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THERMAL ELASTIC-PLASTIC STRESS ANALYSIS OF STEEL FIBER REINFORCED ALUMINUM METAL-MATRIX COMPOSITE ARCHES

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ABSTRACT

A thermal elastic-plastic stress analysis is carried out on steel fiber reinforced aluminum metal-matrix composite arch under uniform temperature distribution. The arch is fixed at both ends. The composite material is assumed to be strain hardening linearly. First yielding temperature, thermal residual stresses and the distribution of these stresses under thermal loading are investigated by using finite element method (FEM). It is found that the orientation angle affects the yield point and the intensity of residual stresses significantly.
Keywords: Elastic-plastic stress analysis, metal-matrix composite, residual stress.
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ÇELİK FİBERLE GÜÇLENDİRİLMİŞ ALÜMİNYUM METAL-MATRİS KOMPOZİT KEMER KİRİŞLERİN ELASTİK-PLASTİK ISIL GERİLME ANALİZİ

ÖZET

Düzgün yayılı sıcaklığa maruz kalan çelik fiberle güçlendirilmiş alüminyum metal-matris kompozit kemer kirişin elastik-plastik ısıl gerilme analizi yapılmıştır. Kemer kiriş iki ucundan ankastre bağlanmıştır. Kompozit malzemenin işlerken doğrusal sertleşen olduğu kabul edilmiştir. İlk akma sıcaklığı, ısıl artık gerilmeler ve bu gerilmelerin ısıl yükleme altındaki dağılımı sonlu elemanlar yöntemi kullanılarak incelenmiştir. Fiber doğrultusunun akma değerini ve artık gerilmelerin şiddetini önemli ölçüde etkilediği görülmüştür.
Anahtar Sözcükler: elastik-plastik gerilme analizi, metal-matris kompozit, artık gerilme.

1. INTRODUCTION

Metal matrix composites are being increasingly used due to their high stiffness, strength-to-weight ratio and temperature performance. When these materials are subjected to thermal loading, they may lead to plastic yielding. Elastic-plastic and residual stresses are very important in failure analysis of metal-matrix laminated plates. Permanent deformations and residual stresses occur in laminated plates when the yield point of the laminate is exceeded.

Residual stresses are defined as stresses that exist in materials or structures in the absence of any external loads. Since residual stresses may arise in all composite materials and influence the properties, and therefore mechanical reliability, of the composite structures significantly, and even may lead to premature failure, it is of utmost importance that the residual thermal stresses are taken into account in both design and analytic modelling of composite

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structures. Understanding the formation of residual stresses, we find ways to decrease their magnitude or use them to our advantage. They could arise due to various reasons, one of which is the nonlinear plastic behavior of material during load/unload.

