Publications Prepared for the Sigma Journal of Engineering and Natural Sciences
2019 International Conference on Applied Analysis and Mathematical Modeling
Special Issue was published by reviewing extended papers

Research Article

NONEXISTENCE AND GROWTH OF SOLUTIONS FOR A PARABOLIC p-LAPLACIAN SYSTEM

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Received: 19.04.2019 Revised: 10.06.2019 Accepted: 14.06.2019

ABSTRACT

In this paper, we investigate the initial boundary problem of a class of doubly nonlinear parabolic systems. We prove a nonexistence of global solutions and exponential growth of solution with negative initial energy.
Keywords: Blow up, exponential growth, parabolic equation, multiple nonlinearities.

1. INTRODUCTION

In this work, we are interested in the blow up and growth of solutions of the following parabolic system:

$$\begin{cases} u_t - div(|\nabla u|^{p-2}\nabla u) + |u|^{q-2}u_t = f_1(u,v), & x \in \Omega,\ t > 0, \\ v_t - div(|\nabla v|^{p-2}\nabla v) + |v|^{q-2}v_t = f_2(u,v), & x \in \Omega,\ t > 0, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega,\ t \geq 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x) & x \in \Omega, \end{cases} \tag{1}$$

where $p,\ q > 2$ are real numbers and Ω is a bounded domain in R^n $(n \geq 1)$ with smooth boundary $\partial\Omega$. $f_i(u,v)$ $(i = 1,2)$ will be given later.

In the case of $p = 2$, Pang and Qiao [1] considered

$$\begin{cases} u_t - \Delta u + |u|^{q-2}u_t = f_1(u,v), \\ v_t - \Delta v + |v|^{q-2}v_t = f_2(u,v), \end{cases} \tag{2}$$

where $q > 2$. They studied the blow up properties of the problem (2) with negative and positive initial energy.

Equation (2) without $|u|^{q-2}u_t$ and $|v|^{q-2}v_t$ term become the following problem

$$\begin{cases} u_t - \Delta u = f_1(u,v), \\ v_t - \Delta v = f_2(u,v). \end{cases} \tag{3}$$

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Problems like equation (3) not only is important from the theoretical viewpoint, but also is much interest in applied science. It appears naturally in the models of physics, chemistry, biology, ecology and so on (see [2-12]). In [13], the authors obtained the global existence solution, blow-up in finite time solu tion, and asymptotic behavior of solution in subcritical energy level and critical energy level, which are divided from potential well theory, respectively. Furthermore, they showed the sufficient conditions of global well posedness with supercritical energy level by combining with comparison principle and semigroup theory.

Recently, In [14] the author also investigated the problem (3). He studied global existence of the solutions by combining the energy method with the Faedo-Galerkin's procedure. Moreover, he discussed the asymptotic stability by using Nakao's technique. Finally he got blow up of solution when initial energy is negative.

The remaining part of this paper is organized as follows: In the next section, we present some notations and statement of assumptions. In section 3, the blow up of the solution is given. In section 4, the growth of solution is given.

2. PRELIMINARIES

In this section, we shall give some assumptions for the proof of our results. Let $\|.\|$, $\|.\|_p$ and $(u,v)=\int_\Omega u(x)v(x)\,dx$ denote the usual $L^2(\Omega)$ norm, $L^p(\Omega)$ norm and inner product of $L^2(\Omega)$, respectively. Throughout this paper, C is used to point out general positive constants.

For the numbers m and q, we suppose that

$$\begin{cases} 2<q<m\le \frac{2(n-1)}{n-2} \ \ if\ n>2, \\ 2<q<m\le +\infty\ \ if\ n=1,2. \end{cases} \tag{4}$$

Regarding the functions $f_1(u,v),\ f_2(u,v)\in C^1$ such that

$$f_1(u,v)=\frac{\partial F(u,v)}{\partial u},f_2(u,v)=\frac{\partial F(u,v)}{\partial v}$$

and

$$\begin{cases} k_0(|u|^m+|v|^m)\le F(u,v)\le k_1(|u|^m+|v|^m), \\ \quad uf_1(u,v)+vf_2(u,v)=(m+1)F(u,v) \end{cases} \tag{5}$$

where k_0,k_1 are positive constants.

