σ Journal of Engineering and Natural Sciences
Mühendislik ve Fen Bilimleri Dergisi
Sigma
2005/4

CHEBYSHEV NETS IN A SUBSPACE OF A GENERALIZED WEYL SPACE

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Geliş/Received: 15.12.2004 Kabul/Accepted: 08.08.2005

ABSTRACT

Let GW_m be m dimentional subspace of an n-dimensional subspace of an n-dimentional generalized Weyl space GW_n and let $\delta = (v_1, v_2, ..., v_m)$ be a net in GW_m. In this paper the necessary and the sufficient condition that the net δ is to be a chebyshev net and a geodesic net relative to GW_m and GW_n is obtained.
Keywords: Generalized Weyl Space, Chebyshev Net, Geodesic Net.
MSC number/numarası: 53A40.

GENELLEŞTİRİLMİŞ WEYL UZAYININ ALT UZAYLARINDA CHEBYSHEV ŞEBEKELERİ

ÖZET

n boyutlu GW_n genellleştirilmiş Weyl uzayının m boyutlu GW_m alt uzayına ait bir şebeke $\delta = (v_1, v_2, ..., v_m)$ olsun. Bu çalışmada δ şebekesinin GW_m ve GW_n e göre Chebyshev şebekesi ve jeodezik şebeke olması için gerek ve yeter koşullar elde edilmiştir.
Anahtar Sözcükler: Genelleştirilmiş weyl uzayı, Chebyshey şebekesi Geodesic şebeke.

1. INTRODUCTION

An m dimentional GW_m is said to be generalized Weyl space if it has an asymmetric conformal metric tensor g_{ij} and an asymmetric connection ∇_k satisfying the compatibility condition given by equation

$$\nabla_k g_{ij} = 2T_k g_{ij} \tag{1.1}$$

where T_k denotes a covariant vector field and ∇_k denotes the usual covariant derivative

Under a renormalization of the fundamental tensor of the form $\breve{g}_{ij} = \lambda^2 g_{ij}$, the complementary vector field T_k is transformed by the law $\breve{T}_k = T_k + \partial_k \ln\lambda$, where λ is a scalar function on GW_m.

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Let L^i_{jk} be denote the coefficients of the asymmetric connection ∇_k. So, a generalized Weyl space is shortly written as $GW_m(L^i_{jk}, g_{ij}, T_k)$.

The main properties of $GW_m(L^i_{jk}, g_{ij}, T_k)$ can be expressed as follows

$$g_{ij} = g_{(ij)} + g_{[ij]} \tag{1.2}$$

$$\nabla_k g_{(ij)} = 2 g_{(ij)} T_k \tag{1.3}$$

$$\nabla_k g_{[ij]} = 2 g_{[ij]} T_k \tag{1.4}$$

$$g_{(ik)} g^{(kl)} = \delta^l_i \tag{1.5}$$

$$\nabla_k g^{(ij)} = -2 T_k g^{(ij)} \tag{1.6}$$

where $g_{(ij)}$ and $g_{[ij]}$ denote symmetric and antisymmetric parts of g_{ij} respectively.

The symmetric part of the connection coefficents L^i_{jk} are given as ([1] , [2], [3], [4])

$$L^i_{(jk)} = W^i_{jk} = \begin{bmatrix} i \\ jk \end{bmatrix} - (\delta^i_j T_k + \delta^i_k T_j - g_{(jk)} g^{(mi)} T_m) \tag{1.7}$$

where $\begin{bmatrix} i \\ jk \end{bmatrix}$ are second kind Christoffel symbols defined by

$$\begin{bmatrix} i \\ jk \end{bmatrix} = \frac{1}{2} g^{(ir)} [\frac{\partial g_{(jr)}}{\partial x^k} + \frac{\partial g_{(kr)}}{\partial x^j} - \frac{\partial g_{(jk)}}{\partial x^r}] \tag{1.8}$$

