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Research Article

Intuitionistic fuzzy hypersoft topology and ıts applications to multi-criteria decision-making

Adem YOLCU[1,*]

[1]*Department of Mathematics, Kafkas University, Kars, 36000, Türkiye*

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ABSTRACT

The aim of this paper is to introduce the concept of intuitionistic fuzzy hypersoft topology. Certain properties of intuitionistic fuzzy hypersoft (IFH) topology like IFH basis, IFH subspace, IFH interior and IFH cloure are investigated. Furthermore, the multicriteria decision making (MCDM) algorithms with aggregation operators based on IFH topology are developed. In Algorithm 1 and Algorithm 2, MCDM problem is applied for IFH sets and IFH topology, respectively. Any real-life implementations of the proposed MCDM algorithms are demonstrated by numerical illustrations.

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INTRODUCTION

Decision-making is an everyday task that can be seen as a method of rating a set of alternatives or choosing the best one(s) from them based on the knowledge given by the decision. Multi criteria decision making (MCDM) refers to a decision-making mechanism in which alternatives are measured on the basis of many attributes, representing the success of alternatives from an individual viewpoint. Its goal is to discern the most precise choice from potential alternatives. A provided option needs to be assessed by the person making the decision by different forms of assessment conditions, such as numbers, intervals, etc. However, it is difficult for one person in a variety of cases, as there are different uncertainties within the results, to choose the right one due to lack of competence or violation. As a result, a wide variety of hypotheses have been proposed to quantify those threats and to track the operation.

Fuzzy set theory, initiated by Zadeh [32] in 1965, is an important mathematical method for modeling and controlling uncertainty based on an incremental approach. The idea of fuzzy sets plays a key role in the field of soft computing, which deals with complexity, partial truth, imprecision and approximation in order to achieve durability, robustness and low solution costs. In 1986, Atanassov [4] suggested the notion of intuitionistic fuzzy sets, defined by both membership and non-membership functions. Intuitionistic fuzzy sets expand fuzzy sets in a meaningful

***Corresponding author.**
*E-mail address: yolcu.adem@gmail.com

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where θ and σ are the membership and non-membership value, respectively such that $0\le\theta_{H(a)}(u),\sigma_{H(a)}(u)\le 1$ and $\theta_{H(a)}(u),\sigma_{H(a)}(u)\in 0,1]$. For sake of simplicity, we write the symbols Δ for $E_1\times E_2\times\ldots\times E_n$, Ω for $A_1\times A_2\times\ldots\times A_n$ and α for an element of the set Γ.

Definition 6 [30] i) An intutionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be null intuitionistic fuzzy hypersoft set and denoted by $0_{(U_{IFH},\Delta)}$ if for all $u\in U$ and $\xi\in\Delta$, $\theta_{H(\xi)}(u)=0$ and $\sigma_{H(\xi)}(u)=1$.

ii) An intutionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be absolute intuitionistic fuzzy hypersoft set and denoted by $1_{(U_{IFH},\Delta)}$ if for all $u\in U$ and $\xi\in\Delta$, $\theta_{H(\xi)}(\alpha)=1$ and $\sigma_{H(\xi)}(u)=0$.

Definition 7 [30] Let U be an initial universe set and (H,Ω_1), (H,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. We say that (H,Ω_1) is an intuitionistic fuzzy hypersoft subset of (G,Ω_2) and denote $(H,\Omega_1)\tilde{\subseteq}(G,\Omega_2)$ if

i) $\Omega_1\subseteq\Omega_2$,

ii) For any $\alpha\in\Omega_1$, $H(\alpha)\subseteq G(\alpha)$,

That is for all $u\in U$ and $\alpha\in\Omega_1$, $\theta_{H(a)}(u)\le\theta_{H(g)}(u)$ and $\sigma_{H(\alpha)}(u)\ge\sigma_{G(\alpha)}(u)$.

Definition 8 [30] The complement of intutionistic fuzzy hypersoft set (H,Ω) over the universe U is denoted by $(H,\Omega)^c$ and defined as $(H,\Omega)^c=(H^c,\Omega)$, where $H^c:E_1\times E_2\times\ldots\times E_n)=\Delta\rightarrow IFP(U)$ and $H^c(\Omega)=(H(\Omega))^c$ for all $\Omega\subseteq\Delta$. Thus if

$$(H,\Omega)=\left\{<\alpha,\left(\frac{u}{\theta_{H(\alpha)}(u),\sigma_{H(\alpha)}(u)}\right)>:u\in U,\alpha\in\Omega\right\},\quad\text{then}$$

$$(H,\Omega)^c=\left\{<\alpha,\left(\frac{u}{\sigma_{H(\alpha)}(u),\theta_{H(\alpha)}(u)}\right)>:u\in U,\alpha\in\Omega\right\}.$$

Definition 9 [30] Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. The union of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\tilde{\cup}(G,\Omega_2)=(K,\Omega_3)$ where $\Omega_3=\Omega_1\cup\Omega_2$ and

$$\theta_{K(\alpha)}(u)=\begin{cases}H(\alpha) & \text{if }\ \alpha\in\Omega_1-\Omega_2\\ G(\alpha) & \text{if }\ \alpha\in\Omega_2-\Omega_1\\ \max\left(H(\alpha),G(\alpha)\right) & \text{if }\ \alpha\in\Omega_1\cap\Omega_2\end{cases}$$

$$\sigma_{K(\alpha)}(u)=\begin{cases}H(\alpha) & \text{if }\ \alpha\in\Omega_1-\Omega_2\\ G(\alpha) & \text{if }\ \alpha\in\Omega_2-\Omega_1\\ \min\left(H(\alpha),G(\alpha)\right) & \text{if }\ \alpha\in\Omega_1\cap\Omega_2\end{cases}$$

Definition 10 [30] Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. The intersection of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\tilde{\cap}(G,\Omega_2)=(K,\Omega_3)$ where $\Omega_3=\Omega_1\cap\Omega_2$,

$$(K,\Omega_3)=\left\{<\xi,\left(\frac{u}{\left(\begin{array}{l}\min\left\{\theta_{H(\xi)}(u),\theta_{G(\xi)}(u)\right\},\\ \max\left\{\theta_{H(\xi)}(u),\theta_{G(\xi)}(u)\right\}\end{array}\right)}\right)>:u\in U,\xi\in\Omega\right\}$$

Definition 11 Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. The difference of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\,\tilde{\setminus}\,(G,\Omega_2)=(K,\Omega_3)$ where $(H,\Omega_1)\tilde{\cap}(G,\Omega_2)^c=(K,\Omega_3)$.

Definition 12 [30] Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. The "AND" operation on them is denoted by $(H,\Omega_1)\wedge(G,\Omega_2)=(K,\Omega_3\times\Omega_2)$ is given as;

$$(K,\Omega_1\times\Omega_2)=\left\{\begin{array}{l}<(\alpha_1,\alpha_2),\left(\dfrac{u}{\theta_{K(\alpha_1,\alpha_2)}(u),\sigma_{K(\alpha_1,\alpha_2)}(u)}\right)\\ >:u\in U,(\alpha_1,\alpha_2)\in\Omega_1\times\Omega_2\end{array}\right\}$$

where

$$\theta_{K(\alpha_1,\alpha_2)}(u)=\min\left\{\theta_{H(\alpha_1)}(u),\theta_{G(\alpha_2)}(u)\right\}$$
$$\sigma_{K(\alpha_1,\alpha_2)}(u)=\max\left\{\sigma_{H(\alpha_1)}(u),\sigma_{G(\alpha_2)}(u)\right\}$$

Definition 13 [30] Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. The "OR" operation on them is denoted by $(H,\Omega_1)\vee(G,\Omega_2)=(K,\Omega_1\times\Omega_2)$ is given as;

$$(K,\Omega_1\times\Omega_2)=\left\{\begin{array}{l}<(\alpha_1,\alpha_2),\left(\dfrac{u}{\theta_{K(\alpha_1,\alpha_2)}(u),\sigma_{K(\alpha_1,\alpha_2)}(u)}\right)\\ >:u\in U,(\alpha_1,\alpha_2)\in\Omega_1\times\Omega_2\end{array}\right\}$$

where

$$\theta_{K(\alpha_1,\alpha_2)}(u)=\max\left\{\theta_{H(\alpha_1)}(u),\theta_{G(\alpha_2)}(u)\right\}$$
$$\sigma_{K(\alpha_1,\alpha_2)}(u)=\min\left\{\sigma_{H(\alpha_1)}(u),\sigma_{G(\alpha_2)}(u)\right\}$$

Theorem 1 [30] Let U be an initial universe set, Ω_1, $\Omega_2\subseteq\Delta$ and (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft sets over the universe U. Then De-Morgan Laws are hold.

i) $((H,\Omega_1)\,\tilde{\cup}\,(G,\Omega_2))^c=(H,\Omega_1)^c\,\tilde{\cap}\,(G,\Omega_2)^c$

ii) $((H,\Omega_1)^c\,\tilde{\cap}\,(G,\Omega_2))^c=(H,\Omega_1)^c\,\tilde{\cup}\,(G,\Omega_2)^c$

INTUITIONISTIC FUZZY HYPERSOFT TOPOLOGICAL SPACES

Definition 14 Let $IFHS(U, \Delta)$ be the set of all intuitionistic fuzzy hypersoft subsets over the universe U and $\tilde{\tau} \subseteq IFHS(U, \Delta)$. Then $\tilde{\tau}$ is called a intuitionistic fuzzy hypersoft topology on U if the following condition hold.

