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A COMPENSATORY APPROACH TO MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEM

Hale GONCE KÖÇKEN[*1], Mehmet AHLATCIOĞLU[2]

[1]*Yildiz Technical University, Faculty of Chemical and Metallurgical Engineering, Department of Mathematical Engineering, Esenler-ISTANBUL*
[2]*Yildiz Technical University, Faculty of Arts and Science, Department of Mathematics, Esenler-ISTANBUL*

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ABSTRACT

The data of real life problems generally cannot be expressed strictly. An efficient way of handling this situation is expressing the data as intervals. Thus this paper focus on the Multi-objective Interval Transportation Problem (MITP) whose parameters i.e. cost coefficients of the objective functions, supply and demand quantities are expressed as intervals. This problem is transformed to a traditional Multi-objective Transportation Problem (MOTP) with crisp parameters. First, interval supply-demand quantities are converted into deterministic ones by means of the convex combination of left and right limits. By an order relation that represents the decision maker's preferences between interval costs, each objective is turned into crisp form within the right limit and centre of the costs. Finally, using Werners' "fuzzy and" operator, a compensatory fuzzy approach to MITP is presented. And to our knowledge, combining compensatory (μ_{and}) operator with MITP has not been published up to now. Our approach generates compromise solutions which are both compensatory and Pareto-optimal. Also a numerical example is given to illustrate the presented approach.
Keywords: Multi-objective transportation problem, fuzzy mathematical programming, interval numbers, compensatory operators.
MSC numbers/numaraları: 90C08, 90C29, 90C70, 65G30, 65640.

ÇOK AMAÇLI ARALIKLI TAŞIMA PROBLEMİNE DENGELEYİCİ BİR YAKLAŞIM

ÖZET

Gerçek hayat problemlerinin verileri genellikle kesin olarak ifade edilemez. Bu durumun ele alınmasının etkili bir yolu verileri aralık şeklinde ifade etmektir. Bu makale, amaç fonksiyonlarının maliyet katsayıları ve arz-talep miktarlarının aralık şeklinde ifade edildiği Çok Amaçlı Aralıklı Taşıma Problemi (MITP) üzerine odaklanmıştır. Bu problem, geleneksel çok amaçlı taşıma problemine dönüştürülmüştür. Öncelikle, aralık arz-talep miktarları, sağ ve sol limitlerinin konveks kombinezonları aracılığıyla determisitik hallerine çevirilmiştir. Aralık maliyetler arasında karar vericinin tercihlerini ifade eden bir sıralama bağıntısı aracılığıyla, her bir amaç, fiyatların sağ limitleri ve merkezleri ile kesin hale dönüştürülmüştür. Son olarak, Werners'in "fuzzy and" operatorü kullanılarak, MITP için dengeleyici bulanık bir yaklaşım sunulmuştur. Bildiğimiz kadarıyla, dengeleyici (μ_{and}) operatorü ile MITP'yi birleştiren bir çalışma şu ana kadar yayımlanmamıştır. Bizim yaklaşımımız hem dengeleyici hem de Pareto-optimal olan uzlaşık çözümler üretmektedir. Ayrıca, sunulan yaklaşımın gösterilmesi için sayısal bir örnek de verilmiştir.
Anahtar Sözcükler: Çok amaçlı taşıma problemi, bulanık matematiksel programlama, aralık yapıdaki sayılar, dengeleyici operatörler.

[*] Corresponding Author/Sorumlu Yazar: e-mail/e-ileti: halegk@gmail.com, tel: (212) 383 46 05

1. INTRODUCTION

Transportation Problem has wide practical applications in logistic systems, manpower planning, personnel allocation, inventory control, production planning, etc. and aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. In many real-life situations, decisions are often made in the presence of multiple, conflicting, incommensurate objectives. Thus MOTP becomes more useful and includes objectives such as distribution cost, quantity of goods delivered, unfulfilled demand, average delivery time of the commodities, reliability of transportation, accessibility to the users, product deterioration, etc. Also in practice, the parameters of MOTP (supply-demand quantities and cost coefficients) are not always exactly known and stable. This imprecision may follow from the lack of exact information, changeable economic conditions, etc. A frequently used way of expressing the imprecision is to use the fuzzy numbers or intervals. In this paper, we assumed that all parameters of MOTP are in form of interval. Expressing the parameters as interval makes Decision Maker (DM) more comfortable and this enables us to consider tolerances for the model parameters in a more natural and direct way. Therefore, MITP seems to be more realistic and reliable according to crisp values. For this problem, Chanas et al. [1] considered that DM can define the supply and demand levels as point (crisp) values, interval values or fuzzy numbers. The links among them are provided, focusing on the case of the Fuzzy Transportation Problem, for which methods of solution are proposed and discussed. Ahlatcioglu and Sivri [2] assumed that the demand parameters are given as interval and proposed a method with two steps to interval transportation problem. By an order relation of intervals, Ahlatcioglu and Sivri [3] proposed a model whose demand quantities and cost coefficients are given as intervals. Das et al. [4] proposed a solution approach based on main idea of interval arithmetic. They converted interval supply-demand constraints to deterministic ones by doubling the numbers of these constraints. M. H. Lohgaonkar and V. H. Bajaj [5] handled the MOTP with interval cost by using a fuzzy programming technique.

In this paper, we focus on the solution procedure of MITP whose supply-demand quantities and cost coefficients are considered as intervals. This problem is transformed to a traditional MOTP with crisp parameters. Supply and demand quantities are converted into their crisp forms by means of the convex combination of left and right limits. By an order relation, each objective is turned into its crisp form within the right limit and centre of the costs. Finally, the obtained traditional MOTP is solved with a fuzzy programming technique by using Werner's μ_{and} operator. Also a numerical example is given to illustrate the approach.

