Sigma Journal of Engineering and Natural Sciences, Vol. 42, No. 5, pp. XX-XX, October, 2024
Yildiz Technical University Press, Istanbul, Türkiye

Higher order theory based analysis of laminated composite plates using fu nctions trigonometric and trigonometric-hyperbolic

Bouziane BESSAIH[1*], Abdelkader LOUSDAD[1] , Abdelaziz LAIREDJ[2], Abdelmalek ABDELMALEK[2]

ABSTRACT

This work studied in detail for the first time the bending of laminated composite plates subjected t mechanical variations by new theory Trigonometric and Trigonometric-Hyperbolic functions of shear defor mation. From the Euler-Lagrange hypothesis and the equations of the shear deformation theory, we will de velop a present method.One of the most important problems of composite plates is the analysis of their be nding behavior. The correct approach used to study their bending behavior includes two trigonometric and t rigonometric-hyperbolic functions satisfying the null shear stress condition at the free edges. In this paper t he bending problem is solved analytically by developing a computational code and numerically solved by F inite Element Method. In order to simplify the study of the bending behavior, an approach taking into con sideration the effect of the transverse shear deformation without the shear coefficient of correction with on ly four unknowns has been developed while requiring five or more unknowns for other theories. Convergen ce analysis has been carried and the results are compared to open literature available for plate bending analysis . The approach proves to be simple and useful in analyzing the bending behavior of composite layered plates.

Keywords: *Composite Plate; Bending; Trigonometry Theory; Hyperbolic Trigonometry; Finite Element Method; Computer Programming*

Introduction

Laminated composites due to their strength and high specific stiffness are increasingly used in various weight sensitive applications such as automotive, aeronautics and aerospace. Most of these applications have to operate in hostile environments; consequently the components of the structures which are subjected to mechanical stresses. In some cases, the mechanical load turns out to be one of the factors governing their design.

Many studies, based on deterministic analysis, have been carried out on the modeling and analysis of plate bending. The formed laminate plates require precise structural analysis to predict the correct bending behavior. Researchers have developed various plate theories to predict the correct bending behavior of thick plates. Kirchhoff's [1] classical plate theory (CPT) is unsuitable for broad plates due to neglect of transverse shear deformation. The first-order shear deformation theory (FSDT) developed by Mindlin [2] is also inappropriate for analysis because it does not gratify zero stress conditions on the top and bottom surfaces of the plate and the shear dependent on the problem-required correction factors.Reissner [3] developed the FSDT, which takes into description the shear deformation effects. Distinct from the FSDT, the HSDT satisfies the equilibrium conditions on the top and bottom surfaces without using a shear correction factor. In addition, Reddy [4] developed a third-order shear deformation theory (TSDT) using polynomial functions for displacement fields. On the other hand, most of the HSDTs are computationally expensive due to the additional unknowns introduced in the theory context.

In recent times, employing the refined form of the shear deformation theories has been the subject of much research. In the intervening time, different forms of polynomial, trigonometric, hyperbolic, and exponential functions are implemented to investigate the mechanical behavior of different structures for displacement fields [5-10].Analyzing the geometrically nonlinear behavior of laminated composite plates using finite element analysis has been studied by a variety of approaches [11-26].For example, but not limited to valuable works on composite

This paper was recommended for publication in revised form by Regional Editor Ahmet Selim Dalkilic
[1] Laboratory mechanics of structure and solids (LMSS),Mechanical engineering department, Faculty of Technology,Universit yof SidiBel Abbes,
[2] AlgeriaLaboratoryof Mechanical, Modeling and Experimentation,(L2ME),Facultyof Technology,University Tahri Mohammed of Bechar,Algeria
**E-mail address: bouzianemath@gmail.com*
Orcid:https://orcid.org/0009-0007-5922-8148 Bouziane Bessaih, 0000-0001-5430-1798 Abdelkader Lousdad, 0000-0001-6639-9884 Abdelaziz Lairedj, 0009-0009-6255-5988 Abdelmalek Abdelmalek
Manuscript Received 13 October 2022, Revised 17 December 2022, Accepted 05 January 2023

Sigma Journal of Engineering and Natural Sciences, Technical Note, Vol. 42, No. 5, pp. XX-XX, October, 2024

materials [27-39] In the present study, two functions have been included and are made to verify the efficiency of the theory of shear deformation of the most minor variable functions for the analysis of bending, cross-folds, and laminated composite plates. These functions in terms of thickness coordinates are used in the kinematics of the theory to account for the effects of shear deformation. The theory applies the distribution of transverse shear stresses and satisfies the conditions for zero shear stress on the top and bottom surfaces.

The theory does not need a problem-dependent shear correction factor. The governing equations and the boundary conditions are obtained. Using a trigonometric solution to solve the variable equations. Finally, the numerical results obtained are compared with exact solutions in the literature to analyze the bending of laminated composite plates.