Combining arguments of [15,12,16], $u(x,t),v(x,t)$ are called a solution of problem (1) on $\Omega\times[0,T)$ if

$$\begin{cases} u,v\in C(0,T;W_0^{1,p}(\Omega))\cap C^1\big(0,T;L^2(\Omega)\big), \\ \quad |u|^{q-2}u_t,\ \ |v|^{q-2}v_t\in L^2(\Omega\times[0,T)) \end{cases} \tag{6}$$

satisfying the initial condition $u(x,0)=u_0(x), v(x,0)=v_0(x)$ and

$$\int_0^t\int_\Omega\,[|\nabla u|^{p-2}\nabla u\nabla w+u_tw+|u|^{q-2}u_tw-f_1(u,v)w]\,dx\,ds=0, \tag{7}$$

$$\int_0^t\int_\Omega\,[|\nabla v|^{p-2}\nabla v\nabla w+v_tw+|v|^{q-2}v_tw-f_2(u,v)w]\,dx\,ds=0 \tag{8}$$

for all $w\in\ C(0,T;W_0^{1,p}(\Omega))$.

The energy functional associated with problem (1) is

$$E(t)=\frac{1}{p}\|\nabla u\|_p^p+\frac{1}{p}\|\nabla v\|_p^p-\int_\Omega\ F(u,v)dx, \tag{9}$$

where $u,v\in W_0^{1,p}(\Omega)$.

Lemma 1 Suppose that (4) and (5) hold. $E'(t)$ is noncreasing function $t>0$ and

$$E'(t) = -\|u_t\|^2 - \|v_t\|^2 - \int_\Omega |u|^{q-2}{u_t}^2 dx - \int_\Omega |v|^{q-2}{v_t}^2\, dx < 0. \tag{10}$$

Proof. Multiplying $Eq.(1)_1$ by u_t and $Eq.(1)_2$ by v_t and integrating over Ω, we obtain

$$\int_0^t E'(\tau)d\tau = -\left[\int_0^t (\|u_t\|^2 + \|v_t\|^2)d\tau + \int_0^t \int_\Omega |u|^{q-2}{u_t}^2 dx d\tau + \int_0^t \int_\Omega |v|^{q-2}{v_t}^2 dx d\tau\right],$$

$$E(t) - E(0) = -\left[\int_0^t (\|u_t\|^2 + \|v_t\|^2)d\tau + \int_0^t \int_\Omega |u|^{q-2}{u_t}^2 dx d\tau + \int_0^t \int_\Omega |v|^{q-2}{v_t}^2 dx d\tau\right]$$

for t>0.

3. BLOW UP OF SOLUTIONS

In this section, we deal with the blow up results of the solution for the problem (1).

Theorem Suppose that (4) holds, $u_0, v_0 \in W_0^{1,p}(\Omega)$ and u, v are local solution of the system (1) and $E(0) < 0$. Then, the solution of the system (1) blows up in finite time.

Proof. We set

$$H(t) = -E(t). \tag{11}$$

From (10) and (11), we have

$$H'(t) = -E'(t) \geq 0. \tag{12}$$

Since $E(0) < 0$, we get

$$H(0) = -E(0) > 0. \tag{13}$$

By the integrate (12), we get

$$0 < H(0) \leq H(t). \tag{14}$$

By using (11) and (9)

$$H(t) - \int_\Omega F(u,v)dx = -\frac{1}{p}\left(\|\nabla u\|_p^p + \|\nabla v\|_p^p\right) < 0. \tag{15}$$

Then, by using (5), we have

$$0 < H(0) \leq H(t) \leq \int_\Omega F(u,v)dx \leq k_1(\|u\|_m^m + \|v\|_m^m). \tag{16}$$

Then, we define

$$\Psi(t) = H^{1-\sigma}(t) + \frac{\varepsilon}{2}\|u\|^2 + \frac{\varepsilon}{2}\|v\|^2, \tag{17}$$

where $\varepsilon > 0$ small to be chosen later and $0 \leq \sigma \leq (m-2)/m$ since $2 < m$. By differentiating (17) and by using (1) and (5), we get

$$\begin{aligned}\Psi'(t) &= (1-\sigma)H^{-\sigma}(t)H'(t) + \varepsilon\int_\Omega u u_t dx + \varepsilon\int_\Omega v v_t dx\\ &= (1-\sigma)H^{-\sigma}(t)H'(t) - \varepsilon\|\nabla u\|_p^p - \varepsilon\|\nabla v\|_p^p\\ &\quad +\varepsilon(m+1)\int_\Omega F(u,v)dx - \varepsilon\int_\Omega |u|^{q-2}u u_t dx - \varepsilon\int_\Omega |v|^{q-2}v v_t dx.\end{aligned} \tag{18}$$

