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Research Article

Reliability estimation of a fault coverage distributed system with replacement options under four different scenarios

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ABSTRACT

In this present study, series-parallel system composed of five subsystems with the following specifications were analyzed: subsystem 1 consists of two dissimilar clients that are connected to a single unit load balancer I which made up subsystem 2, whereas subsystem 3 consist of two active fog node working in parallel, subsystem 4 comprises of a load balancer II and subsystem 5 is made up of two similar units/components of cloud server. Cloud server, load balancer, fog node and clients failure and repair rate are assumed to be exponentially distributed. The system is under four different scenarios as follows: Scenario 1 system with replacement at complete, scenario 2 system with replacement at partial failure and complete, scenario 3 system without failure detection and replacement repair at complete and lastly, scenario 4 system with undetected failure and replacement at complete. This system is susceptible under first order differential difference equation to formulate the expression of availability and MTTF. The steady state availability, MTTF, sensitivity and expected profit based on general were compared and presented. This study is important to system engineers, designers, plant management, developers and maintenance personnel in the suitable designing and analysis of maintenance policy or processes and also in the assessment of performance and safety of the systems in general during and after the burn-in period.

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INTRODUCTION

In many scenarios, computer system utilizes number of distributed networks to provide available and optimal network to the clients. The study of computer network system present its economic and technical feasibility as the best choice for the multipurpose network. However, with the advancement in technology, availability of computer network happens to be subject of research and discussion. Meeting optimal level of availability is of paramount important in information, communication, military and institutional sector. Moreover, reliability could not attend its maximum level, computer network will be very poor.

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High computer system reliability is vital to industrial growth due to the fact that revenue mobilization is proportional to system performance. Due to its importance in industrial, domestic, institutional and manufacturing sector, literature study on dependability, reliability, maintainability and availability modelling of different computer network were developed. However, the developed models are used to address the computer network performance, subject to system failure. The technique of redundancy is thoroughly used to enhance reliability, dependability and availability of the system. In some computer network, the availability and dependability rely on the design of the system and strength of the units. To retain availability and dependability of complex computer network to an optimal level, the structure of the system and its components of optimal availability are required. Generally, system designers can develop technologies in a serial network to improve network availability, dependability and reliability. [1] Explore on performance analysis on computer network system that comprises of centralized database server, load balancer and distributed database server, [2] discuss on reliability metrics of network communication system having receiver, relay and transmitter. [3] Writes on reliability and dependability assessment of complex system having two subsystems on k-out-of-n working under G policy in subsystem 1 and four identical units in active parallel in subsystem 2. [4] Studied reliability of computer network base on genetic algorithm and the optimization technique were developed for better reliability. [5] Presents stochastic performance of computer based test having four subsystems arranged in series namely load balancer, clients, centralized server and database server. [6] Explore on reliability analysis of computer network which comprises of three subsystems: router, workstation and hub. [7] Investigate the performance measures of network with transparent bridge as follows 1-out-of-2: G, 2-out-of-3: F, a bridge unit and D 3-out-of-5: G schemes. [8] Dealt with an article on computer networking systems.

[9] Publish an article on reliability enhancement in intuitionistic fuzzy space, [10] discuss on heterogeneity using rpc in client. [11] Estimate the coliform values of the Tekkekoy deep sea discharge system, which is chosen as an application area, by using a radial-based artificial neural network structure, [12] writes on production-distribution network system for a company, which is active in producing bottled natural spring water was established.

[13] Develop models for the strength and performance analysis of computer network under different maintenance scenario. [14] Writes on reliability measures of database cluster, virtual router redundancy protocol and load balancer, the article analyzed availability by comparing the reliability if load balancer, virtual router redundancy protocol and high availability proxy were put in place, [15] investigated the impact of structure of the system reliability measures of software agent and client server. [16] Defined a Secure Simple Epidemic Algorithm (SSEA) for PSN where a security condition controls the traffic. [17] Classification algorithms were used to classify electromyography and depth sensor data, [18] optimum CW size is defined through meta-heuristic optimization algorithms. [19] Published on OLTP applications with incremental repartitioning of shared-nothing distributed databases, [20] investigates the implication of load balancing of distributed system. [21] Writes on analysis of the FANET TCAs currently in use, along with a brand-new taxonomy of TCAs based on the FANET topology architectures and underlying mathematical models.