Theoretical formulation

Consider the rectangular plate of sides "a" and "b" and of the thickness "h" indicated in Figure1. The plate consists of a number k of homogeneous layers. The plate is subjected to a transverse load q (x, y) on the superior surface of the plate.

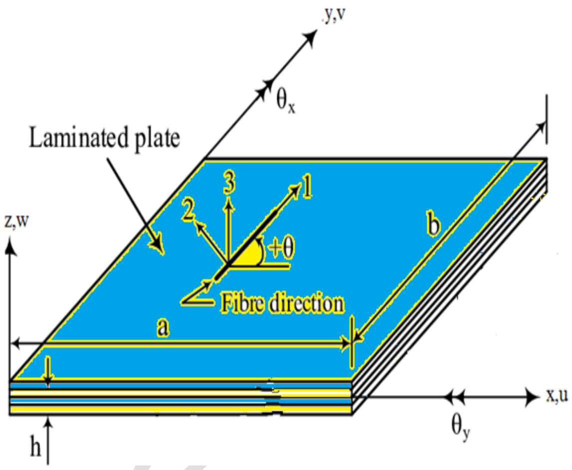

Figure1. Geometry of the laminate plate

The displacements u in x-direction and v in y direction consist of extension (u_0), bending (u_b) and shear components (u_s).

$$u = u_0 + u_b + u_s$$
$$v = v_0 + v_b + v_s \tag{1}$$

The transverse displacement w comprises two components namely: bending (w_b) and shear (w_s)

$$w = w_b + w_s \tag{2}$$

ANALYTICAL SOLUTION

Based on the assumptions mentioned above, the following displacement field associated with the present theory is obtained.

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,z,t) - z\frac{\partial w_b(x,y,t)}{\partial x} - (f(z))\frac{\partial w_s(x,y,t)}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,z,t) - z\frac{\partial w_b(x,y,t)}{\partial y} - (f(z))\frac{\partial w_s(x,y,t)}{\partial y} \\ w(x,y,t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{3}$$

$f(z)$ is replaced by $f_1(z)$ and again by $f_2(z)$

With

$$f_1(z) = z - \frac{h}{\pi}\sin\frac{\pi z}{h}$$
$$g_1(z) = f_1^{'}(z) \tag{3.a}$$
$$f_2(z) = z - \varphi(z)$$
$$g_2(z) = f_2^{'}(z) \tag{3.b}$$

By way of

$$\varphi(z) = z - \frac{2z\sinh\left(\frac{z^2}{h^2}\right)}{2z\sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)} \tag{3.c}$$

The non-zero normal and shear strain components are obtained using the strain displacement relations.

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial x}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial x}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{Bmatrix} - z\begin{Bmatrix}\frac{\partial^2 w_b}{\partial x^2}\\ \frac{\partial^2 w_b}{\partial y^2}\\ 2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix} - f(z)\begin{Bmatrix}\frac{\partial^2 w_s}{\partial x^2}\\ \frac{\partial^2 w_s}{\partial y^2}\\ 2\frac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\\ \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\end{Bmatrix} = g(z)\begin{Bmatrix}\frac{\partial w_s}{\partial y}\\ \frac{\partial w_s}{\partial x}\end{Bmatrix}$$

And

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^k = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}^k \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}^k \quad \text{And} \quad \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}^k = \begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}^k \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^k \tag{5}$$

Where Q_{ij} are the reduced elastic constants of the plane stress in the axes material of the plate, and are defined as:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\vartheta_{21}}, Q_{66} = G_{12}, Q_{55} = G_{13}, Q_{44} = G_{23} \tag{6}$$

Where E_1 , E_2 are the Young's modules along and transverse direction of the fiber G_{12}, G_{13} and G_{23} are the in-plane and transverse shear modules ν_{12} and ν_{21} and the Poisson's ratios. The force and moment resultants of a current theory can be obtained by integrating stresses known by Eq. (5) during the thickness and are as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz; \begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} = \sum_{k=1}^{n} \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z dz;$$

$$\begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \sum_{k=1}^{n} \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} f(z) dz; \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \sum_{k=1}^{n} \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} g(z) dz \tag{7}$$

Where h_k is the thickness ordinate of k layer, the terms (N_x, N_y, N_{xy}) and (M_x^b, M_y^b, M_{xy}^b) are the in-plane force and moment resultants related with the classical plate theory whereas, (Q_x, Q_y) and (M_x^s, M_y^s, M_{xy}^s) are the transverse shear force and moment resultants allied with the transverse shear deformation.

