



Research Article / Araştırma Makalesi

**FUZZY LOGIC CONTROLLER DESIGN BASED ON SUGENO INFERENCE
METHOD FOR NONLINEAR INVERTED PENDULUM DYNAMICAL
SYSTEM**

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ABSTRACT

Fuzzy logic is studied in many multidisciplinary areas such as economy, business and engineering. The design of a fuzzy controller does not require a complicated mathematical model. It only requires a set of rules to form the rule table. Basically all of the rules in the rule table are based on the expert's experience which actually comes from his/her understanding operation of the system. In particular, fuzzy logic controller can be used to control for a nonlinear systems, using rules rather than mathematical model. The sugeno fuzzy inference method provides a more systematic approach to the design of fuzzy logic controller. It works better dynamic performance in multi-state variables (three or more variables). In this study, the aim of control system for the four state variables (angular position of inverted pendulum, angular velocity of inverted pendulum, cart position and cart velocity) is to simultaneously balance the inverted pendulum and place the cart in a desired position with fuzzy logic controller that contains sugeno inference method.

Keywords: Sugeno inference method, fuzzy logic controller, inverted pendulum.

**DOĞRUSAL OLMAYAN TERS SARKAÇ DİNAMİK SİSTEMİ İÇİN SUGENO ÇIKARIM
METODU TABANLI BULANIK MANTIK DENETLEYİCİ TASARIMI**

ÖZ

Bulanık mantık ekonomi, ticaret, mühendislik gibi pek çok disiplinler arası alanda çalışılmaktadır. Bulanık denetleyici tasarımı karmaşık bir matematiksel model gerektirmez. Sadece kural tablosu oluşturmak için bir dizi kural gerektirir. Temelde kural tablosundaki tüm kurallar aslında sistemin çalışmasını bilen uzmanın deneyimine dayanmaktadır. Özellikle, bulanık mantık denetleyici matematiksel model yerine kurallar kullanarak doğrusal olmayan sistemlerin denetimi için kullanılabilir. Sugeno bulanık çıkarım yöntemi, bulanık mantık denetleyici tasarımı için daha sistematik bir yaklaşım sağlar. Çoklu durum değişkeni olan (üç veya daha fazla değişken) sistemlerde daha iyi dinamik performans gösterir. Bu çalışmada denetim sisteminin amacı, sugeno çıkarım metodu bulanık denetleyici ile dört durum değişkeni (ters sarkacın açısal konumu, ters sarkacın açısal hızı, arabanın konumu, arabanın hızı) için aynı anda ters sarkacı dengelemek ve arabayı istenen konuma getirmektir.

Anahtar Sözcükler: Sugeno çıkarım metodu, bulanık mantık denetleyici, ters sarkaç.

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1. INTRODUCTION

In recent years, fuzzy logic based modeling and control methods have become increasingly used applications both in industry and in the academic field. In these type methods, unlike the conventional control system, appropriate control signals are generated without the need for mathematical models of the system. In this generation process, the control algorithm is acting just like a human expert. Control applications of dynamic systems are difficult to realize; because, dynamic systems have usually nonlinear structure. Fuzzy logic controller based on nonlinear transformation capabilities have being able to be successful in the control of these like nonlinear systems. The performance of these controllers can be increased by improving the methods used in the design stage [1]. In this study, in order to demonstrate the success of the sugeno inference method based fuzzy control in the nonlinear systems, the control of four state variables (the angular position of an inverted pendulum, angular velocity of inverted pendulum, cart position and cart velocity) have been realized.

