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Technical Note

Modeling of the effects of non-uniform electric field on resonant tunneling in superlattices

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ABSTRACT

We study superlattices in non-uniform electric field to obtain mid infrared laser. We exploit the transfer matrix method and the transmission coefficient to find the suitable parameters that lead to resonant energy levels that are appropriate for generating three and four-level laser. In particular, we consider superlattice of $GaAs/Al_{0.45}Ga_{0.55}$ As consisting of four barriers, and we apply a graduated electric field. Our calculations predict that under the effect of the electric field equal to 10KV/cm and its graduation step equals to 1.26 KV/cm with the condition of the transition resonant with LO phonon, the obtained electronic transitions are shown to have wavelengths of 23.49 μm and 32.15 μm. We found that the variation of the electric field has an influence on the energy profile of the electron in the superlattice.

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INTRODUCTION

The first realization of the laser in 1960 [1], there were much theoretical and experimental advances in physics and many other fields of science and technology. The first laser was a solid state laser, and later in the 1960s and 1970s lasers based on semiconductors were successfully obtained [2–7]. Quantum cascade lasers (QCLs), which are semiconductor laser based on heterostructures of many layers, were proposed by RF. Kazarinov and RA. Suris in 1971 [8] and it was first realized by Faist et al. in 1994 [9]. Since then, there were much theoretical and experimental effort to improve the performance and efficiency of the QCLs such as operating temperatures, lasing power and wavelength, threshold current density . . . . The modeling and optimization of QCLs depend mainly on the ability of controlling and tuning the quantized electronic energy levels of the superlattice. The key element in finding these energy levels is solving the Schrodinger equation for an electron in the conduction band of the superlattice, and for this, there are many methods such as Runge-kutta methods, finite elements methods, shooting method, Monte Carlo methods, argument principle method, multi-step potential, spectral methods, immersed interface method, . . . [10–24]. In some instances, the transfer matrix method is used with the previous methods to find the transmission coefficient that

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quantifies the tunneling of the electrons across the different barriers of the superlattice. In the past few years there had been much interest in studying superlattices, where varying either the properties of the superlattice or the external electro- magnetic fields is considered [25–37]. In this paper, we use the transfer matrix method to calculate the transmission coefficient of electronic transport in a superlattice made of alternation of layers of two different materials, and use it to extract information about the resonant energy levels as described in [38]. This method is based on the analytical solution of the Schrodinger equation where Airy functions [39, 40] appear due to the presence of external electric field. This later is considered constant in each layer of the superlattice but not necessarily equal in different layers. In particular, we consider a gradually changing electric field in successive layers, then we look for the suitable parameters to have resonant tunneling. We should mention here that resonant tunneling is important to obtain QCLs [41]. Also in this work, we take into consideration the non-radiative transition regarding the longitudinal optical (LO) phonon [9]. In particular, we consider GaAs/AlGaAs superlattice and we search for the suitable electric fields in each layer that give non radiative transition resonant with LO phonon that is equal to 36 meV [33]. In this regard the motivation of our present work is to explore the effect of electric field and their step to find suitable values corresponding to energy separation between first and ground states is on resonant with LO phonon, we also mention here that this condition is very important in order to obtain quantum cascade lasers. We should mention here that similar works to ours were done, but instead of varying the electric fields in different layers, the variation of superlattice material concentration [42, 43] and layer thickness [44] were considered.

THEORY

We consider the superlattice shown in Figure 1-a. We apply a non-uniform electric field F (x) on the superlattice such that the field is constant in every region and zero in first and last regions:

$$F(x) = \begin{cases} F_j & x_j \le x \le x_{j+1} \quad j = 1, \ldots\ldots, 2N-1 \\ 0 & x < x_1 \quad or \qquad x > x_{2N-1} \end{cases} \tag{1}$$

and therefore the potential energy becomes as shown in Figure 1-b. The application of the electric field results in a band bending ΔE_j (with j = 1. . . 2N – 1 and 2N is the number of regions in the structure) given by :

$$\Delta E_j = e(x_{j+1} - x_j)F_j = e d_j F_j \tag{2}$$

where e is the electron's electric charge, F_j is the intensity of the electric field in the j[th] region, x_j is the position of the interface between regions j and j + 1, and d_j is the thickness of the j[th] layer.

