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Research Article

Analzying the relation between soft sets and topological polygroups

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ABSTRACT

In this article, we first provide some examples for soft topological polygroups, then define the closure of a soft topological polygroup and state the necessary condition for the closure of a soft topological polygroup to be a soft topological polygroup itself. We present some more characterizations of soft topological polygroups. In particular, explain the relation between soft sets and complete parts in a polygroup. Furthermore, write the different compositions of the closure of soft topological polygroups. Finally, introduce another definition for soft topological polygroups, which we will work on it in future.

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INTRODUCTION

Classical methods can not be used successfully to solve complex problems in economics, engineering, environment, medicine and etc. due to uncertainties. There are three ways to solve such problem. The first is probability theory, the second is fuzzy set theory, and the third is interval mathematics.

Molodstov in [16] introduced the concept of soft set theory as a mathematical tool for solving problems dealing with ambiguity and uncertainty. He stated that soft set theory has enough parameters to provide an approximate description of the problem. We intend to incorporate soft sets into classical mathematical structures to increase the efficiency of these structures to cover uncertainties.

In this regard, many researchers have worked and were our guiding light, some of which we can mention.

B. Davvaz has provided many example of polygroups in [6] that we have used to construct examples of soft topological polygroups. D. Heidari has defined the topological polygroup in [11] and has established a nice relationship with complete parts. T. Shah in [18] has introduced soft topological groups and studied their properties. G. Oguz in [18] has defined the soft topological polygroups for the first time. In [17], different examples of soft topological polygroups are given.

You can see the concept of soft sets and soft rings in [3] by H. Aktas and N. Cagman. F. Feng and Y. M. Li has specifically identified in [9] soft subsets and soft product operations. T. Hida defines soft topological groups differently in [12], which is inspiring. T. Hida has beautifully introduced the concepts of soft continuous, soft topological polygroups, soft Hausdorff space, soft open covering, soft compact, soft connected similar to classical analysis. Other related results are found in [7,8].

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Preliminaries

Soft sets

Let U be an initial universe and $\mathcal{P}^*(U)$ denotes the power set of U. Suppose that $\mathcal{P}(U)$ be power set without $\emptyset$ and E is a set of parameters. Also A is a subset of E and it is not empty. A pair $(\mathbb{F}, A)$ is said to be a soft set over U, where $\mathbb{F} : A \to \mathcal{P}(U)$ is a map.

Let $(\mathbb{F}, A)$ and $(\mathbb{G}, B)$ are soft sets over U in this case, we have the following compliments:

Use the symbol $(\mathbb{F}, A) \hat{\subset} (\mathbb{G}, B)$ for soft subset if $A \subseteq B$ and $\mathbb{F}(a)$, $\mathbb{G}(a)$ are identical approximations for all $a \in A$ $(F(a) \subseteq G(a))$.

Use the symbol $(\mathbb{F}, A) \hat{\subset} (\mathbb{G}, B)$ for soft superset if $(\mathbb{G}, B) \hat{\subset} (F, A)$.

Use the symbol $(\mathbb{F}, A) \hat{=} (\mathbb{G}, B)$ for soft equal if $(\mathbb{F}, A) \hat{\subset} (\mathbb{G}, B)$ and $(\mathbb{G}, B) \hat{\subset} (\mathbb{F}, A)$.

Use the symbol $(\mathbb{F}, A) \hat{\cap} (\mathbb{G}, B)$ for Bi-intersection if $(\mathbb{F}, A) \hat{\cap} (\mathbb{G}, B) = (\mathrm{H}, C)$, where $C = A \cap B$ and $\mathrm{H}(a) = \mathbb{F}(a) \cap \mathbb{G}(a)$ for all $a \in C$.

Use the symbol $(\mathbb{F}, A) \hat{\cup} (\mathbb{G}, B)$ for soft union if $(\mathbb{F}, A) \cup (\mathbb{G}, B) = (\mathrm{H}, C)$, where $C = A \cup B$ and

$$\mathbb{H}(a) = \begin{cases} \mathbb{F}(a) & a \in A - B \\ \mathbb{G}(a) & a \in B - A \\ \mathbb{F}(a) \cup \mathbb{G}(a) & a \in A \cap B. \end{cases}$$

Use the symbol $(\mathbb{F}, A) \hat{\wedge} (G, B)$ for soft AND if $(\mathbb{F}, A) \hat{\wedge} (\mathbb{G}, B) = (\mathrm{H}, A \times B)$, where $\mathrm{H}((a, b)) = \mathbb{F}(a) \cap \mathbb{G}(b)$ for all $(a, b) \in A \times B$.

Use the symbol $(\mathbb{F}, A) \hat{\vee} (G, B)$ for soft OR if $(\mathbb{F}, A) \hat{\vee} (G, B) = (\mathbb{I}, A \times B)$, where $\mathbb{I}((a, b)) = \mathbb{F}(a) \cup \mathbb{G}(b)$ for all $(a, b) \in A \times B$.

Use the symbol $Supp(\mathbb{F}, A)$ for the support of the soft set $(\mathbb{F}, A)$, where $Supp(\mathbb{F}, A) = \{ a \in A : \mathbb{F}(a) \neq \emptyset\}$. If the support of the soft set $(\mathbb{F}, A)$ is not equal to the empty set we say that $(\mathbb{F}, A)$ is non-null.

If $\mathbb{F}(a) = U$ for all $a \in A$, then we denote $\mathbb{F}$ with $\hat{A}$.

Topological Hyperstructure

Let H be a non-empty set. Then the couple $(H,\circ)$ is called a hypergroupoid if $\circ: H \times H \longmapsto \mathcal{P}(H)$ be a map.

