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Research Article

Performance estimation of parallel Syste under online and offline preventive maintenance

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ABSTRACT

In this paper, the reliability characteristics of a parallel system are investigated. The parallel system under consideration is made up of three active units that run in parallel, with two of them having to be operational in order for the system to work. The main purpose of this study is to quantify/examine the effect of online and offline preventive maintenance. Preventive maintenance is carried out on the systems in two ways: online and offline preventive maintenance. After the first unit of each system fails, online preventive maintenance is performed. Following the failure of the second unit of each system, offline preventive maintenance is performed. Partial and complete failures are the two types of failures that may occur. Both systems can undergo exponential failure and repair. Using supplementary variable technique, Laplace transform, and Copula repair approach, the system of first-order differential equations associated with system effectiveness, which are crucial to this research, is established and resolved. Tables and graphs are used to illustrate the important findings based on assumed numerical values. System designers, programmers, and maintenance supervisors will be able to create and maintain more crucial systems with the assistance of this research paper.

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INTRODUCTION

One of the most important characteristics used to assess system performance in engineering is reliability and is also regarded as the best starting point for any system improvement. Due to lack of adequate probabilistic knowledge, convectional reliability's binary state i.e., performance or failure state assumptions, are inadequate for analysing the reliability of complex industrial systems. The dubiety of each individual parameter in large industrial systems adds to the overall system reliability dubiety.

As a result, a number of researchers have created various industrial systems to investigate reliability, availability, mean

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time to system failure, and cost function, as well as the impact of coverage variables, failure and repair rates. Researchers such as; [1] explored a method for evaluating the reliability and profitability of a warranty-dependent industrial system. [2] focused on evaluating the performance of serial systems with a variety of failure types and repair policies. Using Kolmogrov differential equations, [3] proposed a method for analyzing the reliability of industrial systems. [4] investigated a novel method for reliability analysis of series-parallel systems via credibility theory. [5] presented their research on the availability of standby complex systems in the event of human failure or repair. [6] presented Markov chain profit modeling and evaluation between two dissimilar systems under two forms of failures. [7] have published a paper on the performance assessment of a series-parallel industrial method. [8] presented the findings of an analysis of the multi-objective non-linear programming problem for reliability (GSA) and compared them to results computed using the practice swarm optimization (PSO) methodology. The reliability study of a two-dissimilar unit warm standby repairable system with priority in usage was addressed by [9]. [10] addressed the availability and cost analysis of an engineering system with series-connected subsystems. [11] discussed the evaluation of some reliability characteristics of a single unit system requiring two types of supporting device for operations. [12] used the Gumbel-Hougaard family copula to study the cost assessment of a complex repairable system made up of two subsystems in a series arrangement. [13] used copula distribution to offer a stochastic study of an N-unit plant with various failure kinds. Improving system performance can also be attained either by increasing the reliability of each component/subsystem in the system or by providing proper maintenance (preventive maintenance).

Preventive maintenance is a type of maintenance that is carried out on a piece of equipment on a regular basis to minimize the risk of failing. It is done when the equipment is still running to keep it from breaking down unexpectedly. Cleaning, lubrication, oil changes, modifications, repairs, checking and removing components, as well as partial and full overhauls, are all part of preventive maintenance schedule in operation. Preventive maintenance is used in many industries to help maintenance managers and reliability engineers increase system reliability, availability, and revenue while lowering costs. The primary goal of every manufacturing company or factory is to increase profits by lowering production costs. In today's competitive age, we can't afford to lose production or have facilities fail due to increased demand. As a result, appropriate maintenance procedures must be used to achieve the best results.

Online and offline preventive maintenance techniques are two of the most common and universal maintenance techniques among the many. The aim of online preventive maintenance is to inspect the system or any output on a regular basis. It is a non-interruptible operation, which means the system won't be shut down until it's done. This assists in cost-cutting, reducing equipment downtime, improving asset lifespan and efficiency, and avoids unexpected expenses. Offline preventive maintenance is a type of inspection that is done manually. Regular asset scanning is sufficient for offline preventive maintenance.

Many research papers on system reliability and availability in the presence of preventive maintenance have been published, including the following; [14] investigated cost-effective coal-fired power plant preventive maintenance scheduling. [15] conducted a probabilistic study of two single-unit device reliability models with preventive maintenance beyond the warranty period and deterioration. [16] examined the best preventive maintenance warranty strategy for repairable goods with growing failure rates on a regular basis. [17] proposed a stochastic study of a reheating-furnace system undergoing preventive maintenance and repair. [18] presented a report on a single unit system with preventive maintenance and repair that was subjected to the fastest possible operation and repair times. For single machine framework, [19] suggested an integrated model of production planning and incomplete preventive maintenance strategy. [20] proposed a parallel system reliability assessment with two types of preventive maintenance. According to maximum operation and repair timeframes, [21] developed a single unit system with preventive maintenance and repair. Stochastic modeling of non-identical redundant systems with priority, preventive maintenance, and Weibull failure and repair distributions was presented by [22].

The literature review above reveals that the majority of previous studies have focused on preventive maintenance and other types of repair policies. Preventive maintenance policies, both online and offline under copula distribution approach, have received little or no attention in the literature. As a result, the impact of online and offline preventive maintenance has been highlighted in this paper.

The reminder of the paper is organised such that section 2 contains notations, assumptions, and description of the model under consideration. The formulation and solution of the model were captured in section 3. The model analysis was discussed in section 4. The findings are discussed in section 5, and finally the conclusion part of the paper is given in section 6.

NOTATIONS, ASSUMPTIONS AND DESCRIPTION OF MODEL

Notations

Here, we provide a table with the notations and definitions used in this study.

Assumptions

Below are listed all the presumptions that were made for this study.

i. √Firstly, all subsystems are assumed to be operational.
ii. Two units from each subsystem are needed for operation.
iii. When any component fails, the system's output suffers.
iv. If a subsystem unit fails, it can be repaired when it is still operational, or it can fail completely.