The energy of resonance in the superlattice is determined by solving the one dimensional time independent Schrodinger equation. The Schrodinger equation for one electron in layer j (j = 0, . . ., 2N) is given by.

$$\frac{-\hbar^2}{2}\frac{d}{dx}\left(\frac{1}{m_j^*(x)}\frac{d}{dx}\right)\psi_j(x) + V_j(x)\psi_j(x) = E\psi_j(x) \tag{3}$$

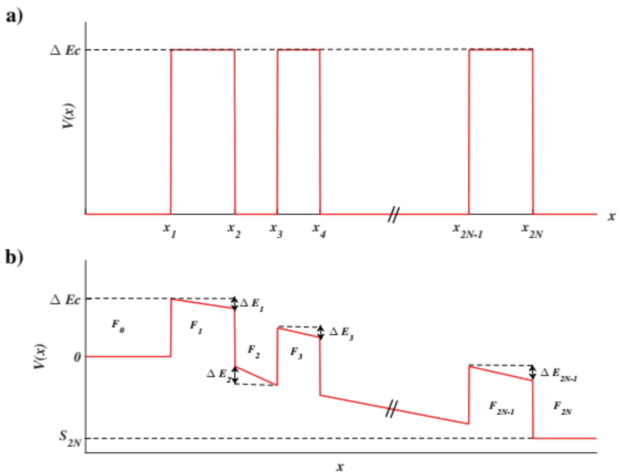

Figure 1. Potential energy structure of the superlattice. a) Unbiased superlattice. b) Biased superlattice: $s_{2N} = -e\sum_{i=1}^{2N-1}(x_{j+1} - x_j)F_j$

where x is the growth axis of the layers in the superlattice, h¯ is the reduced Planck's constant, E is the electron's energy, ψ_j *(x)* is the wave function in layer j, m*(x) is the effective mass of the electron in region j which is equal to:

$$m_j^*(x) = \begin{cases} m_b & j - odd \\ m_\omega & j - oven \end{cases} \tag{4}$$

where m_w and m_b are the effective masses in the well and barrier layers respectively, and V_j *(x)* is the potential energy of the electron as shown in Figure 1-b and it is equal to:

$$V_j(x) = U_j - eF_j(x_{j+1} - x_j) - \sum_{i=1}^{j-1} \Delta E_i \tag{5}$$

where ΔE_i is defined in Eq. (2), and U_j is given by:

$$U_j = \begin{cases} \Delta E_c & j - odd \\ 0 & j - oven \end{cases} \tag{6}$$

where ΔE_c is the conduction band edge offset. The solution of *Eq.* (3) is

$$\psi_j(x) = \begin{cases} A_0 e^{ik_0 x} + B_0 e^{-ik_0 x} & j = 0 \\ A_j Ai(z_i) + B_j Bi(z_i) & 1 \le j \le 2N-1 \\ A_{2N} e^{ik_{2N} x} + B_{2N} e^{-ik_{2N} x} & j = 2N \end{cases} \tag{7}$$

where A_i and B_i are Airy functions of first and second kind respectively [39], A_j and B_j are complex constants that

Table 1. The resonance energy levels of the electron in the superlattice

Resonance energy levels (meV)	*ΔF* = 0	*ΔF* = 2 kV	*ΔF* = −2 kV	Results of [38]
E1	-67.00	-68.40	-65.65	-66.8
E2	-28.85	-57.45	-10.30	-28.2
E3	-10.90	-17.15	31.35	-10.2
E4	30.40	29.30	176.70	31

represent the amplitudes of the wave function, k_0, k_{2N} and z_j are defined as follows:

$$k_0 = \sqrt{\frac{2m_\omega E}{\hbar^2}} \tag{8}$$

$$k_{2N} = \sqrt{\frac{2m_\omega(E+e\sum_{i=1}^{2N-1} d_j F_j)}{\hbar^2}} \tag{9}$$

$$z_j(x) = \left(\frac{2m_j}{\hbar^2}\right)^{\frac{1}{3}} \left[\frac{V_j(x)-E}{(eF_j)^{\frac{2}{3}}}\right] \tag{10}$$

The coefficients A_j and B_j are determined from the continuity conditions of the wave function in the boundary of each layer [45, 46]. Therefore, one finds the relation between the coefficients of layer j and layer j + 1 as follows:

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = M_j \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} \tag{11}$$

This later equation leads to the expression relating the amplitudes of the first region (j = 0) and last region ($j = 2N$).