This paper is organized as follows. After having presented brief information about interval arithmetic in the next section, the mathematical model of MITP and some basic definitions about order relations is given in Section 3. Section 4 introduces the compensatory fuzzy aggregation operators briefly. Section 5 explains our methodology using Werners' compensatory "fuzzy and'' operator. Section 6 gives an illustrative numerical example. Finally, Section 7 includes some results.

2. INTERVAL ARITHMETIC

An extensive research and wide coverage on the interval arithmetic and its applications can be found in [6].

Let $\mathbb{R}$ be the set of all real numbers. An interval in $\mathbb{R}$ is defined by an ordered pair of brackets as

$$A = [a_L, a_R] = \{a : a_L \le a \le a_R, a \in \mathbb{R}\}$$

where a_L and a_R are the left and right limits of A, respectively. The interval is also denoted by its centre and half-width as

$$A = \langle a_C, a_W \rangle = \{a : a_C - a_W \le a \le a_C + a_W, a \in \mathbb{R}\}$$

where $a_C = (a_R + a_L)/2$ and $a_W = (a_R - a_L)/2$ are the centre and half-width of A, respectively.

Interval operations can be defined as follows:

Addition of intervals:

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R] \tag{1.1}$$

or

$$A + B = \langle a_C, a_W \rangle + \langle b_C, b_W \rangle = \langle a_C + b_C, a_W + b_W \rangle \tag{1.2}$$

Multiplication with a real number k:

$$kA = k[a_L, a_R] = \begin{cases} [ka_L, ka_R] & for\ \ k \ge 0, \\ [ka_R, ka_L] & for\ \ k < 0, \end{cases} \tag{2.1}$$

or

$$kA = k\langle a_C, a_W \rangle = \langle ka_C, |k| a_W \rangle \tag{2.2}$$

3. MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEM

The mathematical formulation of MITP can be stated as follows:

$$\min Z^k(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{Lij}^k, c_{Rij}^k\right] x_{ij} \qquad k = 1,2,\ldots,K, \tag{3.1}$$

$$\text{s.t.} \qquad \sum_{j=1}^{n} x_{ij} = \left[a_{Li}, a_{Ri}\right] \quad i = 1,2,\ldots,m, \tag{3.2}$$

$$\sum_{i=1}^{m} x_{ij} = \left[b_{Lj}, b_{Rj}\right] \quad j = 1,2,\ldots,n, \tag{3.3}$$

$$x_{ij} \ge 0 \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n\,. \tag{3.4}$$

x_{ij} is decision variable which refers to product quantity that transported from supply point *i* to demand point *j*. K is the number of the objective functions of MITP. The closed interval $\left[c_{Lij}^k, c_{Rij}^k\right]$ denotes that the unit transportation cost from *i* th supply point to *j* th demand point lies between c_{Lij}^k and c_{Rij}^k for objective *k*. The closed interval $\left[a_{Li}, a_{Ri}\right]$ represent that *i* th supply quantity lies between a_{Li} and a_{Ri}. Similarly, the closed interval $\left[b_{Lj}, b_{Rj}\right]$ represent that *j* th demand quantity lies between b_{Lj} and b_{Rj}. Here, we denote the feasible region of (3) as S. We note that the balance condition $\sum_i a_i = \sum_j b_j$ must hold between supply and demand quantities in (3). This equation will be constructed in Section 5.

3.1. Order Relations Between Intervals

Here, the order relations that represent the decision maker's preferences between interval costs are defined for minimization problems. In this paper, we use the order relations that are defined in [7]. Also, in the literature [3] and [4] used this order relation for MITP.

Let the uncertain costs from two alternatives be represented by intervals A and B. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition 3.1: The order relation $\leq_{LR}$ between $A=[a_L,a_R]$ and $B=[b_L,b_R]$ is defined as

$A \leq_{LR} B$ iff $a_L \leq b_L$ and $a_R \leq b_R$,

$A <_{LR} B$ iff $A \leq_{LR} B$ and $A \neq B$.

This order relation $\leq_{LR}$ represents the decision maker's preference for the alternative with lower minimum cost and lower maximum cost, that is, if $A \leq_{LR} B$, then A is preferred to B. Next the order relation by the center and width of interval is defined in the following definition.

Definition 3.2: The order relation $\leq_{CW}$ between $A=\langle a_C,a_W\rangle$ and $B=\langle b_C,b_W\rangle$ is defined as

$A \leq_{CW} B$ iff $a_C \leq b_C$ and $a_w \leq b_W$,

$A <_{CW} B$ iff $A \leq_{CW} B$ and $A \neq B$.

The order relation represents the decision maker's preference for the alternative with the lower expected cost and less uncertainty, that is $A \leq_{CW} B$, then A is preferred to B.

Since the center and the width of interval can be considered as the expected value and the uncertainty of an interval respectively, this order relation represents the decision maker's preference for the alternative with lower expected value and less uncertainty.

Here we noted that both $\leq_{LR}$ and $\leq_{CW}$ are partial orders which are transitive, reflexive and antisymmetric. And also [7] showed that both of the order relations never conflict with each other in the sense that there is no such pair A and B that $A \neq B$, $A \leq_{LR} B$ and $B \leq_{CW} A$. See [7] for more information about order relations between intervals.