The combination of two subsets A and B of H, it is as follows;

$A \circ B = \cup_{a \in A} a \circ B$ and $a \circ B = \cup_{b \in B} a \circ b$.

In a hypergroupoid $(H,\circ)$ if for every $\mathrm{h} \in H$ we have: $\mathrm{h} \circ H = H = H \circ \mathrm{h}$, then $(H,\circ)$ is called a quasihypergroup and if for every $t, u, w \in H$ we have: $t \circ (u \circ w) = (t \circ u) \circ w$ then $(H,\circ)$ is called a semihypergroup.

The pair $(H,\circ)$ is called a hypergroup if it is a quasihypergroup and a semihypergroup [5,15].

Let $(H,\circ)$ be a semihypergroup and A be a subset of H. Say that A is a complete part of H if for any $n \in \mathrm{N}$ and for all $a_1, \dots, a_n$ of H, the following implication holds:

$$A \cap \prod_{i=1}^{n} a_i \neq \emptyset \Longrightarrow \prod_{i=1}^{n} a_i \subseteq A.$$

The complete parts were introduced for the first time by Koskas [14].

A map $f : G \longrightarrow H$, is called a homomorphism, where $(G,\circ)$ and $(H,*)$ be two hypergroups, if for all x, y in G, we have $f(x \circ y) \subseteq f(x) * f(y)$; and is called strong homomorphism if for all x, y in G, we have $f(x \circ y) = f(x) * f(y)$; f is an isomorphism if f, f^{-1} are strong homomorphisms.

Let $(P,\circ)$ be hypergroup and have other additional features. If there exist unitary operation $^{-1}$ on P and $e \in P$ with the property that for all $p, q, r \in P$, the following items hold.

(i) $e \circ p = p \circ e = p$;

(ii) If $p \in q \circ r$, then $q \in p \circ r^{-1}$ and $r \in q^{-1} \circ p$.

In this case, the hypergroup P is called polygroup. The following results follow from the above axioms:

(i) $e \in p \circ p^{-1} \cap p^{-1} \circ p$,

(ii) $e^{-1} = e$,

(iii) $(p^{-1})^{-1} = p$,

(iv) $(p \circ q)^{-1} = q^{-1} \circ p^{-1}$.

A nonempty subset Q of a polygroup P is called a subpolygroup of P if and only if for all $x, y \in Q$, it follows that $x \circ y \subseteq Q$ and for all $x \in Q$, it follows that $x^{-1} \in Q$.

The subpolygroup N of P is normal in P if and only if $a^{-1} \circ N \circ a \subseteq N$ for all $a \in P$.

Theorem 2.1. [6] If N be a normal subpolygroup of P, then:

(i) $Na = aN$ for all $a \in P$;

(ii) $(aN)(bN) = abN$ for all $a, b \in P$;

(iii) $aN = bN$ for all $b \in aN$.

Example 1: Let P be $\{1, 2\}$ and hyperoperation ø act:

※	1	2
1	1	2
2	2	{1, 2}

With the above multiplication table, P is a polygroup [6].

Let P is polygroup and $(\mathbb{F}, A)$ be a soft set on P. Then $(\mathbb{F}, A)$ is called a (normal)soft polygroup on P if $\mathbb{F}(x)$ is a (normal)subpolygroup of P for all $x \in Supp(\mathbb{F}, A)$.

Example 2. Let $P = \{e, a, b, c\}$ be a set with multiplication table as follows:

∘	e	a	b	c
e	e	a	b	c
a	a	$\{e, a\}$	c	$\{b, c\}$
b	b	c	e	a
c	c	$\{b, c\}$	a	$\{e, a\}$

In conclusion P is a polygroup. Let A be equal P and $(\mathbb{F}, A)$ be a soft set over P. Define $F : A \longmapsto \mathcal{P}(P)$ by $\mathbb{F}(x) = \{ y \in P \mid x\boldsymbol{R}y \Leftrightarrow y \in x^2\}$ for all $x \in A$. Above all $\mathbb{F}(e) = \mathbb{F}(b) = \{ e \}$ and $\mathbb{F}(a) = \mathbb{F}(c) = \{e, a\}$ are subpolygroups of P. In conclusion, $(\mathbb{F}, A)$ is a soft polygroup over P [21].

Suppose that $\mathcal{T}$ is a topology on G, where G is a group, then $(G, \mathcal{T})$ is called a topological group over G if φ and $^{-1}$ are continuous, where:

(1) $\varphi : G \times G \longmapsto G$ is defined by $\varphi(g, \mathrm{h}) = g\mathrm{h}$ and $G \times G$ is endowed with the product topology.

(2) $^{-1} : G \longmapsto G$ is defined by $^{-1}(g) = g^{-1}$ [10].

Let $(\mathbb{F}, A)$ be a soft set defined over G. Then the $(\mathbb{F}, A, \mathcal{T})$ is called soft topological group over G if the following conditions hold:

(1) $\mathbb{F}(a)$ is a subgroup of G for all $a \in A$.

(2) The mapping φ: $(x, y) \longmapsto xy$ of the topological space $\mathbb{F}(a) \times \mathbb{F}(a)$ onto $\mathbb{F}(a)$ is continuous for all $a \in A$.

(3) $^{-1}$: $\mathbb{F}(a) \longmapsto \mathbb{F}(a)$ is defined by $^{-1}(g) = g^{-1}$ is continuous for all $a \in A$.

In [11], it has been proved that condition continuity φ is equivalent to following statement;

If $U \subseteq G$ is open, and $g\mathrm{h} \in U$, then there exist open sets V_g and V_{h} with the property that $g \in V_g$, $\mathrm{h} \in V_{\mathrm{h}}$, and $V_g V_{\mathrm{h}} = \{v_1 v_2 \mid v_1 \in V_g, v_2 \in V_{\mathrm{h}}\} \subseteq U$.

