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Research Article

THE EQUATION OF MOTION OF AXIALLY COMPOSITE BEAMS

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ABSTRACT

We present to obtain the equations of motion of the axially composite beams. The composite beams are produced from two or more different materials. In this study, the material varies along the beam axis. In other words, it is seen that the beam is made of different materials, as the beam proceeds along its axis. The material is homogeneous and the beam is formed by combining the step by step along the beam axis. The mathematical model of this problem can be presented in two different ways. In the first, a multispan beam approach is used. In this approach, the variation of each material is given as one span. The equation of motion is obtained as number of various material and four different transient conditions are written for each material alteration point. In the other, one equation is introduced. This equation contains the discontinuity function. The material variation is modeled with the discontinuity functions. Thus, two different models are obtained for only problem.
Keywords: Composite beam, discontinuity function, Euler-Bernoulli beam theory.

1. KINEMATICS OF EULER-BERNOULLI BEAM THEORY (EBT)

In this section, the model to be used as the application problem is considered and the equations of motion are derived using Green Lagrange tensor. The displacements $\hat{u}_n$ and $\hat{w}_n$ in horizontal and vertical direction in a beam occur according to Euler-Bernolli beam theory. It is assumed that the beam performs the deformation in the efficiency of only simple bending and the influence of shearing force is ignored when these displacements are written. In this case, the deformations in the direction $\hat{x}_1$ and $\hat{x}_3$ are obtained by substituting $\hat{u}_n$ and $\hat{w}_n$ into the three-dimensional Green-Lagrange deformation relation.

From the geometry of the kinematics of Euler-Bernoulli beams [1], the transverse displacements are obtained as follows:

$$\hat{u}_1(\hat{x}_1,t)=\hat{u}_n(\hat{x}_1,t)-\hat{x}_3\frac{\partial\hat{w}_n}{\partial\hat{x}_1};\quad \hat{u}_2(\hat{x}_1,t)=0;\quad \hat{u}_3(\hat{x}_1,t)=\hat{w}_n(\hat{x}_1,t) \tag{1}$$

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The Lagrangian fini te product tensor $\left[L\right]$ is given by [2]

$$\left[L\right]=\frac{1}{2}\begin{bmatrix} 2\frac{\partial u_1}{\partial x_1}+\left(\frac{\partial u_1}{\partial x_1}\right)^2+\left(\frac{\partial u_2}{\partial x_1}\right)^2\left(\frac{\partial u_3}{\partial x_1}\right)^2 & \frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\frac{\partial u_2}{\partial x_1}+\frac{\partial u_3}{\partial x_2}\frac{\partial u_3}{\partial x_1} & \frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_3}\frac{\partial u_2}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}+\frac{\partial u_1}{\partial x_1}\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_1}\frac{\partial u_3}{\partial x_2} & 2\frac{\partial u_2}{\partial x_2}+\left(\frac{\partial u_1}{\partial x_2}\right)^2+\left(\frac{\partial u_2}{\partial x_2}\right)^2\left(\frac{\partial u_3}{\partial x_2}\right)^2 & \frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}+\frac{\partial u_1}{\partial x_3}\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_3}\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\frac{\partial u_3}{\partial x_2} \\ \frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}+\frac{\partial u_1}{\partial x_1}\frac{\partial u_1}{\partial x_3}+\frac{\partial u_2}{\partial x_1}\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\frac{\partial u_3}{\partial x_3} & \frac{\partial u_z}{\partial x_2}+\frac{\partial u_y}{\partial x_3}+\frac{\partial u_x}{\partial x_2}\frac{\partial u_x}{\partial x_3}+\frac{\partial u_y}{\partial x_2}\frac{\partial u_y}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\frac{\partial u_3}{\partial x_3} & 2\frac{\partial u_3}{\partial x_3}+\left(\frac{\partial u_1}{\partial x_3}\right)^2+\left(\frac{\partial u_2}{\partial x_3}\right)^2\left(\frac{\partial u_3}{\partial x_3}\right)^2 \end{bmatrix} \tag{2}$$

These statements are shown as symmetrical tensor components under the name of the unit Green-Lagrange strain tensor. Besides, the Einstein summation is given as

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\frac{1}{2}\left(\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right), \quad k=1,2,3 \tag{3}$$