1. $0_{(U_{IFH}, \Delta)}$, $1_{(U_{IFH}, \Delta)}$ belong to $\tilde{\tau}$,
2. $(H_1,\Omega_1)\tilde{\cap}(G_2,\Omega_2) \in \tilde{\tau}$ implies $(H_1,\Omega_1)\tilde{\cap}(G_2,\Omega_2)$
3. $\{(H_i,\Omega_i): i \in I\} \subseteq \tilde{\tau}$ implies $\tilde{\cup}_{i\in I}(H_i,\Omega_i) \in \tilde{\tau}$.

Then $(U, \tilde{\tau}, \Delta)$ is called an intuitionistic fuzzy hypersoft topological space over U. The members of $\tilde{\tau}$ are said to be intuitionistic fuzzy hypersoft open sets in U.

An intuitionistic fuzzy hypersoft set (H,Ω) over U is said to be an intuitionistic fuzzy hypersoft closed set if its complement $(H,\Omega)^c$ belongs to $\tilde{\tau}$.

Definition 15 Let $IFHS(U, \Delta)$ be the set of all intuitionistic fuzzy hypersoft subsets over the universe U. Then,

1. If $\tilde{\tau} = \{0_{(U_{IFH}, \Delta)}, 1_{(U_{IFH}, \Delta)}\}$, then $\tilde{\tau}$ is called to be intuitionistic fuzzy hypersoft indiscrete topology and $(U, \tilde{\tau}, \Delta)$ is called to be intuitionistic fuzzy hypersoft indiscrete topological space over the universe U.
2. If $\tilde{\tau} = IFHS(U, \Delta)$, then $\tilde{\tau}$ is called to be intuitionistic fuzzy hypersoft discrete topology and $(U, \tilde{\tau}, \Delta)$ is called to be intuitionistic fuzzy hypersoft discrete topological space over the universe U.

Example 1 Let $U = \{u_1,u_2,u_3\}$ be the universe set and E_1,E_2,E_3 be sets of attributes. E_1,E_2,E_3 are defined as follows;

$$E_1=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\},$$
$$E_2=\{\beta_1,\beta_2,\beta_3\},$$
$$E_3=\{\gamma_1,\gamma_2,\gamma_3\}.$$

Suppose that

$$A_1=\{\alpha_2,\alpha_3\},A_2=\{\beta_1,\beta_3\},A_3=\{\gamma_2\},$$
$$B_1=\{\alpha_2\},B_2=\{\beta_2,\beta_3\},B_3=\{\gamma_1,\gamma_2\}$$

are subset of E_i for each $i = 1,2,3$. Let

$$\tilde{\tau}=\left\{0_{(U_{IFH},\Delta)},1_{(U_{IFH},\Delta)},(H_1,\Omega_1),(H_2,\Omega_2),(H_3,\Omega_3),(H_4,\Omega_4)\right\}$$

be a subfamily of $IFHS(U, \Delta)$ where (H_1,Ω_1), (H_2,Ω_2), (H_3,Ω_3), (H_4,Ω_4), are intuitionistic fuzzy hypersoft sets and defined as follows;

$$(H_1,\Omega_1)=\left\{\begin{array}{c} <(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.2,0.7)},\frac{u_2}{(0.5,0.6)},\frac{u_3}{(0.4,0.2)}\}>, \\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{(0.6,0.3)},\frac{u_3}{(0.5,0.4)}\}>, \\ <(\alpha_3,\beta_1,\gamma_2),\{\frac{u_1}{(0.6,0.6)},\frac{u_2}{(0.8,0.5)}\}>, \\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.5,0.7)},\frac{u_2}{(0.6,0.2)},\frac{u_3}{(0.3,0.5)}\}> \end{array}\right\},$$

$$(H_2,\Omega_2)=\left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{(0.5,0.2)},\frac{u_2}{(0.7,0.3)},\frac{u_3}{(0.3,0.8)}\}>, \\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_2}{(0.1,0.2)},\frac{u_3}{(0.3,0.9)}\}>, \\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{(0.9,0.8},\frac{u_2}{(0.5,0.6)},\frac{u_3}{0.8,0.2}\}>, \\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.8,0.1)},\frac{u_2}{(0.5,0.2)}\}> \end{array}\right\},$$

$$(H_3,\Omega_3)=\left\{\begin{array}{c} <(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.2,0.7)},\frac{u_2}{(0.5,0.6)},\frac{u_3}{(0.4,0.2)}\}>, \\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.8,0.1)},\frac{u_2}{(0.6,0.2)},\frac{u_3}{(0.5,0.4)}\}>, \\ <(\alpha_3,\beta_1,\gamma_2),\{\frac{u_1}{(0.6,0.6)},\frac{u_2}{(0.8,0.5)}\}>, \\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.5,0.7)},\frac{u_2}{(0.6,0.2)},\frac{u_3}{(0.3,0.5)}\}>, \\ <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{(0.5,0.2)},\frac{u_2}{(0.7,0.3)},\frac{u_3}{(0.3,0.8)}\}>, \\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_2}{(0.1,0.2)},\frac{u_3}{(0.3,0.9)}\}>, \\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{(0.9,0.8},\frac{u_2}{(0.5,0.6)},\frac{u_3}{0.8,0.2}\}> \end{array}\right\},$$

$$(H_4,\Omega_4)=\left\{<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{(0.5,0.3)}\}>\right\},$$

Then $\tilde{\tau}$ is a intuitionistic fuzzy hypersoft topology and hence $(U, \tilde{\tau}, \Delta)$ is an intuitionistic fuzzy hypersoft topological space over the universe U.

Remark 1 It is clear that each intuitionistic fuzzy hypersoft topology is also intuitionistic fuzzy soft topology. We consider that Example-1. If we select the parameters from a single attribute set such as E_2 while creating fuzzy hypersoft topology, then the resulting topology becomes intuitionistic fuzzy soft topology. So intuitionistic fuzzy hypersoft topology is generalized version of intuitionistic fuzzy soft topology. Therefore intuitionistic fuzzy hypersoft topology is also intuitionistic fuzzy soft topology. But the reverse is not true.

Proposition 1 Let $(U, \tilde{\tau}_1, \Delta)$ and $(U, \tilde{\tau}_2, \Delta)$ be two intuitionistic fuzzy hypersoft topologies over U.

$$\tilde{\tau}_1\tilde{\cap}\tilde{\tau}_2=\{(H,\Omega):(H,\Omega)\in\tilde{\tau}_1 \text{ and } (H,\Omega)\in\tilde{\tau}_2\}$$

Then $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$ an intuitionistic fuzzy hypersoft topology on U.

Proof. Obviously $0_{(U_{IFH}, \Delta)}$, $1_{(U_{IFH}, \Delta)} \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$ Let (H_1,Ω_1), $(H_2,\Omega_2) \in \tilde{\tau}_1$ and $(H_1,\Omega_1),(H_2,\Omega_2) \in \tilde{\tau}_2$ Note that $\tilde{\tau}_1$ and $\tilde{\tau}_2$

are two intuitionistic fuzzy hypersoft topologies on U. Then $(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2) \in \tilde{\tau}_1$ and $(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2) \in \tilde{\tau}_2$ $(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2) \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$. Let $\{(H_i,\Omega_i): i \in I\} \subseteq \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$. Then $(H_i,\Omega_i) \in \tilde{\tau}_1$ and $(H_i,\Omega_i) \in \tilde{\tau}_2$ for any $i \in I$. Since $\tilde{\tau}_1$ and $\tilde{\tau}_2$ are two intuitionistic fuzzy hypersoft topologies on U, $\tilde{\cup}\{(H_i,\Omega_i): i\in I\} \in \tilde{\tau}_1$ and $\tilde{\cup}\{(H_i,\Omega_i): i\in I\} \in \tilde{\tau}_2$ Thus $\tilde{\cup}\{(H_i,\Omega_i): i\in I\} \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$.

Remark 2 The union of two intuitionistic fuzzy hypersoft topologies over U may not be a intuitionistic fuzzy hypersoft topology. This claim is proven by the following example.