Also, condition continuity $^{-1}$ is equivalent to following statement; If $U \subseteq G$ is open, then $U^{-1} = \{ g^{-1} \mid g \in U\}$ is open.

Let $(H, \mathcal{T})$ be a topological space. The following theorem gives us a topology on $\mathcal{P}(H)$ that is induced by $\mathcal{T}$.

Theorem 2.2. Let $(H, \mathcal{T})$ be a topological space, this means that $\mathcal{T}$ is a topology on hypergroup. Then the family β consisting of all sets $S_V = \{U \in \mathcal{P}(H) \mid U \subseteq V, U \in \mathcal{T}\}$ is a base for a topology on $\mathcal{P}(H)$. This topology is denoted by $\mathcal{T}^*$[13].

Let $(H, \mathcal{T})$ be a topological space, where $(H, \circ)$ is a hypergroup. Then the triple $(H, \circ, \mathcal{T})$ is called a topological hypergroup if the following functions are continuous:

(1) φ: $(x, y) \longmapsto x \circ y$, from $H \times H \longmapsto \mathcal{P}(H)$;

(2) ψ: $(x, y) \longmapsto x/y$, from $H \times H \longmapsto \mathcal{P}(H)$, where $x/y = \{ z \in H \mid x \in z \circ y\}$;

Let $(P, \mathcal{T})$ be a topological space, where $(P, \circ, e, ^{-1})$ is a polygroup. Then $(P, \circ, e, ^{-1}, \mathcal{T})$ is called a topological polygroup (in short TP) if the following axioms hold:

(1) The mapping $\circ$: $P \times P \longmapsto \mathcal{P}(P)$ is continuous, where $\circ\,(x, y) = x \circ y$;

(2) The mapping $^{-1} : P \longmapsto P$ is continuous, where $^{-1}(x) = x^{-1}$.

We can combine items (1), (2) and peresent the following case;

The mapping φ: $P \times P \longmapsto \mathcal{P}(P)$, where $\varphi\,(x, y) = x \circ y^{-1}$ be continuous.

Below theorem helps us to recognize continuous hyperoperation. We use the following thorem to test the continuity of the hyperoperation "$\circ$".

Theorem 2.3. Let P is a polygroup. Then the hyperoperation $\circ$: $P \times P \longmapsto \mathcal{P}(P)$ is continuous if and only if $\forall\, a, b \in P$ and $C \in \mathcal{T}$ with the property that $a \circ b \subseteq C$ then there exist $A, B \in \mathcal{T}$ with the property that $a \in A$ and $b \in B$ and $A \circ B \subseteq C$ [11].

SOFT TOPOLOGICAL POLYGROUPS

In this section, we present the definition of soft topological polygroups according to [18] and follow the continuation of contents [17].

Definition 3.1. Let $\mathcal{T}$ be a topology on a polygroup P and $(\mathbb{F}, A)$ be a soft set over P. Then the system $(\mathbb{F}, A, \mathcal{T})$ said to be soft topological polygroup over P if the following axioms hold:

a) $\mathbb{F}(a)$ is a subpolygroup of P for all $a \in A$.

b) The mapping $(x, y) \longmapsto x \circ y$ of the topological space $\mathbb{F}(a) \times \mathbb{F}(a)$ onto $\mathcal{P}(\mathbb{F}(a))$ and the mapping $x \longmapsto x^{-1}$ of the topological space $\mathbb{F}(a) \longmapsto \mathbb{F}(a)$ are continuous for all $a \in A$.

We can express condition (b) of Definition 3.1 as follows;

The mapping $(x, y) \longmapsto x \circ y^{-1}$ of the topological space $\mathbb{F}(a) \times \mathbb{F}(a)$ onto $\mathcal{P}(\mathbb{F}(a))$ is continuous for all $a \in A$. Topology $\mathcal{T}$ on P induces topologies on $\mathbb{F}(a)$, $\mathbb{F}(a) \times \mathbb{F}(a)$ and by Theorem 2.2 on $\mathcal{P}(\mathbb{F}(a))$.

Example 3. In [6] we can see how $\overline{D_4}$ is made, the multiplication table of the polygroup $\overline{D_4}$ is as follows.

$\circ$	C_1	C_2	C_3	C_4	C_5
C_1	C_1	C_2	C_3	C_4	C_5
C_2	C_2	C_1	C_3	C_4	C_5
C_3	C_3	C_3	C_1, C_2	C_5	C_4
C_4	C_4	C_4	C_5	C_1, C_2	C_3
C_5	C_5	C_5	C_4	C_3	C_1, C_2

As an example, we explain how to calculate $C_3 \circ C_3$. To show this product compute the element-wise product of the conjugacy classes $\{r, t\}\{r, t\} = \{ s, 1\} = C_1 \cup C_2$ Furthermore $C_3 \circ C_3$ consists of the two conjugacy classes $\{C_1, C_2\}$. By Theorem 2.3 hyperoperation $\circ$: $\overline{D_4} \times \overline{D_4} \longmapsto \mathcal{P}(\overline{D_4})$ is not continuous with down topologies:

$\mathcal{T}_1 = \{ \emptyset, \overline{D_4}, \{C_1\}\}$ $\mathcal{T}_2 = \{\emptyset, \overline{D_4}, \{C_2\}\}$ $\mathcal{T}_3 = \{ \emptyset, \overline{D_4}, \{C_3\}\}$

$\mathcal{T}_4 = \{ \emptyset, \overline{D_4}, \{C_4\}\}$ $\mathcal{T}_5 = \{ \emptyset, \overline{D_4}, \{C_5\}\}$ $\mathcal{T}_6 = \{ \emptyset, \overline{D_4}, \{ C_1, C_2\}\}$