2. THE INVERTED PENDULUM SYSTEM

The inverted pendulum is a classical control problem. It is a naturally unstable system that is fairly easy to construct. In addition to being unstable, it is also highly nonlinear. The dynamic behavior of this system is completely described by the position and velocity of the cart and the angular position and velocity of the inverted pendulum. As it can be seen figure 1, inverted pendulum system has its length l , its mass m intensive accepted bar endpoints, M mounting a vehicle at point P on the asteroid belt. The angle (Θ) with vertical position rod's passing through the point P and the distance (x) from a reference point on the horizontal axis are represented Θ and x . Rod movement is constrained to the xy plane mounted vehicle able to move only along the x axis [2]. Rod's mass center of gravity(x,y) coordinate plane equation (1) and equation (2) it can be expressed as below:

$$x_G = x + l \sin \theta \quad (1)$$

$$y_G = l \cos \theta \quad (2)$$

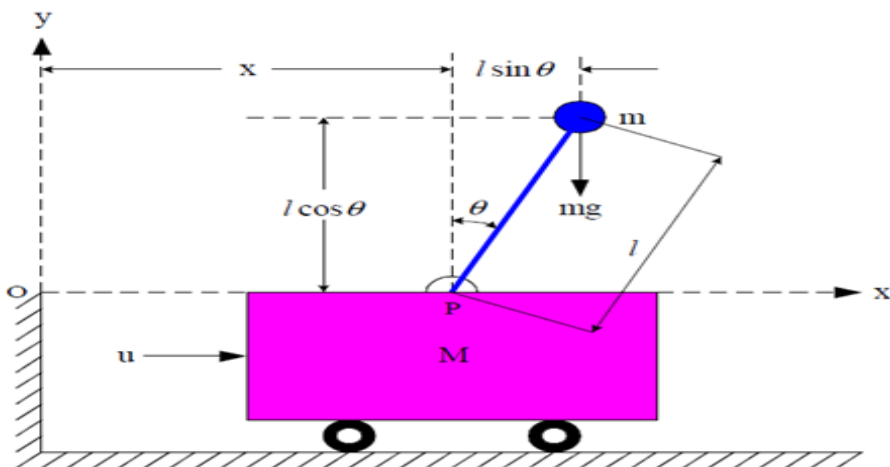


Figure 1. Inverted Pendulum System

When Newton's second law of motion is applied in the x direction, in equation (3) differential equation is obtained.

$$M \frac{d^2 x}{dt^2} + m \frac{d^2 x_G}{dt^2} = u \tag{3}$$

If differential equation in equation (1) is substituted equation (3), equation (4) is obtained.

$$(M+m)\ddot{x} - ml(\sin \theta)\ddot{\theta}^2 + ml(\cos \theta)\ddot{\theta} = u \tag{4}$$

Secondly, applied to the point P m is the mass of Newton's second law of motion around differential equation in equation (5) is obtained.

$$m \frac{d^2 x_G}{dt^2} l \cos \theta - m \frac{d^2 y_G}{dt^2} l \sin \theta = mgl \sin \theta \tag{5}$$

Equation (1) and equation (2) above if instead of written statements, equation(6) is obtained.

$$m \ddot{x} \cos \theta + ml \ddot{\theta} = mg \sin \theta \tag{6}$$

The value of the inverted pendulum system are given in Table 1.

Table 1. Inverted pendulum system values

Definition	Symbol	Value	Unit
Mass of cart	M	3	kg
Mass of inverted pendulum	m	0.5	kg
Length of inverted pendulum rod	l	0.5	m
Acceleration of gravity	g	9.8	m/sn ²

3. FUZZY LOGIC CONTROL BASED ON SUGENO INFERENCE METHOD

Almost, none of the systems are linear in real world. Fuzzy logic control method has emerged as an alternative approach among nonlinear control methods [3]. Fuzzy logic controller based on general structure consists of four basic components as fuzzification, fuzzy inference, knowledge base and defuzzification [4]. Experience of experts in fuzzy logic controller design is very important. Linguistic variables belonging to the system are characterized by membership functions. Expert views on the system is very important when creating rule base[5]. The purpose of creating the rule base, (error) and (exchange error) is to zero. Thus, the control signal would be near zero.[6-7]. Defuzzification in the sugeno model is simple to implement, since the result of each rule is already crisp and the total result is determined by the weight and sum of the result of each rule..Sugeno model also deals with highly nonlinear plants where control actions can be locally described by linear or nonlinear control consequences. In the sugeno model for the four state variables inverted pendulum problem, there are four inputs and each input have five linguistic variables. The total number of rules used for the problem are 50 rules. The mamdani model used more rules for the two state variables problem than the Sugeno model did or a problem of four state variables. From these results, we can conclude that the Mamdani model works well with a two state variable system, but if the system is multi state variables (three or