Many studies have been made on the thermal stress analysis of composite materials. Tsai [1] investigated the thermoelastic behavior of a transversely isotropic material containing a flat toroidal crack. Gardner and Pittman [2] examined residual thermal stresses using a micromechanical model based on the method of cells in filamentary polymer matrix composites containing an elastomeric interphase between the fiber and the matrix. Lorentzen and Clarke [3] measured thermomechanically induced residual strains in Al/SiCp metal-matrix composites using neutron diffraction at room and elevated temperatures. Ponter and Leckie [4] studied the behavior of metal matrix composites subjected to cyclic thermal loading. They investigated the relationship between the observed behavior and the predictions of classical plasticity models for the matrix material. Jiang et. al. [5] developed an analytical model to study the influence of thermal residual stresses on the elastic and yield behaviors of aligned short fiber-reinforced metal matrix composites. Atas and Sayman [6] made an elastic-plastic stress analysis and studied the expansion of plastic zone in layers of stainless steel fiber-reinforced aluminum metal-matrix laminated plates by using FEM and first-order shear deformation theory for small deformations. Huang [7] developed ultimate strength formulae of unidirectional composites including thermal residual stresses due to mismatch of thermal expansion coefficients between fibers and matrix. Meijer et al. [8] investigated the influence of inclusion geometry and thermal residual stresses and strains in particle reinforced metal-matrix composites. Sayman [9] carried out an elastic-plastic thermal stress analysis on steel fiber-reinforced aluminum metal-matrix composite beams with a linearly varying temperature distribution. He investigated the intensities of residual stresses and equivalent plastic strains. Aghdam et al. [10] examined the initial yield and collapse of fiber-reinforced metal-matrix composites by using finite element micromechanical models. They showed that loads to cause collapse of metal matrix composites are higher than those to cause yield first, particularly when the effect of residual stress arising from manufacture is included in the analysis. Sayman and Sayman [11] carried out a thermal elastic-plastic stress analysis on simply supported symmetric cross-ply and angle-ply aluminum metal-matrix composite laminated plates under the uniform temperature distribution across the thickness of the plates. Shabana and Noda [12] performed a thermo-elasto-plastic stress analysis in functionally graded materials subjected to thermal loading taking residual stresses of the fabrication process into consideration. Sayman [13] carried out a thermal elastic-plastic stress analysis in symmetric thermoplastic laminated plates. It was shown that the strength of laminated plates may be increased by using residual stress. Bektaş and Sayman [14] performed an elastic-plastic solution to obtain the residual stresses in a thermoplastic composite laminated plate under linear temperature distribution. They found the residual stresses along the cross sections of the plates. Karakuzu et al. [15] carried out an elastic-plastic stress analysis and examined residual stresses in woven steel fiber reinforced thermoplastic laminated composite plates for transverse uniform loads by using the finite element technique. Recently, Sayman [16] carried out an elastic-plastic stress analysis on symmetric cross-ply and angle-ply aluminum metal-matrix laminated plates under thermal loads varying linearly along the thickness. He obtained residual stress distributions along the thickness of the plates.

In this study, a thermal elastic-plastic stress analysis is performed on steel fiber reinforced aluminum metal-matrix composite arch under uniform temperature distribution. The arch is fixed at both ends. The composite material is orthotropic and assumed to be strain hardening linearly. Elastic, elastic-plastic and residual stresses and the expansion of the plastic zone are investigated by using FEM. The existence of thermal residual stresses will have some effect on the mechanical properties of the composite material. This effect is the investigated in this study.

2. PROPERTIES OF THE COMPOSITE MATERIAL

Stainless steel fiber-reinforced aluminum metal-matrix composite material was manufactured by setting under 30 MPa and at 600°C temperature [17]. Moulds heated by electrical resistance and insulated by glass-fibers were used to fabricate the composite. Under these conditions, the steel fiber and the aluminum matrix were bonded, provided aluminum exceeds the yield strength. Tensile test specimens were obtained from this material and loaded in principal material directions by Instron tensile machine to get mechanical properties and yield strengths of a layer. Strain-gauges were employed for these tests.

The mechanical properties, yield points and plastic parameters of the orthotropic composite material are given in Table 1. X and Y are the yield points in the first and second principal material directions, respectively. S is the yield point for pure shear in the 1-2 plane. The yield point in the third direction Z, is assumed to be equal to Y. Bilinear isotropic hardening model is assumed and the material behavior is described by a stress-strain curve starting at the origin with positive stress and strain values.

Table 1. Mechanical properties and yield points of the material

Longitudinal modulus (E_1)	85 GPa
Transverse modulus (E_2)	74 GPa
Shear modulus (G_{12})	30 GPa
Major Poisson's ratio (ν_{12})	0.3
Axial yield value (X)	230 MPa
Transverse yield value (Y)	24 MPa
Shear yield value (S)	48.9 MPa
Plasticity constant (K)	1250 MPa
Longitudinal thermal expansion coefficient (α_1)	$18.5 \cdot 10^{-6}$ /°C
Transverse thermal expansion coefficient (α_2)	$21.0 \cdot 10^{-6}$ /°C