Table 1. Notations and Definitions

Notations	Definitions
T	Time variable on a time scale
S	Laplace transform variable for all expressions
λ_1	System A's failure rate
λ_2	System B's failure rate
$\emptyset(x)/\ \emptyset(y)$	Repair rate of system A / system B
$\mu_0(x)/\mu_0(y)$	Repair rate for a fully failed states of system A and system B respectively
$p_i(t)$ For i =0 to 10,	The probability that the system is in S_i state at any given time
$\overline{P}(s)$	Laplace transformation of state transition probability $p(t)$
P_i (x, t)	The probability that a system is in state S_i such that for *i*=1..., the system is under repair, and the elapsed repair time is (x, t) with x denoting repair and t denoting time.
P_i (y, t)	The probability that a system is in state S_i such that for *i*=1..., the system is under repair, and the elapsed repair time is (y, t) with y being the repair variable and t being the time variable.
P_i (m, t)	The probability that a system is in state S_i for *i*=1…, the system is under repair, and the elapsed repair time is (m, t) with repair variable being m and time variable being t.
P_i (n, t)	The probability that a system is in state S_i such that for *i*=1…, the system is under repair, and the elapsed repair time is (n, t) with n representing the repair variable and t representing the time variable.
$E_p(t)$	Expected profit over the course of the time interval [0, t)
K_1, K_2	Revenue and service cost per unit time, respectively
$\mu_0(x)$	According to the Gumbel-Hougaard family copula definition, joint probability is expressed as: $c_\theta\left(u_1(x), u_2(x)\right) = exp\left(x^\theta + \{log\,\phi\,(x)^\theta\}^{\frac{1}{\theta}}\right),\ 1 \leq \theta \leq \infty$. Where $\mu_1 = \emptyset(x)$ and $u_2 = e^x$

v. All failure rates are expected to follow exponential distributions and are constant.

vi. Partially failed states are restored using general distribution, while entirely failed state distribution is handled by the Gumbel Hougaard family copula.

vii. The repaired machine unit should function as new and the repair process should have caused no damage.

viii. The load will be ready for the system's successful performance as soon as the failed unit is repaired.

The Model Overviews

In this research, we looked at a mathematical model of parallel system with two subsystems A and B, each consists of three active units in parallel that can fail in two ways: partially or fully. These two systems are separated

Table 2. Description of States

States	Description
S_0	Is the ideal state, in which both systems are fully functional. The system works since one unit is on standby in both systems.
S_1	In this state, one unit in system A has failed, two units in both systems A and B are operational, and one unit in system B is on standby. The system is up and running.
S_3	In this state, one unit in system B has failed, two units in both systems A and B are operational, and one unit in system A is on standby. The system works.
S_4	Here, one unit in system B has previously failed, one unit in system A suddenly failed, and two units in system A are operational. The system is functional.
S_5	Previously, a unit has failed in system A, suddenly a unit failed in system B, then the system is operational.
S_6	Previously, a unit has failed in system B, suddenly a unit failed in system A, then the system is functional.
S_7	After the failure of the first unit in both systems A and B, this state denotes the online preventive maintenance state. In this state, the system is operational.
S_8	This state denotes system's A total failure as a result of two of its unit failing. The system is not functioning.
S_9	This state represents system's B total failure as a result of two of its unit failing. The system is not working.
S_{10}	This state denotes the offline preventive maintenance state where the system is neither down nor working.

from one another and are located in different locations. When two units from both systems A and B fail, the systems continue to function, but any more failure will bring the systems to total failure. In the case of total failure, both systems A and B can be fixed simultaneously using Gumbel-Hougaard family copula, whereas general repair distribution can be used to bring back partially failed states. The systems go through online preventive maintenance immediately after the failure of first unit with a rate of δ_0 and immediately fixed with a rate of $\emptyset(m)$, as well as offline preventive maintenance after the failure of the second unit with a rate of δ_1 and immediately returned with a rate of $\emptyset(n)$. The system has eleven states including perfect state, seven of which are operational and three of which are not. The detailed overview of the states is given below in table 2:

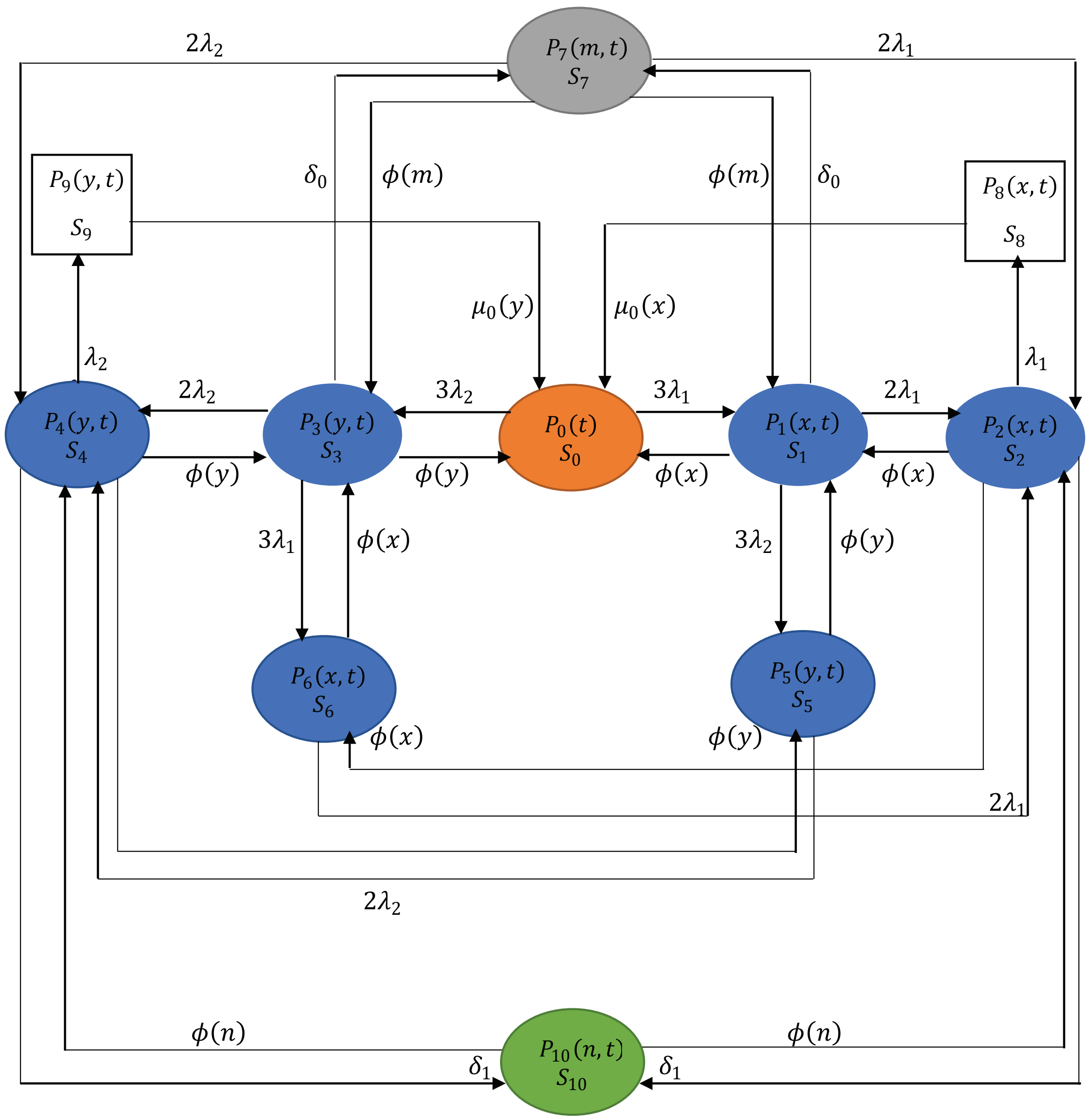
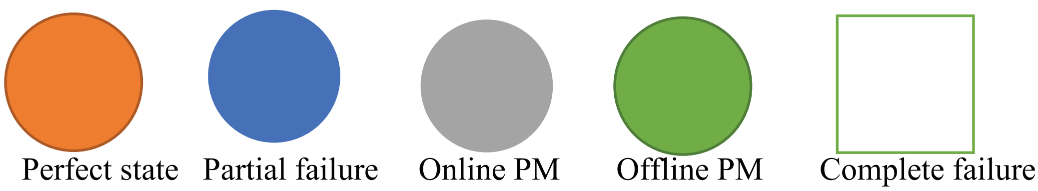