Example 2 We consider that attributes in Example 1. Let

$$\tilde{\tau} = \{0_{(U_{IFH},\Delta)}, 1_{(U_{IFH},\Delta)}, (H_1,\Omega_1)\}$$
$$\tilde{\tau} = \{0_{(U_{IFH},\Delta)}, 1_{(U_{IFH},\Delta)}, (G_1,\Omega_1), (G_2,\Omega_2)\}$$

where

$$(G_1,\Omega_1) = \left\{ \begin{array}{l} <(\alpha_4,\beta_1,\gamma_2), \left\{ \dfrac{u_1}{(0.3,0.8)}, \dfrac{u_2}{(0.4,0.7)} \right\} >, \\ <(\alpha_4,\beta_3,\gamma_2), \left\{ \dfrac{u_2}{(0.5,0.8)}, \dfrac{u_3}{(0.2,0.8)} \right\} >, \\ <(\alpha_1,\beta_3,\gamma_2), \left\{ \dfrac{u_1}{(0.3,0.9)}, \dfrac{u_3}{(0.5,0.2)} \right\} > \end{array} \right\}$$

$$(G_2,\Omega_2) = \left\{ \begin{array}{l} <(\alpha_4,\beta_1,\gamma_2), \left\{ \dfrac{u_1}{(0.5,0.6)}, \dfrac{u_2}{(0.6,0.3)} \right\} >, \\ <(\alpha_1,\beta_3,\gamma_2), \left\{ \dfrac{u_1}{(0.7,0.1)}, \dfrac{u_2}{(0.5,0.5)} \right\} >, \\ <(\alpha_4,\beta_1,\gamma_2), \left\{ \dfrac{u_1}{(0.5,0.4)}, \dfrac{u_2}{(0.7,0.2)} \right\} >, \\ <(\alpha_4,\beta_3,\gamma_2), \left\{ \dfrac{u_1}{(0.6,0.2)}, \dfrac{u_3}{(0.7,0.3)} \right\} > \end{array} \right\}$$

It is clear that $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$ is a intuitionistic fuzzy hypersoft topology. But $(H_1,\Omega_1)\tilde{\cup}(G_2,\Omega_2) \notin \tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2$, then $\tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2$ is not a intuitionistic fuzzy hypersoft topology over the universe U.

Proposition 2 Let $(U, \tilde{\tau}, \Delta)$ be an intuitionistic fuzzy hypersoft topological spaces over U. Then, for any $\alpha \in \Omega$,

$$\tau = \{H(\alpha) : (H,\Omega) \in \tilde{\tau}\}$$

is an intuitionistic fuzzy topology on U.

Proof. (1) $0_{(U_{IFH},\Delta)}$, $1_{(U_{IFH},\Delta)} \in \tilde{\tau}$. In the intuitionistic fuzzy sets, null set $0_{\sim} = (u,(\theta,\sigma)) = (u,(0,1))$ and absolute set $1_{\sim} = (u,(\theta,\sigma)) = (u,(0,1))$. It is clear that the values of null set and absolute set in intuitionistic fuzzy sets equal to the values of null set and absolute set in intuitionistic fuzzy hypersoft sets.Therefore $0_{\sim}$, $1_{\sim} \in \tilde{\tau}$.

(2) Let $G_1, G_2 \in \tilde{\tau}$. Then there exist (H_1,Ω_1), $(H_2,\Omega_2) \in \tilde{\tau}$ such that $G_1 = H_1(\alpha_1)$ and $G_2 = H_2(\alpha_2)$. By $\tilde{\tau}$ be an intuitionistic fuzzy hypersoft topologies on U, $(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2) \in \tilde{\tau}$. Put $(H_3,\Omega_3) = (H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2)$. Then $(H_3,\Omega_3)\in \tilde{\tau}$. Note that $G_1 \cap G_2 = H_1(\alpha_1)\tilde{\cap}H_2(\alpha_2) = (H_3,\Omega_3)$ and $\tau = \{H(\alpha):(H,\Omega) \in \tilde{\tau})\}$. Then $G_1 \cap G_2 \in \tilde{\tau}$.

(3) Let $\{G_i: i \in I\} \subseteq \tilde{\tau}$. Then for every $i \in I$, there exist $(H_i,\Omega_i) \in \tilde{\tau}$ such that $G_i = H_i(\alpha_i)$. By $\tilde{\tau}$ be an intuitionistic fuzzy hypersoft topology on U, $\cup\{(H_i,\Omega_i): i\in I\} \in \tilde{\tau}$. Put $(H,\Omega) = \cup\{(H_i,\Omega_i): i\in I\}$. Then $(H,\Omega) \in \tilde{\tau}$. Note that $\bigcup_{i\in I} G_i = \cup\{(H_i,\Omega_i): i \in I\} = (H,\Omega)$ and $\tau = \{H(\alpha):(H,\Omega) \in \tilde{\tau})\}$. Then $\bigcup_{i\in I} G_i \in \tilde{\tau}$.

Therefore $\tau = \{H(\alpha):(H,\Omega) \in \tilde{\tau})\}$ intuitionistic fuzzy topology on U.

Definition 16 Let $(U, \tilde{\tau}, \Delta)$ be an intuitionistic fuzzy hypersoft topological spaces over U and (H,Ω) be a intuitionistic fuzzy hypersoft set. The intuitionistic fuzzy hypersoft interior of (H,Ω) denoted by $int_{IFH}(H,\Omega)$, is defined by the intuitinistic fuzzy hypersoft union of all intuitionistic fuzzy hypersoft open subsets of (H,Ω).

Clearly, $int_{IFH}(H,\Omega)$ is the largest intuitionistic fuzzy hypersoft open set that is contained in (H,Ω).

Theorem 2 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and (H_1,Ω_1), (H_2,Ω_2) $\in$ IFHS(U,Δ) Then,

1. $int_{IFH}(0_{(U_{IFH},\Delta)}) = 0_{(U_{IFH},\Delta)}$ and $int_{IFH}(1_{(U_{IFH},\Delta)}) = 1_{(U_{IFH},\Delta)}$,
2. $int_{IFH}(H_1,\Omega_1) \tilde{\subseteq} (H_1,\Omega_1)$,
3. (H_1,Ω_1) is an intuitionistic fuzzy hypersoft open set if and only if $int_{IFH}(H_1,\Omega_1)=(H_1,\Omega_1)$,
4. $int_{IFH}(int_{IFH}(H_1,\Omega_1)) = int_{IFH}(H_1,\Omega_1)$,
5. If $(H_1,\Omega_1)\tilde{\subseteq}(H_2,\Omega_2)$, then $int_{IFH}(H_1,\Omega_1) \tilde{\subseteq} int_{IFH}(H_2,\Omega_2)$
6. $int_{IFH}(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2)$, then $int_{IFH}(H_1,\Omega_1)\tilde{\cap}int_{IFH}(H_2,\Omega_2)$.

Proof. 1 and 2 are obvious.

3. Let (H_1,Ω_1) be a intuitionistic fuzzy hypersoft open set. Since $int_{IFH}(H_1,\Omega_1)$ is the largest intuitionistic fuzzy hypersoft open set contained in (H_1,Ω_1), $int_{IFH}(H_1,\Omega_1)=(H_1,\Omega_1)$. Conversely, suppose that $int_{IFH}(H_1,\Omega_1)=(H_1,\Omega_1)$. Since $int_{IFH}(H_1,\Omega_1)$ is an intuitionistic fuzzy hypersoft open set, (H_1,Ω_1) is also intuitionistic fuzzy hypersoft open set.

4. Let $cl_{IFH}(H_1,\Omega_1)=(H_1,\Omega_1)$. Since (H_2,Ω_2) is an intuitionistic fuzzy hypersoft open set $int_{IFH}(H_2,\Omega_2)=(H_2,\Omega_2)$ so $int_{IFH}(int_{IFH}(H_1,\Omega_1)) = int_{IFH}(H_1,\Omega_1)$ is obtained.

5. Let $(H_1,\Omega_1)\tilde{\subseteq}(H_2,\Omega_2)$. $int_{IFH}(H_1,\Omega_1)\tilde{\subseteq}(H_1,\Omega_1)$ and hence $int_{IFH}(H_1,\Omega_1)\tilde{\subseteq}(H_2,\Omega_2)$ also $int_{IFH}(H_2,\Omega_2)$ is the largest intuitionistic fuzzy hypersoft open set contained in (H_2,Ω_2) and $int_{IFH}(H_1,\Omega_1)\tilde{\subseteq} int_{IFH}(H_1,\Omega_1)$.

6. $int_{IFH}(H_1,\Omega_1)\tilde{\subseteq}(H_1,\Omega_1)$ and $int_{IFH}(H_2,\Omega_2)\tilde{\subseteq}(H_2,\Omega_2)$.

Hence $int_{IFH}(H_1,\Omega_1)\tilde{\cap}int_{IFH}(H_2,\Omega_2)\tilde{\subseteq}(H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2)$ Since the largest intuitionistic fuzzy hypersoft open set contained in $(H_1,\Omega_1)\widetilde{\cap}(H_2,\Omega_2)$ is $int_{IFH}[(H_1,\Omega_1)\widetilde{\cap}(H_2,\Omega_2)]$, $int_{IFH}(H_1,\Omega_1)\widetilde{\cap}int_{IFH}(H_2,\Omega_2)\widetilde{\subseteq}int_{IFH}[(H_1,\Omega_1)\widetilde{\cap}(H_2,\Omega_2)]$

Conversely $int_{IFH}(H_1,\Omega_1)\widetilde{\cap}int_{IFH}(H_2,\Omega_2)\widetilde{\cap}int_{IFH}(H_1,\Omega_1)$ and $int_{IFH}(H_1,\Omega_1)\widetilde{\cap}\, int_{IFH}(H_2,\Omega_2)\widetilde{\subseteq}int_{IFH}(H_2,\Omega_2)$

Hence $int_{IFH}[(H_1,\Omega_1)\widetilde{\cap}(H_2,\Omega_2)]\widetilde{\subseteq}int_{IFH}(H_1,\Omega_1)\widetilde{\cap}cl_{IFH}(H_2,\Omega_2)$.