For a linear strain hardening material, the yield stress is given by the Ludwick equation as,

$$\sigma_Y = \sigma_0 + K\varepsilon_p \tag{1}$$

where σ_0 is equal to X which is the yield point in the first principal material direction, K and ε_p are the plasticity constant and equivalent plastic strain, respectively. The initial slope of the stress-strain curve is taken as the elastic modulus of the material. At the specified yield stress, the curve continues along the second slope defined by the tangent modulus (having the same units as the elastic modulus). One may derive the tangent modulus from Eq. (1) by the following operations:

$$\varepsilon_T = \frac{\sigma_0}{E} + \frac{\sigma_Y - \sigma_0}{E_T} \tag{2}$$

$$\varepsilon_e = \frac{\sigma_Y}{E} \tag{3}$$

where ε_T and ε_e are the total strain and elastic strain, respectively; the difference gives the plastic strain as

$$\varepsilon_p = \varepsilon_T - \varepsilon_e = (\sigma_Y - \sigma_0)\frac{E - E_T}{EE_T} \tag{4}$$

from which one obtains the stress-strain relation as

$$\sigma_Y = \sigma_0 + \frac{EE_T}{E - E_T}\varepsilon_p \tag{5}$$

Equating Eq. (5) to Eq. (1) gives the plastic constant as

$$K = \frac{EE_T}{E - E_T} \tag{6}$$

Thus, the tangent modulus is derived as

$$E_T = \frac{KE}{K + E} \tag{7}$$

3. SIMULATION WITH THE FEM

The aluminum metal-matrix composite arch of constant thickness (1mm) is shown in Fig. 1. In this figure, θ represents the angle of fibers. The inner and outer radii are 70mm and 100mm, respectively. The arch has clamped boundary conditions at both ends. In order to simulate elastic, elastic-plastic and residual stress components of σ_x in the arch, the ANSYS finite element analysis software package [18] is used. This software is able to calculate the behavior of composite structures with anisotropic nonlinearities. Four-noded finite strain layered shell element (shell-181) with six degrees of freedom at each node is used for modeling the composite arch with material nonlinearities. In order to mesh the arch for finite element solution, the fine meshing option of ANSYS is chosen for a precise result, and the arch is meshed into 795 elements and 864 nodes. A convergence study is not needed since the mesh is fine more than enough and the CPU time elapsed for the analysis is very short. Schematic finite element model of the arch in ANSYS is shown in Fig. 2. Cartesian coordinates are used for the beam coordinates.

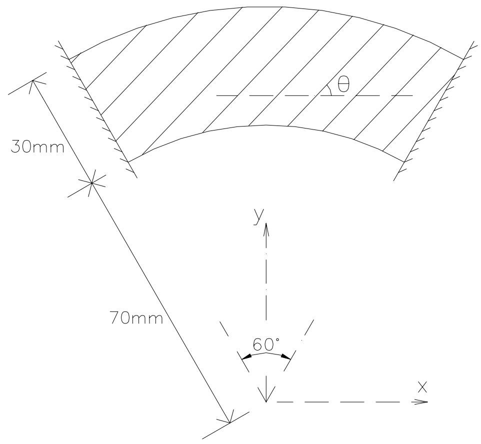

Figure 1. Aluminum metal-matrix composite arch beam.

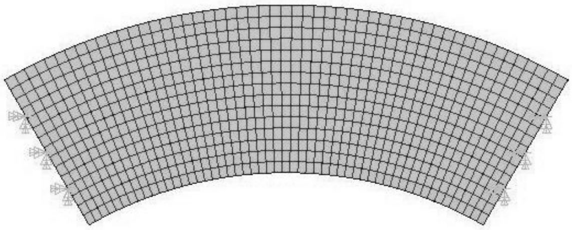

Figure 2. Schematic finite element model in ANSYS

Table 2. Comparison of elastic-plastic, elastic and residual stress components of σ_x in the straight beam (in MPa)