EQUATIONS OF MOTION

The equations of motion governing the coherent variations and the boundary conditions related wit h the existing theory can be derived using the principle of virtual work. The analytical form of the principl e of virtual work can be written as follows:

$$\begin{aligned} &\int_0^a \int_0^b \int_{-h/2}^{h/2} \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} + \tau_{yz} \delta\gamma_{yz} + \tau_{xz} \delta\gamma_{xz} \right] dz dx dy \\ &- \int_0^a \int_0^b q \left(\delta w_b + \delta w_s \right) dy dx \\ &- \int_0^a \int_0^b \left[\begin{array}{l} N_{xx}^0 \dfrac{\partial^2 (w_b + w_s)}{\partial x^2} + N_{yy}^0 \dfrac{\partial^2 (w_b + w_s)}{\partial y^2} \\ + 2 N_{xy}^0 \dfrac{\partial^2 (w_b + w_s)}{\partial x \partial y} \end{array} \right] \left(\delta w_b + \delta w_s \right) dx dy = 0 \end{aligned} \tag{8}$$

Where ∂ is the variation operator .The Integrate of Eq (8), by parts and by collecting the coefficients of $\partial u_0, \partial v_0, \partial w_0$ and ∂w_s the governing equations of equilibrium and the boundary conditions (Euler-Lagrange equations) related to the present theory are obtained by using the fundamental lemma of the calculation of the variation. The equations governing the equilibrium of the plates are as follows:

$$\delta u_0 : \frac{N_x}{\partial x} + \frac{N_{xy}}{\partial y} = 0 \tag{9}$$

$$\delta v_0 : \frac{N_y}{\partial y} + \frac{N_{xy}}{\partial x} = 0 \tag{10}$$

$$\delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_{xx}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial x^2} + N_{yy}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial x \partial y} + q = 0 \tag{11}$$

$$\delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} + N_{xx}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial x^2} + N_{yy}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 \left(w_b + w_s\right)}{\partial x \partial y} + q = 0 \tag{12}$$

By substituting the resultants stress in terms of displacement variables of Eq. (7) in Eqs. (9) – (12), the governing equilibrium equations can be rewritten as follows:

$$\delta u_0 : -A_{11}\frac{\partial^2 u_0}{\partial x^2} - A_{66}\frac{\partial^2 u_0}{\partial y^2} - \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y} + B_{11}\frac{\partial^3 w_b}{\partial x^3} + \frac{\partial Q_{yz}^s}{\partial y} + \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial x \partial y^2} + As_{11}\frac{\partial^3 w_s}{\partial x^3} + \left(As_{12} + 2As_{66}\right)\frac{\partial^3 w_s}{\partial x \partial y^2} = 0 \tag{13}$$

$$\delta v_0 : -\left(A_{12} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y} - A_{22}\frac{\partial^2 v_0}{\partial y^2} - A_{66}\frac{\partial^2 v_0}{\partial x^2} + B_{22}\frac{\partial^3 w_b}{\partial y^3} + \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial y \partial x^2} + As_{22}\frac{\partial^3 w_s}{\partial y^3} + \left(As_{12} + 2As_{66}\right)\frac{\partial^3 w_s}{\partial y \partial x^2} = 0 \tag{14}$$

$$\delta w_b : -B_{11}\frac{\partial^3 u_0}{\partial x^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 u_0}{\partial x \partial y^2} - B_{22}\frac{\partial^3 v_0}{\partial y^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 v_0}{\partial y \partial x^2} + D_{11}\frac{\partial^4 w_b}{\partial x^4} + D_{22}\frac{\partial^4 w_b}{\partial y^4} + 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2 \partial y^2} + Bs_{11}\frac{\partial^4 w_s}{\partial y^4} + 2\left(Bs_{11} + 2Bs_{66}\right)\frac{\partial^4 w_s}{\partial x^2 \partial y^2} Bs_{22}\frac{\partial^4 w_s}{\partial y^2} = q + N_{xx}^0\frac{\partial^2 \left(w_b + w_s\right)}{\partial x^2} + N_{yy}^0\frac{\partial^2 \left(w_b + w_s\right)}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 \left(w_b + w_s\right)}{\partial x \partial y} \tag{15}$$

$$\delta w_s : -As_{11}\frac{\partial^3 u_0}{\partial x^3} - (As_{12}+2As_{66})\frac{\partial^3 u_0}{\partial x \partial y^2} - As_{22}\frac{\partial^3 v_0}{\partial y^3} - (As_{12}+2As_{66})\frac{\partial^3 v_0}{\partial y \partial x^2} + Bs_{11}\frac{\partial^4 w_b}{\partial x^4} +$$
$$Bs_{22}\frac{\partial^4 w_b}{\partial y^4} + 2(Bs_{12}+2Bs_{66})\frac{\partial^4 w_b}{\partial x^2 \partial y^2} + Ass_{11}\frac{\partial^4 w_s}{\partial x^4} + 2(Ass_{12}+2Ass_{66})\frac{\partial^4 w_s}{\partial x^2 \partial y^2} \tag{16}$$
$$+ Ass_{22}\frac{\partial^4 w_s}{\partial y^2} = q + N^0_{xx}\frac{\partial^2 (w_b+w_s)}{\partial x^2} + N^0_{yy}\frac{\partial^2 (w_b+w_s)}{\partial y^2} + 2N^0_{xy}\frac{\partial^2 (w_b+w_s)}{\partial x \partial y}$$