higher) the Mamdani model will be complicated. In that case the sugeno model is better than mamdani to use. The design of most fuzzy controllers has been based on human experience. When the plant needing to be control is complex, the engineer can not always specify linguistically what type of action should be taken; in this situation the sugeno model should be used. Although the Mamdani model has the advantage that no formal mathematical models are needed and dealing system uncertainties, the design of the mamdani model is still based on trial and error procedures. A major criticism of conventional fuzzy logic controller design (Mamdani model) is that there is no way to analytically guarantee stability. On the other hand, sugeno model can analytically guarantee stability [8]. The only difference in the mamdani and sugeno fuzzy inference system; the output membership functions in the sugeno fuzzy inference system as shown in figure 6 is linear [9]. Sugeno inference method output is illustrated in figure 2.

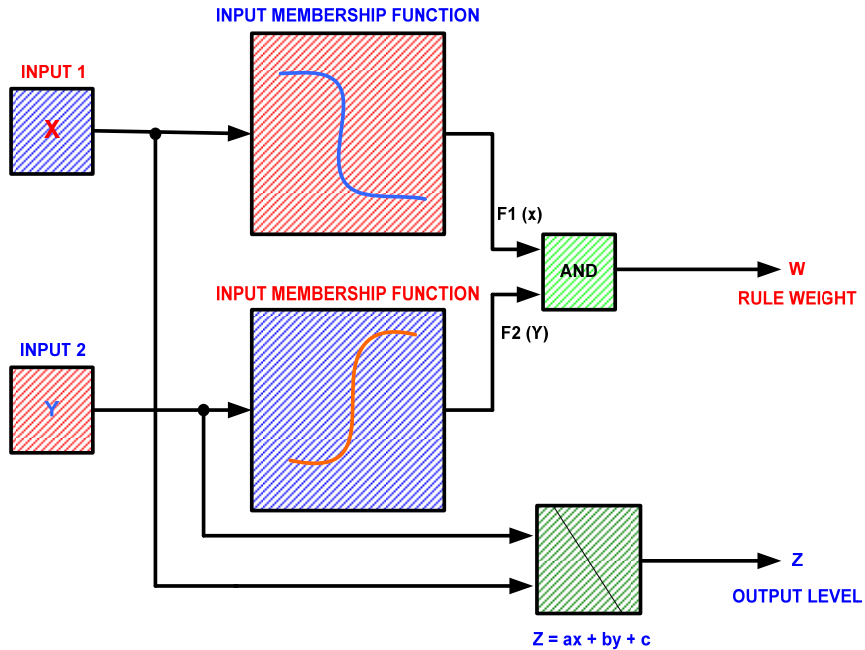


Figure 2. Sugeno inference method output

Mathematical expression that created for sugeno inference method output membership functions is given in equation 7.

$$z = a\theta + b\dot{\theta} + cx + d\dot{x} + r \quad (7)$$

Input and output variables in fuzzy logic controller blur process is converted into a symbolic expression. Linguistic variables used in the designed controller including NB (Negative Big), NS (Negative Small), ZE (Zero), PS (Positive Small), PB (Positive Big) five variables were used. The preference of membership functions for each input supplied to the system is entirely gratuitous. There can be triangle, trapezoid, sinusoid, cauchy, bell, sigmoid, gaussian types. Sigmoidal and gaussian-type membership functions are used for each input to the system. Fuzzy logic controller based on sugeno inference method four state variables (angular position of inverted pendulum, angular velocity of inverted pendulum, cart position, cart velocity) was taken as input.

The membership functions for angular position of inverted pendulum are shown in figure 3.