From the Eq. (3), ε_{11}, ε_{22}, ε_{33} and ε_{12}, ε_{13}, ε_{23} represent the extension ratios and angles of shear, respectively. On the other hand, the derivatives in the following should be firstly calculated

$$\frac{\partial \hat{u}_1}{\partial \hat{x}_1}=\frac{\partial \hat{u}_n}{\partial \hat{x}_1}-\hat{x}_3\frac{\partial^2 w_n}{\partial \hat{x}_1^2}; \quad \frac{\partial \hat{u}_1}{\partial \hat{x}_3}=-\frac{\partial \hat{w}_n}{\partial \hat{x}_1}; \quad \frac{\partial \hat{u}_3}{\partial \hat{x}_1}=\frac{\partial \hat{w}_n}{\partial \hat{x}_1}; \quad \frac{\partial \hat{u}_3}{\partial \hat{x}_3}=0 \tag{4}$$

Substituting the expressions (4) into the unit Green-Lagrange strain tensor yields

$$\varepsilon_{11}=\frac{\partial \hat{u}_n}{\partial \hat{x}_1}+\frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2-\hat{x}_3\left(\frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2}\right) \tag{5.a}$$

$$\varepsilon_{13}=\frac{1}{2}\left[\frac{\partial \hat{u}_1}{\partial \hat{x}_3}+\frac{\partial \hat{u}_3}{\partial \hat{x}_1}+\frac{\partial \hat{u}_1}{\partial \hat{x}_3}\frac{\partial \hat{u}_1}{\partial \hat{x}_1}+\frac{\partial \hat{u}_2}{\partial \hat{x}_3}\frac{\partial \hat{u}_2}{\partial \hat{x}_1}+\frac{\partial \hat{u}_3}{\partial \hat{x}_3}\frac{\partial \hat{u}_3}{\partial \hat{x}_1}\right]\cong 0 \tag{5.b}$$

$$\varepsilon_{33}=\frac{1}{2}\left[2\frac{\partial \hat{u}_3}{\partial x_3}+\left(\frac{\partial \hat{u}_1}{\partial x_3}\right)^2+\left(\frac{\partial \hat{u}_2}{\partial x_3}\right)^2\left(\frac{\partial \hat{u}_3}{\partial x_3}\right)^2\right]\cong 0 \tag{5.c}$$

Substituting the unit Green-Lagrange strain deformations into the unit Von-Karman strain form, one obtains

$$\varepsilon_{11}=x_3^0\varepsilon_{11}^{(0)}+x_3^1\varepsilon_{11}^{(1)}+x_3^2\varepsilon_{11}^{(2)}+\ldots; \ \varepsilon_{13}=x_3^0\varepsilon_{13}^{(0)}+x_3^1\varepsilon_{13}^{(1)}+x_3^2\varepsilon_{13}^{(2)}+\ldots \tag{6.a}$$

$$\varepsilon_{33}=x_3^0\varepsilon_{11}^{(0)}+x_3^1\varepsilon_{33}^{(1)}+x_3^2\varepsilon_{33}^{(2)}+\ldots \tag{6.b}$$

and

$$\varepsilon_{11}^{(0)}=\frac{\partial \hat{u}_n}{\partial \hat{x}_1}+\frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2, \quad \varepsilon_{11}^{(1)}=-\frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2} \tag{7.a}$$

$$\varepsilon_{13}^{(0)}=0,\ \varepsilon_{13}^{(1)}=0,\ \varepsilon_{33}^{(0)}=0,\ \varepsilon_{33}^{(1)}=0 \tag{7.b}$$

Then, the relations in the following are found

$$\varepsilon_{11}=\frac{\partial\hat{u}_n}{\partial\hat{x}_1}+\frac{1}{2}\left(\frac{\partial\hat{u}_n}{\partial\hat{x}_1}\right)^2-\hat{x}_3\left(\frac{\partial^2\hat{w}_n}{\partial\hat{x}_1^2}\right);\quad \varepsilon_{13}\cong 0;\quad \varepsilon_{33}=\varepsilon_{33}^{(0)}\cong 0 \tag{8}$$