3.2. An Order Relation for Minimization Problems

In this subsection, the reformulation of a interval transportation problem as a bi-objective problem is explained. For this subsection, let consider the single objective case of (3) (i.e. $K=1$) and denote this problem as $(3')$. We note that the feasible region of $(3')$ is still denoting with S. Since the objective function $Z(\mathbf{x})$ is an interval function, it is natural that the solution set of $(3')$ should be defined by preference relations between intervals. Therefore using the order relations defined in Section 3.1, which represent the decision maker's preference between interval profits, the solution of $(3')$ can be defined as follows:

Definition 3.3: $\mathbf{x} \in S$ is a solution of $(3')$ iff there is no $\mathbf{x}' \in S$ which satisfies $Z(\mathbf{x}') <_{LR} Z(\mathbf{x})$ or $Z(\mathbf{x}') <_{CW} Z(\mathbf{x})$.

In order to simplify this definition, the order relation $\leq_{RC}$ is defined for the minimization problem as

$A \leq_{RC} B$ iff $a_R \leq b_R$ and $a_C \leq b_C$,

$A <_{RC} B$ iff $A \leq_{RC} B$ and $A \neq B$.

It follows that

$$A \leq_{RC} B \text{ iff } A \leq_{LR} B \text{ or } A \leq_{CW} B , \tag{5.1}$$

$$A <_{RC} B \text{ iff } A <_{LR} B \text{ or } A <_{CW} B . \tag{5.2}$$

Using the order relation $\leq_{RC}$, Definition 3.3 may be simplified as follows:

Definition 3.4: $\mathbf{x} \in S$ is a solution of (3), iff there is no $\mathbf{x}' \in S$ which satisfies $Z(\mathbf{x}') <_{RC} Z(\mathbf{x})$.

The right limit $Z_R(\mathbf{x})$ of the interval objective $Z(\mathbf{x})$ may be calculated from (1.2) and (2.2) as

$$Z_R(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Wij} \left| x_{ij} \right| \tag{6}$$

where c_{Cij} is the centre and c_{Wij} is half-width of the coefficient c_{ij} of the objective $Z(\mathbf{x})$. In the case $x_{ij} \geq 0$, (6) can be modified as

$$Z_R(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Wij} x_{ij} . \tag{7}$$

And the centre of the objective function $Z(\mathbf{x})$:

$$Z_C(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij} x_{ij} . \tag{8}$$

The solution set of $(3')$ defined by (5.2) can be obtained as the Pareto optimal solutions of the following bi-objective problem [7]:

$$\min \left\{ Z_R, Z_C \right\} \tag{9}$$

subject to the feasible region S where Z_R and Z_C are as stated in (7) and (8).

4. COMPENSATORY OPERATORS

There are several fuzzy aggregation operators. The detailed information about them exists in Zimmermann [8] and Tiryaki [9]. The most important aspect in the fuzzy approach is the compensatory or non-compensatory nature of the aggregation operator. Several investigators [8,10,11,12] have discussed this aspect.

Using the linear membership function, Zimmermann [13] proposed the “min” operator model to the Multi-objective linear problem (MOLP). It is usually used due to its easy computation. Although the “min” operator method has been proven to have several nice properties ([11]), the solution generated by min operator does not guarantee compensatory and Pareto-optimality [14,15,16]. The biggest disadvantage of the aggregation operator “min” is that it is non-compensatory. In other words, the results obtained by the "min" operator represent the worst situation and cannot be compensated by other members which may be very good. On the other hand, the decision modeled with average operator is called fully compensatory in the sense that it maximizes the arithmetic mean value of all membership functions.

Zimmermann and Zysno [17] show that most of the decisions taken in the real world are neither non-compensatory (min operator) nor fully compensatory and suggested a class of hybrid compensatory operators with γ compensation parameter.

Basing on the γ-operator, Werners [18] introduced the compensatory "fuzzy and" operator which is the convex combinations of min and arithmetical mean:

$$\mu_{and} = \gamma \ \min_i \left(\mu_i\right) + \frac{(1-\gamma)}{m}\left(\sum_i \mu_i\right), \tag{10}$$

where $0 \le \mu_i \le 1,\ i = 1,\ldots,m$, and the magnitude of $\gamma \in [0,1]$ represent the grade of compensation.

Although this operator is not inductive and associative, this is commutative, idempotent, strictly monotonic increasing in each component, continuous and compensatory. Obviously, when $\gamma = 1$, this equation reduces to $\mu_{and} = \min$ (non-compensatory) operator. In literature, it is showed that the solution generated by Werners' compensatory "fuzzy and" operator does guarantee compensatory and Pareto-optimality for MOLP [9,11,12,16,17,18]. Thus this operator is also suitable for our MITP. Therefore, due to its advantages, in this paper, we used Werners' compensatory "fuzzy and" operator.

5. A COMPENSATORY APPROACH TO MITP

To apply our compensatory approach, MITP is transformed to a traditional Multi-objective Transportation Problem (MOTP). First, interval supply and demand quantities are converted into deterministic ones by means of the convex combination of left and right limits.

$$\left[a_{Li}, a_{Ri}\right] \ \Rightarrow \ a_i = \alpha_i a_{Li} + \left(1-\alpha_i\right) a_{Ri} = a_{Ri} - \left(a_{Ri} - a_{Li}\right)\alpha_i \quad \left(i = 1,2,\ldots,m\right).$$

$$\left[b_{Lj}, b_{Rj}\right] \ \Rightarrow \ b_j = \beta_j b_{Lj} + \left(1-\beta_j\right) b_{Rj} = b_{Rj} - \left(b_{Rj} - b_{Lj}\right)\beta_j \quad \left(j = 1,2,\ldots,n\right).$$

where $\alpha_i, \beta_j \in \left[0,1\right]$.