Figure 1. Transition diagram of the model.



MODELS' FORMULATION AND SOLUTION

Model Formulation

As in Lado and Singh (2019), and Singh and Rawal (2015), we obtain the following differential difference equations as:

$$\left\{\frac{\partial}{\partial t}+3\lambda_1+3\lambda_2\right\}P_0(t)=\int_0^\infty \emptyset(x)P_1(x,t)dx+\int_0^\infty \emptyset(y)P_3(y,t)dy + \int_0^\infty \mu_0(x)P_8(x,t)dx+\int_0^\infty \mu_0(y)P_9(y,t)dy, \quad (1)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+3\lambda_2+\delta_0+\phi(x)\right\}P_1(x,t)=0, \quad (2)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\delta_1+2\phi(x)\right\}P_2(x,t)=0, \quad (3)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+3\lambda_1+2\lambda_2+\delta_0+\phi(y)\right\}P_3(y,t)=0, \quad (4)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\delta_1+2\phi(y)\right\}P_4(y,t)=0, \quad (5)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\lambda_2+\phi(y)\right\}P_5(y,t)=0, \quad (6)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\phi(x)\right\}P_6(x,t)=0, \quad (7)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+2\lambda_1+2\lambda_2+2\phi(m)\right\}P_7(m,t)=0, \quad (8)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right\}P_8(x,t)=0, \quad (9)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right\}P_9(y,t)=0, \quad (10)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+2\phi(n)\right\}P_{10}(n,t)=0. \quad (11)$$

The study's boundary conditions are generated from figure 2 by analyzing the rates of exit from each state in terms of the probability of being in the initial state, $P_0(t)$:

$$P_1(0,t)=3\lambda_1P_0(t), \quad (12)$$

$$P_2(0,t)=\{6\lambda_1^2+6\lambda_1^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1^2\lambda_2\}P_0(t), \quad (13)$$

$$P_3(0,t)=3\lambda_2P_0(t), \quad (14)$$

$$P_4(0,t)=\{6\lambda_2^2+6\lambda_2^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1\lambda_2^2\}P_0(t), \quad (15)$$

$$P_5(0,t)=9\lambda_1\lambda_2P_0(t), \quad (16)$$

$$P_6(0,t)=9\lambda_1\lambda_2P_0(t), \quad (17)$$

$$P_7(0,t)=\{3\lambda_1\delta_0+3\lambda_2\delta_0\}P_0(t), \quad (18)$$

$$P_8(0,t)=\{6\lambda_1^3+6\lambda_1^3\delta_0+6\lambda_1^2\lambda_2\delta_0+18\lambda_1^3\lambda_2\}P_0(t), \quad (19)$$

$$P_9(0,t)=\{6\lambda_2^3+6\lambda_2^3\delta_0+6\lambda_1\lambda_2^2\delta_0+18\lambda_1\lambda_2^3\}P_0(t), \quad (20)$$

$$P_{10}(0,t)=\{6\lambda_1^2\delta_1+6\lambda_1^2\delta_0\delta_1+12\lambda_1\lambda_2\delta_0\delta_1 + 18\lambda_1^2\lambda_2\delta_1+6\lambda_2^2\delta_1+6\lambda_2^2\delta_0\delta_1 + 18\lambda_1\lambda_2^2\delta_1\}P_0(t). \quad (21)$$

Initial conditions

$P_0(0) = 1$, i.e., in a perfect state, the probability of a state transition at time $t = 0$ is zero.

With the above initial condition, we can obtain Laplace transforms of equations (1) to (11) as:

$$\{s+3\lambda_1+3\lambda_2\}\overline{P_0}(s)=1+\int_0^\infty \emptyset(x)\overline{P_1}(x,s)dx+\int_0^\infty \emptyset(y)\overline{P_3}(y,s)dy + \int_0^\infty \mu_0(x)\overline{P_8}(x,s)dx+\int_0^\infty \mu_0(y)\overline{P_9}(y,s)dy, \quad (22)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+3\lambda_2+\delta_0+\phi(x)\right\}\overline{P_1}(x,s)=0, \quad (23)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\delta_1+2\phi(x)\right\}\overline{P_2}(x,s)=0, \quad (24)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+3\lambda_1+2\lambda_2+\delta_0+\phi(y)\right\}\overline{P_3}(y,s)=0, \quad (25)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\delta_1+2\phi(y)\right\}\overline{P_4}(y,s)=0, \quad (26)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\lambda_2+\phi(y)\right\}\overline{P_5}(y,s)=0, \quad (27)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\phi(x)\right\}\overline{P_6}(x,s)=0, \quad (28)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+2\lambda_1+2\lambda_2+2\phi(m)\right\}\overline{P_7}(m,s)=0, \quad (29)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right\}\overline{P_8}(x,s)=0, \quad (30)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right\}\overline{P_9}(y,s)=0, \quad (31)$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+2\phi(n)\right\}\overline{P_{10}}(n,s)=0. \quad (32)$$