Example 3 We consider the attributes in Example 1. Obviously

$$\tau=\{0_{(U_{IFH},\Delta)}=1_{(U_{IFH},\Delta)},(H_1,\Omega_1),(H_2,\Omega_2),(H_3,\Omega_3),(H_4,\Omega_4)$$

is an intuitionistic fuzzy hypersoft topology on U. Suppose that any $(H_5,\Omega_5)\in IFHS(U,\Delta)$ be defined as follow;

$$(H_5,\Omega_5)=\begin{cases}<(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.3,0.5)},\frac{u_2}{(0.7,0.2)},\frac{u_3}{(0.5,0.1)}\}>,\\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.8,0.1)}\frac{u_2}{(0.8,0.1)},\frac{u_3}{(0.6,0.3)}\}>,\\ <(\alpha_3,\beta_1,\gamma_2),\{\frac{u_1}{(0.7,0.5)},\frac{u_2}{(0.8,0.4)}\}>,\\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.6,0.3)},\frac{u_2}{(0.7,0.1)},\frac{u_3}{(0.4,0.1)}\}>,\\ <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{(0.7,0.1)},\frac{u_2}{(0.9,0.2)},\frac{u_3}{(0.4,0.3)}\}>,\\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{(0.9,0.1)},\frac{u_2}{(0.6,0.5)},\frac{u_3}{(0.8,0.1)}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{(0.9,0.1)},\frac{u_2}{(0.7,0.1)},\frac{u_3}{(0.8,0.1)}\}>\end{cases}$$

Then $0_{(U_{IFH},\Sigma)},(H_1,\Omega_1),(H_2,\Omega_2),(H_3,\Omega_3),(H_4,\Omega_4)\widetilde{\subseteq}(H_5,\Omega_5)$. Therefore

$$int_{IFH}(H_5,\Omega_5)=0_{(U_{IFH},\Sigma)}\widetilde{\cup}(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)\widetilde{\cup}(H_3,\Omega_3)\widetilde{\cup}(H_4,\Omega_4)=(H_3,\Omega_3)$$

Definition 17 Let $(U,\widetilde{\tau},\Delta)$ be an intuitionistic fuzzy hypersoft topological spaces over U and (H,Ω), be a intuitionistic fuzzy hypersoft set. The intuitionistic fuzzy hypersoft closure of (H,Ω), denoted by $cl_{IFH}(H,\Omega)$, is defined by the intuitinistic fuzzy hypersoft intersection of all intuitionistic fuzzy hypersoft closed supersets of (H,Ω).

Clearly, $cl_{IFH}(H,\Omega)$ is the smallest intuitionistic fuzzy hypersoft closed set which contain (H,Ω).

Proposition 3 Let $(U,\widetilde{\tau},\Delta)$ be a intuitionistic fuzzy hypersoft topological space over U. Then the following properties are provide.

1. $0_{(U_{IFH},\Delta)},1_{(U_{IFH},\Delta)}$ are intuitionistic fuzzy hypersoft closed sets over U.
2. The intersection of any number of intuitionistic fuzzy hypersoft closed set is a fuzzy hypersoft set over U.
3. The union of any two intuitionistic fuzzy hypersoft closed set is a fuzzy hypersoft closed set over U.

Proof. (1) For $0_{(U_{IFH},\Delta)}=\in\widetilde{\tau}$ is IFH-open set. $(0_{(U_{IFH},\Delta)})^c=1_{(U_{IFH},\Delta)}$ then $1_{(U_{IFH},\Delta)}$ is IFH closed set. Conversely, $1_{(U_{IFH},\Delta)}\in\widetilde{\tau}$ is IFH open set. Then $(1_{(U_{IFH},\Delta)})=0_{(U_{IFH},\Delta)}$ is IFH closed set.

(2) If $(H_\alpha,\Omega_\alpha)^c\in\widetilde{\tau}$ for $\alpha\in I$ then $\bigcup_{\alpha\in I}(H_\alpha,\Omega_\alpha)^c\in\widetilde{\tau}$. So $\bigcup_{\alpha\in I}(H_\alpha,\Omega_\alpha)^c=\left(\bigcap_{\alpha\in I}(H_\alpha,\Omega_\alpha)\right)^c$, we have $\left(\bigcap_{\alpha\in I}(H_\alpha,\Omega_\alpha)\right)^c\in\widetilde{\tau}$. Hence $\bigcap_{\alpha\in I}(H_\alpha,\Omega_\alpha)$ is a IFH close set over U.

(3) Let $(H_1,\Omega_1),(H_2,\Omega_2)\in\widetilde{\tau}^c$. Then $(H_1,\Omega_1)^c,(H_2,\Omega_2)^c\in\widetilde{\tau}$. Also we can write $((H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2))^c=(H_1,\Omega_1)^c\widetilde{\cap}(H_2,\Omega_2)^c$. According to definition of IFH topology, $((H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2))^c\in\widetilde{\tau}$ and hence $(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)\in\widetilde{\tau}^c$.

Theorem 3 Let $(U,\widetilde{\tau},\Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and $(H_1,\Omega_1),(H_2,\Omega_2)\in IFHS(U,\Delta)$ Then,

1. $cl_{IFH}(0_{(U_{IFH},\Delta)})=0_{(U_{IFH},\Delta)}$ and $cl_{IFH}(1_{(U_{IFH},\Delta)})=1_{(U_{IFH},\Delta)}$,
2. $(H_1,\Omega_1)\widetilde{\subseteq}cl_{IFH}(H_1,\Omega_1)$,
3. (H_1,Ω_1) is a intuitionistic fuzzy hypersoft closed set if and only if $(H_1,\Omega_1)=cl_{IFH}(H_1,\Omega_1)$,
4. $cl_{IFH}(cl_{IFH}(H_1,\Omega_1))=cl_{IFH}(H_1,\Omega_1)$,
5. If $(H_1,\Omega_1)\widetilde{\subseteq}(H_2,\Omega_2)$, then $cl_{IFH}(H_1,\Omega_1)\widetilde{\subseteq}cl_{IFH}(H_2,\Omega_2)$,
6. $cl_{IFH}[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]\widetilde{\cup}cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}cl_{IFH}(H_1,\Omega_1)$.

Proof. 1 and 2 are abvious.

3. Let (H_1,Ω_1) be a intuitionistic fuzzy hypersoft closed set. By (2), we have $(H_1,\Omega_1)\widetilde{\subseteq}cl_{IFH}(H_1,\Omega_1)$. Since $cl_{IFH}(H_1,\Omega_1)$ is the smallest intuitionistic fuzzy hypersoft closed set over U which contain (H_1,Ω_1), then $cl_{IFH}(H_1,\Omega_1)\widetilde{\subseteq}(H_1,\Omega_1)$. Hence $(H_1,\Omega_1)=cl_{IFH}(H_1,\Omega_1)$. Conversely, suppose that $(H_1,\Omega_1)=cl_{IFH}(H_1,\Omega_1)$. Since $cl_{IFH}(H_1,\Omega_1)$ is a intuitionistic fuzzy hypersoft closed set, then (H_1,Ω_1) is closed.

4. Let $(H_1,\Omega_1)=cl_{IFH}(H_1,\Omega_1)$. Then, (H_1,Ω_1) is a fuzzy hypersoft closed set. So, we have $cl_{IFH}(cl_{IFH}(H_1,\Omega_1))=cl_{IFH}(H_1,\Omega_1)$.

5. If $H_1,\Omega_1)\widetilde{\subseteq}(H_2,\Omega_2)$, then $(H_2,\Omega_2)=(H_1,\Omega_1)\widetilde{\cup}(H_1,\Omega_1)\Rightarrow cl_{IFH}(H_2,\Omega_2)=[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]=cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}\,cl_{IFH}(H_2,\Omega_2)\Rightarrow cl_{IFH}(H_1,\Omega_1)\widetilde{\subseteq}\,cl_{IFH}(H_2,\Omega_2)$.

6. Since $(H_1,\Omega_1)\widetilde{\subseteq}(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)$ and $(H_2,\Omega_2)\widetilde{\subseteq}(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)$, from the (5), $(H_2,\Omega_2)\,cl_{IFH}[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]$ and $cl_{IFH}(H_1,\Omega_1)\widetilde{\subseteq}\,cl_{IFH}[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]$. Therefore $cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}\,cl_{IFH}(H_2,\Omega_2)\ \widetilde{\subseteq}\ cl_{IFH}[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]$. Conversely, since $(H_1,\Omega_1)\subseteq cl_{IFH}(H_1,\Omega_1)$ and $(H_2,\Omega_2)\subseteq cl_{IFH}(H_2,\Omega_2)$ are intuitionistic fuzzy hypersoft closed sets, $cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}\,cl_{IFH}(H_2,\Omega_2)$ is a intuitionistic fuzzy hypersoft closed set over U being the union of two intuitionistic fuzzy hypersoft fuzzy soft closed sets. Then,

$$cl_{IFH}\left[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)\right]\widetilde{\subseteq}cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}cl_{IFH}(H_2,\Omega_2).$$

Hence $cl_{IFH}[(H_1,\Omega_1)\widetilde{\cup}(H_2,\Omega_2)]=cl_{IFH}(H_1,\Omega_1)\widetilde{\cup}\,cl_{IFH}(H_2,\Omega_2)$ is obtained.