[22] Explore on reliability analysis of computer network which comprises of three subsystems: router, workstation and hub. [23] Dealt with repairable system with reboot delay, one repair policy and imperfect coverage, [24] present work on the reliability measures of coverage factor with a standby system. [25] Investigate parallel system with three types of failure namely human failure, unit failure and major failure. [26] Consider a distributed system with five standby subsystems A (the clients), B (two load balancers), C (two distributed database servers), D (two mirrored distributed database serves) and E (centralized database server) is considered arranged as series-parallel system. [27] Analyzes the advantage of data center network topology by taking reliability and profit requirements into account, with distributed data center network topology having three components as follows: client applications, directory proxy server, and master servers were considered.

This research work further improved the work of previous researchers were five subsystems were considered. Subsystem A consist of 2-clients, subsystem B comprises of a load balancer I, 2-fog node are in subsystem C, subsystem D comprises of load balancer II and lastly, subsystem E consist of 2-cloud server. However, analysis of the model in terms of fault tolerant, general repair and copula were thoroughly investigated. Reliability analysis measures such as availability, MTTF, sensitivity, cost analysis was carried out for different scenarios to check optimality of the entire system with respect both failure and repair rate. Moreover, some practical applications were considered.

This work is structured as follows. Description, assumptions and nomenclatures on the system are presented in section 2, model formulation were discussed in section 3, section 4 consist of results and discussion and lastly section 5 which comprises of the conclusion.

According to the literature review, little research articles on performance estimation of a fault coverage distributed system with replacement options under four different scenarios have been published. Motivated by this fact, we are interested to conducting a research on performance estimation of a fault coverage distributed system with replacement options under four different scenarios in this present work. The impact of the fault tolerance factor, in conjunction with the different scenarios, on the system availability, MTTF, sensitivity and profit were captured. The primary goal of this work is to determine how different scenarios will improve the availability and profit of the system under consideration, followed by a discussion and references, where the paper is concluded.

NOTATIONS, ASSUMPTIONS AND MODEL DESCRIPTION

Notations

v_0 failure rate of load balancer I
v_1 failure rate of clients
v_2 failure rate of fog nodes
v_3 failure rate of cloud servers
v_4 failure rate of load balancer II
ξ_0 Repair rate of load balancer
ξ_1 Replacement rate of clients
ξ_2 Replacement rate of fog nodes
ξ_3 Replacement rate cloud servers
ξ_4 Repair rate of load balancer II
c Fault tolerant (probability of withstanding fault)
$\delta_0 = 1 - c$
$\omega_i(t)$ Probability that a system is in a certain state at a given time.
A_{vk} At time t, the system is available

Assumptions

a. Failure of client is independent to the failure of fog node, load balancer and cloud server and vice vasa.
b. Repair / Replacement is immediate.
c. It is assumed that all the clients are active.
d. Each failure is repairable.
e. Rate of failure and repair obeys exponential distribution.
f. Systems have redundant standby units
g. Switching from standby to operation is perfect

Model Description

Subsystem A is made up of 2-clients in active parallel, subsystem B made up of load balancer I. 2-fog nodes in active parallel made up subsystem C, subsystem D made up of load balancer 2 and lastly, 2-cloud server in active parallel made up subsystem E. Moreover, the entire structure of the system, that is: Client, load balancer, fog node and cloud servers were configured as series-parallel, clients send request to the cloud server which in turn process the result and respond to the request. However, the two load balancers helps in utilization of the information required from the server, in Figure 1 (block diagram of the system), fog node serves as an intermediate between the clients, load balancers and cloud server. Table 1 provides a brief description of the states, while Figure 2 depicts all possible state transition for the model.

