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Research Article

A new method for conversion between pythagorean fuzzy sets and intuitionistic fuzzy sets

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ABSTRACT

Modeling with inconsistent fuzzy information is not possible for some problem types. For such cases, pythagorean fuzzy sets (PFSs) cannot be used in problem formulations and a conversion to another fuzzy set extension is needed. As a new conversion between PFSs and intuitionistic fuzzy sets (IFSs), the projective relation was proposed in the literature and its results were compared with the normalization that is the conversion method used by all. However, projective relation conversion is not valid. This conversion is based on the approach of subtraction of the part causing the inconsistency from the membership, non-membership and indeterminacy grades equally. This is not a proper approach because a negative grade is obtained when one of the membership and non-membership grades of PFS is smalller than the equally substracted part. In this study, the error in the proof of the projective relation has been discussed by presenting a counterexample. A new conversion namely "square-scaled normalization" (SSNORM) which converts PFSs to IFSs by rescaling the grades depending on the relative greatness of their squares has been offered and its score and accuracy functions have been formulated. SSNORM method has been examined on a numerical example from the manufacturing industry and the obtained results have been compared with the normalization. Although both methods obtained results close to each other, SSNORM yielded more cautious results. It reached a bigger score function value but a smaller accuracy function value compared to the normalization. SSNORM method can be preferrable alternative of the normalization if the approximation errors caused by the linear rescaling is high.

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INTRODUCTION

The uncertainty is modeled with the term "membership function" and the sum of membership and non-membership grades of each set element is equal to 1 for standard fuzzy sets (FSs) [2]. However, it may not always be equal to 1. Intuitionistic fuzzy sets (IFSs) were offered by Atanassov [3] as a generalization of FSs. The sum of membership and

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$A_N(x)$ be the support against membership of x in $\tilde{A}$ and, $\theta(x) \in [0, \pi/2]$ be a raidan angle. $A_Y(x)$ and $A_N(x)$ are defined as in Eq. (4) and a Pythagorean membership grade represented with a pair of values $r(x)$ and $d(x)$ for each $x \in X$ while $r(x)$ and $d(x)$ are associated with a pair of $A_Y(x)$ and $A_N(x)$ as in Eqs. (5) and (6) [4].

$$A_Y(x) = r(x) \times \cos(\theta(x)), \quad A_N(x) = r(x) \times \sin(\theta(x)) \quad (4)$$

$$r(x) = \sqrt{\left(A_Y^2(x) + A_N^2(x)\right)} \quad (5)$$

$$d(x) = \frac{\pi - 2\theta(x)}{\pi} \quad (6)$$

Definition 4: Let $\tilde{A}$ be a PFS, $r_{\tilde{A}}(x) \in [0,1]$ be the strength of commitment at x in $\tilde{A}$, $A_Y(x)$ be the support for membership of x in $\tilde{A}$, $A_N(x)$ be the support against membership of x in $\tilde{A}$ and, $\theta \in [0,\pi/2]$ be a raidan angle. PFS $\tilde{A}$ is defined as in Eq. (7) when it is written with the same terminology and letter symbols with IFSs and the indeterminacy grade of an element $x \in X$ ($\pi_{\tilde{A}}(x)$) is defined as in Eq. (8).

$$\tilde{A} = \left\{x, \mu_{\tilde{A}}(x) = A_Y(x), \vartheta_{\tilde{A}}(x) = A_N(x) | x \in X\right\}, \quad \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) \le 1, \quad \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) + \pi_{\tilde{A}}^2(x) = 1 \quad (7)$$

$$\pi_{\tilde{A}}(x) = \sqrt{1 - r_{\tilde{A}}^2(x)} = \sqrt{1 - \left(\mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x)\right)} \quad (8)$$

Definition 5: Let $\tilde{A}$ be a PFS. Denotation of $\tilde{A}$ as a pair of values such that $\alpha = \langle \mu_{\tilde{A}} \in [0,1], \vartheta_{\tilde{A}} \in [0,1] \rangle$, $\mu_{\tilde{A}}^2 + \vartheta_{\tilde{A}}^2 \le 1$ is called as Pythagorean fuzzy number (PFN) [5].

ERROR IN TAO ET AL'S PROOF OF THEOREM

Tao et al. [1] presented Theorem 1 (Theorem 3.1 in the original paper) to propose the "projective relation". Nevertheless, this method yields negative grades in some cases. This problem is caused by an error in the proof of Theorem 1.