A quantity A is called a satellite of weight $\{p\}$ of tensor g_{ij}, if it admits a transformation of the form

$$\breve{A} = \lambda^p A \tag{1.9}$$

The prolonged covariant derivative of a satellite A of the tensor g_{ij} of weight $\{p\}$ is defined by

$$\dot{\nabla}_k A = \nabla_k A - p T_k A \tag{1.10}$$

Let $GW_m(L^i_{jk}, g_{ij}, T_k)$ be a subspace, with coordinates u^i, of the Weyl space $GW_n(L^\alpha_{\beta\gamma}, g_{\alpha\beta}, T_\gamma)$ with coordinates x^α. Suppose that the metrics of $GW_m(L^i_{jk}, g_{ij}, T_k)$ and $GW_n(L^\alpha_{\beta\gamma}, g_{\alpha\beta}, T_\gamma)$ are elliptic and that they are given, respectively, by $g_{ij}\, du^i\, du^j$ and $g_{\alpha\beta}\, du^\alpha\, du^\beta$ which are connected by the relations

$$g_{ij} = g_{\alpha\beta}\, x^\alpha_i x^\beta_j \tag{1.11}$$

where x^α_i denotes the covariant derivative of x^α with respect to u^i.

Let $\underset{r}{v}^{i}$ $(r = 1, 2, ..., m)$ be the contravariant components of the m independent vector fields $\underset{r}{v}$ in GW_m which are normalized by the condition $g_{ij}\,\underset{r}{v}^{i}\,\underset{r}{v}^{j} = 1$. The vector fields $\underset{1}{v}, \underset{2}{v}, ..., \underset{m}{v}$ determine a net $(\underset{1}{v}, \underset{2}{v}, ..., \underset{m}{v})$ in GW_m .

Corresponding to the vector fields $\underset{r}{v}$; the covector fields $\overset{r}{v}$ are defined by the conditions

$$\underset{r}{v}^{i}\,\overset{r}{v}_{j} = \delta^{i}_{j} \quad , \quad \underset{r}{v}^{i}\,\overset{p}{v}_{i} = \delta^{p}_{r} \qquad (1.12)$$

Denote by $\underset{\sigma}{n}^{\alpha}$ $(1, 2, ..., n-m)$, the contravariant components of the $n-m$ linear independent unit vector fields $\underset{\sigma}{n}$ in GW_n normal to GW_m .

The moving frame $\{x^{i}_{\alpha}\ ,\ \overset{\sigma}{n}_{\alpha}\}$ on GW_m, reciprocal to the moving frame $\{x^{\alpha}_{i}\ ,\ \underset{\sigma}{n}^{\alpha}\}$ is defined by the relations

$$\overset{\mu}{n}_{\alpha}\,\underset{\sigma}{n}^{\alpha} = \delta^{\mu}_{\sigma}\ ,\ \ \overset{\sigma}{n}_{\alpha}\,x^{\alpha}_{i} = 0\ \ ,\ x^{i}_{\alpha}\,\underset{\sigma}{n}^{\alpha} = 0\ \ ,\ \ x^{\alpha}_{i}\,x^{j}_{\alpha} = \delta^{j}_{i} \qquad (1.13)$$
$$(\mu = 1, 2, ..., \ n-m)$$

On the other hand, if the components of $\underset{r}{v}$ and $\overset{r}{v}$ relative to GW_n , are respectively denoted by $\underset{r}{v}^{\alpha}$ and $\overset{r}{v}_{\alpha}$, we have

$$\underset{r}{v}^{\alpha} = \underset{r}{v}^{i}\,x^{\alpha}_{i} \quad , \quad \overset{r}{v}_{\alpha} = \overset{r}{v}_{i}\,x^{i}_{\alpha} \qquad (1.14)$$

The derivation formula for $\underset{r}{v}$ and $\overset{r}{v}$ are given by [3], [5]

$$\overset{\bullet}{\nabla}_{k}\,\underset{r}{v}^{i} = \underset{r}{\overset{p}{T}}_{k}\,\underset{p}{v}^{i} \quad , \quad \overset{\bullet}{\nabla}_{k}\,\overset{r}{v}_{i} = -\underset{p}{\overset{r}{T}}_{k}\,\overset{p}{v}_{i} \qquad (1.15)$$