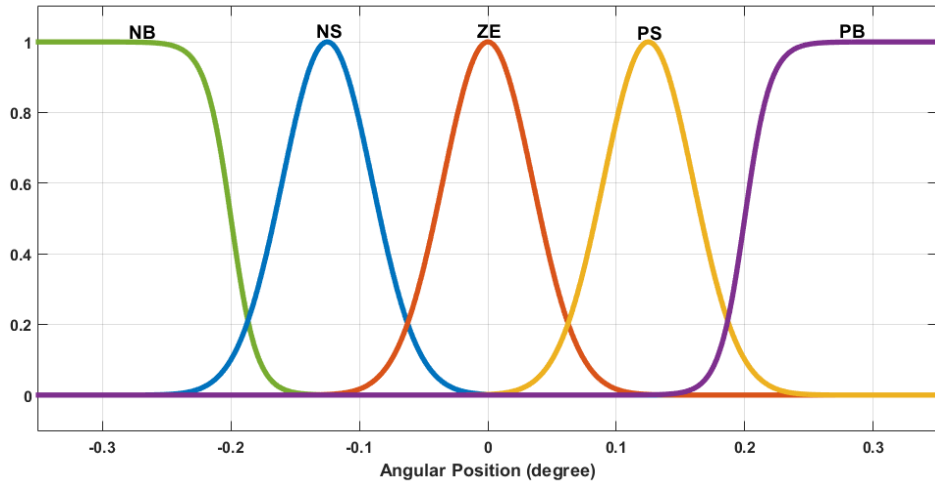


Figure 3. The membership functions for angular position of inverted pendulum

The membership functions for angular velocity of inverted pendulum are shown in figure 4.

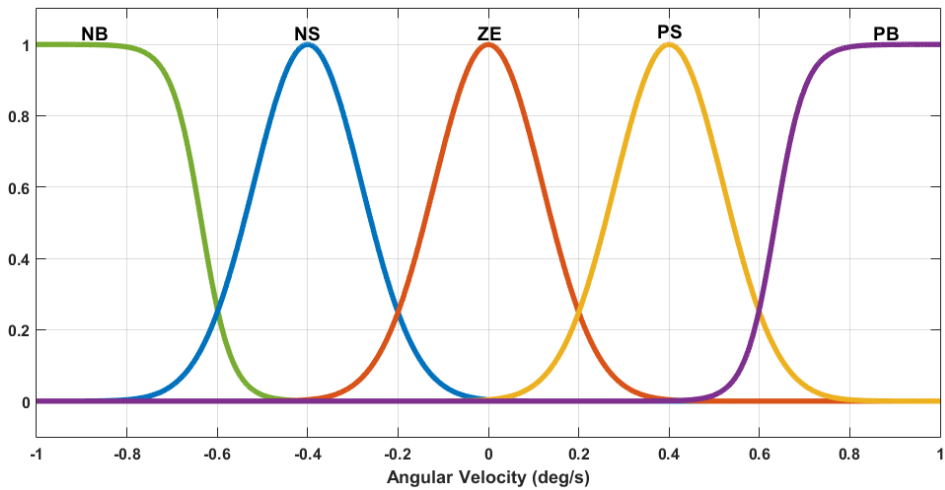


Figure 4. The membership functions for angular velocity of inverted pendulum

The membership functions for cart position are shown in figure 5.