$\mathcal{T}_7 = \{ \emptyset, \overline{D_4}, \{C_1, C_3\}\}$ $\mathcal{T}_8 = \{ \emptyset, \overline{D_4}, \{C_1, C_4\}\}$ $\mathcal{T}_9 = \{ \emptyset, \overline{D_4}, \{ C_1, C_5\}\}$

$\mathcal{T}_{10} = \{ \emptyset, \overline{D_4}\}, \{ C_2, C_3\}\}$ $\mathcal{T}_{11} = \{ \emptyset, \overline{D_4}, \{ C_2, C_4\}\}$ $\mathcal{T}_{12} = \{ \emptyset, \overline{D_4}, \{ C_2, C_5\}\}$

$\mathcal{T}_{13} = \{ \emptyset, \overline{D_4}\}, \{ C_3, C_4\}\}$ $\mathcal{T}_{14} = \{ \emptyset, \overline{D_4}\}, \{ C_3, C_5\}\}$ $\mathcal{T}_{15} = \{ \emptyset, \overline{D_4}, \{ C_4, C_5\}\}$

$\mathcal{T}_{16} = \{ \emptyset, \overline{D_4}, \{C_1, C_2, C_3\}\}$ $\mathcal{T}_{17} = \{ \emptyset, \overline{D_4}, \{C_1, C_2, C_4\}\}$ $\mathcal{T}_{18} = \{ \emptyset, \overline{D_4}\}, \{ C_1, C_2, C_5\}\}$

$\mathcal{T}_{19} = \{ \emptyset, \overline{D_4}, \{C_2, C_3, C_4\}\}$ $\mathcal{T}_{20} = \{ \emptyset, \overline{D_4}, \{C_2, C_3, C_5\}\}$ $\mathcal{T}_{21} = \{ \emptyset, \overline{D_4}, \{ C_3, C_4, C_5\}\}$

$\mathcal{T}_{22} = \{ \emptyset, \overline{D_4}, \{C_1, C_2, C_3, C_4\}\}$ $\mathcal{T}_{23} = \{ \emptyset, \overline{D_4}, \{C_1, C_2, C_3, C_5\}\}$ $\mathcal{T}_{24} = \{ \emptyset, \overline{D_4}, \{C_2, C_3, C_4, C_5\}\}$

Nonetheless $(\overline{D_4}, \mathcal{T}_{dis})$ and $(\overline{D_4}, \mathcal{T}_{ndis})$ are topological polygroups and Subpolygroups of $\overline{D_4}$ are $\emptyset$,

$\overline{D_4}$, $\{C_1\}$, $\{C_1, C_2\}$, $\{C_1, C_2, C_3\}$, $\{C_1, C_2, C_4\}$, $\{C_1, C_2, C_5\}$. Let A be a arbitrary set and $a_1, a_2, a_3, a_4 \in A$. Define a soft set $\mathbb{F}$ by

$$\mathbb{F}(x) = \begin{cases} \{C_1\} & x = a_1 \\ \{C_1, C_2\} & x = a_2 \\ \{C_1, C_2, C_3\} & x = a_3 \\ \{C_1, C_2, C_4\} & x = a_4 \\ \emptyset & otherwise \end{cases}$$

Consider $\mathcal{T}_5 = \{\emptyset, \overline{D_4}, \{C_5\}\}$. In conclusion $(\mathbb{F}, A, \mathcal{T}_5)$ is a soft topological polygroup.

In [6], Davvaz explained a method to get a polygroup from two existing polygroups. If $A = \{e, a_1, a_2, \ldots\}$ and $B = \{e, b_1, b_2, \ldots\}$, we have:

	e	a_1	a_2	$\cdots$	b_1	b_2	$\cdots$
e	e	a_1	a_2	$\cdots$	b_1	b_2	$\cdots$
a_1	a_1	a_1a_1	a_1a_2	$\cdots$	b_1	b_2	$\cdots$
a_2	a_2	a_2a_1	a_2a_2	$\cdots$	b_1	b_2	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
b_1	b_1	b_1	b_1	$\cdots$	$b_1 * b_1$	$b_1 * b_2$	$\cdots$
b_2	b_2	b_2	b_2	$\cdots$	$b_2 * b_1$	$b_2 * b_2$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Let 2 denotes the group Z_2 and let 3 denotes the polygroup $\mathbb{S}_3//\langle(1\ 2)\rangle \cong Z_3/\mathcal{T}$, where $\mathcal{T}$ is the special conjugation with blocks {0}, {1,2}. The multiplication table for 3 is

	0	1
0	0	1
1	1	{0,1}

The system 3[M] is the result of adding a new identity to the polygroup [M]. The system 2[M] is almost as good. Let B be the following form:

	0	1	2
0	0	1	2
1	1	{0,2}	{1,2}
2	2	{1,2}	{0,1}

Example 4. Extension $\mathcal{A}$[B] about B[B] is as follows:

$\circ$	0	1	2	a	b
0	0	1	2	a	b
1	1	{0,2}	{1,2}	a	b
2	2	{1,2}	{0,1}	a	b
a	a	a	a	$\{0,1,2, b\}$	$\{a, b\}$
b	b	b	b	$\{a, b\}$	$\{0,1,2, a\}$

Hyperoperation $\circ$: B[B] $\times$ B[B] $\longmapsto \mathcal{P}$(B[B]) is not continuous with the following topologies:

But $\circ$: B[B] $\times$ B[B] $\longmapsto \mathcal{P}$(B[B]) is continuous with $\mathcal{T}_{22} = \{\emptyset$, B[B], $\{0\}\}$, $\mathcal{T}_{23} = \{\emptyset$, B[B], $\{0,1,2\}\}$. In conclusion