In the literature, [4] assumed that

$$\sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{Lj} \ \text{ and } \ \sum_{i=1}^{m} a_{Ri} = \sum_{j=1}^{n} b_{Rj}.$$

This assumption is very restrictive for real life problems and it is almost impossible to collect the data satisfying these equations from DMs who determine the supply-demand quantities. So in this paper, we handle the balance condition as follows:

$$\sum_i^m a_i = \sum_j^n b_j \qquad \Rightarrow \sum_i^m a_{Ri} - \left(a_{Ri} - a_{Li}\right)\alpha_i = \sum_j^n b_{Rj} - \left(b_{Rj} - b_{Lj}\right)\beta_j$$

$$\Rightarrow \sum_j^n \left(b_{Rj} - b_{Lj}\right)\beta_j - \sum_i^m \left(a_{Ri} - a_{Li}\right)\alpha_i = \sum_j^n b_{Rj} - \sum_i^m a_{Ri}$$

This balance condition provides selection of the best supply-demand quantities for MITP.

Using (9), the objective function (3.1) can be converted to

$$\min Z^k(\mathbf{x}) = \min\left\{Z_R^k(\mathbf{x}), Z_C^k(\mathbf{x})\right\} = \min\left\{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{Rij}^k x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij}^k x_{ij},\right\} \quad (k = 1,2,\ldots,K).$$

Thus (3) is converted to the following traditional MOTP:

$$\min\ Z_R^k(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Rij}^k x_{ij}\ ,\quad k = 1,2,\ldots,K.$$

$$\min\ Z_C^k(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij}^k x_{ij}\ ,\quad k = 1,2,\ldots,K.$$

$$\text{s.t.}\qquad \sum_{j=1}^{n} x_{ij} + \left(a_{Ri} - a_{Li}\right)\alpha_i = a_{Ri}\quad ,\ i = 1,2,\ldots,m,$$

$$\sum_{i=1}^{m} x_{ij} + \left(b_{Rj} - b_{Lj}\right)\beta_j = b_{Rj}\quad ,\ j = 1,2,\ldots,n,$$

$$\sum_{j}^{n}\left(b_{Rj} - b_{Lj}\right)\beta_j - \sum_{i}^{m}\left(a_{Ri} - a_{Li}\right)\alpha_i = \sum_{j}^{n} b_{Rj} - \sum_{i}^{m} a_{Ri}$$

$$x_{ij} \geq 0 \quad i = 1,2,\ldots,m,\quad j = 1,2,\ldots,n,$$

$$\alpha_i, \beta_j \in [0,1]\ ,\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n\ .. \tag{11}$$

Here we denote the feasible region of (11) as S'. Now, the membership functions of objectives will be defined to apply our compensatory approach. Let L_R^k, U_R^k, L_C^k and U_C^k be the lower and upper bounds of the objective function $Z_R^k(\mathbf{x})$ and $Z_C^k(\mathbf{x})$ $(k = 1,2,\ldots,K)$, respectively. These bounds can be determined as follows: solve the MOLP as a single objective linear programming problem using each time only one objective and ignoring all others. Determine the corresponding values for every objective at each solution derived. And find the best and the worst values corresponding to the set of solutions.

Alternatively, by solving $4K$ single-objective linear programming problems, the lower and upper bounds can also be determined for each objective as follows:

$$L_R^k = \min_{\mathbf{x}\in S'} Z_R^k(\mathbf{x}),\ U_R^k = \max_{\mathbf{x}\in S'} Z_R^k(\mathbf{x}),\ L_C^k = \min_{\mathbf{x}\in S'} Z_C^k(\mathbf{x}),\ \ U_C^k = \max_{\mathbf{x}\in S'} Z_C^k(\mathbf{x}), \tag{12}$$

For the sake of simplicity, we used the linear membership functions. The right limit objective's membership function $\mu_{Rk}\left(Z_R^k\right)$ for each $k = 1,2,\ldots,K$:

$$\mu_{Rk}\left(Z_R^k(\mathbf{x})\right) = \begin{cases} 1 & ,Z_R^k < L_R^k, \\ \dfrac{U_R^k - Z_R^k}{U_R^k - L_R^k} & ,L_R^k \leq Z_R^k \leq U_R^k, \\ 0 & ,Z_R^k > U_R^k. \end{cases} \tag{13}$$

Here, $L_R^k \neq U_R^k$, $k = 1,2,...,K$ and in the case of $L_R^k = U_R^k$, $\mu_{Rk}\left(Z_R^k(\mathbf{x})\right) = 1$. The membership function $\mu_{Rk}\left(Z_R^k\right)$ is linear and strictly monotone decreasing for $Z_R^k(\mathbf{x})$ in the interval $\left[L_R^k, U_R^k\right]$. And the center objective's membership function $\mu_{Ck}\left(Z_C^k(\mathbf{x})\right)$ can be constructed similarly:

$$\mu_{Ck}\left(Z_C^k(\mathbf{x})\right)=\begin{cases} 1 & ,Z_C^k < L_C^k, \\ \dfrac{U_C^k - Z_C^k}{U_C^k - L_C^k} & ,L_C^k \le Z_C^k \le U_C^k, \\ 0 & ,Z_C^k > U_C^k. \end{cases} \tag{14}$$

Using Zimmermann's minimum operator ([13]), (11) can be written as:

$$\max_x \min_{Rk,Ck}\left\{\mu_{Rk}(Z_R^k(\mathbf{x})), \mu_{Ck}(Z_C^k(\mathbf{x}))\right\} \tag{15}$$

s.t. $\mathbf{x} \in S'$.