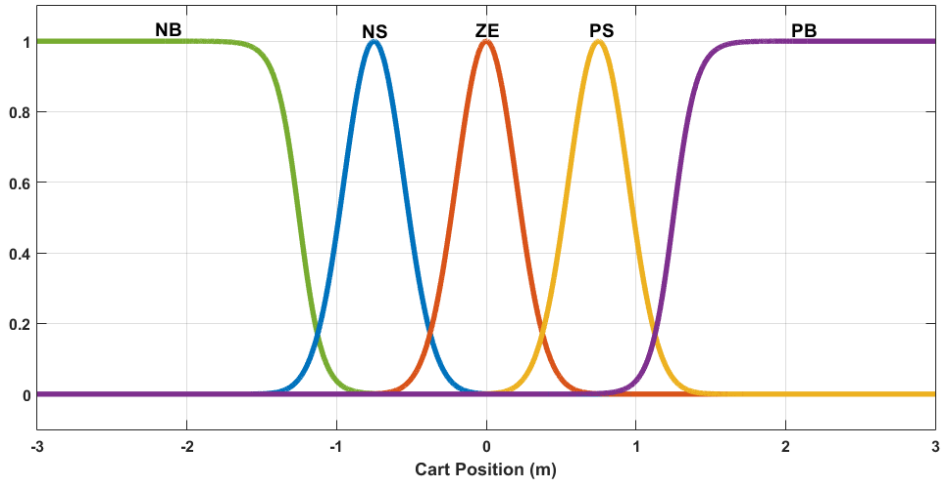


Figure 5. The membership functions for cart position

The membership functions for cart velocity are shown in figure 6.

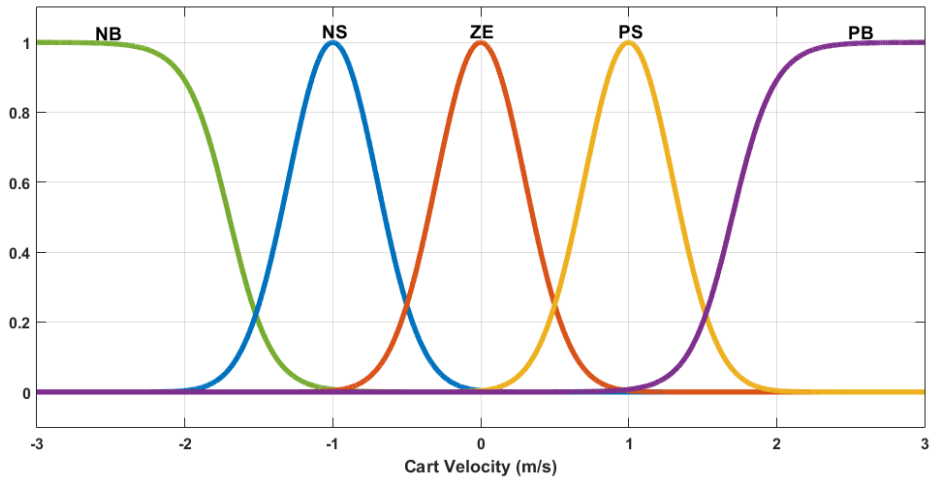


Figure 6. The membership functions for cart velocity

The rule table designed for angular position and angular velocity of the inverted pendulum are shown in table 1.

Table 1. The rule table designed for angular position and angular velocity of the inverted pendulum

u		de(θ)				
		NB	NS	ZE	PS	PB
e(θ)	NB	BS	BS	MS	MS	SS
	NS	BS	MS	MS	SS	SS
	ZE	MS	SS	SS	SS	MS
	PS	SS	SS	MS	MS	BS
	PB	SS	MS	MS	BS	BS

Sugeno outputs created for rules above are given in equation 8.

$$BS = [200 \quad 70 \quad 7 \quad 15 \quad 0]$$

$$MS = [180 \quad 60 \quad 7 \quad 15 \quad 0] \tag{8}$$

$$SS = [155 \quad 55 \quad 7 \quad 15 \quad 0]$$

The rule table designed for cart position and cart velocity of the inverted pendulum are shown in table 2.

Table 2. The rule table designed for cart position and cart velocity.

u		de(x)				
		NB	NS	ZE	PS	PB
e(x)	NB	SB	SB	SM	SM	SS
	NS	SB	SM	SM	SS	SS
	ZE	SM	SS	SS	SS	SM
	PS	SS	SS	SM	SM	SB
	PB	SS	SM	SM	SB	SB

Sugeno outputs created for rules above are given in equation 9.