Taking the prolonged covariant derivative of $(1.13)_4$ with respect to u^{k} and remembering that [5]

$$\overset{\bullet}{\nabla}_{k}\,x^{\alpha}_{i} = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik}\,\underset{\sigma}{n}^{\alpha} + A^{h}_{ik}\,x^{\alpha}_{h} \qquad (1.16)$$

where $\overset{\sigma}{w}_{ik}$ are the components of second fundemental form related to the normal $\overset{\sigma}{n}$ of GW_m defined by

$$\overset{\sigma}{w}_{ik} = \overset{\sigma}{n}_{\alpha}\,\overset{\bullet}{\nabla}_{k}\,x^{\alpha}_{i} \qquad (1.17)$$

and A_{ik}^{h} are defined by

$$A_{ik}^{h} = x_{\alpha}^{h} \overset{\bullet}{\nabla}_{k} x_{i}^{\alpha} \tag{1.18}$$

From $(1.16)_4$, we have

$$\nabla_{k} x_{\alpha}^{j} = \sum_{\sigma=1}^{n-m} \underset{\sigma}{\Omega}{}_{k}^{j} \overset{\sigma}{n}_{\alpha} - A_{hk}^{j} x_{\alpha}^{h} \quad , \quad \underset{\sigma}{\Omega}{}_{k}^{i} = n^{\alpha} \nabla_{k} x_{\alpha}^{j} \tag{1.19}$$

For the definition of the Chebyshev nets of the first and second kind, and that the b-nets and c-nets, the reader is refereed to [3].

2. CHEBYSHEV NETS IN A GENERALIZED WEYL SUBSPACE

Consider the net $(\underset{1}{v}, \underset{2}{v}, ..., \underset{m}{v})$ in GW_m determined by the vector fields $\underset{1}{v}, \underset{2}{v}, ..., \underset{m}{v}$. Let $\underset{rp}{\overline{a}}{}^{\alpha}$, $\underset{rp}{a}{}^{i}$; $\overset{r}{\overline{b}}_{\alpha}$, $\overset{r}{b}_{i}$ and $\underset{r}{\overline{c}}{}^{\alpha}$, $\underset{r}{c}{}^{i}$; be the covariant components of the chebyshev vector fields of the first and the second kind and the geodesic vector fields of the given net relative to GW_n and GW_m, respectively.

Theorem 2.1. For the Chebyshev vector fields of the first and second kind and the geodesic vector fields of the net $(\underset{1}{v}, \underset{2}{v}, ..., \underset{m}{v})$ in GW_m we have

(i)
$$\underset{rp}{\overline{a}}{}^{\alpha} = \underset{p}{v}{}^{\gamma} \overset{\bullet}{\nabla}_{\gamma} A = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{p}{v}{}^{k} \underset{r}{v}{}^{i} \underset{\sigma}{n}{}^{\alpha} + \underset{rp}{a}{}^{h} x_{h}^{\alpha} + A_{ik}^{h} \underset{r}{v}{}^{i} \underset{p}{v}{}^{k} x_{h}^{\alpha}$$

$$\overset{\sigma}{w}_{ik} = g_{(\alpha\beta)} \underset{\sigma}{n}{}^{\beta} \overset{\bullet}{\nabla}_{k} x_{i}^{\alpha} \qquad (r \neq p)$$

(ii)
$$\overset{®}{\overline{b}}_{\alpha} = \underset{®}{v}{}^{\gamma} \overset{\bullet}{\nabla}_{\gamma} \overset{®}{v}_{\alpha} = -\sum_{\sigma=1}^{n-m} \Omega_{k}^{i} \overset{®}{v}_{i} \underset{®}{v}{}^{k} \overset{\sigma}{n}_{\alpha} + b_{h}^{r} x_{\alpha}^{h} + A_{hk}^{i} x_{\alpha}^{h} \overset{®}{v}_{i} \underset{®}{v}{}^{k}$$

$$\underset{\sigma}{\Omega}{}_{k}^{j} = \underset{\sigma}{n}{}^{\alpha} \overset{\bullet}{\nabla}_{k} x_{\alpha}^{i}$$