Example 4 Let us consider the intuitionistic fuzzy hypersoft topology $\tilde{\tau}$ given in Example-1. Suppose that any $(H_5,\Omega_5)\in IFHS(U,\Delta)$ be defined as follow;

$$(H_5,\Omega_5)=\left\{<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.1,0.9)},\frac{u_2}{(0.2,0.6)}\}>\right\}$$

Now we find the complement of intuitionistic fuzzy hypersoft open sets in $\tilde{\tau}$,

$$(H_1,\Omega_1)^c=\left\{\begin{array}{c}<(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.7,0.2)},\frac{u_2}{(0.6,0.5)},\frac{u_3}{(0.2,0.4)}\}>,\\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(1,0)},\frac{u_2}{(0.3,0.6)},\frac{u_3}{(0.4,0.5)}\}>,\\ <(\alpha_3,\beta_1,\gamma_2),\{\frac{u_1}{(0.6,0.6)},\frac{u_2}{(0.5,0.8)},\frac{u_3}{(1,0)}\}>,\\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.7,0.5)},\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.5,0.3)}\}>\end{array}\right\},$$

$$(H_2,\Omega_2)^c=\left\{\begin{array}{c}<(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{(0.2,0.5)},\frac{u_2}{(0.3,0.7)},\frac{u_3}{(0.8,0.3)}\}>,\\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{(1,0)},\frac{u_2}{(0.2,0.1)},\frac{u_3}{(0.9,0.3)}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{(0.8,0.9)},\frac{u_2}{(0.6,0.5)},\frac{u_3}{(0.2,0.8)}\}>,\\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.1,0.8)},\frac{u_2}{(0.2,0.5)},\frac{u_3}{(1,0)}\}>\end{array}\right\},$$

$$(H_3,\Omega_3)^c=\left\{\begin{array}{c}<(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.7,0.2)},\frac{u_2}{(0.6,0.5)},\frac{u_3}{(0.2,0.4)}\}>,\\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.1,0.8)},\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.4,0.5)}\}>,\\ <(\alpha_3,\beta_1,\gamma_2),\{\frac{u_1}{(0.6,0.6)},\frac{u_2}{(0.5,0.8)},\frac{u_3}{(1,0)}\}>,\\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.7,0.5)},\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.5,0.3)}\}>,\\ <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{(0.2,0.5)},\frac{u_2}{(0.3,0.7)},\frac{u_3}{(0.8,0.3)}\}>,\\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{(1,0)},\frac{u_2}{(0.2,0.1)},\frac{u_3}{(0.9,0.3)}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{(0.8,0.9)},\frac{u_2}{(0.6,0.5)},\frac{u_3}{(0.2,0.8)}\}>\end{array}\right\},$$

$$(H_4,\Omega_4)^c=\left\{<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(1,0)},\frac{u_2}{(0.3,0.5)},\frac{u_3}{(1,0)}\}>\right\},$$

$$\left(0_{(U_{FH},\Sigma)}\right)^c=1_{(U_{FH},\Sigma)},\left(1_{(U_{FH},\Sigma)}\right)^c=0_{(U_{FH},\Sigma)}$$

Obviously, $\left(0_{(U_{FH},\Sigma)}\right)^c,\left(1_{(U_{FH},\Sigma)}\right)^c,(\Theta,\Gamma_1)^c,(\Theta,\Gamma_2)^c$ are all fuzzy hypersoft closed sets over $(U, \tilde{\tau}, \Sigma)$. Then $(H_5,\Omega_5)=(0_{(U_{IFH},\Sigma)})^c(H_1,\Omega_1)^c,(H_2,\Omega_2)^c,(H_3,\Omega_3)^c,(H_4,\Omega_4)^c$. Therefore

$$cl_{IFH}(H_5,\Omega_5)=\left\{<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.1,0.8)},\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.4,0.5)}\}>\right\}$$

Theorem 4 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and $(H,\Omega)IFHS(U,\Delta)$. Then,

1. $(cl_{IFH}(H,\Omega))^c = int_{IFH}((H,\Omega)^c)$,
2. $(int_{IFH}(H,\Omega))^c = cl_{IFH}((H,\Omega)^c)$.

Proof. 1.

$$cl_{IFH}(H,\Omega)=\tilde{\cap}\left\{(H,\Omega)\in\tilde{\tau}^c:(H_2,\Omega_2)\tilde{\subset}(H,\Omega)\right\}$$
$$\Rightarrow\left(cl_{IFH}(H,\Omega)\right)^c=\left(\tilde{\cap}\left\{(H,\Omega)\in\tilde{\tau}^c:(H_2,\Omega_2)\tilde{\subset}(H,\Omega)\right\}\right)^c$$
$$=\tilde{\cup}\left\{(H,\Omega)\in\tilde{\tau}:(H,\Omega)^c\tilde{\subset}(H_2,\Omega_2)^c\right\}=int_{IFH}\left((H,\Omega)^c\right)$$
$$int_{FH}(\Theta,\Gamma)=\tilde{\cup}\left\{(H,\Omega)\in\tilde{\tau}:(H,\Omega)\tilde{\subset}(H_2,\Omega_2)\right\}$$
$$\Rightarrow\left(int_{IFH}(H,\Omega)\right)^c=\left(\tilde{\cup}\left\{(H,\Omega)\in\tilde{\tau}:(H,\Omega)\tilde{\subset}(H_2,\Omega_2)\right\}\right)^c$$
$$=\tilde{\cap}\left\{(H,\Omega)\in\tilde{\tau}^c:(H_2,\Omega_2)^c\tilde{\subset}(H,\Omega)^c\right\}=cl_{IFH}\left((H,\Omega)^c\right).$$

Definition 18 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and $\tilde{B}\subseteq\tilde{\tau}$. $\tilde{B}$ is called a intuitionistic fuzzy hypersoft basis for the intuitionistic fuzzy hypersoft topology $\tilde{\tau}$ if every element of $\tilde{\tau}$ can be written as the intuitionistic fuzzy hypersoft union of elements of $\tilde{B}$.

Proposition 4 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and $\tilde{B}$ be intuitionistic fuzzy hypersoft basis for $\tilde{\tau}$. Then $\tilde{\tau}$ equals the collection of intuitionistic fuzzy hypersoft union of elements of $\tilde{B}$.

Proof. The proof is clear from definition of intuitionistic fuzzy hypersoft basis.

Example 5 We consider that the example 1. Then

$$\tilde{B}=\{0_{(U_{IFH},\Delta)},1_{(U_{IFH},\Delta)},(H_1,\Omega_1),(H_2,\Omega_2),(H_4,\Omega_4)\}$$

is a intuitionistic fuzzy hypersoft basis for the intuitionistic fuzzy hypersoft topology $\tilde{\tau}$.

Theorem 5 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and (H,Ω) be a intuitionistic fuzzy hypersoft set over U. Then the collection $\tilde{\tau}_{(H,\Omega)} = \{(H,\Omega)\tilde{\cap}(G_i,\Gamma_i):(G_i,\Gamma_i)\in\tilde{\tau}$ for $i\in I\}$ is a intuitionistic fuzzy hypersoft topology on the intuitionistic fuzzy hypersoft subset (Θ,Γ) relative parameter set Γ.

Proof. $0_{(U_{IFH},\Delta)},1_{(U_{IFH},\Delta)}\in\tilde{\tau}_{(H,\Omega)}$ Besides,

$$\bigcap_{i=1}^{n}((H,\Omega)\tilde{\cap}(G_i,\Gamma_i)) = \left(\bigcap_{i=1}^{n}(G_i,\Gamma_i)\right)\tilde{\cap}(H,\Omega)$$

and

$$\bigcup_{i=I}((H,\Omega)\tilde{\cap}(G_i,\Gamma_i)) = \left(\bigcup_{i=I}(G_i,\Gamma_i)\right)\tilde{\cap}(H,\Omega)$$

for $\tilde{\tau}_{(H,\Omega)} = \{(G_i,\Gamma_i):)i\in I\}$. Therefore, the intuitionistic fuzzy hypersoft union of any number of intuitionistic fuzzy hypersoft set in $\tilde{\tau}_{(H,\Omega)}$ belong to $\tilde{\tau}_{(H,\Omega)}$ and the finite intuitionistic fuzzy hypersoft intersection of intuitionistic fuzzy hypersoft set in $\tilde{\tau}_{(H,\Omega)}$ belong to $\tilde{\tau}_{(H,\Omega)}$. Hence is an intuitionistic fuzzy hypersoft topology on (H,Ω).