By introducing an auxiliary variable λ, (15) can be transformed into the following equivalent conventional LP problem:

$$\begin{aligned} &\max \quad \lambda \\ &\text{s.t.} \quad \mu_{Rk}(Z_R^k(\mathbf{x})) \ge \lambda ,\ k=1,...,K \\ &\qquad \mu_{Ck}(Z_C^k(\mathbf{x})) \ge \lambda ,\ k=1,...,K \\ &\qquad \mathbf{x} \in S' \\ &\lambda \in [0,1]. \end{aligned} \tag{16}$$

It is pointed out that Zimmermann's min operator model doesn't always yield a Pareto-optimal solution [14,15,16]. By using Werners' μ_{and} operator, (16) is converted to as follows:

$$\max \mu_{and} = \lambda + \frac{(1-\gamma)}{2K}\left[\sum_{k=1}^{K}\lambda_{Rk} + \sum_{k=1}^{K}\lambda_{Ck}\right]$$

s.t.:

$$\sum_{j=1}^{n} x_{ij} + \left(a_{Ri} - a_{Li}\right)\alpha_i = a_{Ri} \quad , \quad i=1,2,\dots,m,$$

$$\sum_{i=1}^{m} x_{ij} + \left(b_{Rj} - b_{Lj}\right)\beta_j = b_{Rj} \quad , \quad j=1,2,\dots,n,$$

$$\sum_{j}^{n}\left(b_{Rj} - b_{Lj}\right)\beta_j - \sum_{i}^{m}\left(a_{Ri} - a_{Li}\right)\alpha_i = \sum_{j}^{n} b_{Rj} - \sum_{i}^{m} a_{Ri}$$

$$\mu_{Rk}(Z_R^k(\mathbf{x})) \ge \lambda + \lambda_{Rk} ,\ k=1,...,K$$

$$\mu_{Ck}(Z_C^k(\mathbf{x})) \ge \lambda + \lambda_{Ck} ,\ k=1,...,K$$

$$\lambda + \lambda_{Rk} \le 1 ,\ k=1,2,\dots,K$$

$$\lambda + \lambda_{Ck} \le 1 ,\ k=1,2,\dots,K$$

$$\gamma, \lambda, \lambda_{Rk}, \lambda_{Ck} \in [0,1] ,\ k=1,2,\dots,K$$

$$\alpha_i, \beta_j \in [0,1] ,\ i=1,2,\dots,m,\ j=1,2,\dots,n .$$

$$x_{ij} \geq 0, \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n. \tag{17}$$

So, our compensatory model generates compensatory compromise Pareto-optimal solutions for MITP.

We shall give this assertion in the following theorem.

Theorem: If $(\mathbf{x},\boldsymbol{\lambda}^{\mathbf{x}})$ is an optimal solution of problem (17), then $\mathbf{x}$ is a Pareto-optimal solution for MITP, where $\boldsymbol{\lambda}^{\mathbf{x}} = (\lambda^{\mathbf{x}}, \lambda_{R1}^{\mathbf{x}}, \lambda_{R2}^{\mathbf{x}}, \ldots, \lambda_{RK}^{\mathbf{x}}, \lambda_{C1}^{\mathbf{x}}, \lambda_{C2}^{\mathbf{x}}, \ldots, \lambda_{CK}^{\mathbf{x}})$.

If required, the proof of the theorem can be found in [19]. Also, Pareto-optimality test ([19]) can be applied to the solutions of (17) and it could be seen that these solutions are Pareto-optimal for MITP.

6. A NUMERICAL EXAMPLE

Consider the following MITP:

$$\min Z^1(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 x_{ij} \quad , \quad \min Z^2(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2 x_{ij}$$

s.t.

$$\sum_{j=1}^{4} x_{1j} = [5,9] \ , \ \sum_{j=1}^{4} x_{2j} = [17,21] \ , \ \sum_{j=1}^{4} x_{3j} = [16,18] \ ,$$

$$\sum_{i=1}^{3} x_{i1} = [10,12] \ , \ \sum_{i=1}^{3} x_{i2} = [2,10] \ , \ \sum_{i=1}^{3} x_{i3} = [13,18] \ , \ \sum_{i=1}^{3} x_{i4} = [15,17] \ ,$$

$$x_{ij} \geq 0 \quad i = 1,2,3 \quad j = 1,2,3,4 .$$

where
$$c_{ij}^1 = \begin{bmatrix} [1,5] & [1,7] & [5,10] & [4,6] \\ [1,5] & [7,12] & [2,4] & [3,10] \\ [7,14] & [6,7] & [3,8] & [5,10] \end{bmatrix}, \ c_{ij}^2 = \begin{bmatrix} [3,8] & [2,8] & [2,7] & [1,8] \\ [4,10] & [7,12] & [7,13] & [9,15] \\ [4,6] & [1,5] & [3,8] & [1,4] \end{bmatrix}.$$

Here, the total supply quantity is in the interval $a_i \in [38,48]$ and total demand quantity is in the interval $b_j = [40,57]$. Thus $\sum_{j}^{n} b_{Rj} - \sum_{i}^{m} a_{Ri} = 9$. We also note that

$$\sum_{i=1}^{m} a_{Li} \neq \sum_{j=1}^{n} b_{Lj} \ \text{ and } \ \sum_{i=1}^{m} a_{Ri} \neq \sum_{j=1}^{n} b_{Rj} .$$

Using (11), problem can be converted to the following MOTP with crisp parameters:

$$\min Z_R^1(\mathbf{x}) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Rij}^1 x_{ij} \quad , \quad \min Z_C^1(\mathbf{x}) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Cij}^1 x_{ij}$$

$$\min Z_R^2(\mathbf{x}) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Rij}^2 x_{ij} \quad , \quad \min Z_C^2(\mathbf{x}) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Cij}^2 x_{ij}$$

s.t.