(B[3], $\mathcal{T}_{dis}$) and (B[3], $\mathcal{T}_{ndis}$) and (B[3], $(\mathcal{T}_i)_{i=22,23}$) are topological polygroups. Subpolygroups of B[B] are

$\emptyset$, B[B], {0}, {0,1,2}. Let A be a arbitrary set and $a_1, a_2, a_3 \in A$ and define a soft set $\mathbb{F}$ by

$$\mathbb{F}(x) = \begin{cases} \{0\} & x = a_1 \\ \{0,1,2\} & x = a_2 \\ \mathcal{B}[\mathcal{B}] & x = a_3 \\ \emptyset & otherwise \end{cases}$$

Then, $(\mathbb{F}, A, (\mathcal{T}_i)_{i=22,23})$ is a soft topological polygroup. Notably, if we define soft set $\mathbb{F}$ by

$$\mathbb{F}(x) = \begin{cases} \{0\} & x = a_1 \\ \emptyset & x = a_2 \\ \{0,1,2\} & otherwise \end{cases}$$

then $(\mathbb{F}, A, (\mathcal{T}_i)_{i=3,4,7,8,19,20})$ are soft topological polygroups.

Theorem 3.2. [18] Let $(\mathbb{F}, A)$ be a soft polygroup over P and $(P, \mathcal{T})$ be a topological polygroup. Then $(\mathbb{F}, A, \mathcal{T})$ is a soft topological polygroup over P.

Theorem 3.3. [18] Let $(\mathbb{F}, A, \mathcal{T})$ and $(\mathbb{G}, B, \mathcal{T})$ be soft topological polygroups over P. Then $(\mathbb{F}, A, \mathcal{T}) \hat{\cap} (\mathbb{G}, B, \mathcal{T})$ and $(\mathbb{F}, A, \mathcal{T}) \cap_E (\mathbb{G}, B, \mathcal{T})$ are soft topological polygroups over P.

Theorem 3.4. [18] If $(\mathbb{F}_i, A_i, \mathcal{T})$ be a nonempty family of soft topological polygroups, then $\hat{\cap}_{i\in I} (\mathbb{F}_i, A_i, \mathcal{T})$ is a soft topological polygroup over P.

Theorem 3.5. [18] Let $(\mathbb{F}, A, \mathcal{T})$ and $(\mathbb{G}, B, \mathcal{T})$ be soft topological polygroups over P. Then $(\mathbb{F}, A, \mathcal{T}) \hat{\wedge} (\mathbb{G}, B, \mathcal{T})$

$\mathcal{T}_1 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1\}\}$ $\mathcal{T}_2 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{2\}\}$ $\mathcal{T}_3 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{a\}\}$

$\mathcal{T}_4 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{b\}\}$ $\mathcal{T}_5 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,1\}\}$ $\mathcal{T}_6 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,2\}\}$

$\mathcal{T}_7 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0, a\}\}$ $\mathcal{T}_8 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0, b\}\}$ $\mathcal{T}_9 = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1,2\}\}$

$\mathcal{T}_{10} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1, a\}\}$ $\mathcal{T}_{11} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1, b\}\}$ $\mathcal{T}_{12} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{2, a\}\}$

$\mathcal{T}_{13} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{2, b\}\}$ $\mathcal{T}_{14} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,1, a\}\}$ $\mathcal{T}_{15} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,1, b\}\}$

$\mathcal{T}_{16} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1,2, a\}\}$ $\mathcal{T}_{17} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1,2, b\}\}$ $\mathcal{T}_{18} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{2, a, b\}\}$

$\mathcal{T}_{19} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,1,2, a\}\}$ $\mathcal{T}_{20} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{0,1,2, b\}\}$ $\mathcal{T}_{21} = \{\emptyset, \mathcal{B}[\mathcal{B}], \{1,2, a, b\}\}$

and $(\mathbb{F}, A, \mathcal{T})\hat{\cup}(\mathbb{G}, B, \mathcal{T})$ are soft topological polygroups over P.

Theorem 3.6. [18] Let $(\mathbb{F}_i, A_i, \mathcal{T})$ be a nonempty family of soft topological polygroups over P. Then $\hat{\wedge}_{i\in I}(\mathbb{F}_i, A_i, \mathcal{T})$ and $\hat{\cup}_{i\in I}(\mathbb{F}_i, A_i, \mathcal{T})$ are soft topological polygroups over P.

Definition 3.7. Let $(\mathbb{F}, A, \mathcal{T})$ is a soft topological polygroup. Then $(\mathbb{F}, A, \mathcal{T})$ is called soft trivial if $\mathbb{F}(a) = \{e\}$ for all $a \in A$ and whole if $\mathbb{F}(a) = P$ for all $a \in A$.

Definition 3.8. Let $(\mathbb{F}, A, T)$ be a soft topological polygroup over P. Then $(\mathbb{G}, B, \mathcal{T})$ is called a soft topological subpolygroup (resp. normal subpolygroup) of $(\mathbb{F}, A, \mathcal{T})$ if the following items hold:

a) B subset of A and $\mathbb{G}(b)$ is a subpolygroup (resp. normal subpolygroup) of $\mathbb{F}(b)$ for every $b \in supp(\mathbb{G}, B)$.

b) The mapping $(x, y) \longmapsto x \circ y^{-1}$ of the topological space $\mathbb{G}(b) \times \mathbb{G}(b)$ onto $\mathcal{P}(\mathbb{G}(b))$ is continuous for every $b \in supp(\mathbb{G}, B)$.