The model of Euler Bernoulli beam can be obtained by considering the unit deformations. On the other hand, the extended Hamilton's principle is used to obtain the equations of motion. The extended Hamilton's principle is given as

$$\int_{t_2}^{t_1}(-\delta K+\delta U+\delta V)dt=0 \tag{9}$$

where $\delta\mathrm{K}$, δU and δV denotes the variation of kinetic energy, the variation of potential energy and the variation of the virtual work performed by loads, respectively. On the other hand, we assume that the cross-sectional area A_n and the density of the beam material ρ_n are constant for each span. Then,

$$\delta K=\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\int_{A_n}\rho_n\dot{\hat{u}}_n\delta\dot{\hat{u}}_n\,dA_n\,d\hat{x}_1 \tag{10}$$

where N is the the number of the span. Thus, the variation of potential energy can be written as

$$\delta U=\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\int_{A_n}\sigma_{11}\,\delta\varepsilon_{11}\,dA_n\,d\hat{x}_1 \tag{11}$$

where $\sigma_{11}=E_n\,\varepsilon_{11}$. The equation of the virtual work performed by the external loads is given as

$$\delta V=-\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\left(\hat{P}\frac{\partial\hat{w}_n}{\partial\hat{x}_1}\frac{\partial\delta\hat{w}_n}{\partial\hat{x}_1}+\hat{f}\,\delta\hat{u}_n+\hat{q}\,\delta\hat{w}_n\right)d\hat{x}_1 \tag{12}$$

where $\hat{P}$ represents the dimensional axial load, $\hat{f}$ and $\hat{q}$ are the distributed loads in horizontal and vertical directions, respectively. Considering the Eqs. (10) and (11) in detail, then

$$\delta K=\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\int_{A_n}\rho_n\left[\left(\dot{\hat{u}}_n-\hat{x}_3\frac{\partial\dot{\hat{w}}_n}{\partial\hat{x}_1}\right)\left(\delta\dot{\hat{u}}_n-\hat{x}_3\frac{\partial\delta\hat{w}_n}{\partial\hat{x}_1}\right)+\dot{\hat{w}}_n\delta\dot{\hat{w}}_n\right]dA_n\,d\hat{x}_1 \tag{13}$$

$$\delta U=\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\int_{A_n}\sigma_{11}\left(\frac{\partial\delta\hat{u}_n}{\partial\hat{x}_1}-\hat{x}_3\frac{\partial^2\delta\hat{w}_n}{\partial\hat{x}_1^2}\right)dA_n\,d\hat{x}_1 \tag{14}$$

Applying integration by parts to the Eqs. (12)-(13), the coefficients of $\delta\hat{u}_n$ and $\delta\hat{w}_n$ introduce the equations of motion. Besides, the applications of boundary values appeared during integration by parts reveal the boundary conditions. Applying the extended Hamilton's principle to these equations, the equations of motion are obtained as

$$m^{(0)}\ddot{\hat{u}}_n - \frac{\partial M_{11}^{(0)}}{\partial \hat{x}_1} - m^{(1)}\frac{\partial \ddot{\hat{w}}_n}{\partial \hat{x}_1} = \hat{f} \tag{15}$$

$$m^{(0)}\ddot{\hat{w}}_n + m^{(1)}\frac{\partial \ddot{\hat{u}}_n}{\partial \hat{x}_1} - m^{(2)}\frac{\partial^2 \ddot{\hat{w}}_n}{\partial \hat{x}_1^2} + \frac{\partial}{\partial \hat{x}_1}\left(M_{11}^{(0)}\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right) + \frac{\partial^2 M_{11}^{(1)}}{\partial \hat{x}_1^2} + \hat{P}\frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2} = \hat{q} \tag{16}$$

and the boundary conditions

$$\delta\hat{u}_n : m^{(0)}\dot{\hat{u}}_n - m^{(1)}\frac{\partial \dot{\hat{w}}_n}{\partial \hat{x}_1} + M_{11}^{(0)} \tag{17.a}$$

$$\delta\hat{w}_n : m^{(1)}\ddot{\hat{u}}_n + M_{11}^{(0)}\frac{\partial \hat{w}_n}{\partial \hat{x}_1} + \frac{\partial M_{11}^{(1)}}{\partial \hat{x}_1} - \hat{P}\frac{\partial \hat{w}_n}{\partial \hat{x}_1} \tag{17.b}$$

$$\frac{\partial \delta\hat{w}_n}{\partial x_1} : -m^{(1)}\dot{\hat{u}}_n + m^{(2)}\frac{\partial \dot{\hat{w}}_n}{\partial \hat{x}_1} - M_{11}^{(1)} \tag{17.c}$$