Definition 19 Let $(U, \tilde{\tau}, \Delta)$ be a intuitionistic fuzzy hypersoft topological space over U and (H,Ω) be a intuitionistic fuzzy hypersoft set over U. Then the intuitionistic fuzzy hypersoft topology $\tilde{\tau}_{(H,\Omega)} = \{(H,\Omega)\tilde{\cap}(G_i,\Gamma_i):(G_i,\Gamma_i)\in \tilde{\tau}$ for $i \in I\}$is called intuitionistic fuzzy hypersoft subspace topology and $((H,\Omega)\tilde{\tau}_{(H,\Omega)},\Omega)$ is called a intuitionistic fuzzy hypersoft subspace of $(U, \tilde{\tau}, \Delta)$.

Example 6 Let $(U, \tilde{\tau}, \Delta)$ be an intuitionistic fuzzy hypersoft topological space over U and $(H,\Omega)\in IFHS(U,\Delta)$. We consider the intuitionistic fuzzy hypersoft topology in Example 1 and (H,Ω) be defined as follow;

$$(H,\Omega)=\left\{\begin{matrix} <(\alpha_2,\beta_1,\gamma_2),\{\frac{u_1}{(0.2,0.7)},\frac{u_3}{(0.4,0.2)}\}>, \\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{(0.2,0.1)},\frac{u_3}{(0.2,0.6)}\}>, \\ <(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{(0.2,0.6)},\frac{u_2}{(0.5,0.2)},\frac{u_3}{(0.2,0.6)}\}>, \\ <(\alpha_3,\beta_3,\gamma_2),\{\frac{u_1}{(0.6,0.8)},\frac{u_3}{(0.3,0.1)}\}> \end{matrix}\right\}$$

Then the collection

$$\tilde{\tau}_{(H,\Omega)}=\left\{\begin{matrix} 0_{(U_{IFH},\Delta)}\tilde{\cap}(H,\Omega),1_{(U_{IFH},\Delta)}\tilde{\cap}(H,\Omega), \\ (H_1,\Omega_1)\tilde{\cap}(H,\Omega), \\ (H_2,\Omega_2)\tilde{\cap}(H,\Omega),(H_3,\Omega_3)\tilde{\cap}(H,\Omega), \\ (H_4,\Omega_4)\tilde{\cap}(H,\Omega) \end{matrix}\right\}$$

is a intuitionistic fuzzy hypersoft subspace topology and $((H,\Omega)\tilde{\tau}_{(H,\Omega)},\Omega)$ is a intuitionistic fuzzy hypersoft topological subspace of $(U, \tilde{\tau}, \Delta)$.

MCDM PROBLEM BASED ON IFH-TOPOLOGY

There are various kinds of decision-making strategies for selecting the right option. It is sometimes very difficult to choose an effective decision-making strategy in our real life issues with a similar scenario. However, the IFH-topology based MCDM approach plays an enthusiastic role in our everyday lives and this is very helpful in selecting the best alternative. In this section, firstly, the problem is solved using IFH set structure. Later, these IFH sets were accepted as sub-base and a topology was created and the problem was solved again by using the open sets of this topology. As a result, the role of topology in MDCM was obtained by comparing the findings obtained in two ways.

Definition of the problem

Facility location selection is one of the first and most important problems of not only hospitals but also all businesses during the establishment phase. Facility location is a situation that will affect many units, especially suppliers. Therefore, the establishment locations of hospitals are at least as important as having good internal equipment. In this problem, it will be tried to choose the most suitable place for a hospital to be opened.

Let $U = \{u_1,u_2,u_3,u_4,u_5\}$ be a universe set where $u_i \cdot i = \overline{1,5}$ represent locations considered for the hospital establishment site and E_1,E_2,E_3 be the set of attributes. E_1,E_2,E_3 are defined as follows;

E_1 = Land/building cost = $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$,
E_2 = Population density = $\{\beta_1, \beta_2, \beta_3\}$
E_3 = Distance to suppliers = $\{\gamma_1, \gamma_2, \gamma_3\}$

Suppose that A_i and $B_i \cdot i = 1,2,3$ sets be the sets of the choices they have formed based on basic criteria by eliminating different decision makers from all attributes.

$$A_1 = \{\alpha_1\},\ A_2 = \{\beta_3\},\ A_3 = \{\gamma_2,\gamma_3\},$$
$$B_1 = \{\alpha_1,\alpha_2\}, B_2 = \{\beta_3\}, B_3 = \{\gamma_2\}$$

are subset of E_i for each $i = 1,2,3$.

Solving the problem with IFHSs

Algorithm 1

Step-1 : Input the IFH sets $(H_1,\Omega_1),(H_2,\Omega_2)$ over U.
Step-2 : Find resultant intuitionistic fuzzy hypersoft set $(H_1,\Omega_1)\vee(H_2,\Omega_2)$
Step-3 : Construct comparison table of intuitionistic fuzzy hypersoft set and compute row sum (r_i) and column sum (t_i)
Step-4 : Calculate the resulting score R_i of u_i, $\forall i$.
Step-5 : Optimal choice is u_j, that has $\max\{R_i\}$.

Figure-1 shows a brief flow-chart of Algorithm 1.

Suppose that intuitionistic fuzzy hypersoft sets (H_1,Ω_1) and (H_2,Ω_2) defined as follows:

$$(H_1,\Omega_1)=\left\{\begin{matrix} <(\alpha_1,\beta_3,\gamma_2),\{\frac{u_1}{(0.3,0.5)},\frac{u_2}{(0.2,0.8)}\}>, \\ <(\alpha_1,\beta_3,\gamma_3),\{\frac{u_2}{(0.6,0.1)},\frac{u_4}{(0.7,0.5)},\frac{u_5}{(0.3,0.1)}\}> \end{matrix}\right\}$$

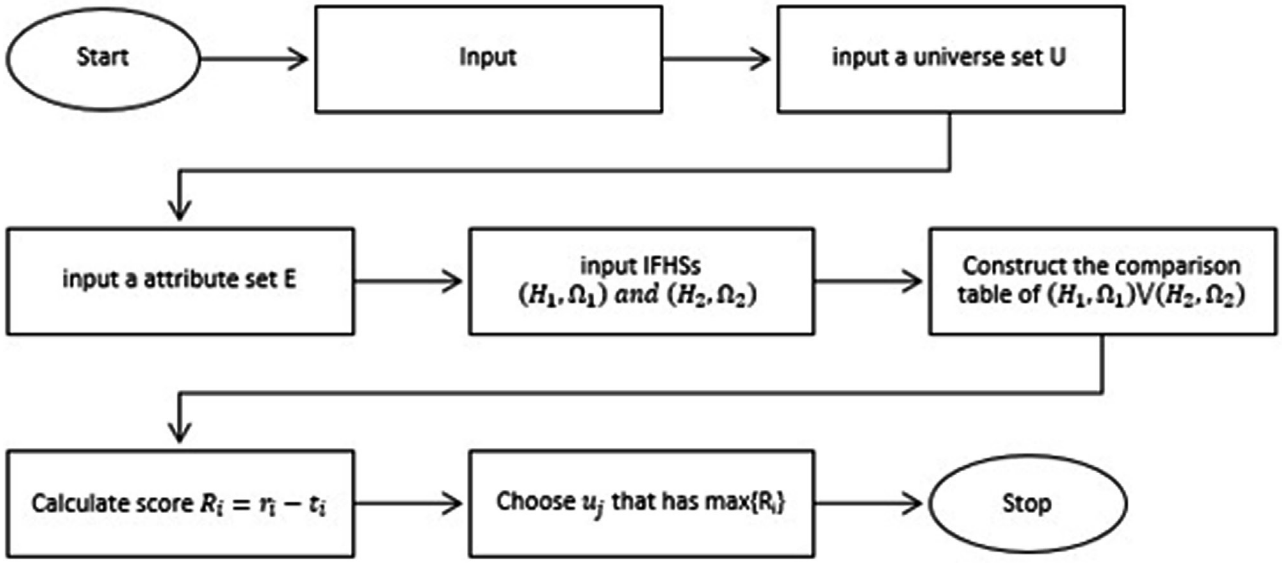

Figure 1. Graphical representation of Algorithm-1.

Table 1: Tabular form of (H_1,Ω_1)

(H_1,Ω_1)	u_1	u_2	u_3	u_4	u_5
$(\alpha_1,\beta_3,\gamma_2) = a_1$	(0.3,0.5)	(0.2,0.8)	(0,1)	(0,1)	(0,1)
$(\alpha_1,\beta_3,\gamma_3) = a_2$	(0,1)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)

Table 2: Tabular form of (H_2,Ω_2)

(H_2,Ω_2)	u_1	u_2	u_3	u_4	u_5
$(\alpha_1,\beta_3,\gamma_2) = b_1$	(0.5,0.7)	(0.4,0.1)	(0,1)	(0,1,0.7)	(0,1)
$(\alpha_2,\beta_2,\gamma_2) = b_2$	(0,1,0.7)	(0.1,0.7)	(0,1)	(0.7,0.3)	(0.4,0.1)

Table 3: Tabular form of $(H_1,\Omega_1) \vee (H_2,\Omega_2)$

$(H_1,\Omega_1)\vee(H_2,\Omega_2)$	u_1	u_2	u_3	u_4	u_5
$(a_1\times b_1) = x_1$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0,1,0.7)	(0,1)
$(a_1\times b_2) = x_2$	(0,3,0.5)	(0.2,0.8)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(a_2\times b_1) = x_3$	(0.5,0.7)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(a_2\times b_2) = x_4$	(0.1,0.7)	(0.5,0.5)	(0,1)	(0.7,0.3)	(0.4,0.1)

$$(H_2,\Omega_2)=\begin{Bmatrix} <(\alpha_1,\beta_3,\gamma_2),\{\frac{u_1}{(0.5,0.7)},\frac{u_2}{(0.4,0.1)},\frac{u_4}{(0.1,0.7)}\}>, \\ <(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{(0.1,0.7)},\frac{u_4}{(0.7,0.3)},\frac{u_5}{(0.4,0.1)}\}>, \end{Bmatrix}$$

The tabular representations of (H_1,Ω_1) and (H_2,Ω_2) are shown in below.