In order to estimate the last terms in (18), we use the following Young's inequality

$$ab \leq \delta^{-1}a^2 + \delta b^2,$$

so we have

$$\begin{aligned}\int_\Omega |u|^{q-2}u u_t dx &\leq \int_\Omega |u|^{\frac{q-2}{2}} u_t |u|^{\frac{q-2}{2}} dx\\ &\leq \delta^{-1}\int_\Omega |u|^{q-2}{u_t}^2 dx + \delta\int_\Omega |u|^q\, dx.\end{aligned}$$

In the same way, we get

$$\int_\Omega |v|^{q-2} v v_t dx \le \delta^{-1} \int_\Omega |v|^{q-2} {v_t}^2 dx + \delta \int_\Omega |v|^q \, dx,$$

where δ are constant depending on the time t and specified later. So, (18) becomes

$$\begin{aligned} \Psi'(t) \ge (1-\sigma) H^{-\sigma}(t) H'(t) - \varepsilon \|\nabla u\|_p^p - \varepsilon \|\nabla v\|_p^p \\ + \varepsilon(m+1)(\|u\|_m^m + \|v\|_m^m) - \varepsilon\delta\left(\|u\|_q^q + \|v\|_q^q\right) \\ - \varepsilon\delta^{-1} \int_\Omega |u|^{q-2} {u_t}^2 dx - \varepsilon\delta^{-1} \int_\Omega |v|^{q-2} {v_t}^2 dx. \end{aligned} \tag{19}$$

From the definition $H(t)$, it follows that

$$\|\nabla u\|_p^p + \|\nabla v\|_p^p = -pH(t) + p \int_\Omega F(u,v) dx,$$

$$\begin{aligned} \Psi'(t) \ge (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon(m+1-p)(\|u\|_m^m + \|v\|_m^m) \\ - \varepsilon\delta\left(\|u\|_q^q + \|v\|_q^q\right) + \varepsilon p H(t) \\ - \varepsilon\delta^{-1} \int_\Omega |u|^{q-2} {u_t}^2 dx - \varepsilon\delta^{-1} \int_\Omega |v|^{q-2} {v_t}^2 dx. \end{aligned} \tag{20}$$

As the embedding $L^m \hookrightarrow L^q \hookrightarrow L^2, m > q > 2$, we have

$$\begin{cases} \|u\|_q^q \le C \|u\|_m^q \le C(\|u\|_m^m)^{\frac{q}{m}}, \\ \|v\|_q^q \le C \|v\|_m^q \le C(\|v\|_m^m)^{\frac{q}{m}}. \end{cases} \tag{21}$$

Since $0 < \frac{q}{m} < 1$, now applying the following inequality

$$x^l \le (x+1) \le \left(1 + \frac{1}{z}\right)(x+z),$$

which holds for all $x \ge 0$, $0 \le l \le 1$, $z > 0$, especially, taking $x = \|u\|_m^m$, $l = \frac{q}{m}$, $z = H(0)$, we get

$$C(\|u\|_m^m)^{\frac{q}{m}} \le \left(1 + \frac{1}{H(0)}\right)\left(\|u\|_m^m + H(0)\right),$$

Similarly

$$C(\|v\|_m^m)^{\frac{q}{m}} \le \left(1 + \frac{1}{H(0)}\right)\left(\|v\|_m^m + H(0)\right).$$

Then, from (16) and (21), we get

$$\begin{aligned} \|u\|_q^q + \|v\|_q^q \le C\left(\|u\|_m^q + \|v\|_m^q\right) \\ \le C_1(\|u\|_m^m + \|u\|_m^m). \end{aligned} \tag{22}$$

Insert (22) into (20), it follows that

$$\begin{aligned} \Psi'(t) \ge (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon p H(t) + \varepsilon c'(\|u\|_m^m + \|v\|_m^m) \\ - \varepsilon\delta^{-1} \int_\Omega |u|^{q-2} {u_t}^2 dx - \varepsilon\delta^{-1} \int_\Omega |v|^{q-2} {v_t}^2 dx, \end{aligned} \tag{23}$$

where we pick δ small enough such that $c' = m + 1 - p - C_1\delta > 0$ and taking $\delta^{-1} = kH^{-\sigma}(t)$ (23) follows that

$$\begin{aligned} \Psi'(t) \ge (1-\sigma-k\varepsilon) H^{-\sigma}(t) H'(t) + \varepsilon p H(t) + \varepsilon c'(\|u\|_m^m + \|v\|_m^m) \\ \ge \beta(H(t) + \|u\|_m^m + \|v\|_m^m), \end{aligned} \tag{24}$$

where $\beta = min\{\varepsilon p, \ \varepsilon c'\}$ and we pick ε small enough such that $1 - \sigma - k\varepsilon \ge 0$.