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = M \begin{pmatrix} A_{2N} \\ B_{2N} \end{pmatrix} \tag{12}$$

where the 2 × 2 complex matrix M is the transfer matrix [47, 48] which contains all the physical information about the structure shown in Figure 1, and it is equal to:

$$M = \prod_{j=1}^{2N} M_j \tag{13}$$

The energy levels of the electron in the superlattice that give resonant tunneling [47–49] coincide with energies that give maximum tunneling [38] (see Figure 2). This later is quantified by the transmission coefficient $T(E)$ which is given by the following expression [14]:

$$T(E) = \frac{k_{2N}\ m_0}{k_0\ m_{2N}} |M_{11}|^{-2} \tag{14}$$

where M_{11} is the matrix element of first row and first column of the transfer matrix M.

RESULTS AND DISCUSSION

In this study, we consider superlattice structure of four barriers ($2N = 8$) produced by many alternating layers of $GaAs$ and $Al_xGa_{1-x}As$. This structure was considered in the literature such as [11, 38, 48, 50]. We put this structure in an electric field $F(x)$ as defined in *Eq.* (1) with the electric field in each layer F_j (j = 1, . . ., 7) is chosen to be equal to:

$$F_j = F_1 + (j-1)\Delta F \tag{15}$$

where F_1 is the electric field in region 1, and ΔF is the graduation step of the applied electric field.

As said in the previous section, the energy levels of the electron in the superlattice are the peaks of the transmission coefficient when plotted against electron energy. In Figure 2 the variation of the transmission coefficient as function of the electron energy is shown for electric field intensity $F_1 = 100\ kV/cm$. The energy levels of the electron in the considered structure are the peaks of the transmission coefficient [38] and are given in Table 1.

We are interested in obtaining laser from the considered superlattice structure of Figure 1-b. It was shown in [33]

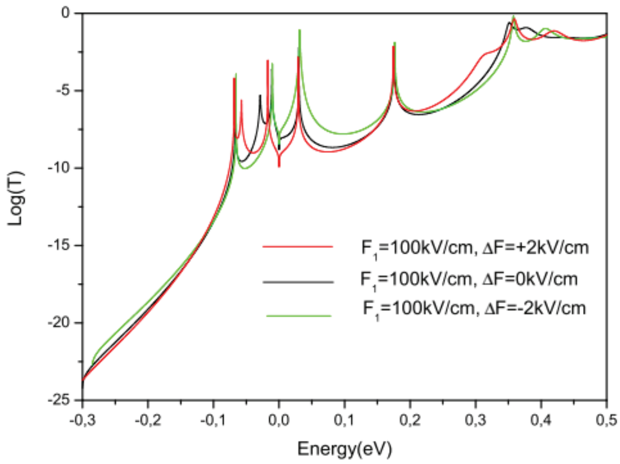

Figure 2. Energy dependence of the transmission coefficient for different values of the electric field. Thickness of layers are d_1 = 3.5 nm, d_2 = 11 nm, d_3 =3.5 nm, d_4 = 2.82 nm, d_5 = 3.5 nm, d_6 = 2.2 nm and d_7 = 3.5 nm. Effective mass of the electron in well and barrier regions are m_w = 0.067 m_e and m_b =0.1043 m_e respectively, with m_e is the electron mass. Conduction band offset ΔE_c = 0.477 eV. Electric field graduation step ΔF = ±2 kV/cm.

that in order to obtain the laser, the transition from the first excited state to the ground state must be resonant with the longitudinal optical phonon. For *GaAs*, we must have:

$$E_2 - E_1 \sim 36\, meV \tag{16}$$

The idea is to find a suitable graduation step ΔF that satisfies condition (16). For this, we fixed F at *10 kV/cm* then change ΔF and we calculate the energy difference $E_2 - E_1$.

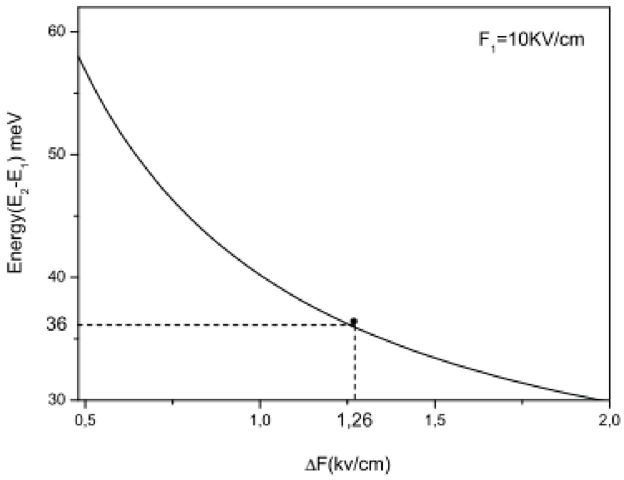

Figure 3. Variation of the energy of the intersubband transition $E_2 - E_1$ as function of the graduation step of the electric field.