Laplace transform of boundary conditions:

$$\bar{P}_1(0,s)=3\lambda_1\overline{P_0}(s), \quad (33)$$

$$\overline{P_2}(0,s)=\{6\lambda_1^2+6\lambda_1^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1^2\lambda_2\}\overline{P_0}(s), \quad (34)$$

$$\overline{P_3}(0,s)=3\lambda_2\overline{P_0}(s), \quad (35)$$

$$\bar{P}_4(0,s)=\{6\lambda_2^2+6\lambda_2^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1\lambda_2^2\}\overline{P_0}(s), \quad (36)$$

$$\overline{P_5}(0,s)=9\lambda_1\lambda_2\overline{P_0}(s), \quad (37)$$

$$\overline{P_6}(0,s)=9\lambda_1\lambda_2\overline{P_0}(s), \quad (38)$$

$$P_7(0,s)=\{3\lambda_1\delta_0+3\lambda_2\delta_0\}P_0(s), \quad (39)$$

$$P_8(0,s)=\{6\lambda_1^3+6\lambda_1^3\delta_0+6\lambda_1^2\lambda_2\delta_0+18\lambda_1^3\lambda_2\}P_0(s), \quad (40)$$

$$P_9(0,s)=\{6\lambda_2^3+6\lambda_2^3\delta_0+6\lambda_1\lambda_2^2\delta_0+18\lambda_1\lambda_2^3\}P_0(s), \quad (41)$$

$$\begin{aligned}P_{10}(0,s)=\{&6\lambda_1^2\delta_1+6\lambda_1^2\delta_0\delta_1+12\lambda_1\lambda_2\delta_0\delta_1\\&+18\lambda_1^2\lambda_2\delta_1+6\lambda_2^2\delta_1+6\lambda_2^2\delta_0\delta_1\\&+18\lambda_1\lambda_2^2\delta_1\}P_0(s).\end{aligned} \quad (42)$$

Solution of the models

Combining equation (22) and (32) with the aid of (33) to (42), we get:

$$\bar{P}_1(s)=3\lambda_1\left\{\frac{1-S_\emptyset(s+2\lambda_1+3\lambda_2+\delta_0)}{(s+2\lambda_1+3\lambda_2+\delta_0)}\right\}\overline{P_0}(s), \quad (43)$$

$$\overline{P_2}(s)=M_1\left\{\frac{1-\overline{S_{2\emptyset}}(s+\lambda_1+\delta_1)}{(s+\lambda_1++\delta_1)}\right\}\overline{P_0}(s), \quad (44)$$

$$\overline{P_3}(s)=3\lambda_2\left\{\frac{1-\overline{S_\emptyset}(s+3\lambda_1+2\lambda_2+\delta_0)}{(s+3\lambda_1+2\lambda_2+\delta_0)}\right\}\overline{P_0}(s), \quad (45)$$

$$\bar{P}_4(s)=M_2\left\{\frac{1-S_{2\emptyset}(s+\lambda_2+\delta_1)}{(s+\lambda_2+\delta_1)}\right\}\overline{P_0}(s), \quad (46)$$

$$\overline{P_5}(s)=9\lambda_1\lambda_2\left\{\frac{1-S_\emptyset(s+2\lambda_2)}{(s+2\lambda_2)}\right\}\overline{P_0}(s), \quad (47)$$

$$\overline{P_6}(s)=9\lambda_1\lambda_2\left\{\frac{1-S_\emptyset(s+2\lambda_1)}{(s+2\lambda_1)}\right\}\overline{P_0}(s), \quad (48)$$

$$\overline{P_7}(s)=\{3\lambda_1\delta_0+3\lambda_2\delta_0\}\left\{\frac{1-\overline{S_{2\emptyset}}(s+2\lambda_1+2\lambda_2)}{(s+2\lambda_1+2\lambda_2)}\right\}\overline{P_0}(s), \quad (49)$$

$$\overline{P_8}(s)=M_3\left\{\frac{1-\overline{S_{\mu_0}}(s)}{s}\right\}\overline{P_0}(s), \quad (50)$$

$$\bar{P}_9(s)=M_4\left\{\frac{1-S_{\mu_0}(s)}{s}\right\}\overline{P_0}(s), \quad (51)$$

$$\overline{P_{10}}(s)=M_5\left\{\frac{1-\overline{S_{2\emptyset}}(s)}{s}\right\}\overline{P_0}(s), \quad (52)$$

From (21), we obtain:

$$\overline{P_0}(s)=\frac{1}{\Delta(s)}, \quad (53)$$

Where:

$$\begin{aligned}\Delta(s)=&(s+3\lambda_1+3\lambda_2)-3\lambda_1\overline{S_\emptyset}(s+2\lambda_1+3\lambda_2+\delta_0)\\&-3\lambda_2\overline{S_\emptyset}(s+3\lambda_1+2\lambda_2+\delta_0)\text{-}(M_3+M_4)\overline{S_{\mu_0}}(s),\end{aligned}$$

$$M_1=\{6\lambda_1^2+6\lambda_1^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1^2\lambda_2\},$$

$$M_2=\{6\lambda_2^2+6\lambda_2^2\delta_0+6\lambda_1\lambda_2\delta_0+18\lambda_1\lambda_2^2\},$$

$$M_3=\{6\lambda_1^3+6\lambda_1^3\delta_0+6\lambda_1^2\lambda_2\delta_0+18\lambda_1^3\lambda_2\},$$

$$M_3=\{6\lambda_2^3+6\lambda_2^3\delta_0+6\lambda_1\lambda_2^2\delta_0+18\lambda_1\lambda_2^3\},$$

$$\begin{aligned}M_5=\{&6\lambda_1^2\delta_1+6\lambda_1^2\delta_0\delta_1+12\lambda_1\lambda_2\delta_0\delta_1+18\lambda_1^2\lambda_2\delta_1\\&+6\lambda_2^2\delta_1+6\lambda_2^2\delta_0\delta_1+18\lambda_1\lambda_2^2\delta_1\},\end{aligned}$$