such that

$$m^{(0)} = \rho_n A_n,\ m^{(1)} = 0,\ m^{(2)} = \rho_n I_n \tag{18}$$

$$M_{11}^{(0)} = E_n A_n\left[\frac{\partial \hat{u}_n}{\partial \hat{x}_1} + \frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2\right],\ M_{11}^{(1)} = -E_n I_n \frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2} \tag{19}$$

where I_n, $n = 1, 2, \ldots$ represents the moment of inertia for each span. However, the cross section and moment of inertia depend on the displacement of the beam. Here, it is assumed that the beam is compose of homogeneous, isotropic and elastic material. Then, the relation in the following is found

$$\left(m^{(0)}, m^{(1)}, m^{(2)}\right) = \int_{A_n} \rho_n\left(1, \hat{x}_3, \hat{x}_3^2\right) d\hat{A}_n \tag{20}$$

Substituting $m^{(0)}, m^{(1)}, m^{(2)}$ and $M_{11}^{(0)}, M_{11}^{(1)}$ into the Eqs. (15)-(16), the equations of motion are obtained as

$$\rho_n A_n \ddot{\hat{u}}_n - \frac{\partial}{\partial x_1}\left[E_n A_n\left(\frac{\partial \hat{u}_n}{\partial \hat{x}_1} + \frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2\right)\right] = \hat{f} \tag{21.a}$$

$$\rho_n A_n \ddot{\hat{w}}_n + \frac{\partial}{\partial \hat{x}_1}\left(\left[E_n A_n\left(\frac{\partial u_n}{\partial \hat{x}_1} + \frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2\right)\right].\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right) + E_n I_n \frac{\partial^4 \hat{w}_n}{\partial \hat{x}_1^4} - \rho_n I_n \frac{\partial \ddot{\hat{w}}_n}{\partial \hat{x}_1^2} + \hat{P}\frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2} = \hat{q} \quad (21.b)$$

$(n = 1, 2, \ldots)$. Since the transverse displacement is equal to zero and there is not the transverse distributed load, the relation in the following can be written as

$$\hat{u}_n \ll \hat{w}_n, \qquad \ddot{\hat{u}}_n \approx 0, \qquad \hat{f} = 0 \quad (22)$$

Substituting this relations to Eq. (21.a) yields

$$-\frac{\partial}{\partial x_1}\left[E_n A_n\left(\frac{\partial \hat{u}_n}{\partial \hat{x}_1} + \frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2\right)\right] = 0 \quad (23)$$

Integrating both sides of the equality, one obtains

$$-E_n A_n\left(\frac{\partial \hat{u}_n}{\partial \hat{x}_1} + \frac{1}{2}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2\right) = g(t) \quad (24)$$

The Eq. (24) shows that the Eq. (21.a) does not depend on space. Integrating both sides of the Eq. (24) on domain for span N, the resulting equation is

$$g(t) = -\frac{E_n A_n}{2L}\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2 d\hat{x}_1 \quad (25)$$

Substituting the Eq. (25) to the Eq. (21.b), the equation of motion with transverse vibrations is obtained as [4]

$$\rho_n A_n \ddot{\hat{w}}_n - \frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2}\left[\sum_{n=1}^{N}\frac{E_n A_n}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2 d\hat{x}_1\right] + E_n I_n \frac{\partial^4 w_n}{\partial \hat{x}_1^4} - \rho_n I_n \frac{\partial \ddot{\hat{w}}_n}{\partial \hat{x}_1^2} + \hat{P}\frac{\partial^2 \hat{w}_n}{\partial \hat{x}_1^2} = \hat{q} \quad (26)$$

Depending on the various situations appeared on the structures, the discontinuities of the structure elements are encountered. For example, the structures such as cracks in a beam, supports placed at certain intervals along the beam, singular forces on the beam or intermittently placed concentrated masses lead to discontinuities in the beam element. For each case which causes the discontinuity, the transient conditions are handled one by one in the following.

2. THE TRANSIENT CONDITIONS FOR THE BEAMS HAVING THE DIFFERENT TYPE DISCONTINUITIES

2.1. The number N of Simply Supported Beam

In this section, the beam with the number N of simply support is considered. For example, the beams placed on a column in a building are modelled in this way.