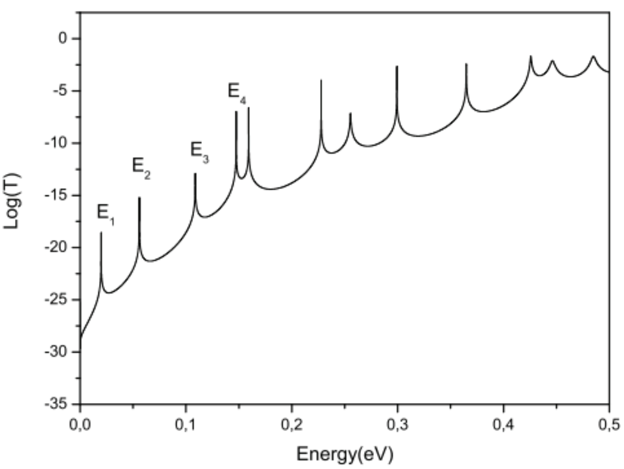

Figure 4. Transmission coefficient as function of electron energy for electric field F_1 = 10 kV/cm and field graduation step ΔF = 1.26 kV/cm. Thickness of layers are d_1 = 3.5 nm, d_2 = 11 nm, d_3 = 3.5 nm, d_4 = 2.82 nm, d_5 =3.5 nm, d_6 = 2.2 nm and d_7 = 3.5 nm. Effective mass of the electron in well and barrier regions are $m_w = 0.067\, m_e$ and m_b = 0.1043 me respectively, with m_e is the electron mass. Conduction band offset ΔE_c = 0.477 eV.

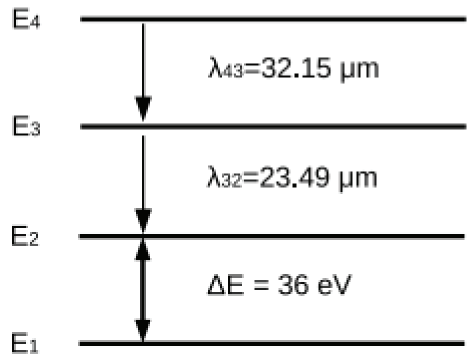

Figure 5. Schematic of intersubband transitions for F_1 = 10 kV/cm and ΔF =1.26 kV/cm.

The results are plotted in Figure 3. It is clear that increasing the step ΔF leads to smoothly and inversely decreasing energy separation between ground and first excited energy levels that means that when the step increases leads the ground and first excited levels to getting closer each other, consequently the step has a considerable affect in properties of the structure, according to the Figure 3, we see that condition (16) is satisfied for ΔF *=1.26 kV/cm*. In this regard in Figure 4. we represent the transmission coefficient *log(T(E))* which is illustrated as a function of various values of the tunneling energy with electric field equals to *F =10 kV/cm* and ΔF *=1.26 kV/cm*, then we extract the energy levels from the peaks as explained before. The found energy levels give the transitions shown in Figure 5. The obtained wave lengths 23.49µm and 32.15 µm are in the range of mid-infrared laser [51]. This range of wavelength has many applications in fields such as bio-chemical detection, remote sensing, astronomy, communication, biology, nondestructive materials evaluation and medicine [52].

CONCLUSION

We studied the effects of changing the electric field in every layer of the super- lattice (Figure 1-b) on the wavelength of the laser resulting from different electronic energy transitions. In particular, we considered a superlattice based on $GaAs/Al_{0.45}Ga_{0.55}As$ with base electric field F_1 = 10 kV/cm and graduation step ΔF = 1.26 kV/cm. We found that the step graduation ΔF has a significant effect on resonance energy. The requirement that difference of energy between the ground and first exiting levels must be compatible with the optical phonon for the considered semiconductor material. These parameters enable us to obtain electronic transitions of wavelengths 23.49 µm and 32.15 µm respectively. This is very important condition in the design of optoelectronic devices based on resonant tunneling, e.g. quantum cascade lasers. Other values for the wavelength can be obtained by tuning system configuration such as electric field profile and the heterostructure geometry and material properties. Being able to tune the wavelength is very important and useful in many potential technological applications.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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