The probability that the system is operational is:

$$\begin{aligned}P_{up}(s)=&P_0(s)+P_1(s)+P_2(s)+P_3(s)+P_4(s)\\&+P_5(s)+P_6(s)+P_7(s),\\ \bar{P}_{up}(s)=&\frac{1}{\Delta(s)}\Big\{1+3\lambda_1\left(\frac{1-\overline{S_\emptyset}(s+2\lambda_1+3\lambda_2+\delta_0)}{(s+2\lambda_1+3\lambda_2+\delta_0)}\right)\\&+M_1\left(\frac{1-\overline{S_{2\emptyset}}(s+\lambda_1+\delta_1)}{(s+\lambda_1++\delta_1)}\right)+3\lambda_2\left(\frac{1-\overline{S_\emptyset}(s+3\lambda_1+2\lambda_2+\delta_0)}{(s+3\lambda_1+2\lambda_2+\delta_0)}\right)\\&+M_2\left(\frac{1-\overline{S_{2\emptyset}}(s+\lambda_2+\delta_1)}{(s+\lambda_2+\delta_1)}\right)+9\lambda_1\lambda_2\left(\frac{1-\overline{S_\emptyset}(s+2\lambda_2)}{(s+2\lambda_2)}\right)+9\lambda_1\lambda_2\left(\frac{1-\overline{S_\emptyset}(s+2\lambda_1)}{(s+2\lambda_1)}\right)\\&+(3\lambda_1\delta_0+3\lambda_2\delta_0)\left(\frac{1-\overline{S_{2\emptyset}}(s+2\lambda_1+2\lambda_2)}{(s+2\lambda_1+2\lambda_2)}\right)\Big\}.\end{aligned} \quad (54)$$

MODEL ANALYSIS

Availability Analysis

To make a distinction, we looked at the system's availability in two different ways, viz:

When repair follows Gumbel-Hougaard Family copula distribution

When the repair follows the copula distribution, the system availability can be determined from (54) for $\overline{S_\emptyset}(s) = \frac{\emptyset}{s+\emptyset}$, $\frac{1-\overline{S_\emptyset}(s)}{s}$, $\bar{S}_{\mu_0}(s) = \bar{S}_{exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}(S) = \frac{exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}{s+exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}$, with all repairs set to 1, i.e., $\emptyset(x) = \emptyset(y) = \mu_0(x) = \mu_0(y) = \mu_0(m) = \mu_0(n) = 1$, and $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\delta_0 = 0.03$, and $\delta_1 = 0.04$ fixed. Then if we take Laplace transforms, we get:

$$P_{up}(s) = \{0.00005276490045e^{-2.718445492t} - 0.002813764515e^{-2.060000000t} - 0.007805509296e^{-1.185732893t} - 0.00001540029829e^{-1.106406617t} + 1.012602413e^{-0.007714997787t} - 0.0007566858053e^{-1.040000000t} - 0.0003698765456e^{-2.050000000t} - 0.0008939418739e^{-2.060000000t}\}. \quad (55)$$

Table 3 and figure 2 illustrate the system's availability for copula repair distribution when $t = 0,10,20,\ldots\ldots\ldots,100$.

Table 3. Availability against time *t* with respect to copula distribution

Time	0	10	20	30	40	50	60	70	80	90	100
Availability	1.0000	0.9374	0.8678	0.8034	0.7437	0.6885	0.6374	0.5901	0.5463	0.5057	0.4681

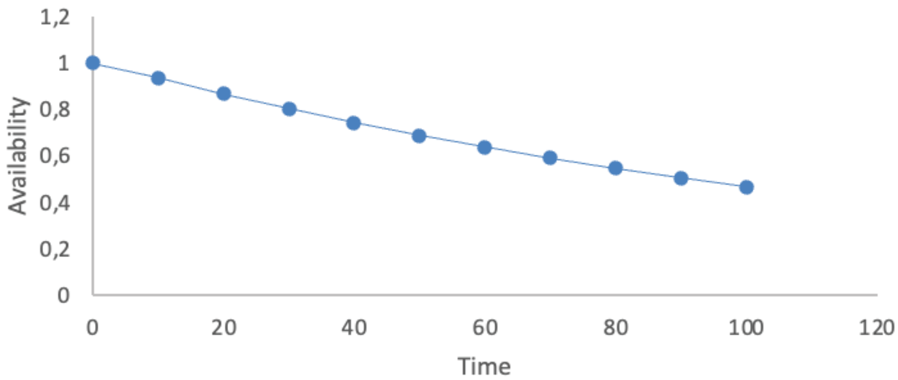


Figure 2. Availability against time with respect to copula distribution.

When repair follows general distribution

To obtain the availability of the system when the repair is through general distribution, we set $\overline{S_\emptyset}(s) = \frac{1}{s+\emptyset}$ and all repairs to 1, i.e., $\emptyset(x) = \emptyset(y) = \mu_0(x) = \mu_0(y) = \mu_0(m) = \mu_0(n) = 0$, and $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\delta_0 = 0.03$, and $\delta_1 = 0.04$, then taking the Laplace transform of equation (54), we get the expression for availability as:

$$P_{up}(s) = \{0.00008717398954e^{-1.000077209t} + 1.012518714e^{-0.007714366521t} - 0.002814812304e^{-2.060000000t} - 0.0007578535962e^{-1.040000000t} - 0.0003700136500e^{-2.050000000t} - 0.0008971075757e^{-1.020000000t} - 0.007751104760e^{-1.185801404t} - 0.00001499582430e^{-1.106407020t}\}. \quad (56)$$

Table 4 and figure 3 present the system's availability for general repair distribution when $t = 0,10,20,\ldots\ldots\ldots,100$.

Table 4. Availability against time *t* in terms of general repair

Time	0	10	20	30	40	50	60	70	80	90	100
Availability	1.0000	0.9374	0.8678	0.8033	0.7437	0.6885	0.6374	0.5900	0.5462	0.5057	0.4681

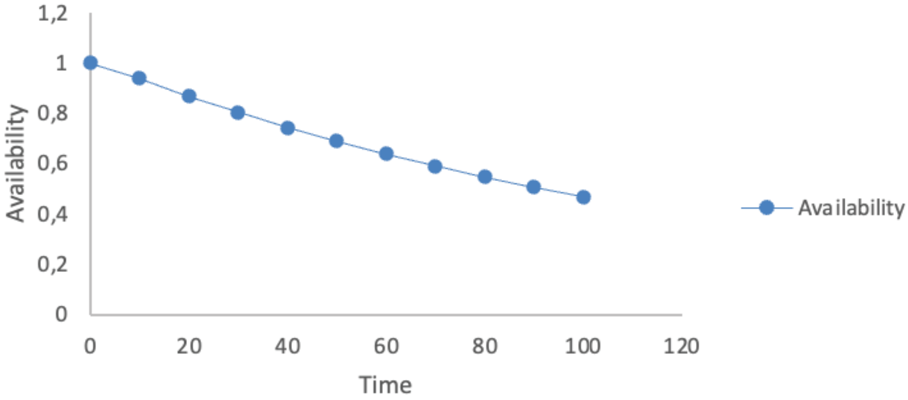


Figure 3. Availability against time in terms of general repair.