Where A_{ij}, B_{ij}, As_{ij}, D_{ij}, Bs_{ij}, Ass_{ij}, Acc_{ij} are the stiffness coefficients of the laminate which are given as:

$$\{A_{ij}, B_{ij}, As_{ij}, D_{ij}\} = \sum_{k=1}^{n} \overline{Q}_{ij}^{k} \int_{-h/2}^{h_k} \{1, z, f(z), z^2\} dz; (i = j = 1,2,6)$$
$$\{Bs_{ij}, Ass_{ij}\} = \sum_{k=1}^{n} \overline{Q}_{ij}^{k} \int_{-h/2}^{h_k} f(z)\{z, f(z)\} dz; (i = j = 1,2,6) \tag{17}$$
$$\{Acc_{ij}\} = \sum_{k=1}^{n} \overline{Q}_{ij}^{k} \int_{-h/2}^{h_k} [g(z)]^2 dz; (i = j = 4,5)$$

Where

$$g(z) = 1 - f'(z) \tag{18}$$

FLEXURAL ANALYSIS OF LAMINATED COMPOSITE PLATES

The Navier's solution technique is used for bending to analyze the laminated composite plates simply supported on the four edges satisfying the following boundary conditions:

$$\text{At } x = 0 \text{ and } x = a : v_0 = w_b = w_s = M^b_x = M^s_x = 0 \tag{19}$$
$$\text{At } y = 0 \text{ and } y = b : u_0 = w_b = w_s = M^b_y = M^s_y = 0 \tag{20}$$

Following the technique of the Navy's solution, the governing equations of the laminate simply supported by the composite plates in the case of bending analysis are obtained by eliminating the compression loads in the plane $(N^0_{xx}, N^0_{yy}, N^0_{yx})$ resulting from the equations. (13) - (16).

$$-A_{11}\frac{\partial^2 u_0}{\partial x^2} - A_{66}\frac{\partial^2 u_0}{\partial y^2} - (A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x \partial y}$$
$$+ B_{11}\frac{\partial^3 w_b}{\partial x^3} + (B_{12}+2B_{66})\frac{\partial^3 w_b}{\partial x \partial y^2} + \tag{21}$$
$$As_{11}\frac{\partial^3 w_s}{\partial x^3} + (As_{12}+2As_{66})\frac{\partial^3 w_s}{\partial x \partial y^2} = 0$$

$$-\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}-A_{22}\frac{\partial^2 v_0}{\partial y^2}-A_{66}\frac{\partial^2 v_0}{\partial x^2}+B_{22}\frac{\partial^3 w_b}{\partial y^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_b}{\partial y\partial x^2}+ \\ As_{22}\frac{\partial^3 w_s}{\partial y^3}+\left(As_{12}+2As_{66}\right)\frac{\partial^3 w_s}{\partial y\partial x^2}=0 \tag{22}$$

$$-B_{11}\frac{\partial^3 u_0}{\partial x^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2} \\ -B_{22}\frac{\partial^3 v_0}{\partial y^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial y\partial x^2}+D_{11}\frac{\partial^4 w_b}{\partial x^4} \\ +D_{22}\frac{\partial^4 w_b}{\partial y^4}+2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}+Bs_{11}\frac{\partial^4 w_s}{\partial x^4} \\ +2\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}+Bs_{22}\frac{\partial^4 w_s}{\partial y^4}=q \tag{23}$$

$$-As_{11}\frac{\partial^3 u_0}{\partial x^3}-\left(As_{12}+2As_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}-As_{22}\frac{\partial^3 v_0}{\partial y^3}-\left(As_{12}+2As_{66}\right)\frac{\partial^3 v_0}{\partial y\partial x^2} \\ +Bs_{11}\frac{\partial^4 w_b}{\partial x^4}+Bs_{22}\frac{\partial^4 w_b}{\partial y^4}+2\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}+Ass_{11}\frac{\partial^4 w_s}{\partial x^4} \\ +2\left(Ass_{12}+2Ass_{66}\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}+Ass_{22}\frac{\partial^4 w_s}{\partial y^4}=q \tag{24}$$

The plate is subjected to a transverse load q (x, y) on the upper surface, i.e. $z=-h/2$. The transverse load is presented in double trigonometric series as shown in Eq. (25).

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\boldsymbol{\alpha}x\sin\boldsymbol{\beta}y \tag{25}$$

As $\boldsymbol{\alpha}=m\boldsymbol{\pi}/a$, $\boldsymbol{\beta}=n\boldsymbol{\pi}/b$ and q_{mn} is the Fourier expansion coefficient.

If $(m=1, n=1)\Rightarrow$ sinusoidal distributed load $q_{mn}=q_0$.

While q_0 is the maximum load at the center of the plate. The following solution form is assumed for the variables of unknown displacement u_0, v_0, w_b, w_s satisfying exactly the boundary conditions of simply supported plates.