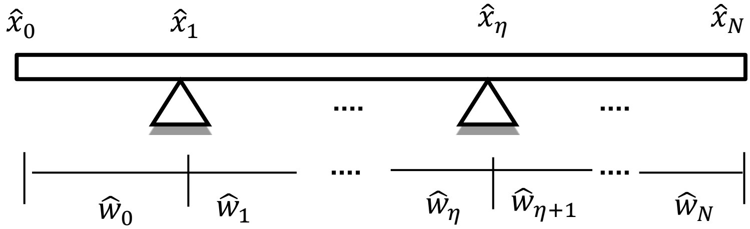

Figure 1. Simply supported multispan nonlinear beam

Transient conditions

$$\hat{w}_\eta(\hat{x}_\eta)=0, \quad \hat{w}_{\eta+1}(\hat{x}_\eta)=0$$

$$\hat{w}'_\eta(\hat{x}_\eta)-\hat{w}'_{\eta+1}(\hat{x}_\eta)=0, \quad \hat{w}''_\eta(\hat{x}_\eta)-\hat{w}''_{\eta+1}(\hat{x}_\eta)=0$$

2.2. The Beam Supported with the number N of Spring

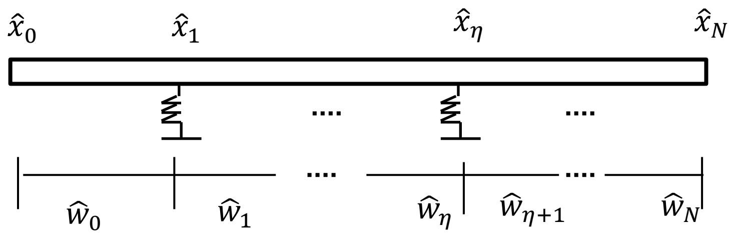

Figure 2. The multispan nonlinear beam supported with spring

Transient conditions

$$\hat{w}_\eta\left(\hat{x}_\eta\right)=\hat{w}_{\eta+1}\left(\hat{x}_\eta\right), \quad \hat{w}'_\eta\left(\hat{x}_\eta\right)=\hat{w}'_{\eta+1}\left(\hat{x}_\eta\right), \quad \hat{w}''_\eta\left(\hat{x}_\eta\right)=\hat{w}''_{\eta+1}\left(\hat{x}_\eta\right)$$

$$\hat{k}\,\hat{w}_\eta\left(\hat{x}_\eta\right)-\hat{E}_\eta\hat{I}_\eta\hat{w}'''_\eta\left(\hat{x}_\eta\right)-\hat{w}'_\eta\left(\hat{x}_\eta\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_n\hat{A}_n}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_n}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]$$

$$+\hat{E}_{\eta+1}\hat{I}_{\eta+1}\hat{w}'''_{\eta+1}\left(\hat{x}_{\eta+1}\right)+\hat{w}'_{\eta+1}\left(\hat{x}_{\eta+1}\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_{n+1}\hat{A}_{n+1}}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_{n+1}}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]=0$$

2.3. The Beam with the number N of Crack

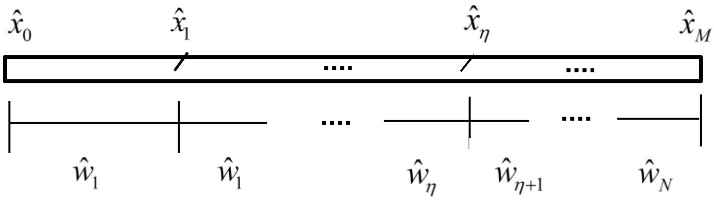

Figure 3. The multispan nonlinear beam with cracks

Transient conditions

$$\hat{w}_\eta\left(\hat{x}_\eta\right)=\hat{w}_{\eta+1}\left(\hat{x}_\eta\right),\quad \hat{w}'_\eta\left(\hat{x}_\eta\right)=\hat{w}'_{\eta+1}\left(\hat{x}_\eta\right),\quad \hat{w}''_\eta\left(\hat{x}_\eta\right)=\hat{w}''_{\eta+1}\left(\hat{x}_\eta\right)$$

$$\hat{E}_\eta\hat{I}_\eta\hat{w}'''_\eta\left(\hat{x}_\eta\right)+\hat{w}'_\eta\left(\hat{x}_\eta\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_n\hat{A}_n}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_n}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]$$
$$=\hat{E}_{\eta+1}\hat{I}_{\eta+1}\hat{w}'''_{\eta+1}\left(\hat{x}_{\eta+1}\right)+\hat{w}'_{\eta+1}\left(\hat{x}_{\eta+1}\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_{n+1}\hat{A}_{n+1}}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_{n+1}}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]$$

