0$.

For pure complex perturbations B^* the inner-algorithm determines the local optima for the maximization problem (3.2)

$$\lambda(\varepsilon) = \arg\max |\lambda|. \quad (3.2)$$

Here in Equ. (3.2), $\lambda \in \Lambda^{B^*}(M)$ which then cause the approximation of lower bound for μ-value $\mu_{B^*}(M)$.

For more details on the construction of both inner algorithm and outer algorithm, we refer interested readers to [16] and the references therein.

4. AN ANTI-ALIASING FILTERS

An Anti-aliasing filters are being used in the flight control electronics in order to minimize the distortion effects caused by the digital sampling. The following example is taken from [12] which consider a low-pass Sallen-Key filter [15]. The input and output transfer function for the Sallen-Key filter is given as

$$F(r) = \frac{1}{K_1 K_2 B_1 B_2 r^2 + B_2(K_1 + K_2)r + 1}. \quad (4.1)$$

In order to compare μ-values lower bounds by means of Low rank ODE's with the once obtained by GBA and LMI, we have collected the required data for transfer function matrices from [12].

5. COMPARISON OF μ-VALUE LOWER BOUNDS

In this section, we present the comparison of the numerical computation of μ-values lower bounds approximated by mussv and μ-values lower bounds computed by algorithm [16]. Each of the following 5-dimensional matrix is obtained from [12].

Case-I:

The matrix $M5$ is computed by using the MATLAB command $M = freqresp(sys, 2)$.

$$\begin{bmatrix} -.2179-.0136i & -2.0632-.2383i & -.2179-.0136i & -2.5790-.2979i & -2.8866-.1804i \\ .0008-.0125i & .0136-.2179i & .0008-.0125i & 0.1420-.2724i & .0103-.1649i \\ -.0066+.0058i & -.1158+.1021i & -.0066+.0058i & -0.1447+.1277i & -.0876+.0773i \\ .0016-.0249i & .0272-.4358i & .0016-.0249i & 0.0340-.5447i & .0206-.3299i \\ -.0235-.0015i & -.4112-.0257i & -.0235-.0015i & -0.5140-.0321i & -.5613-.0195i \end{bmatrix}$$

We take the uncertainty set of block diagonal matrices as:

$$\Delta_B = \{diag(\Delta_1) : \Delta_1 \in \mathbb{C}^{5,5}\}.$$

Using Matlab function mussv, we obtain the perturbation Δ with

$$\begin{bmatrix} -.0101+.0006i & .0002+.0009i & -.0004-.0004i & .0017i & -.0020+i \\ -.0990+.0112i & .0023+.0084i & -.0045-.0039i & .0013+.0168i & -.0195+.0014i \\ -.0101+.0006i & .0002+.0009i & -.0004-.0004i & 0.0017i & -.0020+.0000i \\ -.1238+.0144i & .0029+.0105i & -.0057-.0048i & .0017+.0210i & -.0244+.0018i \\ -.1355+.0085i & .0026+.0116i & -.0059-.0056i & .0006+.0229i & -.0266+.0006i \end{bmatrix},$$

while $\|\Delta\|_2 = 0.2180$. The upper bound is obtained as 4.5869 while the same lower bound is approximated as 4.5869.

The algorithm [16] computes the admissible perturbation $E^*\Delta^*$ with

$$\begin{bmatrix} -0.0458+0.0030i & 0.0009+0.0039i & -0.0020-0.0019i & 0.0003+0.0078i & -0.0090+0.0003i \\ -0.4568+0.0522i & 0.0112+0.0387i & -0.0211-0.0178i & 0.0067+0.0778i & -0.0898+0.0069i \\ -0.0458+0.0030i & 0.0009+0.0039i & -0.0020-0.0019i & 0.0003+0.0078i & -0.0090+0.0003i \\ -0.5696+0.0699i & 0.0144+0.0481i & -0.0266-0.0220i & 0.0092+0.0970i & -0.1120+0.0095i \\ -0.6173+0.0344i & 0.0120+0.0528i & -0.0270-0.0255i & 0.0029+0.1049i & -0.1211+0.0022i \end{bmatrix}$$

while $\|\Delta^*\|_2 = 1$. The lower bound is approximated as $\mu_{lower} = 4.5867$.