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} u_{mn}\cos\boldsymbol{\alpha}x\sin\boldsymbol{\beta}y \\ v_{mn}\sin\boldsymbol{\alpha}x\cos\boldsymbol{\beta}y \\ w_{bmn}\sin\boldsymbol{\alpha}x\sin\boldsymbol{\beta}y \\ w_{smn}\sin\boldsymbol{\alpha}x\sin\boldsymbol{\beta}y \end{Bmatrix} \tag{26}$$

Also

$u_{mn}, v_{mn}, w_{bmn}, w_{smn}$ are the unknown constants to be determined In case of sinusoidal distributed load, the positive integers are unity $(m=1, n=1)$ .The Substitution of this form of solution and the transverse load $q(x,y)$ in the governing equations (21) - (24) leads to the set of algebraic equations which can be written in matrix form as follows.

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{12} & p_{22} & p_{23} & p_{24} \\ p_{13} & p_{23} & p_{33} & p_{34} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{bmn} \\ w_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_0 \\ q_0 \end{Bmatrix} \tag{27}$$

Where the elements of the stiffness matrix $[P]$ are the following: $P_{12} = (A_{12} + A_{66})\alpha\beta$,

$$\begin{aligned} & p_{11} = A_{11}\alpha^2 + A_{66}\beta^2,\ p_{12} = (A_{12} + A_{66})\alpha\beta,\ p_{13} = -\left[\alpha^3 B_{11} + (B_{12} + 2B_{66})\alpha\beta^2\right], \\ & p_{14} = -\left[\alpha^3 As_{11} + (As_{12} + 2As_{66})\alpha\beta^2\right],\ p_{22} = A_{22}\beta^2 + A_{66}\alpha^2,\ p_{23} = -\left[\beta^3 B_{22} + (B_{12} + 2B_{66})\beta\alpha^2\right] \\ & p_{24} = -\left[\beta^3 As_{22} + (As_{12} + 2As_{66})\beta\alpha^2\right],\ p_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4, \\ & p_{34} = Bs_{11}\alpha^4 + 2(Bs_{12} + 2Bs_{66})\alpha^2\beta^2 + Bs_{22}\beta^4, \\ & p_{44} = Ass_{11}\alpha^4 + 2(Ass_{12} + 2Ass_{66})\alpha^2\beta^2 + Ass_{22}\beta^4 + Acc_{55}\alpha^2 + Acc_{44}\beta^2, \\ & p_{21} = p_{12},\ p_{31} = p_{13},\ p_{32} = p_{23},\ p_{41} = p_{14},\ p_{42} = p_{24},\ p_{43} = p_{34}. \end{aligned} \tag{28}$$

By Opening the solution of Eq. (27), unknown constants $u_{mn}, v_{mn}, w_{bmn}, w_{smn}$ can be obtained.
By means of the constitutive relations (3) - (5). The transverse shear stresses τ_{xy}, τ_{yz} are obtained.
The following material properties are used for bending analysis of simply supported laminated composite plates subjected to a sinusoidal distributed load

$$E_1 = 25E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.25E_2, \quad \nu_{12} = 0.25, \quad \nu_{21} = \frac{E_2}{E_1}\nu_{12} \tag{29}$$

The Displacements and stresses are presented in the following non-dimensional form:

$$\begin{aligned} & \bar{u}\left(0,\frac{b}{2},-\frac{h}{2}\right) = \frac{uE_2h^2}{q_0a^3},\ \bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{100wh^3}{q_0a^4},\ \bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{\sigma_x h^2}{q_0a^2},\ \bar{\sigma}_y\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{\sigma_y h^2}{q_0a^2} \\ & \bar{\tau}_{xy}\left(0,0,-\frac{h}{2}\right) = \frac{\tau_{xy}h^2}{q_0a^2},\ \bar{\tau}_{xz}\left(0,\frac{b}{2},0\right) = \frac{\tau_{xz}h}{q_0a},\ \bar{\tau}_{yz}\left(\frac{a}{2},0,0\right) = \frac{\tau_{yz}h}{q_0a} \end{aligned} \tag{30}$$

Comparative analysis

In this step, based on the mathematical formulations, a computer program with the MATLAB language is developed. In this work we have chosen the Shell 99 element and a 40x40 mesh for symmetry reasons, we modeled only 1/4 of the plate or the Ansys library [40] (version 14.0) offers more than 150 elements of different types defining an application category. These standard elements are differentiated by the number of degrees of freedom applied to each node of the test structure the field of use (structural, mechanical, magnetic, thermal, electrical, etc.) or even if the elements are defined in a 2D or 3D space.

To study the bending behavior of simply supported laminated composite plates using two different function theories, we are interested in comparing the results obtained from two-ply laminated plates of the same thickness and chosen orientation with results available in the literature, illustrated in Table 1 and Figures (2 to 8)

Table 1. Comparison of non-dimensional displacements and stresses for the two layers $\left[0^{\circ}/90^{\circ}\right]$ square composite laminated plate $(b=a)$ subjected to a sinusoidal distributed load.