2.4. The Beam with the number N of concentrated mass

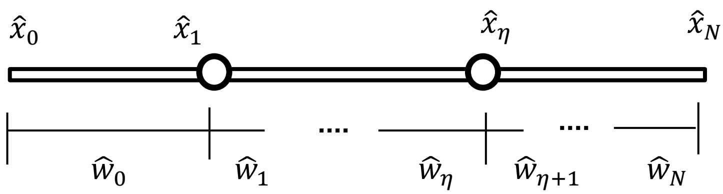

Figure 4. The multispan nonlinear beam with concentrated mass

Transient conditions

$$\hat{w}_\eta\left(\hat{x}_\eta\right)=\hat{w}_{\eta+1}\left(\hat{x}_\eta\right),\quad \hat{w}'_\eta\left(\hat{x}_\eta\right)=\hat{w}'_{\eta+1}\left(\hat{x}_\eta\right),\quad \hat{w}''_\eta\left(\hat{x}_\eta\right)=\hat{w}''_{\eta+1}\left(\hat{x}_\eta\right)$$

$$\hat{E}_\eta\hat{I}_\eta\hat{w}'''_\eta\left(\hat{x}_\eta\right)+\hat{w}'_\eta\left(\hat{x}_\eta\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_n\hat{A}_n}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_n}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]$$
$$-\hat{E}_{\eta+1}\hat{I}_{\eta+1}\hat{w}'''_{\eta+1}\left(\hat{x}_{\eta+1}\right)-\hat{w}'_{\eta+1}\left(\hat{x}_{\eta+1}\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_{n+1}\hat{A}_{n+1}}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial\hat{w}_{n+1}}{\partial\hat{x}_1}\right)^2 d\hat{x}_1\right]-M\,\ddot{\hat{w}}_\eta\left(\hat{x}_\eta\right)=0$$

(M represents a mass on the beam)

2.5. The Stepped Beam

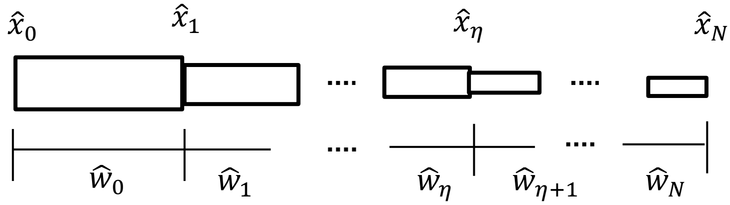

Figure 5. The multispan nonlinear stepped beam

Transient conditions

$$\hat{w}_\eta\left(\hat{x}_\eta\right)=\hat{w}_{\eta+1}\left(\hat{x}_\eta\right),\quad \hat{w}'_\eta\left(\hat{x}_\eta\right)=\hat{w}'_{\eta+1}\left(\hat{x}_\eta\right),\quad E_\eta I_\eta \hat{w}''_\eta\left(\hat{x}_\eta\right)-E_{\eta+1}I_{\eta+1}\hat{w}''_{\eta+1}\left(\hat{x}_\eta\right)=0$$

$$\hat{E}_\eta \hat{I}_\eta \hat{w}'''_\eta\left(\hat{x}_\eta\right)+\hat{w}'_\eta\left(\hat{x}_\eta\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_n\hat{A}_n}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial \hat{w}_n}{\partial \hat{x}_1}\right)^2 d\hat{x}_1\right]$$

$$=\hat{E}_{\eta+1}\hat{I}_{\eta+1}\hat{w}'''_{\eta+1}\left(\hat{x}_{\eta+1}\right)+\hat{w}'_{\eta+1}\left(\hat{x}_{\eta+1}\right)\left[\hat{P}-\sum_{n=1}^{N}\frac{\hat{E}_{n+1}\hat{A}_{n+1}}{2L}\int_{x_{n-1}}^{x_n}\left(\frac{\partial \hat{w}_{n+1}}{\partial \hat{x}_1}\right)^2 d\hat{x}_1\right]$$