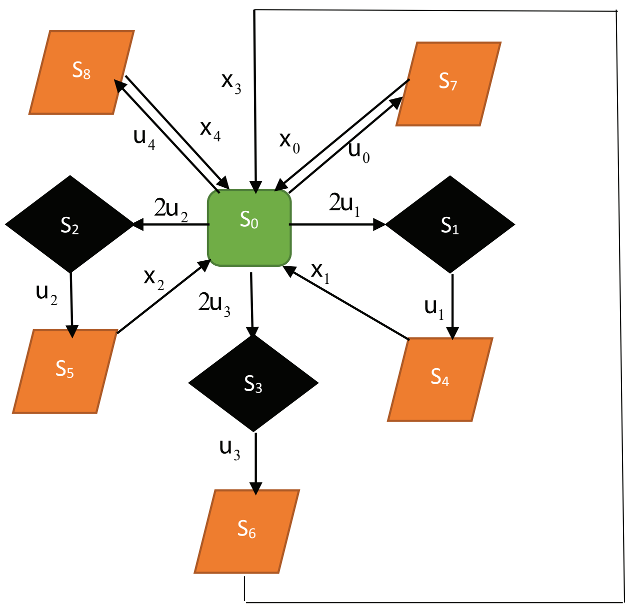


Figure 2. Transition diagram of scenario 1 system with replacement at complete.

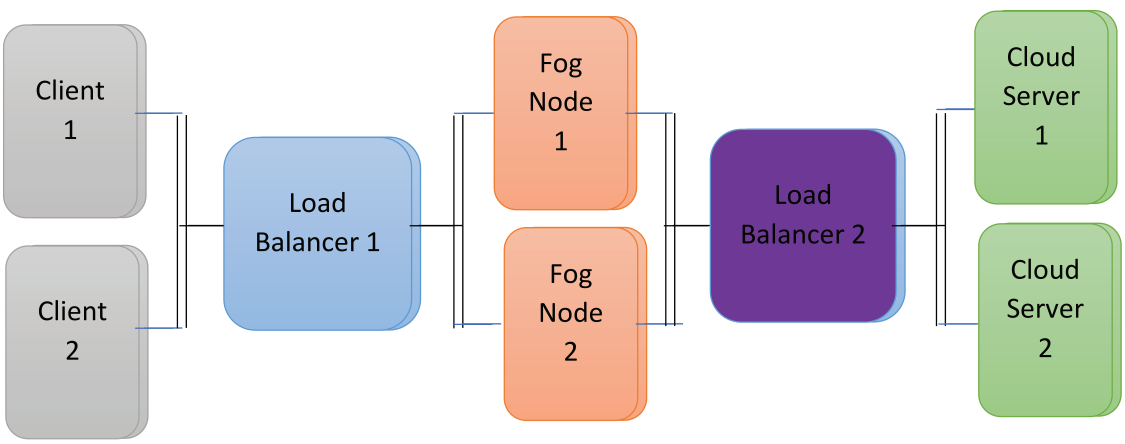


Figure 1. Performance block diagram of the system.

Table 1. State description of the system

State	Description	System Status
S_0	The clients, fog nodes, load balancer and cloud servers are working.	System is operational
S_1	Two fog nodes, One client failed, another client, load balancer and two cloud servers are working.	System is operational
S_2	One fog node, another fog node, two clients, load balancer and two cloud servers are working.	System is operational
S_3	One cloud server failed, another cloud server, two clients, two fog nodes are working	System is operational
S_4	One client failed, another client has failed.	System is down
S_5	One fog node failed, another fog node failed	System is down
S_6	One cloud server failed, another cloud server failed	System is down
S_7	Load balancer I failed	System is down
S_8	Load balancer II failed	System is down

Subsystem A: System with Replacement at Complete Failure State

Maintenance staff performs a perfect repair (repair as new) when a cloud server or client experiences a partial hardware failure. In the event of a complete failure over time, the component will be completely replaced. The system's Markov chain-based state transitions are shown in Figure 2 below.

Subsystem B: System with Replacement at Partial Failure and Complete State

The analysis is carried out as follows: in the event a system component fails due to hardware failure, maintenance personnel are charged with the responsibility of replacing the problematic part of the system to ensure that the system can still function. Figure 3 below shows the markov chain transition diagram

Subsystem C: System without Failure Detection and Replacement Repair at Complete State

The underlying premise is that whenever a fault manifests itself, whether at the cloud server or client side, the failure detection device were not in place to verify the failed component, as a result the failed component is therefore being replaced in order to avoid the failure occurring again anytime soon. Figure 4 below shows a diagram of a Markov chain transition.