$$SB = [155 \quad 55 \quad 20 \quad 40 \quad 0]$$

$$SM = [155 \quad 55 \quad 12 \quad 20 \quad 0] \tag{9}$$

$$SS = [155 \quad 55 \quad 7 \quad 15 \quad 0]$$

4. SIMULATION RESULTS

Inverted pendulum system is conventional materials known in the literature to test the dynamic nonlinear controller types. The problem that will examine aims using a control input four output state variables (angular position of inverted pendulum, angular velocity of inverted pendulum, cart position and cart velocity) around unstable equilibrium point fuzzy logic control with sugeno inference method. The depth of the problem constitutes simultaneous control of the four state variables. In the fuzzy control approach using sugeno inference method fifty rules with two sigmoidal membership functions (NB & PB) and three gaussian membership functions (NS & ZE & PS) for each input variable have been used. Initial angular position of the inverted pendulum has been taken 8.5 (degree) and reference angular position adjusts to 0 degree. Initial cart position of the cart has been taken 0.2 m and desired cart position has been set to 1 (m). The control model for fuzzy control of inverted pendulum system using sugeno inference method for nonlinear plant model is shown in figure 7.

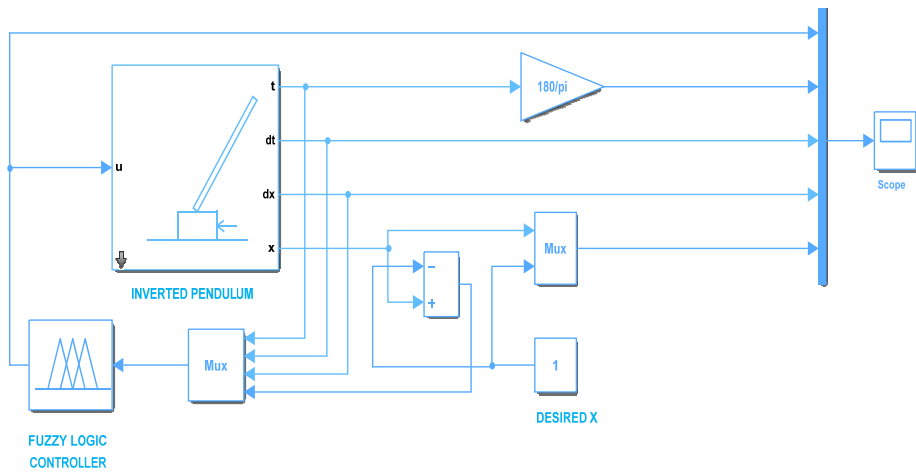


Figure 7. Matlab model of inverted pendulum system using sugeno inference method

The simulation results of fuzzy control using sugeno inference method are shown in figures 8-11 respectively. Here also all the state variables (angular position of inverted pendulum, angular velocity of inverted pendulum, cart position and cart velocity), and control signal has been drawn.

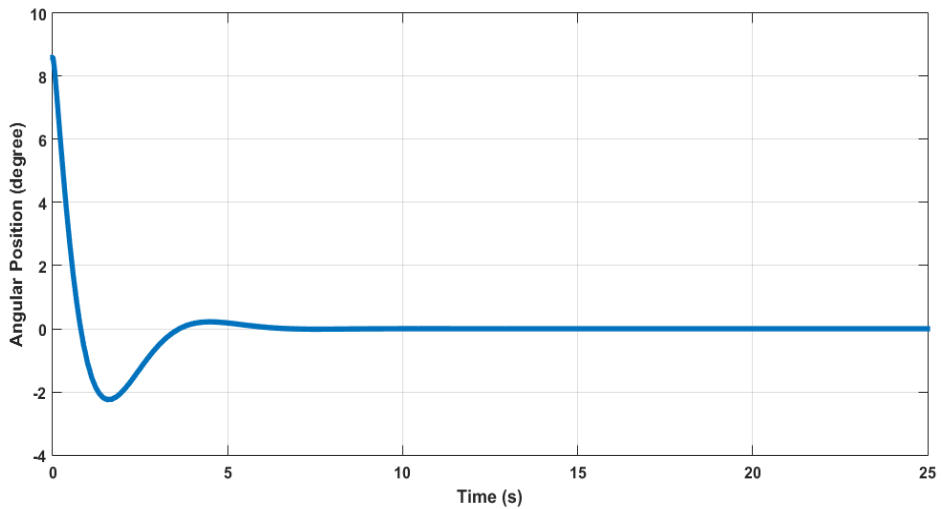


Figure 8. The response of fuzzy logic controller for angular position(θ) control of inverted pendulum