θ	Temp	Sayman [9]						Present FEM solution					
		At the lower surface			At the upper surface			At the lower surface			At the upper surface		
	(°C)	$(\sigma_x)_p$	$(\sigma_x)_e$	$(\sigma_x)_r$	$(\sigma_x)_p$	$(\sigma_x)_e$	$(\sigma_x)_r$	$(\sigma_x)_p$	$(\sigma_x)_e$	$(\sigma_x)_r$	$(\sigma_x)_p$	$(\sigma_x)_e$	$(\sigma_x)_r$
0	159.56	-230.31	247.52	17.21	0	1.62	1.62	-230.05	247.23	17.18	-1.58	3.39	1.81
	175.52	-230.69	261.73	31.05	0	6.48	6.48	-230.40	263.04	32.64	-1.71	8.43	6.72
30	50.73	-71.80	77.20	5.39	0	0.58	0.58	-71.64	75.65	4.01	-0.82	1.68	0.86
	55.80	-71.81	81.61	9.80	0	2.14	2.14	-71.66	82.12	10.46	-1.08	3.56	2.48
45	30.67	-43.09	46.32	3.22	0	0.37	0.37	-42.97	45.79	2.82	-0.56	1.31	0.75
	33.74	-43.09	48.95	5.86	0	1.32	1.32	-42.97	49.27	6.30	-0.70	2.19	1.49
60	21.89	-30.82	33.12	2.30	0	0.27	0.27	-30.73	32.53	1.80	-0.41	1.39	0.98
	24.07	-30.82	35.00	4.18	0	0.95	0.95	-30.73	35.22	4.49	-0.49	2.09	1.60
90	16.84	-24.00	25.79	1.79	0	0.20	0.20	-23.92	24.91	0.99	-0.38	1.04	0.66
	18.53	-24.00	27.25	3.25	0	0.75	0.75	-23.92	27.40	3.48	-0.46	1.39	0.93

For finite element analysis, Tsai-Hill theory is used as the yield criterion due to the same yield points in tension and compression. Since $Y = Z$, the yield condition for this criterion can be written as [19]

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1 \sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \tag{8}$$

To find the residual stresses $(\sigma_x)_r$, a completely elastic stress system due to the temperature $-T$ is superposed on the elastic-plastic stress system due to temperature $+T$. The superposition of the elastic and elastic-plastic solutions gives the residual stress components as follows:

$$(\sigma_x)_r = (\sigma_x)_p + (\sigma_x)_e \tag{9}$$

where $(\sigma_x)_p$ and $(\sigma_x)_e$ are the elastic-plastic and (only) elastic stress components of σ_x .

4. VERIFICATION OF THE FEM SOLUTION

In order to validate the accuracy and applicability of the present FEM solution, the numerical results for an aluminum metal-matrix composite straight beam are compared to the results of Sayman [9] who has performed an analytical solution. The material properties of the beam are as given in Table 1. The beam is fixed by two rigid plates at both ends. Its temperature varies linearly and is zero and T_0 at the upper and lower surfaces, respectively. The length of the beam is 100mm.

The composite beam is analyzed using the four-noded finite strain layered shell element (shell-181) that is also used in the analysis of the composite arch. Comparison of elastic-plastic, elastic and residual stress components of σ_x are given in Table 2. The agreement between the two sets of results is well within the engineering accuracy, as can be seen.

5. RESULTS AND DISCUSSION

It is assumed that uniform temperature acts on the whole composite. First, the temperature was increased from zero to the point at which the first plastic strain occurs; then it was increased by 5°C increments. For all orientations of the fibers, plastic yielding begins at or near the midpoint of the lower surface of the arch. If the arch is fixed to the rigid planes at 0°C without any stress, the temperature (T_0) at which plastic yielding starts at the lower surface is given in Table 3. As seen from the table, it is maximum for 0° orientation angle as 93.90°C due to the highest stiffness and yield point of the beam for this orientation angle. When the orientation angle is increased, the temperature at which plastic yielding starts decreases. It is minimum for 90° orientation angle as 10.31°C.