Subsystem D: System with Detected Failure and Replacement at Complete State

In this subsystem, the units were considered fault tolerant in the sense that even when a fault occurs in one or more host components, they continue to operate without malfunctioning. Fault tolerance device is the property that allows a system to continue operating properly on the

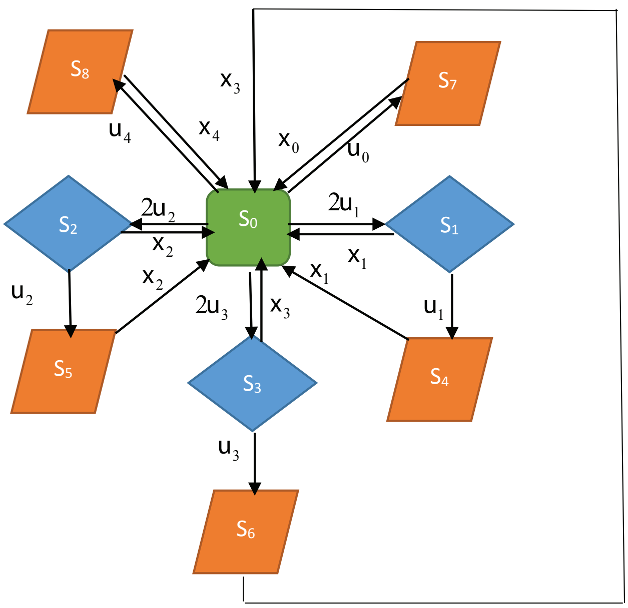


Figure 3. Transition diagram of scenario 2 system with replacement at partial failure and complete.

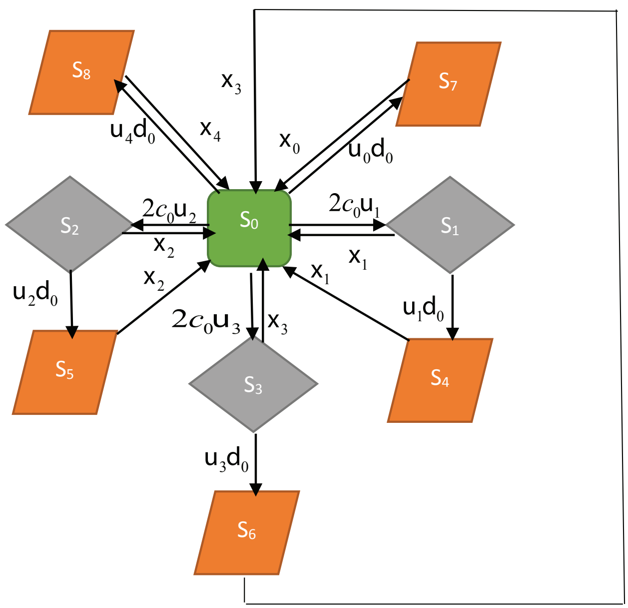


Figure 4. Transition diagram of scenario 3 system without failure detection and replacement repair at complete.

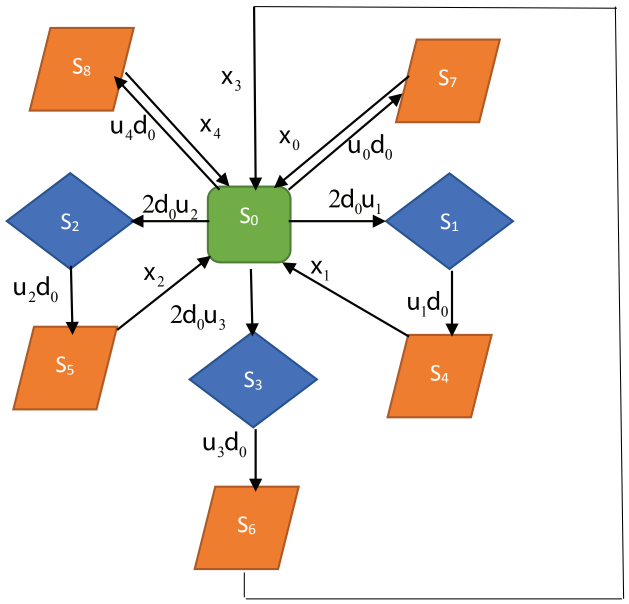


Figure 5. Transition diagram of scenario 4 system with detected failure and replacement at complete.

occurrence of a failure. The fault tolerant system, however, cannot withstand catastrophic failures, which results in system failure and requires replacement. The system's state transitions are shown in Figure 5 below using the Markov chain model.