RELIABILITY ANALYSIS

By reducing all repair rates to zero i.e., $\emptyset(x) = \emptyset(y) = \mu_0(x) = \mu_0(y) = \mu_0(m) = \mu_0(n) = 0$, and taking $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\delta_0 = 0.03$, and $\delta_1 = 0.04$ in equation (54), then using the inverse Laplace transform, the system's reliability can be expressed as:

$$R(t) = \{0.176000000e^{-0.06000000000t} + 0.03600000000e^{-0.04000000000t} - 6e^{-0.1000000000t} + 0.01725000000e^{-0.05000000000t} + 8.245035714e^{-0.09000000000t} - 1.500000000e^{-0.1100000000t} + 0.02571428571e^{-0.02000000000t}\}. \quad (57)$$

Reliability can be calculated using equations (57) for different values of time t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100, as shown in Table 5 and Figure 4.

Mean Time to Failure (MTTF) Analysis

The expression for MTTF can be obtained by setting all repairs to zero and restricting as *s* reaches zero:

$$\lim_{s\to 0} \Delta(s) = 3\lambda_1 + 3\lambda_2,$$

$$MTTF = \lim_{s\to 0} \overline{P_{up}}(s) = \frac{1}{\Delta(s)}\left\{1 + \frac{3\lambda_1}{2\lambda_1+3\lambda_2+\delta_0} + \frac{M_1}{\lambda_1++\delta_1} + \frac{3\lambda_2}{3\lambda_1+2\lambda_2+\delta_0} + \frac{M_2}{\lambda_2+\delta_1} + \frac{9\lambda_1}{2} + \frac{9\lambda_2}{2} + \frac{3\lambda_1\delta_0+3\lambda_2\delta_0}{2\lambda_1+2\lambda_2}\right\}. \quad (58)$$

Setting $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\delta_0 = 0.03$, $\delta_1 = 0.04$ and varying λ_1, λ_2, δ_0 and δ_1 one as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 respectively in equation (58), one may

Table 5. Variation of reliability corresponding to time *t*

Time	0	10	20	30	40	50	60	70	80	90	100
Reliability	1.0000	0.7978	0.4774	0.2579	0.1340	0.0695	0.0370	0.0206	0.0123	0.0079	0.0054

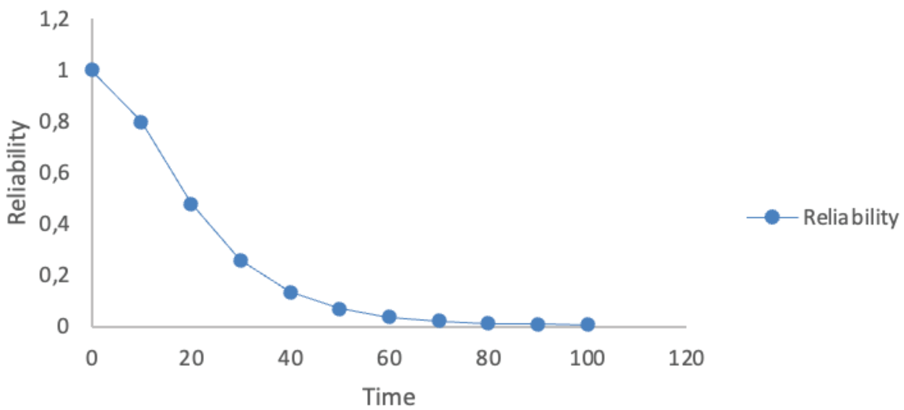


Figure 4. Variation of reliability corresponding to time.

Table 6. MTTF variation in terms of failure rates

Failure Rate	λ_1	λ_2	δ_0	δ_1
0.01	23.4391	31.8526	25.4245	24.1469
0.02	18.6473	23.4391	24.3056	23.7803
0.03	15.8481	19.0529	23.4391	23.5730
0.04	13.9870	16.3130	22.7579	23.4391
0.05	12.6479	14.4229	22.2165	23.3453
0.06	11.6322	13.0340	21.7833	23.2759
0.07	10.8323	11.9676	21.4356	23.2224
0.08	10.1844	11.1217	21.1566	23.1799
0.09	9.6480	10.4337	20.9336	23.1453

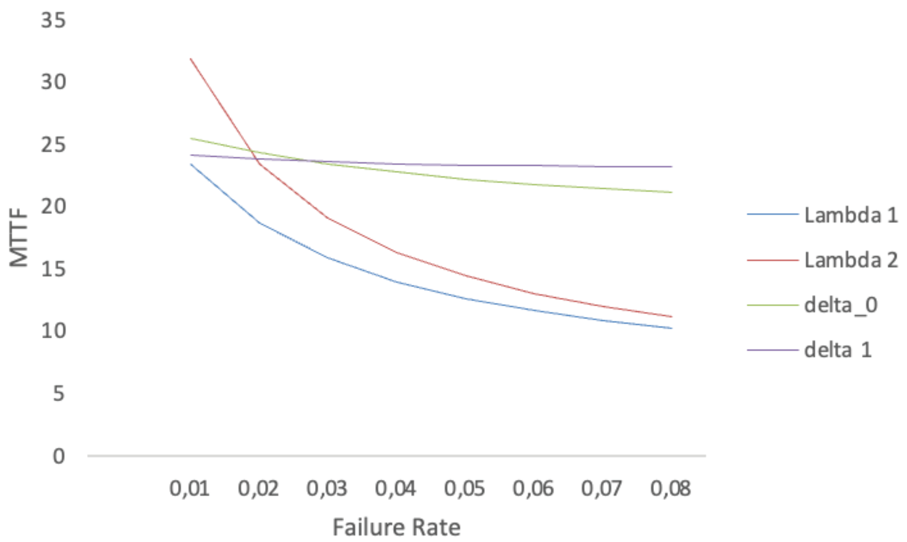


Figure 5. MTTF variation in terms of failure rates.

obtain table 6 and figure 5 which show the variation of MTTF in terms of failure rates.

Sensitivity Analysis

MTTF sensitivity can be determined using the partial derivative of MTTF with respect to failure rates in equation (58). Table 7 and Figure 6 display the MTTF sensitivity for fixed values of failure rates as $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\delta_0 = 0.03$, $\delta_1 = 0.04$.