Table 3. Temperature starting plastic yielding in the arch

θ °	0°	30°	45°	60°	90°
T_0 (°C)	93.90	25.36	16.664	12.63	10.31

Table 4. Elastic, plastic and residual stress components at the lower and upper surfaces of the arch

		At the lower surface			At the upper surface		
θ °	T (°C)	$(\sigma_x)_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_x)_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_x)_r$ (MPa)
0	95	-228,84	233,50	4,66	-0,33	15,30	14,97
	100	-230,09	245,79	15,70	1,66	16,11	17,77
	105	-230,35	258,08	27,73	3,73	16,91	20,64
	110	-230,68	270,37	39,69	6,24	17,72	23,96
	115	-231,14	282,66	51,52	9,02	18,53	27,55
	120	-231,99	294,95	62,96	11,85	19,33	31,18
	125	-232,59	307,24	74,65	14,75	20,14	34,89
30	30	-55,16	63,83	8,67	-2,52	5,24	2,72
	35	-55,29	74,47	19,18	-1,48	6,11	4,63
	40	-55,18	85,11	29,93	0,10	6,99	7,09
	45	-55,03	95,75	40,72	1,62	7,86	9,48
	50	-55,01	106,39	51,38	2,70	8,74	11,44
45	20	-41,43	46,93	5,50	-1,33	3,72	2,39
	25	-41,45	58,66	17,21	0,29	4,65	4,94
	30	-41,49	70,39	28,90	1,58	5,59	7,17
	35	-41,51	82,12	40,61	2,15	6,52	8,67
	40	-41,52	93,85	52,33	2,38	7,45	9,83
	45	-41,54	105,58	64,04	2,52	8,38	10,90
	50	-41,55	117,31	75,76	2,68	9,50	12,18
60	15	-28,70	34,27	5,57	-0,76	2,96	2,20
	20	-28,73	45,70	16,97	1,56	3,95	5,51
	25	-28,75	57,12	28,37	3,26	4,93	8,19
	30	-28,76	68,54	39,78	4,05	5,92	9,97
	35	-28,77	79,96	51,19	4,31	6,91	11,22
90	15	-23,93	35,69	11,76	1,16	3,13	4,29
	20	-23,93	47,59	23,66	4,86	4,17	9,03
	25	-23,94	59,49	35,55	8,47	5,21	13,68
	30	-23,95	71,39	47,44	11,88	6,25	18,13
	35	-23,95	83,28	59,33	15,20	7,30	22,50

Elastic, elastic-plastic and residual stress components at the lower and upper surfaces of the arch for 0°, 30°, 45°, 60°, and 90° orientation angles are given in Table 4. As seen from the table, the residual stress component at the lower surface is always greater than that at the upper surface. If the magnitude of the residual stress component of σ_x at T = 30°C is compared for different orientation angles (except for $\theta = 0°$), one may observe that they increase with greater orientation angles at both lower and upper surfaces. When the temperature is increased, the intensity of residual stress increases at the lower and upper surfaces. It is tensile in all cases.

If the plastic region is increased further, the intensity of residual stress component of σ_x becomes greater. When the stiffness of the beam is further increased, it further resists the thermal expansion and thus, it produces higher residual stresses.

6. CONCLUSIONS

Elastic-plastic thermal stress analysis of composite arch was carried out and the following conclusions were made:

1. Thermal stresses have significance in the design since they cause plastic yielding and failure of the material.
2. In each case plastic yielding occurs first in the lower surface and the intensity of residual stress component of σ_x becomes maximum in the lower surface.
3. The orientation angle θ affects the yield point significantly; the higher the orientation angle, the lower the temperature that causes plastic yielding.
4. Residual stresses can be obtained from the plastic and elastic solutions.
5. Residual stresses are always tensile.

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