MODEL FORMULATION

From Figure 1 above to derive the system of linear differential equation, the explicit expression of system availability can be obtained by solving the equations below. The results of the state probability equations for the system's operational states can then be used to determine the system availability. In order to analyse the system availability of the system, we define the $\omega_i(t)$ to be the probability that the system is in state i at time t and that we have $\omega(t) = \left[\omega_1(t), \omega_2(t), \ldots, \omega_8(t)\right]$ be the probability row vector with initial conditions.

$$\omega_k(0) = \begin{cases} 1, & k = 0 \\ 0, & k = 1,2,3,4,5,6,7,8 \end{cases} \tag{1}$$

The steady state probability of systems availability can be obtained from the solutions for $\omega_i(t), i = 0,1,2,3,4,5,6,7,8$ State 0,1,2 and 3 are the only working states of all the scenarios in Figure 1, thus the steady state availability $A_{vi}(\infty)$ at time bility that the system is in state i at time t and that we have is

$$A_{Ti}(\infty) = \omega_0(\infty) + \omega_1(\infty) + \omega_2(\infty) + \omega_3(\infty) \tag{2}$$

From Figure 2, the corresponding set of differential difference equations for Subsystem 1 are

$$\begin{pmatrix} \omega_0'(t) \\ \omega_1'(t) \\ \omega_2'(t) \\ \omega_3'(t) \\ \omega_4'(t) \\ \omega_5'(t) \\ \omega_6'(t) \\ \omega_7'(t) \\ \omega_8'(t) \end{pmatrix} = \begin{pmatrix} -\eta_1 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2v_1 & -v_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_2 & 0 & -v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_3 & 0 & 0 & -v_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix}$$

Availability of Subsystem 1 is

$$A_{T1}(\infty) = \omega_0(\infty) + \omega_1(\infty) + \omega_2(\infty) + \omega_3(\infty) \tag{3}$$

Setting $\omega_k'(t) = 0$ as $t \to \infty$ in steady state, to obtained

$$\begin{pmatrix} -\eta_1 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2v_1 & -v_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_2 & 0 & -v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_3 & 0 & 0 & -v_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

the normalizing condition is

$$\sum_{k=0}^{8} \omega_k(\infty) = 1 \tag{4}$$

Using (4) to give the explicit expressions for the steady-state availability of Subsystem 1 given in (3) is now

$$A_{T1}(\infty) = \frac{7\xi_0\xi_1\xi_2\xi_3\xi_4}{Q_1} \tag{5}$$

where

$Q_1 = 7\xi_0\xi_1\xi_2\xi_3\xi_4 + v_4\xi_0\xi_1\xi_2\xi_3 + 2v_3\xi_0\xi_1\xi_2\xi_4 + 2v_3\xi_0\xi_1\xi_2\xi_4 + 2v_2\xi_0\xi_1\xi_3\xi_4 + 2v_1\xi_0\xi_2\xi_3\xi_4 + v_0\xi_1\xi_2\xi_3\xi_4$ and $\eta_1 = \left(v_0 + 2v_1 + 2v_2 + 2v_3 + v_4\right)$.

To evaluate the $MTTF_1$, the rows and columns of the absorbing (failure) states from the above matrix were deleted and transposed to obtain the new matrix L_1.

$$L_1 = \begin{pmatrix} -\eta_1 & 2v_1 & 2v_2 & 2v_3 \\ 0 & -v_1 & 0 & 0 \\ 0 & 0 & -v_2 & 0 \\ 0 & 0 & 0 & -v_3 \end{pmatrix}$$

$$MTTF_1 = \frac{7}{v_0 + 2v_1 + 2v_2 + 2v_3 + v_4} \tag{6}$$

$$PF_1 = AT_1C_0 - Cr_1Br_1 \tag{7}$$

From Figure 3, the corresponding set of differential difference equations for Subsystem 2 are

$$\begin{pmatrix} \omega_0^{'}(t) \\ \omega_1^{'}(t) \\ \omega_2^{'}(t) \\ \omega_3^{'}(t) \\ \omega_4^{'}(t) \\ \omega_5^{'}(t) \\ \omega_6^{'}(t) \\ \omega_7^{'}(t) \\ \omega_8^{'}(t) \end{pmatrix} = \begin{pmatrix} -\eta_1 & \xi_1 & \xi_2 & \xi_3 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2v_1 & -\eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_2 & 0 & -\eta_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_3 & 0 & 0 & -\eta_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix}$$