We now estimate $\Psi^{\frac{1}{1-\sigma}}(t)$. From definition of $\Psi(t)$

$$\Psi^{\frac{1}{1-\sigma}}(t) = \left(H^{1-\sigma}(t) + \frac{\varepsilon}{2}\|u\|^2 + \frac{\varepsilon}{2}\|v\|^2\right)^{\frac{1}{1-\sigma}}. \tag{25}$$

As the embedding $L^m \hookrightarrow L^2,\ m > 2$, we have

$$\Psi^{\frac{1}{1-\sigma}}(t) \le C\left(H(t) + \|u\|_m^{2/1-\sigma} + \|v\|_m^{2/1-\sigma}\right). \tag{26}$$

Now, by the inequality $x^l \le (x+1) \le \left(1+\frac{1}{z}\right)(x+z)$ for $x = \|u\|_m^m$, $l = 2/m(1-\sigma) < 1$, since $\sigma < (m-2)/m,\ z = H(0)$, we get

$$\begin{aligned}\|u\|_m^{2/1-\sigma} &\le (\|u\|_m^m)^{2/m(1-\sigma)} \\ &\le \left(1+\frac{1}{H(0)}\right)\left(\|u\|_m^m + H(0)\right) \\ &\le C\|u\|_m^m.\end{aligned} \tag{27}$$

In the same way, we get

$$\|v\|_m^{2/1-\sigma} \le C\|v\|_m^m. \tag{28}$$

Therefore, (26) becomes that

$$\Psi^{\frac{1}{1-\sigma}}(t) \le C(H(t) + \|u\|_m^m + \|v\|_m^m). \tag{29}$$

By associatining of (24) and (29) we reach

$$\Psi'(t) \ge \xi \Psi^{\frac{1}{1-\sigma}}(t), \tag{30}$$

where $\xi > 0$ is a constant. A simple integration (30) from 0 to t yields that

$$\Psi^{\frac{\sigma}{1-\sigma}}(t) \ge \frac{1}{\Psi^{-\frac{\sigma}{1-\sigma}}(0) - \frac{\xi\sigma t}{1-\sigma}},$$

which implies that the solution blows up in a finite time T^*, with

$$T^* \le \frac{1-\sigma}{\xi\sigma\Psi^{\frac{\sigma}{1-\sigma}}(0)}.$$

4. EXPONENTIAL GROWTH OF SOLUTIONS

In this section, we state and prove exponential growth result.

Theorem Suppose that (4) holds, $u_0, v_0 \in W_0^{1,p}(\Omega)$ and $E(0) < 0$. Then, the solution of the system (1) grows exponentially.

Proof. Let us define the functional

$$\Phi(t) = H(t) + \frac{\varepsilon}{2}\|u\|^2 + \frac{\varepsilon}{2}\|v\|^2, \tag{31}$$

where $H(t) = -E(t)$. By differentiating (31) and using Eq.(1), we get

$$\begin{aligned}\Phi'(t) &= H'(t) + \varepsilon\left(\int_\Omega u u_t dx + \int_\Omega v v_t dx\right) \\ &= \|u_t\|^2 + \|v_t\|^2 - \varepsilon\|\nabla u\|_p^p - \varepsilon\|\nabla v\|_p^p + \varepsilon(m+1)\int_\Omega [u f_1(u,v) + v f_2(u,v)]dx \\ &\quad + \int_\Omega |u|^{q-2} {u_t}^2 dx + \int_\Omega |v|^{q-2} {v_t}^2 dx - \varepsilon\int_\Omega |u|^{q-2} u u_t dx - \varepsilon\int_\Omega |v|^{q-2} v v_t dx \\ &= \|u_t\|^2 + \|v_t\|^2 - \varepsilon\|\nabla u\|_p^p - \varepsilon\|\nabla v\|_p^p + \varepsilon(m+1)\int_\Omega F(u,v)dx \\ &\quad + \int_\Omega |u|^{q-2} {u_t}^2 dx + \int_\Omega |v|^{q-2} {v_t}^2 dx - \varepsilon\int_\Omega |u|^{q-2} u u_t dx \\ &\quad - \varepsilon\int_\Omega |v|^{q-2} v v_t dx.\end{aligned} \tag{32}$$

In order to estimate the last two terms in the right-hand side of (32), we use the following Young's inequality,

$$ab \le \delta^{-1}a^2 + \delta b^2,$$

so we have

$$\begin{aligned}\int_\Omega |u|^{q-2}uu_t dx &\le \int_\Omega |u|^{\frac{q-2}{2}}u_t|u|^{\frac{q-2}{2}}dx\\ &\le \delta^{-1}\int_\Omega |u|^{q-2}{u_t}^2dx+\delta\int_\Omega |u|^q\,dx.\end{aligned}$$