		Cas1	a/h =4				
Parameter	Ansys Present	Trigonometric Function Present	trigonometric Hyperbolic Function Present	Exact Pagano	HSDT Reddy	FSDT Mindlin	CPT Kirchhoff
$\overline{u}$	0.0120	0.0115	0.0117	……..	0.0114	0.0088	0.0088
$\overline{w}$	2.0911	2.0706	2.0728	2.0670	2.0256	1.9682	1.0636
$\overline{\sigma}_x$	1.0312	1.0139	1.0271	0.8410	0.9172	0.7157	0.7157
$\overline{\sigma}_y$	0.1200	0.1108	0.1129	0.1090	0.0932	0.0843	0.0843
$\overline{\tau}_{xy}$	0.0910	0.0801	0.0819	0.0591	0.0713	0.0525	0.0525
$\overline{\tau}_{xz}$	0.1322	0.1290	0.1301	0.1200	0.1270	0.0910	…….
$\overline{\tau}_{yz}$	0.1452	0.1366	0.1389	0.1350	0.1270	0.0910	……..
		Cas 2	a/h =10				
$\overline{u}$	0.0121	0.0099	0.0101	………	0.0095	0.0088	0.0088
$\overline{w}$	1.2828	1.2483	1.2528	1.2250	1.2479	1.2083	1.0636
$\overline{\sigma}_x$	0.8510	0.8008	0.8100	0.7302	0.7652	0.7157	0.7157
$\overline{\sigma}_y$	0.1100	0.0909	0.1045	0.0886	0.0889	0.0843	0.0843
$\overline{\tau}_{xy}$	0.0811	0.0698	0.0709	0.0535	0.0680	0.0525	0.0525
$\overline{\tau}_{xz}$	0.1414	0.1351	0.1386	0.1210	0.1310	0.0910	………
$\overline{\tau}_{yz}$	0.1416	0.1343	0.1370	0.1250	0.1310	0.0910	……….

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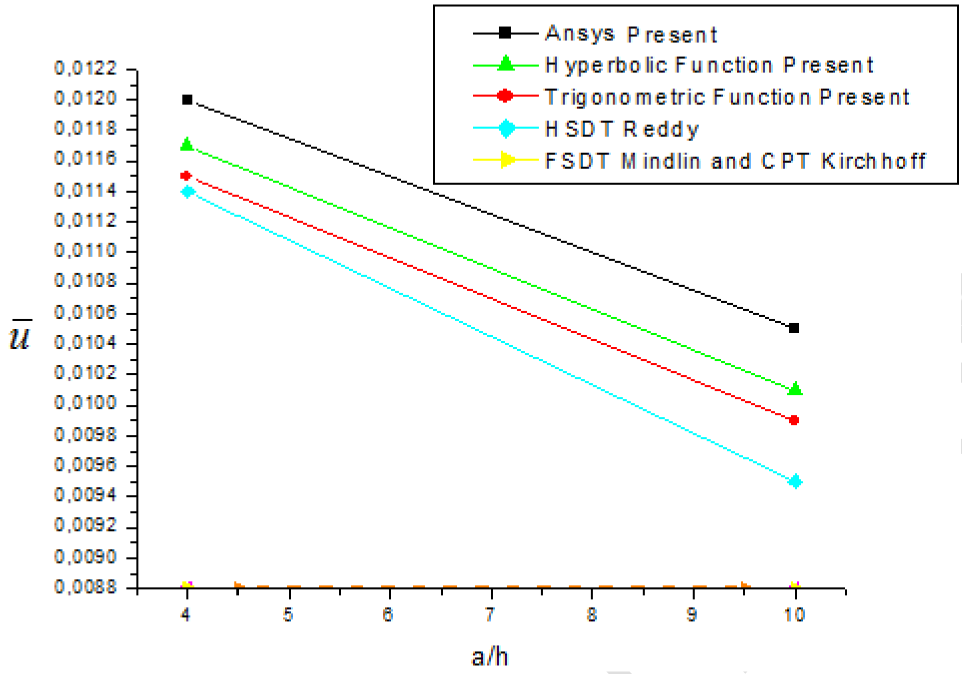

Figure2. Comparison of non-dimensional displacement $\bar{u}$ for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