Cost Function Analysis

Case 1: Expression for cost function/Expected profit when the repair follows copula distribution

If the service facility is open at all times, the formula below will determine the estimated profit for the interval [0, t).

$$E_p(t) = K_1 \int_0^t P_{up}\,(t)dt - K_2 t - K_3 t \tag{59}$$

Where K_1, K_2 and K_3 in the interval [0, t) are the revenue generated, service cost per unit time, and, service cost due to online and offline maintenance, respectively. Equation (60) can be obtained for the same set of parameters in equation (55).

$$\begin{aligned} E_P(t) = K_1\{&0.0001804275832e^{-2.050000000t} - 0.00001940995345e^{-2.718445492t} \\ &+ 0.006582856343e^{-1.185732893t} + 0.00001391920299e^{-1.106406617t} \\ &- 131.2511605e^{-0.007714997787t} + 0.0008764136019e^{-1.020000000t} \\ &+ 0.001365905104e^{-2.060000000t} + 0.0007275825051e^{1.040000000t} \\ &+ 131.2414328\} - (K_2 - K_3)t. \end{aligned} \tag{60}$$

Fixing $K_1 = 1$, $K_2 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, and, $K_3 = 0.1$. Table 8 and figure 7 can be obtained by using different values of the time variable, such as $t = 0, 10, 20, \ldots\ldots\ldots 100$.

Case 2: when the repair follows general distribution, the cost function is expressed as:

Table 7. Sensitivity of the MTTF to changes in failure rate

Failure rate	$\frac{\partial}{\partial\lambda_1}(MTTF)$	$\frac{\partial}{\partial\lambda_2}(MTTF)$	$\frac{\partial}{\partial\delta_0}(MTTF)$	$\frac{\partial}{\partial\delta_1}(MTTF)$
0.01	-654.5725	-1243.4183	-127.5855	-51.0185
0.02	-352.1928	-573.6695	-97.9045	-26.4351
0.03	-223.0796	-336.5049	-76.4815	-16.2583
0.04	-155.5404	-223.4422	-60.5112	-11.0296
0.05	-115.4175	-160.0163	-48.2868	-7.9799
0.06	-89.4430	-120.6018	-38.7212	-6.0437
0.07	-71.5630	-94.3224	-31.0950	-4.7370
0.08	-58.6773	-75.8745	-24.9171	-3.8131
0.09	-49.0550	-62.4030	-19.8423	-3.1358

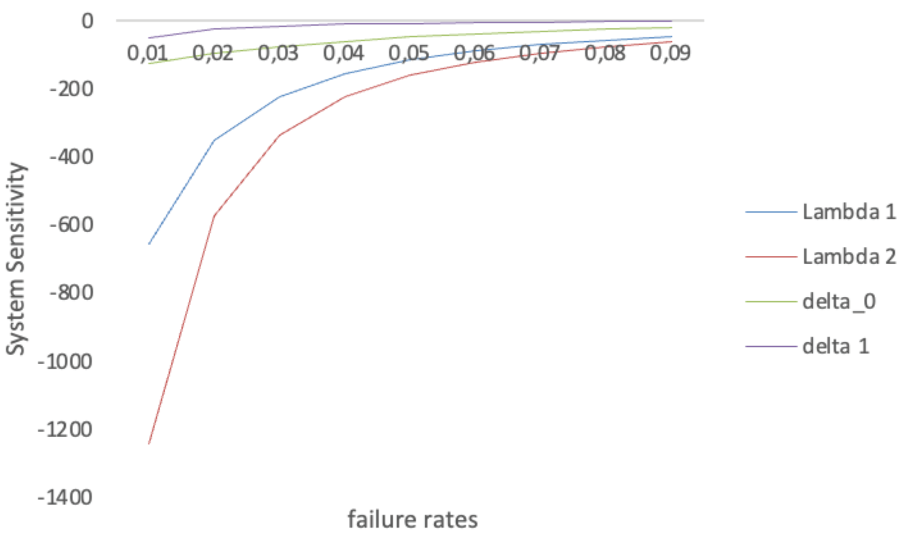


Figure 6. Sensitivity vs failure rate.

Table 8. Cost function/Expected profit versus time for copula repair

Time	E_p(t) $K_2 = 0.1$	E_p(t) $K_2 = 0.2$	E_p(t) $K_2 = 0.3$	E_p(t) $K_2 = 0.4$	E_p(t) $K_2 = 0.6$	E_p(t) $K_2 = 0.6$
0	0	0	0	0	0	0
10	7.7355	6.7355	5.7355	4.7355	3.7355	2.7355
20	14.7571	12.7571	10.7571	8.7571	6.7571	4.7571
30	21.109	18.109	15.109	12.109	9.109	6.109
40	26.8407	22.8407	18.8407	14.8407	10.8407	6.8407
50	32.9984	27.9984	22.9984	17.9984	12.9984	7.5678
60	37.6246	31.6246	25.6246	19.6246	13.6246	7.6246
70	42.7588	35.7588	28.7588	21.7588	14.7588	7.7588
80	48.4375	40.4375	32.4375	24.4375	16.4375	8.4375
90	53.6946	44.6946	35.6946	26.6946	17.6946	8.6946
100	59.5614	49.5614	39.5614	29.5614	19.5614	9.5614

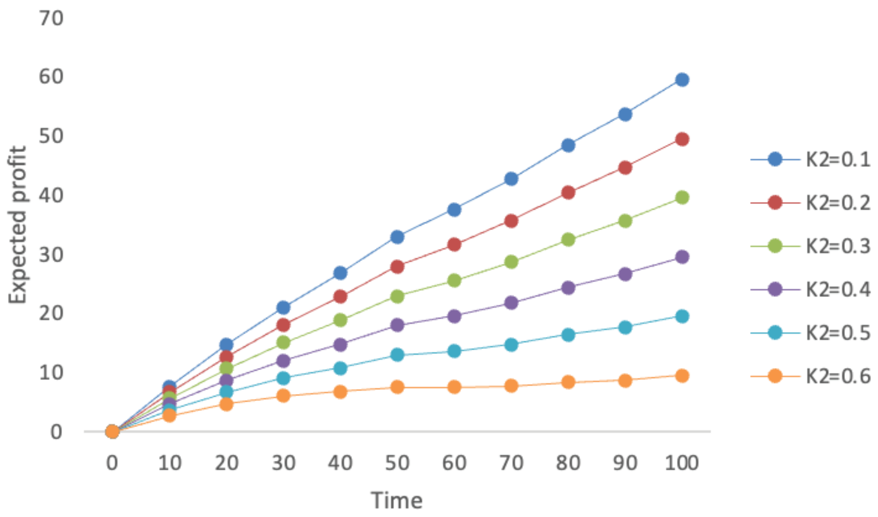


Figure 7. Cost function/Expected profit versus time for copula repair.