(iii)
$$\underset{r}{\overline{c}}{}^{\alpha} = \underset{r}{v}{}^{\gamma} \overset{\bullet}{\nabla}_{\gamma} v_{r}^{\alpha} = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} \underset{\sigma}{n}{}^{\alpha} + x_{h}^{\alpha} c_{r}^{h} + A_{ik}^{h} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} x_{h}^{\alpha}$$

where the symbol ® indicates that the summation on R will not be made.

Proof. (i) Taking the prolonged covariant derivative of each side of $(1.14)_1$ with respect to u^k and using (1.15) and (1.16), we get

$$\overset{\bullet}{\nabla}_{k} \underset{r}{v}{}^{\alpha} = \underset{r}{v}{}^{i} (\overset{\bullet}{\nabla}_{k} x_{i}^{\alpha}) + x_{i}^{\alpha} (\overset{\bullet}{\nabla}_{k} \underset{r}{v}{}^{i}) = \underset{r}{v}{}^{i} (\sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} n_{\sigma}^{\alpha} + A_{ik}^{h} x_{h}^{\alpha}) + x_{i}^{\alpha} (\underset{r}{\overset{p}{T}}{}_{k} \underset{p}{v}{}^{i}) \tag{2.1}$$

$$\underset{l}{v}^{k}\,\overset{\bullet}{\nabla}_{k}\,\underset{r}{v}^{\alpha}=\sum_{\sigma=1}^{n-m}\overset{\sigma}{w}_{ik}\,\underset{r}{v}^{i}\,\underset{l}{v}^{k}\,n^{\alpha}_{\sigma}+A^{h}_{ik}\,\underset{r}{v}^{i}\,\underset{l}{v}^{k}\,x^{\alpha}_{h}+x^{\alpha}_{i}\,(\underset{r}{\overset{p}{T}}_{k}\,\underset{l}{v}^{k})\,\underset{p}{v}^{i} \tag{2.2}$$

In view of

$$\underset{l}{v}^{k}\,\overset{\bullet}{\nabla}_{k}=\underset{l}{v}^{\gamma}\,\overset{\bullet}{\nabla}_{\gamma}\quad,\quad \underset{r}{\overset{p}{T}}_{k}\,\underset{l}{v}^{k}=\underset{rl}{\overset{p}{\tau}} \tag{2.3}$$

the last relation becomes

$$\underset{l}{v}^{k}\,\overset{\bullet}{\nabla}_{\gamma}\,\underset{r}{v}^{\alpha}=\sum_{\sigma=1}^{n-m}\overset{\sigma}{w}_{ik}\,\underset{r}{v}^{i}\,\underset{l}{v}^{k}\,\underset{\sigma}{n}^{\alpha}+A^{h}_{ik}\,\underset{r}{v}^{i}\,\underset{l}{v}^{k}\,x^{\alpha}_{h}+x^{\alpha}_{i}\,(\underset{rl}{\overset{p}{\tau}}\,\underset{p}{v}^{i}) \tag{2.4}$$

where the functions $\underset{rl}{\overset{p}{\tau}}$ $(r\neq l)$ are the chebyshev curvatures of the first kind of the curves of the net

$$\underset{rp}{a}=\underset{rp}{\overset{s}{\tau}}\,\underset{s}{v}^{i} \tag{2.5}$$

So that (2.4) reduces to

$$\underset{rl}{\overline{a}}^{\alpha}=\underset{l}{v}^{\gamma}\,\overset{\bullet}{\nabla}_{\gamma}\,A=\sum_{\sigma=1}^{n-m}\overset{\sigma}{w}_{ik}\,\underset{l}{v}^{k}\,\underset{r}{v}^{i}\,\underset{\sigma}{n}^{\alpha}+\underset{rl}{a}^{h}\,x^{\alpha}_{h}+A^{h}_{ik}\,\underset{r}{v}^{i}\,\underset{l}{v}^{k}\,x^{\alpha}_{h}\qquad(r\neq l) \tag{2.6}$$