Theorem 1: Eq. (9) provides a conversion from a Pythagorean fuzzy number (PFN) $\langle \mu_P, \vartheta_P, \pi_P \rangle$ to and intuitionistic fuzzy number (IFN) $\langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ [1].

$$\mu_I^P = \mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \quad \vartheta_I^P = \vartheta_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \quad \pi_I^P = \pi_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \quad (9)$$

Theorem 1 assumes that Eq. 10) is verified for all PFSs. However, it is not satisfied if $\vartheta_P + \pi_P - 1$ is greatet IB O2μ_P.

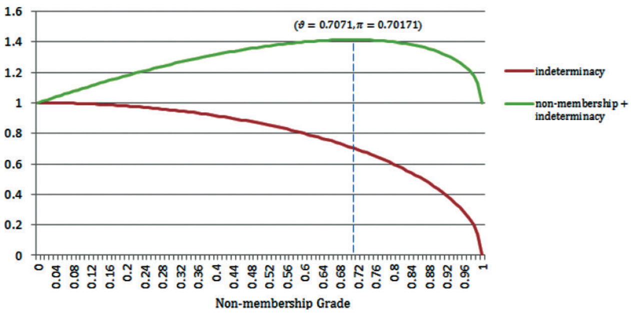

Figure 1. Non-membership and indeterminacy grades when membership is equal to 0.

$$\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3} \ge \frac{2\mu_P - \vartheta_P^2 - \pi_P^2 + 1}{3} \quad (10)$$

Disproof of Theorem 1 has been presented below. Only the problematic part of the theorem has been considered to earn from the space. According to Tao et al [1], Eq. (10) is satif-sied for all PFNs $\tilde{P}$:$\langle \mu_P, \vartheta_P, \pi_P \rangle$.

Disproof: In order to find the lower limit of the statement shown in Eq. (9), membership grade should be minimized, and the sum of the non-membership and inde-terminacy grades should be maximized. Figure 1 shows the maximum value of the sum of the non-membership and indeterminacy grades is equal to 1.4142 when the member ship grade is equal to 0.

$$\lim_{\substack{\mu_P \to 0 \\ (\vartheta_P + \pi_P) \to 2}} \left(\mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}\right) = \lim_{\substack{\mu_P \to 0 \\ (\vartheta_P + \pi_P) \to 2}} \left(\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3}\right) = \frac{0 - 0.7071 - 0.7071 + 1}{3} = -0.138$$

The upper limit of the statement is found by maximizing the membership grade.

$$\lim_{\substack{\mu_P \to 1 \\ (\vartheta_P + \pi_P) \to 0}} \left(\mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}\right) = 1 - \frac{1 + 0 + 0 + 1}{3} = 1$$

Thus, we have:

$$-0.138 \le \mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3} \le 1$$

As seen above, the statement shown in Eq 9 can take negative values. This result shows, the Theorem 1 pre-sented by Tao et all. [1] is not valid. Example 1 supports this finding.

Example 1: Let $\tilde{P} = \langle \mu_P = 0.0866, \vartheta_P = 0.95, \pi_P = 0.3 \rangle$ be a PFN satisfying $0 \le \mu_P^2 + \vartheta_P^2 + \pi_P^2 \le 1$. The projection of $\tilde{P}$ is found as $\langle \mu_I^P = -0.0256, \vartheta_I^P = 0.8378, \pi_I^P = 0.1878 \rangle$ depending on the Eq. (fi). Negative membership grade is unpermitted for IFNs, so the projection of $\tilde{P}$ is not an

IFS. In addition, it does not ensure Eq. (10) too as shown below:

$$\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3} = -0.0256 \ngeq 0.06 = \frac{2\mu_P - \vartheta_P^{\,2} + \pi_P^{\,2} + 1}{3}$$

A NEW CONVERSIONS BETWEEN PYTHAGOREAN FUZZY SETS AND INTUITIONISTIC FUZZY SETS

Normalization is a popular approach to convert non-standard FSs into standard ones. For example, Smarandache [6] has offered to use the normalization to convert the Neutrosophic sets (NSs) into IFSs. Tao et al. [1] have suggested to use it for conversion between PFS and IFSs. The l ogic b ehind t he n ormalization i s s imple. It rescales the membership, non-membership, and indeterminacy grades with a ratio to satisfy the condition given in Eq. (2). Normalization is formulated as in Definition 6.