Now, we find resultant intuitionistic fuzzy hypersoft set $(H_1,\Omega_1) \vee (H_2,\Omega_2)$.

Table 4: Comparison table of intuitionistic fuzzy hypersoft set $(H_1,\Omega_1) \vee (H_2,\Omega_2)$

$(H_1,\Omega_1)\vee(H_2,\Omega_2)$	u_1	u_2	u_3	u_4	u_5
u_1	4	1	4	1	1
u_2	2	4	4	0	1
u_3	0	0	4	0	1
u_4	3	1	4	4	1
u_5	2	1	4	0	4

Now we find the comparison table of intuitionistic fuzzy hypersoft set $(H_1,\Omega_1) \vee (H_2,\Omega_2)$ by using the algorithm which is given by Roy and Maji [24]. Comparison table is a square table in which the number of rows and number of columns are equal, rows and columnsboth are labelled by the object names $u_1,u_2,\ldots,u_n$ on of the universe, and the entries are x_i, $i = 1,2,\ldots,n$ given by x_i = the number of parameters for which the membership value of oi exceeds or equal to the membership value of u_i. The comparison table is given below.

Here we calculate the column sum (t_i) and row sum (r_i) after that we calculate the score R_i for each u_i, $i = 1,2,3,4,5$.

According to Table-5, it is clear that the most suitable location for the hospital is u_4. In the next section, the same problem will be solved by constructing the IFH topology and the results will be discussed.

Solving the problem with IFHS-topology

Algorithm 2

Step-1 : Consider a universe of U.

Step-2 : A set E of attributes.

Step-3 : Construct the IFH sets $(H_1,\Omega_1),(H_2,\Omega_2)$ over U.

Step-4 : Write IFH-topology $\bar{\tau}$ in which (H_1,Ω_1) and (H_2,Ω_2) are open IFHs in $\bar{\tau}$.

Step-5 : Find resultant intuitionistic fuzzy hypersoft set $(H_1,\Omega_1)\vee(H_2,\Omega_2)$ and other open IFHs in $\bar{\tau}$ with "OR" operation.

Step-6 : Construct comparison table of intuitionistic fuzzy hypersoft set and compute row sum (r_i) and column sum (t_i)

Step-7 : Calculate the resulting score R_i of u_i, $\forall i$.

Step-8 : Optimal choice is u_j that has $\max\{R_i\}$.

Table 5: Tabular form of score value

	Rowsum(r_i)	Columnsum(t_i)	Score($R_i = r_i - t_i$)
u_1	11	11	0
u_2	11	7	4
u_3	5	20	−15
u_4	12	5	8
u_5	11	8	3

Figure-2 shows a brief flow-chart of Algorithm 2.

Let's build the IFH topology now. We have (H_1,Ω_1), (H_2,Ω_2). Let's create a topology so that these sets are open sets.

$$\tilde{\tau} = \{0_{(U_{IFH},\Delta)}, 1_{(U_{IFH},\Delta)}, (H_1,\Omega_1),(H_2,\Omega_2), (H_3,\Omega_3),(H_4,\Omega_4)\}$$

where $(H_3,\Omega_3),(H_4,\Omega_4)$ defined as follows.

$$(H_3,\Omega_3) = (H_1,\Omega_1)\tilde{\cup}(H_2,\Omega_2) = \left\{\begin{array}{l} <(\alpha_1,\beta_3,\gamma_2),\{\frac{u_1}{(0.5,0.5)},\frac{u_2}{(0.4,0.1)},\frac{u_4}{(0.1,0.7)}\}>, \\ <(\alpha_1,\beta_3,\gamma_3),\{\frac{u_2}{(0.6,0.1)},\frac{u_4}{(0.7,0.5)},\frac{u_5}{(0.3,0.1)}\}>, \\ <(\alpha_1,\beta_2,\gamma_2),\{\frac{u_1}{(0.1,0.7)},\frac{u_4}{(0.7,0.3)},\frac{u_5}{(0.4,0.1)}\}> \end{array}\right\}$$

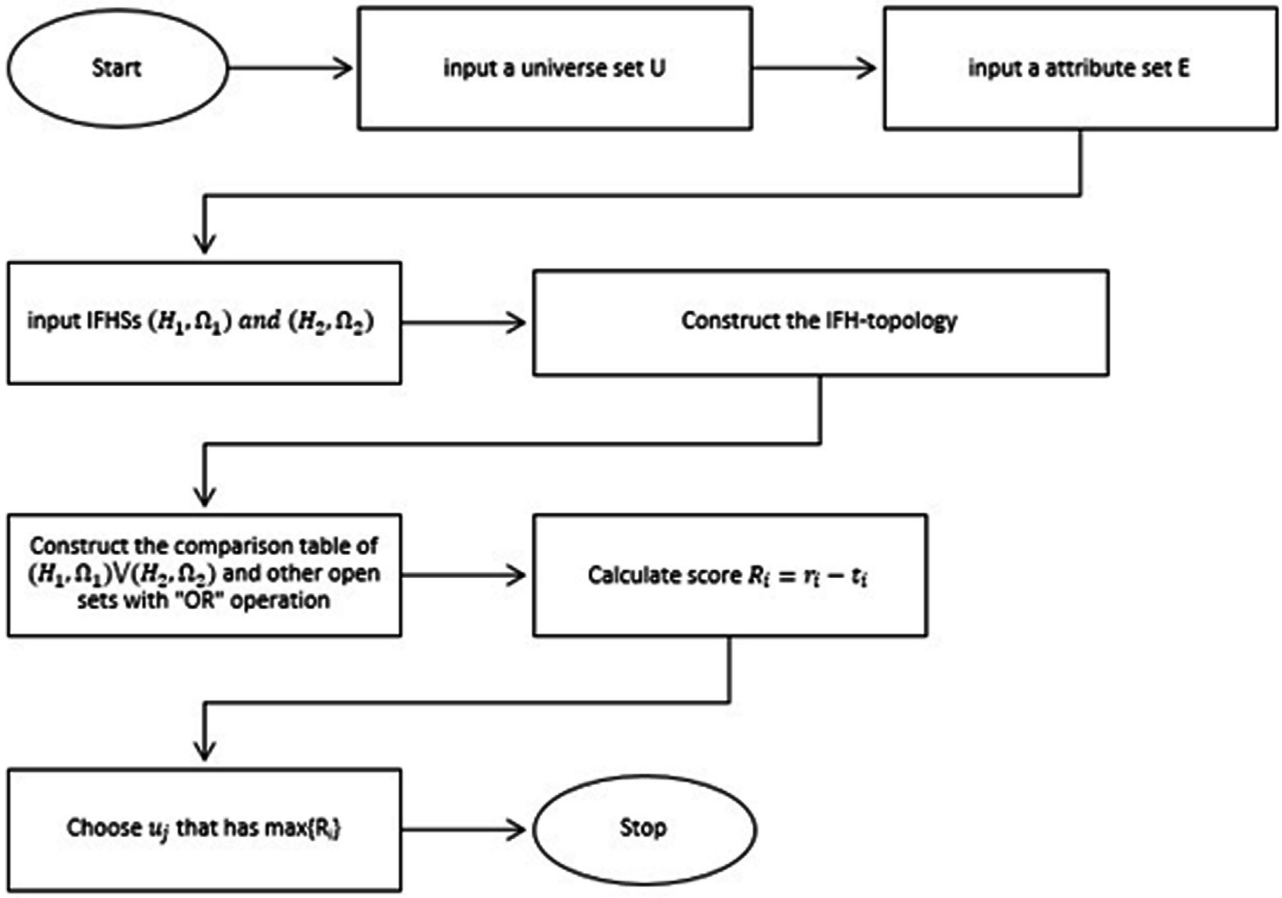

Figure 2. Graphical representation of Algorithm-2.