$$\sum_{j=1}^{4} x_{1j} + 4\alpha_1 = 9 \ , \ \sum_{j=1}^{4} x_{2j} + 4\alpha_2 = 21 \ , \ \sum_{j=1}^{4} x_{3j} + 2\alpha_3 = 18 \ ,$$

$$\sum_{i=1}^{3} x_{i1} + 2\beta_1 = 12 \ , \ \sum_{i=1}^{3} x_{i2} + 8\beta_2 = 10 \ , \ \sum_{i=1}^{3} x_{i3} + 5\beta_3 = 18 \ , \ \sum_{i=1}^{3} x_{i4} + 2\beta_4 = 17 \ ,$$

$$2\beta_1+8\beta_2+5\beta_3+2\beta_4-4\alpha_1-4\alpha_2-2\alpha_3=9$$

$$x_{ij}\geq 0 \quad i=1,2,3 \quad j=1,2,3,4\ ,$$

$$\alpha_i,\beta_j\in[0,1]\ ,\ i=1,2,3 \quad j=1,2,3,4\ .$$

where $c^1_{Rij}=\begin{bmatrix}5 & 7 & 10 & 6\\ 5 & 12 & 4 & 10\\ 14 & 7 & 8 & 10\end{bmatrix}$, $\quad c^1_{Cij}=\begin{bmatrix}3 & 4 & 7.5 & 5\\ 3 & 9.5 & 3 & 6.5\\ 10.5 & 6.5 & 5.5 & 7.5\end{bmatrix}$,

$c^2_{Rij}=\begin{bmatrix}8 & 8 & 7 & 8\\ 10 & 12 & 13 & 15\\ 6 & 5 & 8 & 4\end{bmatrix}$, $\quad c^2_{Cij}=\begin{bmatrix}5.5 & 5 & 4.5 & 4.5\\ 7 & 9.5 & 10 & 12\\ 5 & 3 & 5.5 & 2.5\end{bmatrix}$.

Using (12), the lower and upper bounds of the objectives are determined to construct the membership functions as follows:

Table 1. Bound values of objectives.

	Z^1_R	Z^1_C	Z^2_R	Z^2_C
L_k	262	192	303	210.5
U_k	536	391	518	390.5

Table 1 implies that, under the assumption of having only the objective Z^1, the objective value lies in the interval $Z^1\in[122,262]$ and similarly $Z^2\in[118,303]$.

Using (17), the compensatory model is constructed as follows:

$$\max \mu_{and}=\lambda+\frac{(1-\gamma)}{4}\left[\lambda_{R1}+\lambda_{R2}+\lambda_{C1}+\lambda_{C2}\right]$$

s.t.: $\quad \sum_{j=1}^{4}x_{1j}+4\alpha_1=9\ ,\ \sum_{j=1}^{4}x_{2j}+4\alpha_2=21\ ,\ \sum_{j=1}^{4}x_{3j}+2\alpha_3=18\ ,$

$$\sum_{i=1}^{3}x_{i1}+2\beta_1=12\ ,\ \sum_{i=1}^{3}x_{i2}+8\beta_2=10\ ,\ \sum_{i=1}^{3}x_{i3}+5\beta_3=18\ ,\ \sum_{i=1}^{3}x_{i4}+2\beta_4=17\ ,$$

$$2\beta_1+8\beta_2+5\beta_3+2\beta_4-4\alpha_1-4\alpha_2-2\alpha_3=9$$

$$Z^1_R(\mathbf{x})+274\left(\lambda+\lambda_{R1}\right)\leq 536\ ,\ Z^2_R(\mathbf{x})+215\left(\lambda+\lambda_{R2}\right)\leq 518$$

$$Z^1_C(\mathbf{x})+199\left(\lambda+\lambda_{C1}\right)\leq 391\ ,\ Z^2_C(\mathbf{x})+180\left(\lambda+\lambda_{C2}\right)\leq 390.5$$

$$\lambda+\lambda_{Rk}\leq 1\ ,\ k=1,2 \qquad \lambda+\lambda_{Ck}\leq 1\ ,\ k=1,2$$

$$\gamma,\lambda,\lambda_{Rk},\lambda_{Ck}\in[0,1]\ ,\ k=1,2$$

$$x_{ij}\geq 0\ ,\ \alpha_i,\beta_j\in[0,1]\ ,\ i=1,2,3\ ,\ j=1,2,3,4\ . \tag{18}$$

By solving (18), the results for different 11 values of the compensation parameter γ with 0.1 increment are obtained and given in Table 2. The results are: the values of objective functions Z_R^k and Z_C^k $(k=1,2)$, the satisfactory levels of the objectives corresponding to solution $\mathbf{x}$, (i.e. the values of membership functions) μ_{Rk} and μ_{Ck} $(k=1,2)$; the most basic satisfactory level λ; the compensation satisfactory level μ_{and}, respectively.

So, our compensatory model generates the following compensatory compromise Pareto-optimal solutions $\mathbf{X}^{1*}$, $\mathbf{X}^{2*}$ and $\mathbf{X}^{3*}$ for our MITP.