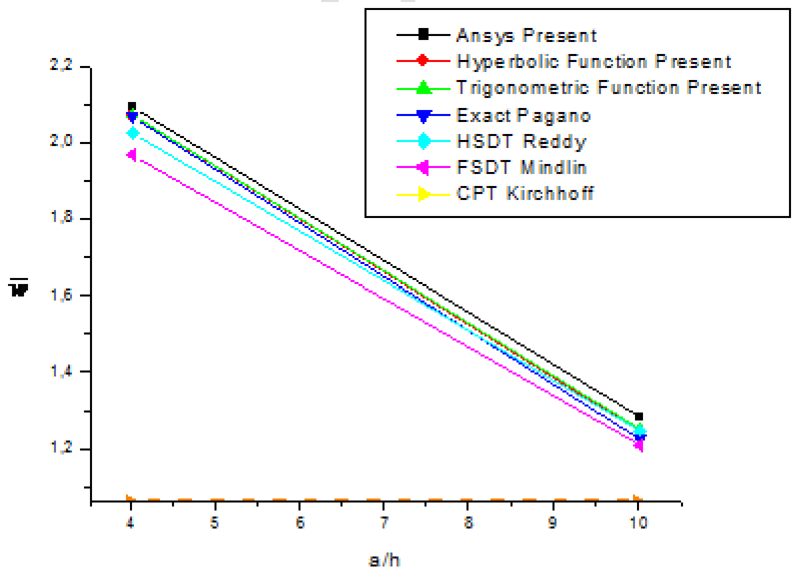

Figure3. Comparison of non-dimensional displacement $\bar{w}$ for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

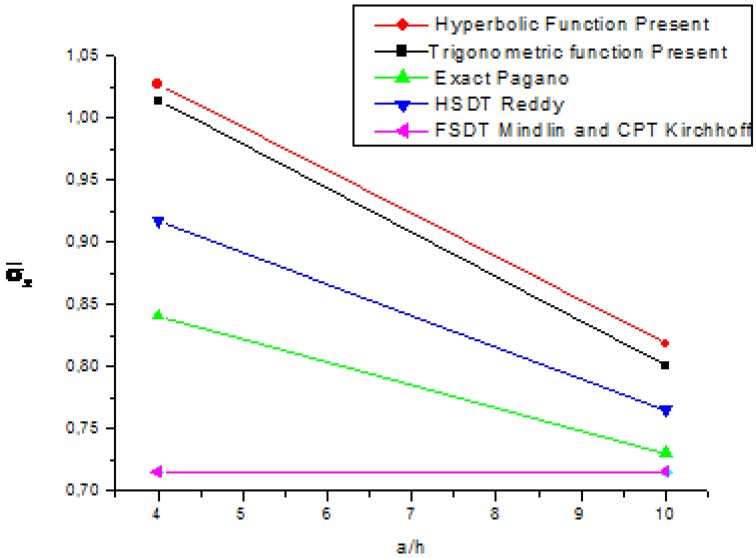

Figure4. Comparison of non-dimensional stress for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

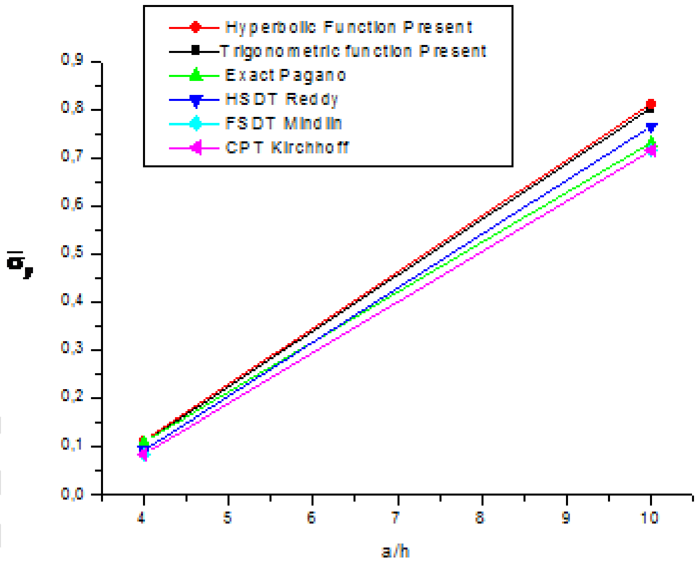

Figure5. Comparison of non-dimensional stress for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

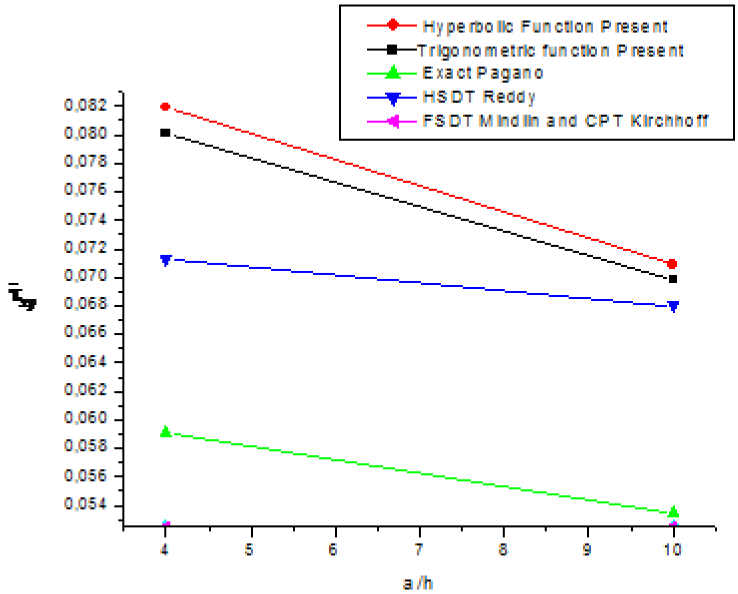

Figure 6: Comparison of non-dimensional stress for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