Similarly,

$$\int_\Omega |v|^{q-2}vv_t dx \le \delta^{-1}\int_\Omega |v|^{q-2}{v_t}^2dx+\delta\int_\Omega |v|^q\,dx.$$

Then, (32) becomes

$$\begin{aligned}\Phi'(t) \ge \|u_t\|^2+\|v_t\|^2-\ \varepsilon\|\nabla u\|_p^p-\varepsilon\|\nabla v\|_p^p+\varepsilon(m+1)(\|u\|_m^m+\|v\|_m^m)\\ -\varepsilon\delta\big(\|u\|_q^q+\|v\|_q^q\big)+(1-\varepsilon\delta^{-1})\int_\Omega |u|^{q-2}{u_t}^2dx\\ +(1-\varepsilon\delta^{-1})\int_\Omega |v|^{q-2}{v_t}^2dx. \qquad (33)\end{aligned}$$

By using follows equality that

$$-\|\nabla u\|_p^p-\|\nabla v\|_p^p=pH(t)-p\int_\Omega\ F(u,v)dx.$$

Hence, (33) becomes

$$\begin{aligned}\Phi'(t) \ge \varepsilon pH(t)+\|u_t\|^2+\|v_t\|^2+\varepsilon(m+1-p)(\|u\|_m^m+\|v\|_m^m)\\ -\varepsilon\delta\big(\|u\|_q^q+\|v\|_q^q\big)+(1-\varepsilon\delta^{-1})\int_\Omega |u|^{q-2}{u_t}^2dx\\ +(1-\varepsilon\delta^{-1})\int_\Omega |v|^{q-2}{v_t}^2dx. \qquad (34)\end{aligned}$$

Then, from (22) we obtain

$$\begin{aligned}\Phi'(t) \ge \varepsilon pH(t)+\|u_t\|^2+\|v_t\|^2+\varepsilon a_1(\|u\|_m^m+\|v\|_m^m)\\ +(1-\varepsilon\delta^{-1})\int_\Omega |u|^{q-2}{u_t}^2dx+(1-\varepsilon\delta^{-1})\int_\Omega |v|^{q-2}{v_t}^2dx,\end{aligned}$$

where δ small enough such that $a_1=m+1-p-\delta C_1>0$ and taking ε and δ small enough such that $1-\varepsilon\delta^{-1}>0$, then

$$\Phi'(t)\ge C(H(t)+\|u_t\|^2+\|v_t\|^2+\|u\|_m^m+\|v\|_m^m). \qquad (35)$$

On the other hand, by definition of $\Phi(t)$ and Poincare's inequality, we get

$$\begin{aligned}\Phi(t) &= H(t)+\frac{\varepsilon}{2}\|u\|^2+\frac{\varepsilon}{2}\|v\|^2\\ &\le C(H(t)+\|\nabla u\|^2+\|\nabla v\|^2).\end{aligned}$$

Now, we estimate

$$\begin{aligned}\|\nabla u\|^2 &\le C\|\nabla u\|_p^2\\ &= C\big(\|\nabla u\|_p^p\big)^{\frac{2}{p}}\\ &\le \left(1+\frac{1}{H(0)}\right)\left(\|\nabla u\|_p^p+H(0)\right)\\ &\le C\left(\|\nabla u\|_p^p+H(t)\right). \qquad (36)\end{aligned}$$

Similarly,

$$\|\nabla v\|^2\le C\left(\|\nabla v\|_p^p+H(t)\right).$$

So we have

$$\Phi(t)\le C\big(H(t)+\|\nabla u\|_p^p+\|\nabla v\|_p^p\big).$$

From definition of $H(t)$, we get

$$\Phi(t)\le C(H(t)+\|u\|_m^m+\|v\|_m^m)$$

$$\leq C(H(t)+\|u\|_m^m+\|v\|_m^m+\|u_t\|^2+\|v_t\|^2). \quad (37)$$

From (35) and (37), we arrive at

$$\Phi'(t) \geq r\Phi(t), \quad (38)$$

where r is a positive constant.

Integration of (38) over $(0,t)$ gives us

$$\Phi(t) \geq \Phi(0)\exp(rt).$$

From (37) and (16), we get

$$\Phi(t) \leq H(t) \leq \|u\|_m^m+\|v\|_m^m.$$

Consequently, we show that the solution in the L_m-norm growths exponentially.

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