Using the same argument above, availability expression of Subsystem 2 is

$$A_{T2}(\infty) = \omega_0(\infty) + \omega_1(\infty) + \omega_2(\infty) + \omega_3(\infty) \tag{8}$$

Setting $\omega_k^{'}(t) = 0$ as $t \to \infty$ in steady state, to obtained

$$\begin{pmatrix} -\eta_1 & \xi_1 & \xi_2 & \xi_3 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2v_1 & -\eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_2 & 0 & -\eta_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2v_3 & 0 & 0 & -\eta_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

the normalizing condition is

$$\sum_{k=0}^{8} \omega_k(\infty) = 1 \tag{9}$$

Using (8) to give the explicit expressions for the steady-state availability of Subsystem 2 given in (8) is now

$$A_{T2}(\infty) = \frac{2(\xi_0\xi_1\xi_2\xi_3\xi_4 v_2 + \xi_0\xi_2\xi_3\xi_4 v_1 v_2)}{Q_2} \tag{10}$$

Where $Q_2 = 2\xi_0\xi_2^2\xi_3 v_1^2 + \xi_0\xi_4\xi_2^2\xi_3 + 2v_2\xi_2\xi_3\xi_4 + 2v_3\xi_0\xi_1\xi_2\xi_4 + 2v_2^2\xi_0\xi_1\xi_3\xi_4 + v_4\xi_0\xi_1\xi_2^2\xi_3 + 2v_3\xi_0\xi_1\xi_2^2\xi_4$

and $\eta_1 = (v_0 + 2v_1 + 2v_2 + 2v_3 + v_4), \eta_2 = (\xi_1 + v_1), \eta_3 = (\xi_2 + v_2), \eta_4 = (\xi_3 + v_3)$

To evaluate the MTTF_2, the rows and columns of the absorbing (failure) states from the above matrix were deleted and transposed to obtain the new matrix L_2.

$$L_2 = \begin{pmatrix} -\eta_1 & 2v_1 & 2v_2 & 2v_3 \\ 0 & -\eta_2 & 0 & 0 \\ 0 & 0 & -\eta_3 & 0 \\ 0 & 0 & 0 & -\eta_4 \end{pmatrix}$$

$$MTTF_2 = \frac{2(v_3\xi_1\xi_2 + v_3 v_2\xi_1 + v_2 v_3\xi_2 + v_1 v_2 v_3)}{2(v_2 v_3^2\xi_1 + 2v_3 v_2^2\xi_1 + 2v_1 v_3^2\xi_2 + 2v_1 v_2^2\xi_3 + 2v_2 v_1^2\xi_3)} \tag{11}$$

$$PF_2 = AT_2C_0 - Cr_2Br_2 - Cr_1Br_{22} \tag{12}$$

From Figure 4, the corresponding set of differential difference equations for Subsystem 3 are

$$\begin{pmatrix} \omega_0^{'}(t) \\ \omega_1^{'}(t) \\ \omega_2^{'}(t) \\ \omega_3^{'}(t) \\ \omega_4^{'}(t) \\ \omega_5^{'}(t) \\ \omega_6^{'}(t) \\ \omega_7^{'}(t) \\ \omega_8^{'}(t) \end{pmatrix} = \begin{pmatrix} -\eta_5 & \xi_1 & \xi_2 & \xi_3 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2c_0v_1 & -\eta_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2c_0v_2 & 0 & -\eta_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2c_0v_3 & 0 & 0 & -\eta_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_0v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_0v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_0v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ \delta_0v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ \delta_0v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix}$$

Using the same argument above, availability expression of Subsystem 3 is

$$A_{T3}(\infty) = \omega_0(\infty) + \omega_1(\infty) + \omega_2(\infty) + \omega_3(\infty) \tag{13}$$