Table 9. Cost function/Expected profit versus time for general repair

Time	$E_p(t)$ $K_2 = 0.1$	$E_p(t)$ $K_2 = 0.2$	$E_p(t)$ $K_2 = 0.3$	$E_p(t)$ $K_2 = 0.4$	$E_p(t)$ $K_2 = 0.6$	$E_p(t)$ $K_2 = 0.6$
0	0	0	0	0	0	0
10	7.7348	6.7348	5.7348	4.7348	3.7348	2.7348
20	14.7558	12.7558	10.7558	8.7558	6.7558	4.7558
30	21.1071	18.1071	15.1071	12.1071	9.1071	6.1071
40	26.8384	22.8384	18.8384	14.8384	10.8384	6.8384
50	32.9956	27.9956	22.9956	17.9956	12.9956	7.5678
60	37.6215	31.6215	25.6215	19.6215	13.6215	7.6215
70	42.7555	35.7555	28.7555	21.7555	14.7555	7.7555
80	48.4340	40.4340	32.4340	24.4340	16.4340	8.4340
90	53.6910	44.6910	35.6910	26.6910	17.6910	8.6910
100	59.5576	49.5576	39.5576	29.5576	19.5576	9.5576

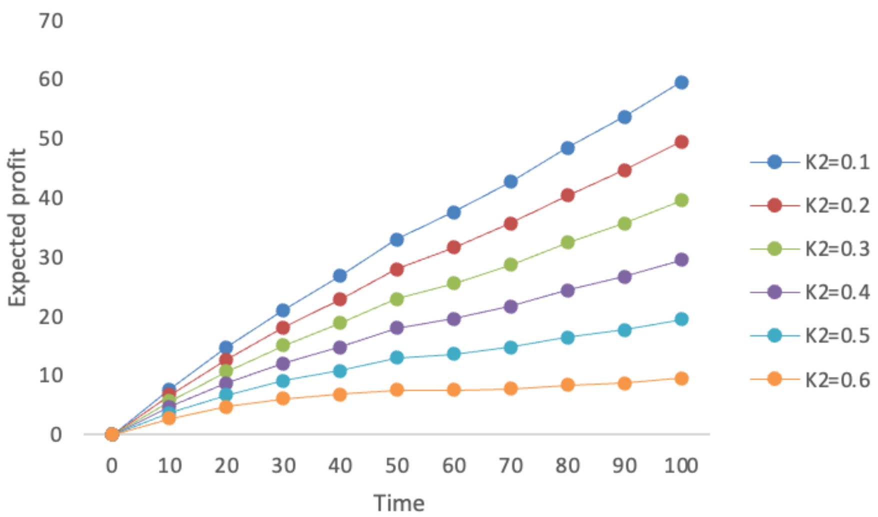


Figure 8. Cost function/Expected profit versus time for copula repair.

$$\begin{aligned}E_P(t) = K_1\{&0.0001804944634e^{-2.050000000t} + 0.006536596039e^{-1.185801404t}\\&+ 0.00001355362330e^{-1.106407020t} - 0.00008716725944e^{-1.000077209t}\\&- 131.2510510e^{-0.007714366521t} + 0.0008795172311e^{-1.020000000t}\\&+ 0.001366413740e^{-2.060000000t} + 0.0007287053810e^{1.040000000t}\\&+ 131.2413316\} - (K_2 - K_3)t.\end{aligned} \tag{61}$$

Table 9 and figure 8 are obtained using the same parameters as in equation (60).

RESULTS AND DISCUSSION

For investigating the effects of online and offline preventive maintenance, we calculate the system availability, reliability, mean time to failure (MTTF), MTTF sensitivity, and cost function for the established models numerically. When failure rates are set at various values, figure 2 shows how availability changes over time. The figure depicts how the availability of repairable systems declines over time, but eventually stabilizes at zero after a sufficiently long interval of time. The graphical representation of the model can predict the future behavior of a complex system for a given set of parameters at any time. The values of availability when repair follows a general distribution are similar to the values of availability when repair follows the Gumbel-Hougaard family copula distribution; this can be seen in figure 2 and 3. This is attributable to the system's online and offline preventive maintenance. However, there is no denying that using copula repair increases system's availability over general repair.

Figure 4 shows the difference in reliability over time. When compared to system availability, it can be seen that system reliability plummets. This is due to lack of system repairs. The general consensus is that the less the repairs, the lower the reliability. This illustrates what failure to manage the structure/system entails.

Figure 5 illustrates the differences in the value of the system's mean time to failure (MTTF) for failure rates λ_1, λ_2, δ_0 and δ_1 respectively, fixing other parameters constant. The MTTF values with respect to δ_0 and δ_1 is much higher than the MTTF values corresponding to λ_1 and λ_2. According to this sensitivity study, online and offline preventive maintenance failure rates are more responsible for proper system functioning. Figure 5 illustrates this.

The information on the sensitivity analysis conducted in this paper is shown in figure 6.

Figures 7 and 8 demonstrate the difference in effective benefit over time when the repair follows a copula distribution or a general distribution, respectively, with revenue cost per unit time is set at 1.0, cost of online and offline maintenance set at 0.1, the service cost is varied and the failure rates constant. Figures 7 and 8 show that when the repair follows the copula distribution, the expected gain is higher than when the repair follows the general distribution. Because of online and offline preventive maintenance, the expected benefit when repairing with copula distribution is comparable to the expected profit when repairing with general repair. According to this sensitivity review, the importance of using copula repair would be decreased if online and offline preventive maintenance is mandated. However, copula repair is still recommended for the system's proper operation.

CONCLUSION

The reliability metrics for different failure values and repair rates are critically analyzed in order to measure the performance of the systems under consideration. The system's transient probabilities and reliability metrics such as availability, reliability, mean time to failure and cost are calculated using the Markovian process, Laplace transformation, and supplementary variable approaches. The influence of time and other system characteristics on reliability metrics was simulated using MATLAB. Numerical experiments were used to obtain and validate the fundamental expressions such as availability, reliability, mean time to failure, sensitivity, and cost function. On the basis of the numerical results obtained for a particular case in figures 2-8 and tables 1-7, it is clear that the value, i.e., the importance of copula repairs has decreased with online and offline preventive maintenance. This means by implementing online and offline preventive maintenance, system failure would be greatly reduced, resulting in improved output and revenue generation. It is generally known that system failure will reduce production efficiency and may even result in a tragedy. This research paper will help system engineers, programmers, and maintenance managers design more important systems and maintain them in the best interests of humanity. Future iterations of this work may combine the Gumbel-Hougaard family Copula with the Bivariate Gumbel-Hougaard family Copula.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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