Theorem 3.9. Let $(\mathbb{F}, A, \mathcal{T})$ be a soft topological polygroup over P, and $(\mathbb{G}_i, B_i, \mathcal{T})_{i\in I}$ be a non-empty family of (normal) soft topological subpolygroups of $(\mathbb{F}, A, \mathcal{T})$. Then

(1) If $\cap_{i\in I} B_i \neq \emptyset$, then $\hat{\cap}_{i\in I}(\mathbb{G}_i, B_i, \mathcal{T})$ is a (normal) soft subpolygroup of $(\mathbb{F}, A, \mathcal{T})$.

(2) If $B_i \cap B_j = \emptyset$ for all $i, j \in I$ and $i \neq j$, then $(\cap_E)_{i\in I}(\mathbb{G}_i, B_i, \mathcal{T})$ is a (normal) soft subpolygroup of $(\mathbb{F}, A, \mathcal{T})$.

(3) If $B_i \cap B_j = \emptyset$ for all $i, j \in I$ and $i \neq j$, then $\hat{\cup}_{i\in I}(\mathbb{G}_i, B_i, \mathcal{T})$ is a (normal) soft subpolygroup of $(\mathbb{F}, A, \mathcal{T})$.

(4) $\hat{\wedge}_{i\in I}(\mathbb{G}_i, B_i, \mathcal{T})$ is a (normal) soft subpolygroup of the soft polygroup $\hat{\wedge}_{i\in I}(\mathbb{F}, A, \mathcal{T})$.

Proof.

(1) Suppose that $C = \cap_{i\in I}(B_i)$ and $\mathrm{H}(c) = \cap_{i\in I}(\mathbb{G}_i(c))$ Furthermore, $C \subseteq A$ and $\mathrm{H}(c)$ is a (normal) soft subpolygroup of A and the mapping in Definition 3.8 (b) is continuous on $\mathrm{H}(c)$.

(2) Take $C = \cup_{i\in I}(B_i)$, $\mathrm{H}(c) = \mathbb{G}_i(c)$ where $c \in B_i$ and $\mathrm{H}(c)$ is a (normal) soft subpolygroup of $\mathbb{F}(c)$ and the mapping in Definition 3.8 (b) is continuous on $\mathrm{H}(c)$.

(3) Take $C = \cup_{i\in I}(B_i)$, $\mathrm{H}(c) = \mathbb{G}_i(c)$, where $c \in B_i$ thus $B_i \subseteq A$ notably $\cup_{i\in I}(B_i) \subseteq A$ in conclusion $\mathrm{H}(c) = \mathbb{G}_i(c)$ is a (normal) soft subpolygroup of $\mathbb{F}(c)$ and the mapping in Definition 3.8 (b) is continuous on $\mathrm{H}(c)$.

(4) Select $C = \times_{i\in I}(B_i)$, $\mathrm{H}((c_i)_{i\in I}) = \cap_{i\in I}\mathbb{G}_i((c_i)_{i\in I})$ and $\mathbb{G}_i(c_i)$ is a (normal) soft subpolygroup of $\times_{i\in I}\mathbb{F}(c_i)$ in conclusion the mapping in Definition 3.8 (b) is continuous on $\mathrm{H}((c_i)_{i\in I})$.

Definition 3.10. Let $(\mathbb{F}, A, \mathcal{T})$ and $(\mathbb{G}, B, \xi)$ be the soft topological polygroups over P_1 and P_2, where $\mathcal{T}$ and ξ are topologies defined over P_1 and P_2 respectively. Let $f: P_1 \longmapsto P_2$ and $g: A \longmapsto B$ be two mappings. Then the pair (f, g) is called a soft topological polygroup homomorphism if the following condition hold.

(a) f is strong epimorphism and g is surjection.

(b) $f(\mathbb{F}(a)) = \mathbb{G}(g(a))$.

(c) $f_a: (\mathbb{F}(a), \mathcal{T}_{\mathbb{F}(a)}) \longmapsto (\mathbb{G}(g(a)), \xi_{\mathbb{G}(g(a))})$ is continuous.

Then $(\mathbb{F}, A, \mathcal{T})$ is said to be soft topologically homomorphic to $(\mathbb{G}, B, \xi)$ and denoted by $(\mathbb{F}, A, \mathcal{T}) \sim (\mathbb{G}, B, \xi)$. If f is a polygroup isomorphism, g is bijective and f_a is continuous as well as open, then the pair (f, g) is called a soft topological polygroup isomorphism. In this case $(\mathbb{F}, A, \mathcal{T})$ is soft topologically isomorphic to $(\mathbb{G}, B, \xi)$, which is denoted by $(\mathbb{F}, A, \mathcal{T}) \simeq (\mathbb{G}, B, \xi)$.

Theorem 3.11. If $(\mathbb{F}, A, \mathcal{T}) \sim (\mathbb{G}, B, \xi)$ and $(\mathbb{F}, A, \mathcal{T})$ is a normal soft polygroup over P, then $(\mathbb{G}, B, \xi)$ is a normal soft polygroup over Q, where $(\mathbb{F}, A, \mathcal{T})$ and $(\mathbb{G}, B, \xi)$ are soft topological polygroups over P and Q.

Proof.

Let (f, g) be a soft topological homomorphism from $(\mathbb{F}, A)$ to $(\mathbb{G}, B)$. For all $x \in supp(\mathbb{F}, A)$, $\mathbb{F}(x)$ is a normal subpolygroup of P; then $f(\mathbb{F}(x))$ is a normal subpolygroup of Q. For all $y \in supp(\mathbb{G}, B)$, there exists $x \in supp(\mathbb{F}, A)$ with the property that $g(x) = y$. In conclusion $\mathbb{G}(y) = \mathbb{G}(g(x)) = f(\mathbb{F}(x))$ is a normal subpolygroup of Q. Thus, $(\mathbb{G}, B)$ is a normal soft polygroup on Q.