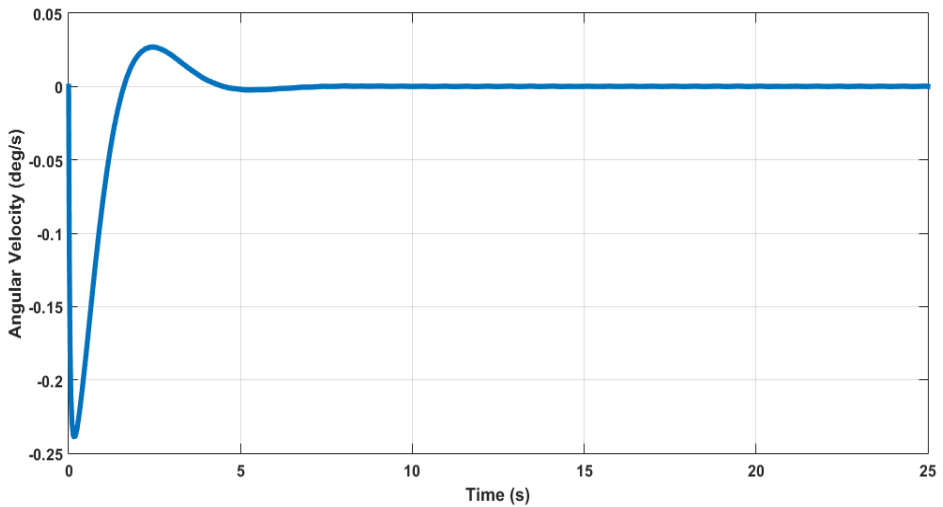


Figure 9. The response of fuzzy logic controller for angular velocity($\dot{\theta}$) control of inverted pendulum