(ii) Differentiating covariantly both sides of $(1.14)_2$ with respect to u^k and taking (1.19) and $(1.15)_2$ into consideration we obtain

$$\begin{aligned}\overset{\bullet}{\nabla}_{k}\,\overset{r}{v}_{\alpha}&=\overset{\bullet}{\nabla}_{k}\,(\overset{r}{v}_{i}\,x^{i}_{\alpha})=(\overset{\bullet}{\nabla}_{k}\,\overset{r}{v}_{i})\,x^{i}_{\alpha}+\overset{r}{v}_{i}\,(\overset{\bullet}{\nabla}_{k}\,x^{i}_{\alpha})\\&=(\sum_{\sigma=1}^{n-m}\underset{\sigma}{\Omega}^{i}_{k}\,\overset{\sigma}{n}_{\alpha}-A^{i}_{hk}\,x^{h}_{\alpha})\,\overset{r}{v}_{i}-\underset{p}{\overset{r}{T}}_{k}\,\overset{p}{v}_{i}\,x^{i}_{\alpha}\end{aligned} \tag{2.7}$$

Multiplying (2.7) by $\underset{®}{v}^{k}$ and remembering that $\overset{\bullet}{\nabla}_{k}\,\overset{r}{v}_{\alpha}=x^{\gamma}_{k}\,\overset{\bullet}{\nabla}_{\gamma}\,\overset{r}{v}_{\alpha}$, we have

$$\underset{®}{v}^{\gamma}\,\overset{\bullet}{\nabla}_{\gamma}\,\overset{®}{v}_{\alpha}=\sum_{\sigma=1}^{n-m}\underset{\sigma}{\Omega}^{i}_{k}\,\overset{\sigma}{n}_{\alpha}\,\overset{®}{v}_{i}\,\underset{®}{v}^{k}-A^{i}_{hk}\,\overset{®}{v}_{i}\,\underset{®}{v}^{k}\,x^{h}_{\alpha}-\underset{p}{\overset{r}{T}}_{k}\,\overset{p}{v}_{i}\,\underset{r}{v}^{k}\,x^{i}_{\alpha} \tag{2.8}$$

Since the Chebyshev curvatures $\underset{l}{\overset{r}{\rho}}$ of the second kind of the curves belonging to the net $(\underset{1}{v},\underset{2}{v},\ldots,\underset{m}{v})$ are defined by

$$\underset{l}{\overset{r}{\rho}}=\underset{p}{\overset{®}{T}}_{k}\,\underset{®}{v}^{k} \tag{2.9}$$

The last relation takes the form

$$\underset{®}{v}^{\gamma}\,\overset{\bullet}{\nabla}_{\gamma}\,\overset{®}{v}_{\alpha}=\sum_{\sigma=1}^{n-m}\underset{\sigma}{\Omega}^{i}_{k}\,\overset{\sigma}{n}_{\alpha}\,\overset{®}{v}_{i}\,\underset{®}{v}^{k}-A^{i}_{hk}\,\overset{®}{v}_{i}\,\underset{®}{v}^{k}\,x^{h}_{\alpha}-(\underset{l}{\overset{r}{\rho}}\,\overset{l}{v}_{i})\,x^{i}_{\alpha} \tag{2.10}$$

or, in terms of the chebyshev vector fields of the second kind, (2.10) becomes

$$\overset{r}{b}_{\alpha} = -\sum_{\sigma=1}^{n-m} \Omega^{i}_{k} \underset{\sigma}{v_i} \overset{\circledR}{v}{}^{k} \overset{\sigma}{n}_{\alpha} - b^{r}_{h} x^{h}_{\alpha} - A^{i}_{hk} x^{h}_{\alpha} \overset{\circledR}{v_i} \underset{\circledR}{v}{}^{k} \tag{2.11}$$

where $\overset{r}{b}_i$ is defined by $\overset{r}{b}_i = \overset{r}{\underset{s}{\rho}} \overset{s}{v}_i$.