Theorem 3.12. Let N be a normal subpolygroup of P, and $(\mathbb{F}, A, \mathcal{T})$ be a soft topological polygroup over P. Then $(\mathbb{F}, A, \mathcal{T}) \sim (\mathbb{G}, A, \mathcal{T})$, where $\mathbb{G}(x) = \mathbb{F}(x) / N$ for all $x \in A$, and $N \subseteq \mathbb{F}(x)$ for all $x \in supp(\mathbb{F}, A)$.

Proof.

Firstly $supp(\mathbb{G}, A) = supp(\mathbb{F}, A)$ and we know that P/N is a factor polygroup. Since for every $x \in supp(\mathbb{F}, A)$, $\mathbb{F}(x)$ is a subpolygroup of P and $N \subseteq \mathbb{F}(x)$, it follows that $\mathbb{F}(x)/N$ is also a factor polygroup, which is a subpolygroup of P/N. Thus $(\mathbb{G}, A)$ is a soft polygroup over P/N. Therefore $f: P \longmapsto P/N$, $\mathbb{F}(a) = aN$. Clearly, f is a strong epimorphism. In other words $g: A \longmapsto A$, $g(x) = x$. Then g is a surjective mapping. For all $x \in supp(\mathbb{F}, A)$, $f(\mathbb{F}(x)) = \mathbb{F}(x)/N = \mathbb{G}(x) = \mathbb{G}(g(x))$. For all $x \in A - supp(\mathbb{F}, A)$, notably $f(\mathbb{F}(x)) = \emptyset = \mathbb{G}(g(x))$. Therefore, (f, g) is a soft topological homomorphism, and $(\mathbb{F}, A, \mathcal{T}) \sim (\mathbb{G}, B, \xi)$.

Definition 3.13. Let $(\mathbb{F}, A, \mathcal{T})$ be a soft topological polygroup over P. Then the closure of $(\mathbb{F}, A, \mathcal{T})$ denoted by $(\bar{\mathbb{F}}, A, \mathcal{T})$ and define as $\bar{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$ where $\overline{\mathbb{F}(a)}$ is the closure of $\mathbb{F}(a)$ in topology defined on P.

Theorem 3.14. [11] Let P be a topological polygroup with the property that every open subset of P is a complete part. Then:

(1) If K is a subsemihypergroup of P, then as well as $\bar{K}$.

(2) If K is a subpolygroup of P, then as well as $\bar{K}$.

Theorem 3.15. Let $(\mathbb{F}, A, \mathcal{T})$ be a soft topological polygroup over a topological polygroup $(P, \mathcal{T})$ and every open subset of P is a complete part. Then the following are true.

(1) $(\mathbb{F}, A, \mathcal{T})$ is also a soft topological polygroup over $(P, \mathcal{T})$.

(2) $(\mathbb{F}, A, \mathcal{T}) \hat{\subset} (\mathbb{F}, A, \mathcal{T})$.

Proof.

(1) By Theorem 3.14 $\overline{\mathbb{F}(a)}$ is subpolygroup P and since $(P, \mathcal{T})$ is a topological polygroup, it follows that condition

(b) of Definition 3.1 holds on $\overline{\mathbb{F}(a)}$.

(2) It is clear.

Definition 3.16. Let $(\mathbb{F}, A)$, $(\mathbb{G}, B)$ be soft set over the polygroup $< P, e, \circ, ^{-1} >$. Define $(\mathbb{F}, A) \hat{\circ} (\mathbb{G}, B) = (H, C)$

where $C = A \cup B$ for all a ∈ C, and

$$H(a) = \begin{cases} \mathbb{F}(a) & a \in A - B \\ \mathbb{G}(a) & a \in B - A \\ \mathbb{F}(a) \circ \mathbb{G}(a) & a \in A \cap B \end{cases}$$

Theorem 3.17. [11] Let A and B be subsets of a topological polygroup P with the property that every open subset of P is a complete part. Then:

(1) $\bar{A} \circ \bar{B} \subseteq \overline{(A \circ B)}$.

(2) $(\bar{A})^{-1} = \overline{(A^{-1})}$.

Theorem 3.18. [11] In every topological space $(X, \mathcal{T})$ if $A, B \subseteq X$ we have:

(1) $\bar{A} \cup \bar{B} \subseteq \overline{(A \cup B)}$.

(2) $\bar{A} \cap \bar{B} \subseteq \overline{(A \cap B)}$.

Theorem 3.19. Let $(\mathbb{F}, A, \mathcal{T})$, $(\mathbb{G}, B, \mathcal{T})$ be soft topological polygroups over a topological polygroup $(P, \mathcal{T})$ and every open subset of P is a complete part Then:

(1) $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\cup}(\bar{\mathbb{G}}, B, \mathcal{T}) = \overline{(\mathbb{F},A,\mathcal{T})\hat{\cup}(\mathbb{G},B,\mathcal{T})}$.

(2) $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\cap}\ (\bar{\mathbb{G}}, B, \mathcal{T}) = \overline{(\mathbb{F},A,\mathcal{T})\hat{\cap}(\mathbb{G},B,\mathcal{T})}$.

(3) $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\wedge}(\bar{\mathbb{G}}, B, \mathcal{T}) = \overline{(\mathbb{F},A,\mathcal{T})\hat{\wedge}(\mathbb{G},B,\mathcal{T})}$.

(4) $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\circ}\ (\bar{\mathbb{G}}, B, \mathcal{T}) \subseteq \overline{(\mathbb{F},A,\mathcal{T})\hat{\circ}(\mathbb{G},B,\mathcal{T})}$.

(5) $(\bar{\mathbb{F}}, A, \mathcal{T})\ \bigcap_E(\bar{\mathbb{G}}, B, \mathcal{T}) = \overline{(\mathbb{F},A,\mathcal{T})\bigcap_E(\mathbb{G},B,\mathcal{T})}$.