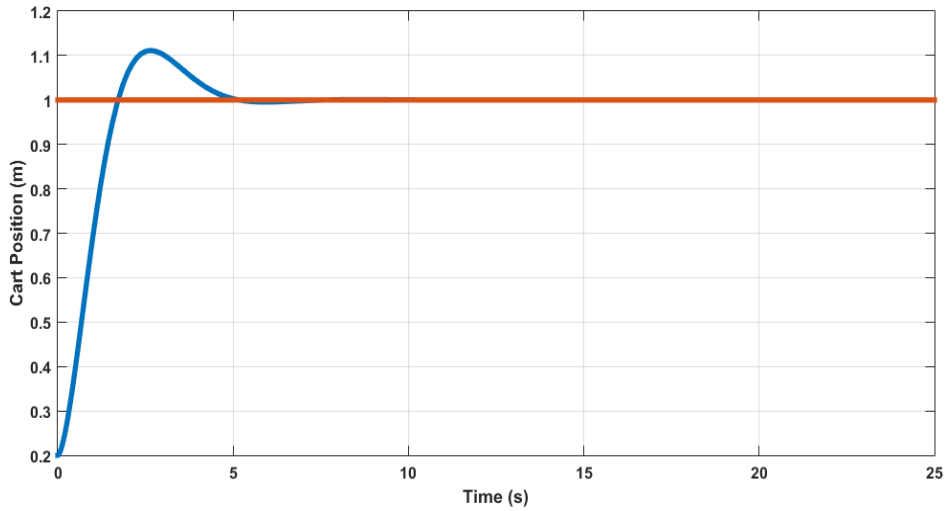


Figure 10. The response of fuzzy logic controller for cart position(x) control

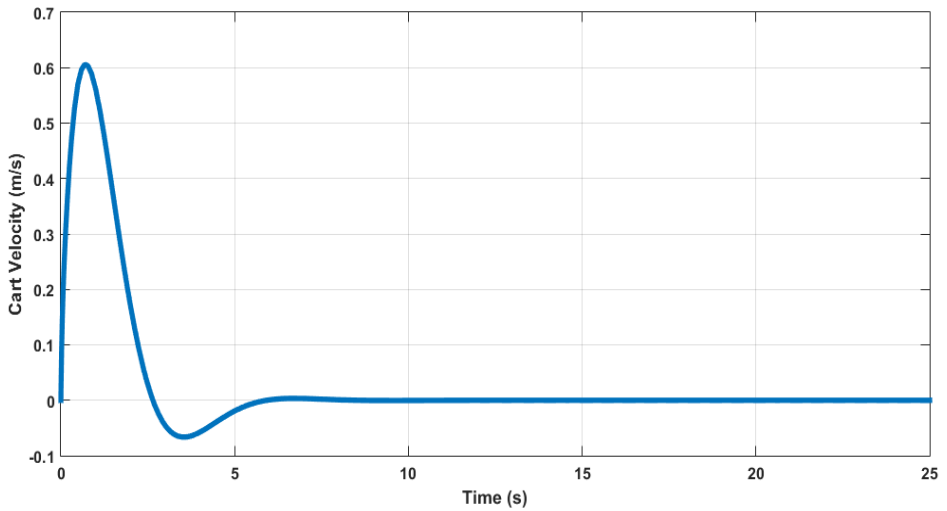


Figure 11. The response of fuzzy logic controller for cart velocity (\dot{x}) control

Control signal applied to the inverted pendulum system is given figure 12.

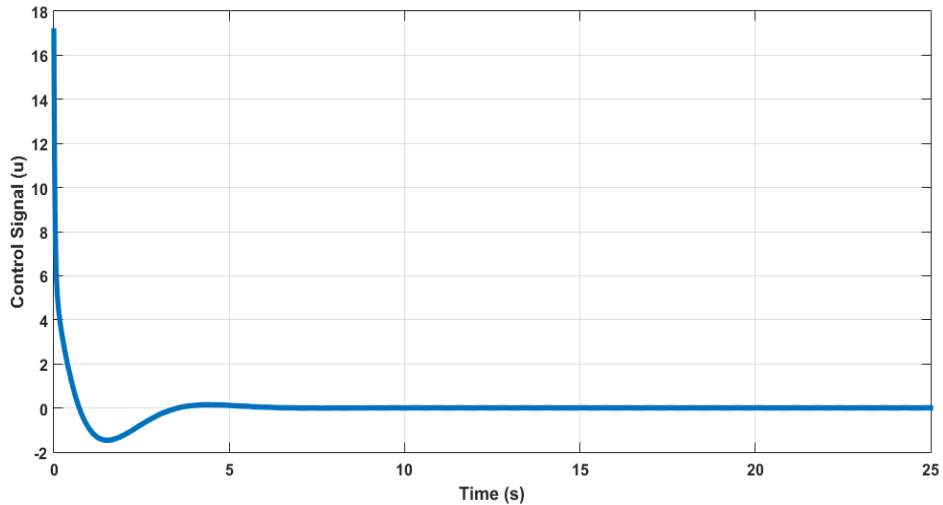


Figure 12. Control signal applied to the inverted pendulum system

Fuzzy logic controller based sugeno inference method response for angular position of inverted pendulum is shown figure 8. It has been observed that the angular position of inverted pendulum reaches zero (degree) in vertically upright position quickly in about five seconds. The angular velocity of inverted pendulum approaches zero (degree/second) quickly in nearly five seconds. The cart position reaches to the desired position of 1 (m) with 11% overshoot in about five seconds. The cart velocity follows zero (m/sn) in about five seconds. The inverted pendulum stabilizes in vertically upright position after five seconds.

5. CONCLUSION

Classical control systems cannot show desired performance features in the changing state conditions. By modeling these systems to changing state conditions a proper classical control system design is quite difficult. The design of the fuzzy controller does not require complicated mathematical model. It requires only a set of basic rules to form the rule table. Basically, all of the rules in the rule table are based on the expert's experience. In order to control the four state variables, sugeno model has good performance results using the completed nonlinear model. The simulation of fuzzy logic controller based on sugeno inference method with five membership functions on each input demonstrate that the fuzzy controller effectively controls the inverted pendulum. The performance investigation of this control approach with fuzzy logic controller based sugeno inference method parameters using optimization algorithms may be realized as future studies.

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