(iii) If we multiply (2.1) by $\underset{r}{v}{}^{k}$ and sum for k , we find

$$\underset{r}{v}{}^{k} \overset{\bullet}{\nabla}_{k} \underset{r}{v}{}^{\alpha} = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} \underset{\sigma}{n}{}^{\alpha} + (\overset{p}{\underset{r}{T}}_{k} \underset{r}{v}{}^{k}) \underset{p}{v}{}^{i} x^{\alpha}_{i} + A^{h}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} x^{\alpha}_{h} \tag{2.12}$$

or using $(1.14)_1$ and the fact that $\overset{\bullet}{\nabla}_{k} = x^{\gamma}_{k} \overset{\bullet}{\nabla}_{\gamma}$, we transform the last equation into

$$\underset{r}{v}{}^{\gamma} \overset{\bullet}{\nabla}_{\gamma} \underset{r}{v}{}^{\alpha} = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} n^{\alpha} + \overset{p}{\underset{r}{S}} \underset{p}{v}{}^{i} x^{\alpha}_{i} + A^{h}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} x^{\alpha}_{h} \tag{2.13}$$

where $\overset{p}{\underset{r}{S}}$ are geodesic curvatures of the curves belonging to the net considered and are defined by $\overset{p}{\underset{r}{S}} = \overset{p}{\underset{r}{T}}_{k} \underset{r}{v}{}^{k}$.

In terms of the geodesic vector fields of the net, (2.13) reduces to

$$\underset{r}{\bar{c}}{}^{\alpha} = \sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} n^{\alpha} + x^{\alpha}_{h} \underset{r}{c}{}^{h} + A^{h}_{ik} \underset{r}{v}{}^{i} \underset{r}{v}{}^{k} x^{\alpha}_{h} \tag{2.14}$$

where $\underset{r}{c}{}^{i}$ is the geodesic vector field, relative to GW_m , of the $r-$th family of the net defined by $\underset{r}{c}{}^{i} = \overset{p}{\underset{r}{S}} \underset{p}{v}{}^{i}$.

The relations (2.6), (2.11), (2.14), allow us to state the following theorem:

Theorem 2.2. (i) A neccessary and the sufficient condition the net $\delta = (\underset{1}{v}, \underset{2}{v}, ..., \underset{n}{v})$ in GW_m which is a chebyshev net of the first kind with respect to GW_n to be a chebyshev net of the first kind with respect to GW_m is that

$$\sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{l}{v}{}^{i} \underset{l}{v}{}^{i} = 0 \quad \text{and} \quad A^{h}_{ik} \underset{r}{v}{}^{i} \underset{l}{v}{}^{k} = 0$$

(ii) A neccessary and the sufficient condition the net $\delta = (\underset{1}{v}, \underset{2}{v}, ..., \underset{n}{v})$ in GW_m which is a chebyshev net of the second kind with respect to GW_n to be a chebyshev net of the second kind with respect to GW_m is that

$$\sum_{\sigma=1}^{n-m} \Omega^{i}_{k} \underset{\sigma}{v_i} \underset{\circledR}{\overset{\circledR}{v}}{}^{k} = 0 \quad \text{and} \quad A^{i}_{hk} \overset{\circledR}{v_i} \underset{\circledR}{v}{}^{k} = 0$$

(iii) A neccessary and the sufficient condition the net $\delta = (\underset{1}{v}, \underset{2}{v}, ..., \underset{n}{v})$ in GW_m which is a geodesic net relative GW_n to be a geodesic net relative to GW_m is that

$$\sum_{\sigma=1}^{n-m} \overset{\sigma}{w}_{ik} \underset{r}{v}^i \underset{r}{v}^k = 0 \quad \text{and} \quad A_{ik}^h \underset{r}{v}^i \underset{r}{v}^k = 0$$

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