Proof

(1) Let a be element of $A - B$. then $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\cup}(\bar{\mathbb{G}}, B, \mathcal{T})(a) = (\bar{\mathbb{F}}, A, \mathcal{T})(a) = \overline{\mathbb{F}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\cup}(\mathbb{G},B,\mathcal{T})}(a) = \bar{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$.

Let a be element of $B - A$. Then $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\cup}(\bar{\mathbb{G}}, B, \mathcal{T})(a) = (\bar{\mathbb{G}}, A, \mathcal{T})(a) = \overline{\mathbb{G}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\cup}(\mathbb{G},B,\mathcal{T})}(a) = \bar{\mathbb{G}}(a) = \overline{\mathbb{G}(a)}$.

Let a be element of $A \cap B$. Then $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\cup}(\bar{\mathbb{G}}, B, \mathcal{T})(a) = \overline{\mathbb{F}(a)} \cup \overline{\mathbb{G}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\cup}(\mathbb{G},B,\mathcal{T})}(a) = \overline{\mathbb{F}(a) \cup \mathbb{G}(a)}$. By Theorem 3.18, the proof completes.

(4) Let a be element of $A - B$. Then $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\circ}\ (\bar{\mathbb{G}}, B, \mathcal{T})(a) = (\bar{\mathbb{F}}, A, \mathcal{T})(a) = \overline{\mathbb{F}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\circ}(\mathbb{G},B,\mathcal{T})}(a) = \bar{\mathbb{F}}(a) = \overline{\mathbb{F}(a)}$.

Let a be element of $B - A$. Then $(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\circ}\ (\bar{\mathbb{G}}, B, \mathcal{T})(a) = (\bar{\mathbb{G}}, A, \mathcal{T})(a) = \overline{\mathbb{G}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\circ}(\mathbb{G},B,\mathcal{T})}(a) = \mathbb{G}(a) = \overline{\mathbb{G}(a)}$.

Let a be element of $A \cap B$. Then$(\bar{\mathbb{F}}, A, \mathcal{T})\ \hat{\circ}\ (\bar{\mathbb{G}}, B, \mathcal{T})(a) = \overline{\mathbb{F}(a) \circ \mathbb{G}(a)}$ In conclusion,
$\overline{(\mathbb{F},A,\mathcal{T})\hat{\circ}(\mathbb{G},B,\mathcal{T})}(a) = \overline{\mathbb{F}(a) \circ \mathbb{G}(a)}$. By Theorem 3.17 the proof completes. Other items are similar to either (1) or (4).

CONCLUSION

In the previous sections, we were introduced to a definition of a soft topological polygroup and examined its examples and results. Some of the results of this paper can be generalized to K-algebras [2] and Li algebras[1]. Now we can define soft topological polygroup in different ways.

[4,19] A family of soft sets over U is called a soft topology on U if the following axioms hold:

(1) $\hat{\emptyset}$ and $\hat{U}$ are in $\mathcal{T}$,

(2) $\mathcal{T}$ is closed under finite soft intersection,

(3) $\mathcal{T}$ is closed under (arbitrary) soft union.

We will use the symbol $\mathbb{F}^{\hat{c}}$ to denote soft complement of F and is defined by $\mathbb{F}^{\hat{c}}(e) = U \setminus \mathbb{F}(e)(e \in E)$ and will use the symbol $(U, \mathcal{T}, E)$ to denote a soft topological space and soft set F is called a soft close set if $\mathbb{F}^{\hat{c}}$ is soft open set, where each member of $\mathcal{T}$ said to be a soft open set.

A soft set $\mathbb{F}$ said to be a soft neighborhood of x if there exists a soft open set $\mathbb{G}$ with the property that $x \hat{\in} \mathbb{G} \hat{\subseteq} \mathbb{F}$, where x be an element of the universe U. The soft neighborhood system of x we will consider the collection of all soft neighborhoods of x.

Let V be a subset of the universe U. A soft set $\mathbb{F}$ said to be a soft neighborhood of V if there exists a soft open set $\mathbb{G}$ with the property that $V \hat{\subseteq} \mathbb{G} \hat{\subseteq} \mathbb{F}$. (i.e $\forall\ e \in E$: $V \subseteq \mathbb{G}(e) \subseteq \mathbb{F}(e)$).

The collection of all soft neighborhoods of V said to be the soft neighborhood system of V.

Definition 4.1. Let P_1, P_2 be polygroups and $(P_1, \mathcal{T}_1, E)$, $(P_2, \mathcal{T}_2, E)$ be soft topological spaces. The function φ: $(P_1, \mathcal{T}_1, E) \longmapsto (P_2, \mathcal{T}_2, E)$ said to be a soft continuous function if for all soft open set $\mathbb{F}' \in \mathcal{T}_2$, the inverse image $\varphi^{-1}(\mathbb{F}')$ is also soft open.

Definition 4.2. Let $(P, \circ, e, ^{-1})$ be a polygroup and $\mathcal{T}$ be a soft topology on P with a parameter set E. then $(P, \mathcal{T}, E)$ is a soft topological polygroup if the following items true:

(i) For each soft neighborhood $\mathbb{F}$ of $p \circ q$, where $(p, q) \in P \times P$ there exist soft neighborhoods $\mathbb{F}_p$ and $\mathbb{F}_q$ of p and q with the property that $\mathbb{F}_p \circ \mathbb{F}_q \hat{\subseteq} \mathbb{F}$.

(ii) The inversion function $^{-1}$: $P \longmapsto P$ is soft continuous.

We can draw interesting results from this definition. Needless to say, the main idea is the recent definition of Hida in [12].

AUTHORSHIP CONTRIBUTIONS

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DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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