Definition 6: Let $\tilde{P} = \langle \mu_p, \vartheta_p, \pi_p \rangle$ be a PFN, $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN. Normalization of $\tilde{P}$ defined by Eq. (11) provides a conversion from $\tilde{P}$ to $\tilde{A}$ [1].

$$\mu_I^P = \frac{\mu_P}{\mu_P + \vartheta_P + \pi_P}, \quad \vartheta_I^P = \frac{\vartheta_P(x)}{\mu_P + \vartheta_P + \pi_P}, \quad \pi_I^P = \frac{\pi_P(x)}{\mu_P + \vartheta_P + \pi_P} \tag{11}$$

Normalization scale downs the grades by dividing them with the same ratio. For this reason, it protects the relative greatness of the grades between each other and always produces an IFS. While the main condition of PFSs shown in Eq. (7) is considering the squares of the grades, the normalization rescales the grades by using the sum of them. Accuracy of the normalization approach is negotiable because protecting the relative greatness of the grades. This approach may not always be acceptable for some situations related to PFSs. The definition space [which is limited by Eq. (7)] consumption increases exponentially while a grade is getting bigger. Based on this, rescaling can be done by considering the relative greatness of the squares of the grades as an alternative approach to the normalization. In this way, the rescaling procedure reduces large grades more than small grades. This approach is formulated as in Definition 7.

Definition 7: Let $\tilde{P} = \langle \mu_p, \vartheta_p, \pi_p \rangle$ be a PFN, $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN. Square-scaled normalization of $\tilde{P}$ defined by Eq. (12) provides a conversion from $\tilde{P}$ to $\tilde{A}$.

$$\begin{aligned} \mu_I^P &= \mu_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \mu_P^{\,2} \\ &= \mu_P\left(1 - \mu_P(\mu_P + \vartheta_P + \pi_P - 1)\right), \\ \vartheta_I^P &= \vartheta_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \vartheta_P^{\,2} \\ &= \vartheta_P\left(1 - \vartheta_P(\mu_P + \vartheta_P + \pi_P - 1)\right), \\ \pi_I^P &= \pi_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \pi_P^{\,2} \\ &= \pi_P\left(1 - \pi_P(\mu_P + \vartheta_P + \pi_P - 1)\right) \end{aligned} \tag{12}$$

Theorem 2: Eq. (12) produces an IFN for all PFNs.
Proof: $\mu_I^P + \vartheta_I^P + \pi_I^P = 1$ should be satisfied for all IFNs.

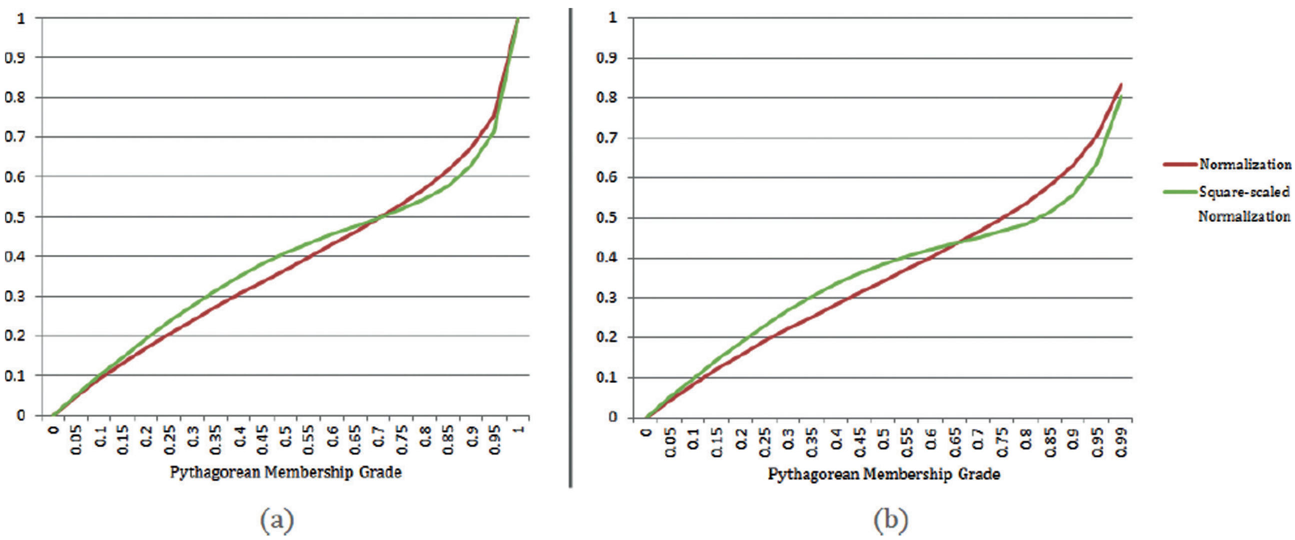

Figure 2. Membership grades for standard and square-scaled normalizations when (a) $\boldsymbol{\pi = 0}$ and (b) $\boldsymbol{\pi = 0.1}$.

company in this section. The company produces pens; and the body of plastic pens are purchased from a supplier. Acceptance sampling procedure has been applied to the pen bodies. Pen bodies might have different types of defects having different significances. For example, while the screw area defects are considered totally defective, major color defects and minor form defects are considered acceptable. The defectiveness levels also vary inside the defect classes thus deciding the defectiveness class of the pen bodies may not be possible because of indetermination.