Table 6: Tabular form of (H_3,Ω_3)

(H_3,Ω_3)	u_1	u_2	u_3	u_4	u_5
$(\alpha_1,\beta_3,\gamma_2) = a_1$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0,1,0.7)	(0,1)
$(\alpha_1,\beta_3,\gamma_3) = a_2$	(0,1)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(\alpha_1,\beta_2,\gamma_2) = a_3$	(0,1,0.7)	(0,1)	(0,1)	(0.7,0.3)	(0.4,0.1)

Table 7: Tabular form of (H_4,Ω_4)

(H_4,Ω_4)	u_1	u_2	u_3	u_4	u_5
$(\alpha_1,\beta_3,\gamma_2) = b_1$	(0.3,0.7)	(0.2,0.8)	(0,1)	(0,1)	(0,1)

Table 8: Tabular form of (H_3,Ω_3)

$(H_3,\Omega_3)\vee(H_4,\Omega_4)$	u_1	u_2	u_3	u_4	u_5
$(a_1 \times b_1) = y_1$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0,1,0.7)	(0,1)
$(a_2 \times b_1) = y_2$	(0,3,0.7)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(a_3 \times b_1) = y_3$	(0.3,0.7)	(0.2,0.8)	(0,1)	(0.7,0.3)	(0.4,0.1)

Table 9: Tabular form of $(H_1,\Omega_1) \vee (H_2,\Omega_2) \vee (H_3,\Omega_3) \vee (H_4,\Omega_4)$

$(H_1,\Omega_1)\vee(H_2,\Omega_2) \vee (H_3,\Omega_3)\vee(H_4,\Omega_4)$	u_1	u_2	u_3	u_4	u_5
$(x_1 \times y_1)$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0,1,0.7)	(0,1)
$(x_1 \times y_2)$	(0.5,0.5)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(x_1 \times y_3)$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_2 \times y_1)$	(0.5,0.5)	(0.4,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_2 \times y_2)$	(0,3,0.5)	(0.6,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_2 \times y_3)$	(0,3,0.5)	(0.2,0.8)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_3 \times y_1)$	(0.5,0.7)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(x_3 \times y_2)$	(0.5,0.7)	(0.6,0.1)	(0,1)	(0.7,0.5)	(0.3,0.1)
$(x_3 \times y_3)$	(0.5,0.5)	(0.6,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_4 \times y_1)$	(0,3,0.7)	(0.6,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_4 \times y_2)$	(0,3,0.7)	(0.6,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)
$(x_4 \times y_3)$	(0.3,0.7)	(0.6,0.1)	(0,1)	(0.7,0.3)	(0.4,0.1)

$$(H_4,\Omega_4) = (H_1,\Omega_1)\tilde{\cap}(H_2,\Omega_2) = \left\{ < (\alpha_1,\beta_3,\gamma_2), \{\frac{u_1}{(0.3,0.7)}, \frac{u_2}{(0.2,0.8)}\} > \right\}$$

The tabular representations of (H_3,Ω_3) and (H_4,Ω_4) are shown in below.

Table 10: Comparison table of intuitionistic fuzzy hypersoft set $(H_1,\Omega_1) \vee (H_2,\Omega_2) \vee (H_3,\Omega_3) \vee (H_4,\Omega_4)$

$(H_1,\Omega_1)\vee(H_2,\Omega_2)$	u_1	u_2	u_3	u_4	u_5
u_1	12	1	12	1	1
u_2	8	12	12	1	11
u_3	0	0	12	0	1
u_4	11	1	12	12	1
u_5	4	3	12	0	12

Table 11: Tabular form of score value

	Rowsum(r_i)	Columnsum(t_i)	Score($R_i = r_i - t_i$)
u_1	27	35	−8
u_2	44	17	27
u_3	13	60	−4
u_4	37	14	23
u_5	31	25	6

Now we calculate $(H_1,\Omega_1) \vee (H_2,\Omega_2) \vee (H_3,\Omega_3) \vee (H_4,\Omega_4)$. We have $(H_3,\Omega_3) \vee (H_4,\Omega_4)$. Then we find $(H_1,\Omega_1) \vee (H_2,\Omega_2)$ now.

Now, we find $(H_1,\Omega_1) \vee (H_2,\Omega_2) \vee (H_3,\Omega_3) \vee (H_4,\Omega_4)$.

Now we find the comparison table of intuitionistic fuzzy hypersoft set $(H_1,\Omega_1) \vee (H_2,\Omega_2) \vee (H_3,\Omega_3) \vee (H_4,\Omega_4)$. The comparison table is given below.

Here we calculate the column sum (t_i) and row sum (r_i) after that we calculate the score R_i for each u_i, $i = 1,2,3,4,5$.

According to Table 11, it is clear that the most suitable location for the hospital is u_2.

RESULT

In this problem, the most suitable place is sought for the selection of the hospital location. When the problem is solved according to Algorithm 1, it is seen that u_4 location is the most suitable place. Then the topology was built so that the sets used in Algorithm 1 are open sets in Algorithm 2. Here, by creating a topology, the finite combination of sets and arbitrary intersection was used first. Therefore, the decision-making problem has been enriched in terms of both parameters and other examined items. While the location is selected over 4 different parameters according to Algorithm 1, the number of parameters handled with the help of the topology in Algorithm 2 has increased to 12. Thus, the problem has been studied in more depth. The findings obtained in Algorithm 1 and Algorithm 2 are compared in the graph below.

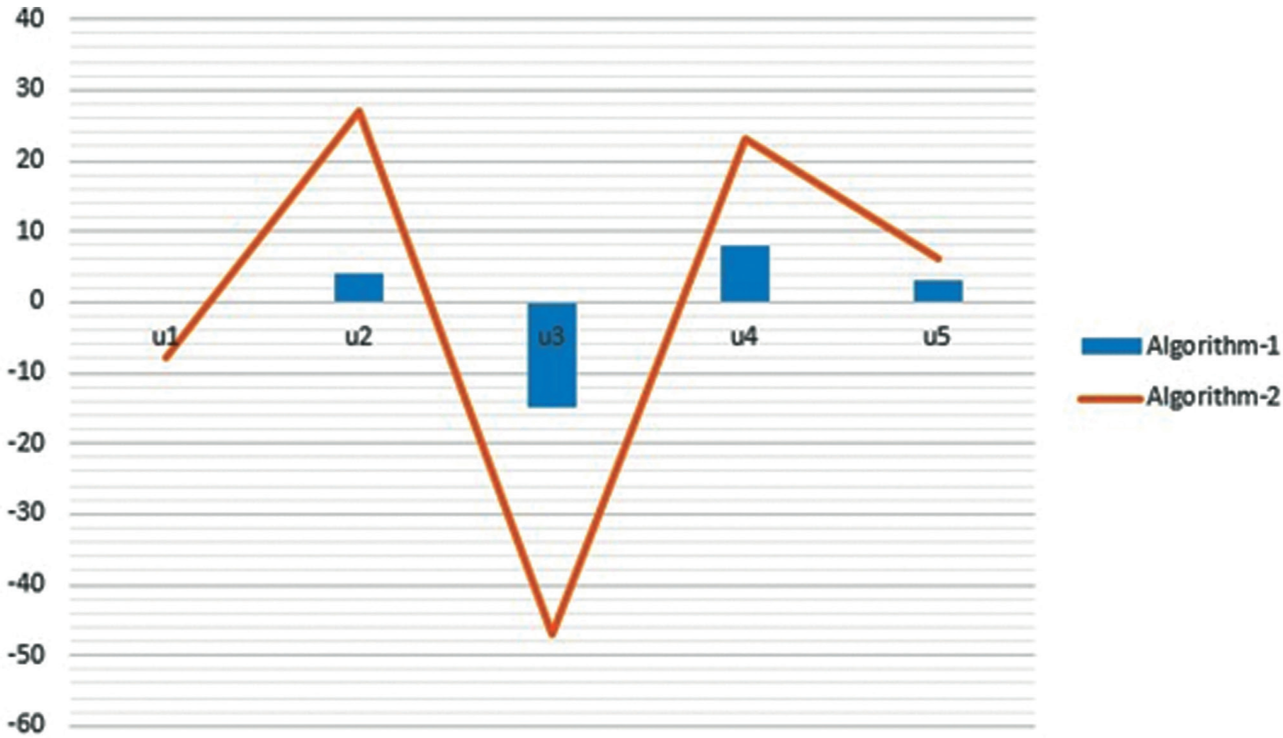

Figure 3. Comparison of Algorithm-1 and Algorithm-2.

When all these findings are examined, it may be more appropriate to choose u_2 as the most suitable location for the hospital.

CONCLUSION

The analysis of hypersoft topological spaces is of considerable significance because it offers a general structure composed of parametrized classical topological spaces. The aim of present paper is to study the concept of intuitionistic fuzzy hypersoft topological spaces. We investigated some properties of intuitionistic fuzzy hypersoft topological spaces and we introduced some notions like that interior, closure, basis, subspace topology on intuitionistic fuzzy hypersoft topological spaces. These are illustrated qith appropriate examples. Additionally, the concept of intuitionistic fuzzy hypersoft topology is extended tp develop multi criteria decision making problems. To better understand the importance of the study, firstly, Algorithm 1 solved the problem using the IFH set structure. Then, the findings obtained by solving the problem with IFH topology (Algorithm 2) were compared. Such a final result was achieved. Continuity, connectedness, compactness and many other topological concepts can be studied on IFH topological spaces as a continuation of this work in the future.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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