Case-II:

The matrix $M5$ computed by using the MATLAB command $M = freqresp(sys, 3)$.

$$\begin{bmatrix} -.2179-.0136i & -2.0632-.2383i & -.2179-.0136i & -2.5790-.2979i & -2.8866-.1804i \\ .0008-.0125i & .0136-.2179i & .0008-.0125i & .1420-.2724i & .0103-.1649i \\ -.0066+.0058i & -.1158+.1021i & -.0066+.0058i & -.1447+.1277i & -.0876+.0773i \\ .0016-.0249i & .0272-.4358i & .0016-.0249i & .0340-.5447i & .0206-.3299i \\ -.0235-.0015i & -.4112-.0257i & -.0235-.0015i & -.5140-.0321i & -.5613-.0195i \end{bmatrix}.$$

We take the uncertainty set of block diagonal matrices as:

$$\Delta_B = \{diag(\Delta_1, \Delta_2) : \Delta_1 \in \mathbb{C}^{3,3}, \Delta_2 \in \mathbb{C}^{2,2}\}.$$

Using Matlab function mussv, we obtain the perturbation Δ with

$$\begin{bmatrix} -.1067-.0290i & -.0116+.0198i & -.0016-.0089i & 0 & 0 \\ -.6439-.5091i & -.1369+.1018i & .0147-.0654i & 0 & 0 \\ -.1067-.0290i & -.0116+.0198i & -.0016-.0089i & 0 & 0 \\ 0 & 0 & 0 & -.3526+.2289i & -.1292-.1172i \\ 0 & 0 & 0 & -.3617+.5637i & -.2645-.0853i \end{bmatrix}.$$

We obtain an upper bound 1.1681 while the lower bound is approximated as 1.1680.

The algorithm [16] computes the admissible perturbation $E^*\Delta^*$ with

$$\begin{bmatrix} -.1241-.0342i & -.0136+.0227i & -.0018-.0103i & 0 & 0 \\ -.7563-.5901i & -.1584+.1178i & .0168-.0758i & 0 & 0 \\ -.1241-.0342i & -.0136+.0227i & -.0018-.0103i & 0 & 0 \\ 0 & 0 & 0 & -.4093+.2667i & -.1509-.1356i \\ 0 & 0 & 0 & -.4288+.6560i & -.3092-.1016i \end{bmatrix},$$

while $\|\Delta^*\|_2 = 1$. The lower bound is approximated as $\mu_{lower} = 1.1680$ which is same as the one approximated by mussv.

Case-III:

The matrix $M5$ computed by using the MATLAB command $M = freqresp(sys, 4)$.

$$\begin{bmatrix} -.1042+.0064i & -.0732+.1126i & -.1042+.0064i & -.0915+.1407i & -1.3802+.0852i \\ -.0074-.0031i & -.1289-.0548i & -.0074-.0031i & -.0361-.0685i & -.0976-.0415i \\ .0007+.0017i & .0123+.0306i & .0007+.0017i & .0154+.0383i & .0093+.0232i \\ -.0147-.0063i & -.2578-.1096i & -.0147-.0063i & -.3222-.1370i & -.1951-.0830i \\ -.0007+.0015i & -.0115+.0270i & -.0007+.0015i & -.0144+.0338i & -.2587+.0205i \end{bmatrix}.$$

We take the uncertainty set of block diagonal matrices as:

$$\Delta B = \{diag(\delta_1 I_2, \delta_2 I_2, \Delta_1) : \delta_1, \delta_2, \in \mathrm{R}, \Delta_1 \in \mathrm{C}^{1,1}\}.$$

Using Matlab function mussv, we obtain the perturbation Δ with

$$\begin{bmatrix} -2.9492 & 0 & 0 & 0 & 0 \\ 0 & -2.9492 & 0 & 0 & 0 \\ 0 & 0 & -2.0538 & 0 & 0 \\ 0 & 0 & 0 & -2.0538 & 0 \\ 0 & 0 & 0 & 0 & -2.9499+0.0064i \end{bmatrix},$$

while $\|\Delta^*\|_2 = 2.9499$. We obtain the upper bound 0.3449 while the lower bound is approximated as 0.3390.

The algorithm [16] computes the admissible perturbation $E^*\Delta^*$ with

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -.6965 & 0 & 0 \\ 0 & 0 & 0 & -.6965 & 0 \\ 0 & 0 & 0 & 0 & -1+.0023i \end{bmatrix},$$

while $\|\Delta^*\|_2 = 1$. The lower is approximated as $\mu_{lower} = 0.3390$ which is same as the one approximated by mussv.