The parameters of the traditional ASPs are decided by using a single numerical acceptable quality level (AQL) [20]. For this reason, they do not give the ability to model this variability of item defectiveness. Multiple defect types and relevant AQL levels are defined, and multiple separate ASP procedure are conducted in practice as an alternative solution to overcome this limitation [21]. However, there is no need to organize multiple ASPs because the ASP formulation based on IFS proposed by Işık & Kaya [22] fits well with this scenario.

Definition 11: Let $\tilde{p}$ be the fuzzy defective item proportion and, $\tilde{q}$ be the fuzzy non-defective item proportion in a lot, n be the sample size, c be the maximum allowed defective item count, and τ be the maximum allowed indeterminate item count. An intuitionistic fuzzy ASP is a set of rules, in which the defectiveness of the items is an IFS such that $\tilde{A} = \{x, \mu_{\tilde{A}}(x) = \tilde{P}, \vartheta_{\tilde{A}}(x) = \tilde{q} \mid x \in X\}$ satisfying $\tilde{p} + \tilde{q} \leq 1$ with the hesitancy/non-determinacy degree $\tilde{\pi} = 1 - \tilde{p} - \tilde{q}$ [22].

Definition 12: Lot acceptance probability ($\tilde{P}$) and lot rejection probability ($\tilde{P}_r$) for intuitionistic fuzzy ASPs are calculated as shown in Eqs. (15) and (16) [22].

$$\tilde{P}_a = \sum_{d=0}^{c} \binom{n}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{\tau} \binom{n \Theta d}{i} \otimes \left(1 \Theta \tilde{p} \Theta \tilde{q}\right)^i \otimes \tilde{q}^{(n \Theta i \Theta d)} \right] \quad (15)$$

$$\tilde{P}_r = \sum_{d=c \oplus 1}^{n} \binom{n}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{n \Theta d} \binom{n \Theta d}{i} \otimes \left(1 \Theta \tilde{p} \Theta \tilde{q}\right)^i \otimes \tilde{q}^{(n \Theta i \Theta d)} \right] \quad (16)$$

The symbols ⊕, Θ and ⊗ have been used for addition, subtraction, and multiplication operations, respectively related to FSs.

Definition 13: Average outgoing quality $(\widetilde{AOQ})$ and average total inspection $(\widetilde{ATI})$ are calculated for the lots having N items as in Eqs. (17) and (18) [22].

$$\widetilde{AOQ} = \tilde{P}_a \otimes \tilde{P} \quad (17)$$

$$\widetilde{ATI}_{(0)} = n \oplus \left(1 \Theta \tilde{P}_a\right) \otimes \left(N \Theta n\right) \quad (18)$$

In some cases that require working with multiple experts such as purchasing the pen bodies from an alternative supplier in case of urgency, the assessments for the item defectiveness may contain inconsistency. In such cases, the ASP formulation offered by Işık & Kaya [22] cannot be used and conversion to the IFSs is required.

Assume that the company wants to perform ASP having parameters $n = 50$, $c = 5$ and $\tau = 4$ for the lots having 500 items. The experts assessed the defective item proportion as 0.15, non-defective item proportion as 0.98 and indeterminate item proportion as 0.13. For this assessment, the main condition of the PFSs presented in Eq. (7) is satisfied. Membership, non-membership, and indeterminacy grades are found as in Table 1 for NORM and SSNORM.

As seen in Table 1, the defective item proportion have been obtained close to each other for two conversion meth-ods. However, the difference of defectiveness and non-defec-tiveness grades have been obtained bigger for NORM in comparison with SSNORM. The ASP results for the conver-sion methods have been presented in Table 2. SSNORM has

Table 1. Membership, non-membership and indeterminacy grades for the conversion methods

Conversion Method	Defectiveness	Non-defectiveness	Indeterminacy
Normalization	0.1190	0.7778	0.1032
Square-scaled Normalization	0.1441	0.7303	0.1256

Table 2. ASP results for the conversion methods

Conversion Method	Acceptance Probability	Rejection Probability	Average Outgoing Quality	Average Total Inspection
Normalization	0.1598	0.5561	0.2842	0.0190
Square-scaled Normalization	0.0453	0.7453	0.2093	0.0065

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