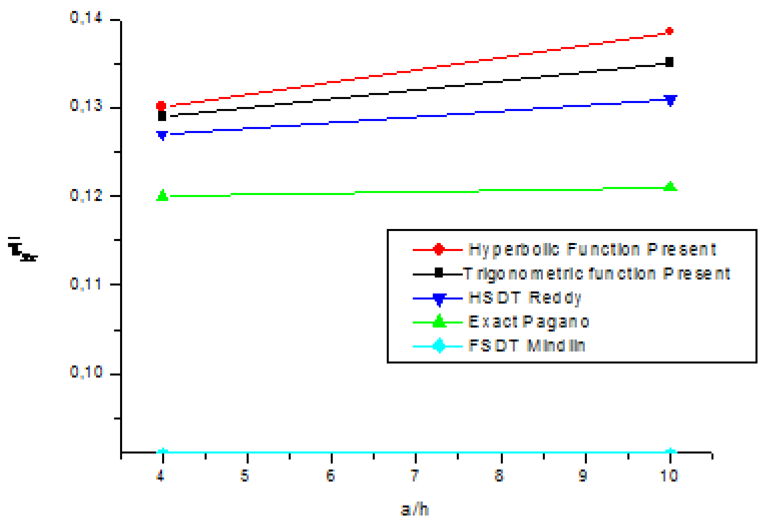

Figure7. Comparison of non-dimensional stress for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

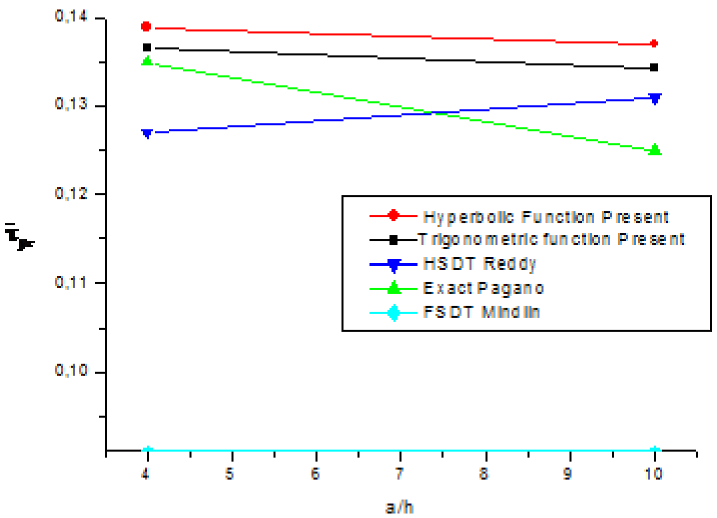

Figure8. Comparison of non-dimensional stress for the two layers$[0^{\circ}/90^{\circ}]$ square composite laminated plate $(b = a)$ subjected to a sinusoidal distributed load.

Discussion

The applicability of the proposed method for analyzing plates laminated with one is demonstrated, using a $\left[0^{\circ}/90^{\circ}\right]$ laminated plate under several sets of boundary conditions. The plate has a length / thickness ratio a/h and an equality of width / length ratio $(b = a)$, and is subjected to a sinusoidal transverse load distribution as defined in the equation. Note that simple types of supports are used in these examples. The results mentioned above indicate excellent agreement between the current results and those obtained by other solutions from authors indicated on the figures. Many analyzes are performed in this study by using a finite element model of the plate . The model was developed using linear layered structural shell elements in ANSYS 14.0. From the results of a simply supported two-ply symmetrical laminated composite plate it was observed that the bending is greater for this chosen modulus ratio .A comparison of the same with that of the literature values of Reddy, Pagno and Mindlin in respect of normal displacement are in good agreement. The present solution gives about 0.5% higher values in comparison with the results of Reddy, Pagno and Mindlin.

Conclusion

The study conducted in this article sheds light on the mechanical behavior of laminated plates subjected to bending. The approach developed and the results obtained significantly contribute to the study of the bending of laminated plates made of composite materials having an anisotropic mechanical behavior. Results for deflections and stresses of the laminated composite plate as a function of thickness ratios are obtained.

The calculations of the approximate solutions (displacement and stresses) are carried out by a program developed in MATLAB. For the second case by the numerical approach, the checking and the validation of the results are made by the computer code (ANSYS).

The absence of taking into account the transverse shearing also constitutes an important effect on the behavior in bending the plates. The results obtained were compared with the literature and it can be said that they are in good agreement.

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