Case-IV:

The matrix $M5$ computed by using the MATLAB command $M = freqresp(sys, 5)$.

$$\begin{bmatrix} -.1019+.0033i & -.0339+.0572i & -.1019+.0033i & -.0423+.0715i & -1.3504+.0433i \\ -.0071-.0021i & -.1236-.0369i & -.0071-.0021i & -.0295-.0461i & -.0936-.0279i \\ .0006+.0011i & .0108+.0201i & .0006+.0011i & .0136+.0251i & .0082+.0152i \\ -.0141-.0042i & -.2473-.0737i & -.0141-.0042i & -.3091-.0921i & -.1872-.0558i \\ -.0002+.0008i & -.0043+.0146i & -.0002+.0008i & -.0054+.0182i & -.2533+.0110i \end{bmatrix}.$$

We take the uncertainty set of block diagonal matrices as:

$$\Delta_B = \{diag(\delta_1 I_2, \delta_2 I_2, \delta_3 I_1) : \delta_1, \delta_2, \in \mathbb{C}, \delta_3 \in \mathbb{R}\}.$$

Using the Matlab function mussv, we obtain the perturbation Δ with

$$\begin{bmatrix} -2.6613+.1902i & 0 & 0 & 0 & 0 \\ 0 & -2.6613+.1902i & 0 & 0 & 0 \\ 0 & 0 & -2.6056+.5738i & 00 & \\ 0 & 0 & 0 & -2.6056+.5738i & 0 \\ 0 & 0 & 0 & 0 & -2.6681 \end{bmatrix},$$

while $\|\Delta^*\|_2 = 2.6681$. We have approximated an upper bound 0.3768 while the lower bound is approximated as 0.3748.

The algorithm [16] computes the admissible perturbation $E^*\Delta^*$ with

$$\begin{bmatrix} -.8083+.5888i & 0 & 0 & 0 & 0 \\ 0 & -.8083+.5888i & 0 & 0 & 0 \\ 0 & 0 & -.9448+.3276i & 00 & \\ 0 & 0 & 0 & -.9448+.3276i & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $\Delta^*\ 2 = 1$. In this case lower bound is approximated as $\mu_{lower} = 0.3768$, which is much better than the lower bound approximated by mussv.

Case-V:

The matrix M_5 computed by using the MATLAB command $M = freqresp(sys, 6)$.

$$\begin{bmatrix} -.1012+.0019i & -.0202+.0331i & -.1012+.0019i & -.0253+.0414i & -1.3400+.0251i \\ -.0068-.0016i & -.1196-.0275i & -.0068-.0016i & -.0245-.0344i & -.0905-.0208i \\ .0005+.0008i & .0092+.0147i & .0005+.0008i & .0115+.0184i & .0070+.0111i \\ -.0137-.0031i & -.2391-.0551i & -.0137-.0031i & -.2989-.0688i & -.1810-.0417i \\ -.0001+.0005i & -.0021+.0090i & -.0001+.0005i & -.0026+.0113i & -.2516+.0068i \end{bmatrix}.$$

We take the uncertainty set of block diagonal matrices as:

$$\Delta_B = \{diag(\Delta_1, \Delta_2, \delta_1 I_1) : \Delta_1, \Delta_2 \in \mathbb{C}^{2,2}, \delta_1 \in \mathbb{C}\}.$$

Using the Matlab function mussv, we obtain the perturbation Δ

$$\begin{bmatrix} -.4759-.0961i & -.0821+.2203i & 0 & 0 & 0 \\ -1.0461-1.6845i & -.8850+.3725i & 0 & 0 & 0 \\ 0 & 0 & -.0064-.0331i & -.2680+.3950i & 0 \\ 00 & .0861-.1305i & -2.1668+.4433i & 0 & \\ 0 & 0 & 0 & 0 & -2.2359-.3815i \end{bmatrix}$$

while $\|\Delta^*\|_2 = 2.2683$. Whave approximated the upper bound as 0.4409 while the same lower bound is obtained.

The algorithm [16] computes the admissible perturbation $E^*\Delta^*$ with

$$\begin{bmatrix} -.2108 - .0482i & -.0447 + .0952i & 0 & 0 & 0 \\ -.4926 - .7206i & -.3935 + .1594i & 0 & 0 & 0 \\ 0 & 0 & -.0021 - .0149i & -.1280 + .1696i & 0 \\ 0 & 0 & .0382 - .0573i & -.9558 + .1907i & 0 \\ 0 & 0 & 0 & 0 & -.9834 - .1813i \end{bmatrix},$$

while $\|\Delta^*\|_2 = 1$. In this case the obtained lower bound is $\mu_{lower} = 0.4409$, same as the one approximated by mussv.

6. CONCLUSION

In this article we have presented the approximation of μ-values for the family of matrices obtained from [12] for an anti-aliasing filter. The numerical experimentations show that how lower bounds of μ-value approximated by MATLAB function mussv and the one approximated by algorithm [16] are related to each other.

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