3. THE MATHEMATICAL MODEL WITH DISCONTINUITY FUNCTION

The nonlinear mathematical model for multispan beams is considered, so far [12]. Thus, the transient conditions except for the boundary conditions are given for each discontinuity point observed during solution. Then, the problem can be modelled by one equation including discontinuity function instead of a set of equations. The governing equation is obtained as

$$\rho(\hat{x})A(\hat{x})\ddot{\hat{w}}-\frac{1}{2\int_0^L \frac{1}{E(\hat{x})A(\hat{x})}d\hat{x}}\left[\int_0^L\left(\hat{w}'\right)^2 d\hat{x}\right]\frac{\partial^2\hat{w}}{\partial\hat{x}^2}+\frac{\partial^2}{\partial\hat{x}^2}\left[E(\hat{x})I(\hat{x})\frac{\partial^2\hat{w}}{\partial\hat{x}^2}\right]-\rho(\hat{x})I(\hat{x})\frac{\partial\ddot{\hat{w}}}{\partial\hat{x}^2}$$
$$+\hat{P}(\hat{t})\frac{\partial^2\hat{w}}{\partial\hat{x}^2}+\hat{k}(\hat{x})\hat{w}=q(\hat{x},\hat{t}) \qquad (27)$$

with similar processes where $\hat{k}(\hat{x})$ represents the general spring constant. For practicability, the variable x in stead of $\hat{x}_1$ in the Eq. (27) is considered. Now, we consider the variation of the beams having the different discontinuities in the Section 2 such that the mathematical model have the discontinuity function.

3.1. Simply Supported Beam

The spring constant $\hat{k}$ in the Eq. (27) vanishes if the beam has two simply supports. Besides, the boundary conditions are

$$\hat{w}\left(\hat{x}_0,\hat{t}\right)=\hat{w}\left(\hat{x}_N,\hat{t}\right)=0 \qquad (28)$$

3.2. The Beam Supported with the number N of Spring

The spring constant $\hat{k}$ in the Eq. (27) becomes

$$\hat{k}(x) = \left[\hat{k}_0\,\delta(\hat{x}-\hat{x}_1) + \hat{k}_1\,\delta(\hat{x}-\hat{x}_2) + \dots\right] \tag{29}$$

or

$$\hat{k}(x) = \sum_{i=1}^{N} \hat{k}_i\,\delta(\hat{x}-\hat{x}_i) \tag{30}$$

where δ is defined as the Dirac delta function.

3.3. The Beam with the number N of Crack

For the cracked beam, the cross-sectional area $A(\hat{x})$ and moment of inertia $I(\hat{x})$ become

$$A(\hat{x}) = A_0\left[1 - \sum_{i=1}^{N} a_i\,\delta(\hat{x}-\hat{x}_i)\right] \tag{31}$$

$$I(\hat{x}) = I_0\left[1 - \sum_{i=1}^{N} b_i\,\delta(\hat{x}-\hat{x}_i)\right] \tag{32}$$

where $a_i \le 1$ and $b_i \le 1$ changes as depending on the depth of crack.

3.4. The Beam with the number N of concentrated mass

The coefficient of ρA becomes

$$m + \sum_{i=1}^{N} M_i\,\delta(\hat{x}-\hat{x}_i) \tag{33}$$

where m denotes the mass of cross sectional area and M_i is each mass on the beam.

3.5. The Stepped Beam

For stepped beam, the coefficients in the Eq. (27) is

$$A(\hat{x}) = A_0\left[1 \mp \sum_{i=1}^{N} a_i\,H(\hat{x}-\hat{x}_i)\right] \tag{34}$$

$$I(\hat{x}) = I_0\left[1 \mp \sum_{i=1}^{N} b_i\,H(\hat{x}-\hat{x}_i)\right] \tag{35}$$

where function H corresponds to the Heaviside step function (a_i describes ratio of cross-sectional area in $i.$ step to initial cross-sectional area. b_i represents ratio of moment of inertia in $i.$ step to initial moment of inertia.

4. CONCLUSIONS

In this study, the derivation of equation for the axially composite beams is introduced. The composite beams occur different materials. We present two different model to observe the variation of material in the beam. Firstly, we give the nonlinear mathematical model as multispan beams. For this kind of equations, the transient conditions except for the boundary conditions must be written for each discontinuity point observed during solution. On the other hand, one equation having the discontinuity function is proposed for the different type of discontinuities in the beam. In the first model, it becomes increasingly difficult to make a solution since the number of the written equation and the transient conditions rise as the number of discontinuous point increases. The analytical or semi-analytical solution can be obtained for a small number of discontinuous points. Therefore, second mathematical model is more favourable to overcome these difficulties encountered in solution.

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