For the value of $\gamma = 0$:

$$\mathbf{X}^{1*} = \begin{Bmatrix} x_{11}=0, x_{12}=0, x_{13}=0, x_{14}=5 \\ x_{21}=10, x_{22}=0, x_{23}=7, x_{24}=0 \\ x_{31}=0, x_{32}=2, x_{33}=6, x_{34}=10 \end{Bmatrix},$$

$Z_R^1\left(\mathbf{X}^{1*}\right)=268$, $Z_C^1\left(\mathbf{X}^{1*}\right)=197$, $Z_R^2\left(\mathbf{X}^{1*}\right)=329$, $Z_C^2\left(\mathbf{X}^{1*}\right)=226.5$

For the value of $\gamma = 0.1 — \gamma = 0.7$:

$$\mathbf{X}^{2*} = \begin{Bmatrix} x_{11}=1.9163, x_{12}=0, x_{13}=1.8943, x_{14}=1.1893 \\ x_{21}=8.0837, x_{22}=0, x_{23}=8.9163, x_{24}=0 \\ x_{31}=0, x_{32}=2, x_{33}=2.1893, x_{34}=13.8107 \end{Bmatrix},$$

$Z_R^1\left(\mathbf{X}^{2*}\right)=279.3654$, $Z_C^1\left(\mathbf{X}^{2*}\right)=205.5241$, $Z_R^2\left(\mathbf{X}^{2*}\right)=317.611$, $Z_C^2\left(\mathbf{X}^{2*}\right)=222.7327$

For the value of $\gamma = 0.8 — \gamma = 1.0$:

$$\mathbf{X}^{3*} = \begin{Bmatrix} x_{11}=1.8117, x_{12}=0.2506, x_{13}=1.8795, x_{14}=1.0582 \\ x_{21}=8.1883, x_{22}=0, x_{23}=8.8117, x_{24}=0 \\ x_{31}=0, x_{32}=1.7494, x_{33}=2.3088, x_{34}=13.9418 \end{Bmatrix},$$

$Z_R^1\left(\mathbf{X}^{3*}\right)=280.53$, $Z_C^1\left(\mathbf{X}^{3*}\right)=205.4578$, $Z_R^2\left(\mathbf{X}^{3*}\right)=317.5402$, $Z_C^2\left(\mathbf{X}^{3*}\right)=222.6732$.

These solutions imply following interval objective values for our MITP:

$Z^1\left(\mathbf{X}^{1*}\right)=[126,268]$

$Z^1\left(\mathbf{X}^{2*}\right)=[131.6828\ ,\ 279.3654]$

$Z^1\left(\mathbf{X}^{3*}\right)=[130.3856\ ,\ 280.53]$

$Z^2\left(\mathbf{X}^{1*}\right)=[124,329]$

$Z^2\left(\mathbf{X}^{2*}\right)=[127.8544\ ,\ 317.611]$

$Z^2\left(\mathbf{X}^{3*}\right)=[127.8062\ ,\ 317.5402]$

All of these solutions pointed out that for all possible values of $c_{ij}^k\ (i=1,2,3\ ;\ j=1,2,3,4\ ;\ k=1,2)$, the certainly transported amounts are:

$\{x_{22}=0, x_{24}=0, x_{31}=0\}$.

And also, the least transported amount are:

$$\left\{\begin{array}{l} x_{14} \geq 1.0582, x_{21} \geq 8.0837, \\ x_{23} \geq 7, x_{32} \geq 1.7494, \\ x_{33} \geq 2.1893, x_{34} \geq 10 \end{array}\right\}.$$

For $\gamma = 0$, μ_{and} equals to average (full-compensatory) operator that is $\mu_{and} = \min \frac{1}{4}\left(\mu_{R1} + \mu_{R2} + \mu_{C1} + \mu_{C2}\right) = 0.9358$ and gives the solution $\mathbf{X}^{1^*}$.

For $\gamma = 1$, μ_{and} equals to min (non-compensatory) operator that is $\mu_{and} = \min_{k}\left(\mu_{Rk}, \mu_{Ck}\right) = 0.9324$ and gives the solution $\mathbf{X}^{3^*}$. This solution remains the same for $\gamma = [0.8,1]$.

These solutions and the values of all membership functions are offered to DM. If DM is not satisfied with the proposed solution then he/she could assign the weights w_k, $\left(w_k > 0, \sum_{k=1}^{2} w_k = 1\right)$ on his/her objectives Z^k, $k = 1,2$. In this case, the weights w_k are inserted to the compensatory model as the following manner [9]:

$$\frac{\mu_{Rk}\left(Z_R^k\right)}{w_k} \geq \lambda + \lambda_{Rk} \ , \quad \forall\ k = 1,2$$

$$w_k\left(\lambda + \lambda_{Rk}\right) \leq 1, \quad \forall\ k = 1,2.$$

instead of the constraints

$$\mu_{Rk}(Z_R^k(\mathbf{x})) \geq \lambda + \lambda_{Rk}\ , \ \forall\ k = 1,2$$

$$\lambda + \lambda_{Rk} \leq 1, \quad \forall\ k = 1,2.$$

Similar inequalities could be formed for $\mu_{Ck}(Z_C^k)$, $k = 1,2$.

We note here that our model has the capability of handling the MITP with unbalanced interval supply-demand quantities unlike [4]. In addition, while the fuzzy models given in [1] and [4] use Zimmermann's "min" operator, by means of using Werners' "fuzzy and" operator, the results of our model contains the results of "min" operator.

7. CONCLUSION

In this paper, we deal with MITP whose costs and supply-demand quantities are given as intervals. The interval supply-demand quantities are handled by means of the convex combination of left and right limits. And for interval costs, MITP is reduced to the question of determining Pareto-optimal solutions of a bi-criteria linear programming problem for each objective in the following way: the minimization problem with the interval objective function is converted into a traditional MOLP whose objectives are to minimize the right limit and the center of the interval objective function. These two objectives can be considered as the minimization of the worst case and the average case. After obtaining the traditional MOLP, Werners' "fuzzy and" operator is used to aggregate the objectives. Our compensatory approach generates compromise solution which is both compensatory and Pareto-optimal. It is known that Zimmerman's "min" operator is not compensatory and also does not guarantee to generate the Pareto-optimal solutions. Werners' μ_{and} operator is useful about computational efficiency and always generates Pareto-optimal solutions. And to our knowledge, combining compensatory (μ_{and}) operator with MITP has not been published up to now.

REFERENCES / KAYNAKLAR

[1] S. Chanas, M. Delgado, J. L. Verdegay, M. A. Vila, 1993, Interval and fuzzy extensions of classical transportation problems, Transportation Planning and Technology, Volume 17, Issue 2, pages 203-218.

[2] Ahlatçıoğlu M., Sivri M., 1998, "Interval Transportation Problem and A Solution Proposal", Marmara University, Journal of Sciences and Technology, Vol.14, pages 85-90.

[3] Ahlatçıoğlu M., Sivri M., 1999, "Fiyatların ve Talep Merkezleri Taleplerinin Belirli Aralıklar Arasında Olması Durumunda Taşıma Problemine Bir Çözüm Önerisi", Marmara Üniversitesi, Journal of Sciences and Technology, Vol.15, pages 139-144.

[4] Das, S.K., Goswami, A. ve Alam, S.S., 1999, Multiobjective transportation problem with interval cost, source and destination parameters, EJOR, 117 : 100-112.

[5] M. H. Lohgaonkar and V. H. Bajaj, 2010, Fuzzy Multi-objective Transportation Problem with Interval Cost, International Journal of Agricultural and Statistical, Vol 6 (1), pages 187–196.

[6] Fiedler, M., Nedoma, J., Ramik, J., Rohn, J., Zimmermann, K., 2006, Linear Optimization Problems with Inexact Data, Springer Science+Business Media, Inc., NewYork, USA.

[7] Ishibuchi, H., Tanaka, H. ,1990, Multiobjective programming in optimization of the interval objective functions. EJOR, 48: 219-225.

[8] Zimmermann, H.-J., 1991, Fuzzy Set Theory and Its Applications, Second Revised Edition, Kluwer Academic Publishers, Sixth Printing, Boston, Dordrecht, London.

[9] Tiryaki, F., 2006, "Interactive compensatory fuzzy programming for decentralized multi-level linear programming (DMLLP) problems", Fuzzy Sets and Systems, Volume 157, Pages 3072-3090.

[10] Y.J. Lai, C.L. Hwang, 1996, Fuzzy Multiple Objective Decision Making: Methods and Applications, Springer, Berlin, Heidelberg, New York, (second corrected printing).

[11] M.K. Luhandjula, 1982, "Compensatory operator in fuzzy linear programming with multiple objective", Fuzzy Sets and Systems, Vol 8, pp 245-252.

[12] H.S. Shih, E.S. Lee, 2000, Compensatory fuzzy multiple level decision making, Fuzzy Sets and Systems, 114, 71–87.

[13] H..J. Zimmermann, 1978, "Fuzzy programming and linear programming with several objective functions", Fuzzy Sets and Systems, Vol 1, pp 45-55.

[14] S. M. Guu, Y.K. Wu, 1997, "Weighted coefficients in two-phase approach for solving the multiple objective programming problems", Fuzzy Sets and Systems, 85, pp 45-48.

[15] E.S. Lee, R.J. Li, 1993, "Fuzzy multiple objective programming and compromise programming with Pareto optimum", Fuzzy Sets and Systems, 53, pp 275-288.

[16] Y.K. Wu, S.M. Guu, 2001, "A compromise model for solving fuzzy multiple objective problems", Journal of the Chinese Institute of Industrial Engineers, 18,5 , pp 87-93.

[17] H.J. Zimmermann, P. Zysno, 1980, "Latent connectives in human decision making", Fuzzy Sets and Systems, 4 (1), pp 37-51.

[18] B.M. Werners, 1988, "Aggregation models in mathematical programming", in (G. Mitra, Ed.), Mathematical Models for Decision Support, Springer, Berlin, pp 295-305.

[19] Ahlatçioğlu M., Tiryaki F., 2007, "Interactive fuzzy programming for decentralized two-level linear fractional programming (DTLLFP) problems", Omega, 35: 432-450.

Table 2. The results of our compensatory model

	$\gamma = 0$*	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.6$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
Z_R^1	268	279.3654	279.3654	279.3654	279.3654	279.3654	279.3654	279.3654	280.53	280.53	280.53
Z_C^1	197	205.5241	205.5241	205.5241	205.5241	205.5241	205.5241	205.5241	205.4578	205.4578	205.4578
Z_R^2	329	317.611	317.611	317.611	317.611	317.611	317.611	317.611	317.5402	317.5402	317.5402
Z_C^2	226.5	222.7327	222.7327	222.7327	222.7327	222.7327	222.7327	222.7327	222.6732	222.6732	222.6732
μ_{R1}	0.978102	0.936623	0.936623	0.936623	0.936623	0.936623	0.936623	0.936623	0.932372	0.932372	0.932372
μ_{C1}	0.974874	0.93204	0.93204	0.93204	0.93204	0.93204	0.93204	0.93204	0.932373	0.932373	0.932373
μ_{R2}	0.87907	0.932042	0.932042	0.932042	0.932042	0.932042	0.932042	0.932042	0.932371	0.932371	0.932371
μ_{C2}	0.911111	0.932041	0.932041	0.932041	0.932041	0.932041	0.932041	0.932041	0.932371	0.932371	0.932371
λ	0.8791	0.932	0.932	0.932	0.932	0.932	0.932	0.932	0.9324	0.9324	0.9324
μ_{and}	0.9358	0.9331	0.933	0.9328	0.9327	0.9326	0.9325	0.9324	0.9324	0.9324	0.9324

* Alternate optimal solutions exist.