Setting $\omega_k^{'}(t) = 0$ as $t \to \infty$ in steady state, to obtained

$$\begin{pmatrix} -\eta_5 & \xi_1 & \xi_2 & \xi_3 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2c_0v_1 & -\eta_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2c_0v_2 & 0 & -\eta_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2c_0v_3 & 0 & 0 & -\eta_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_0v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_0v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_0v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ \delta_0v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ \delta_0v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

the normalizing condition is

$$\sum_{k=0}^{8} \omega_k(\infty) = 1 \tag{14}$$

Using (14) to give the explicit expressions for the steady-state availability of Subsystem 3 given in (14) is now

$$A_{T3}(\infty) = \frac{2(c_0\xi_0\xi_1\xi_2^2\xi_3\xi_4 v_1 + c_0\xi_0\xi_2\xi_3\xi_4 v_1 v_2)}{Q_3} \tag{15}$$

Where $Q_3 = \xi_0\xi_1^2\xi_2^2\xi_3\xi_4 + v_2\xi_0\xi_1^2\xi_2\xi_3\xi_4 + 2c_0v_2^2\xi_0\xi_1^2\xi_2\xi_3 + 2c_0v_3\xi_0\xi_1\xi_2^2\xi_4 + \delta_0v_4\xi_0\xi_1^2\xi_2^2\xi_3 + \delta_0v_0\xi_4\xi_1^2\xi_2^2\xi_3$

and $\eta_5 = (\delta_0v_0 + 2c_0v_1 + 2c_0v_2 + 2c_0v_3 + \delta_0v_4)$, $\eta_6 = (\xi_1 + \delta_0v_1)$, $\eta_7 = (\xi_2 + \delta_0v_2), \eta_8 = (\xi_3 + \delta_0v_3)$

To evaluate the MTTF_3, the rows and columns of the absorbing (failure) states from the above matrix were deleted and transposed to obtain the new matrix L_3.

$$L_3 = \begin{pmatrix} -\eta_5 & 2c_0v_1 & 2c_0v_2 & 2c_0v_3 \\ \xi_1 & -\eta_6 & 0 & 0 \\ \xi_2 & 0 & -\eta_7 & 0 \\ \xi_3 & 0 & 0 & -\eta_8 \end{pmatrix}$$

$$MTTF_3 = \frac{2c_0(v_3\xi_1\xi_2 + \delta_0 v_3 v_2 \xi_1 + \delta_0 v_1 v_3 \xi_2 + 2\delta_0 v_1 v_2 v_3)}{2c_0\delta_0^2 v_2 v_3^2 \xi_1 + 2c_0\delta_0^2 v_3 v_2^2 \xi_1 + 2c_0\delta_0^2 v_1 v_3^2 + 2v_1 v_2 v_3^2 + 2v_1 v_2^2 v_3 + 2v_2 v_1^2 v_3} \quad (16)$$

$$PF_3 = AT_2C_0 - Cr_2Br_3 - Cr_1Br_{33} \quad (17)$$

From Figure 5, the corresponding set of differential difference equations for Subsystem 4 are

$$\begin{pmatrix} \omega_0'(t) \\ \omega_1'(t) \\ \omega_2'(t) \\ \omega_3'(t) \\ \omega_4'(t) \\ \omega_5'(t) \\ \omega_6'(t) \\ \omega_7'(t) \\ \omega_8'(t) \end{pmatrix} = \begin{pmatrix} -\eta_9 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2\delta_0 v_1 & -v_1\delta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\delta_0 v_2 & 0 & -v_2\delta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\delta_0 v_3 & 0 & 0 & -v_3\delta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_0 v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_0 v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_0 v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ \delta_0 v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ \delta_0 v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix}$$

Using the same argument above, availability expression of Subsystem 4 is

$$A_{T3}(\infty) = \omega_0(\infty) + \omega_1(\infty) + \omega_2(\infty) + \omega_3(\infty) \quad (18)$$

Setting $\omega_k'(t) = 0$ as $t \to \infty$ in steady state, to obtained

$$\begin{pmatrix} -\eta_9 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 & \xi_0 & \xi_4 \\ 2\delta_0 v_1 & -v_1\delta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\delta_0 v_2 & 0 & -v_2\delta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\delta_0 v_3 & 0 & 0 & -v_3\delta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_0 v_1 & 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_0 v_2 & 0 & 0 & -\xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_0 v_3 & 0 & 0 & -\xi_3 & 0 & 0 \\ \delta_0 v_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_0 & 0 \\ \delta_0 v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi_4 \end{pmatrix} \begin{pmatrix} \omega_0(t) \\ \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \\ \omega_5(t) \\ \omega_6(t) \\ \omega_7(t) \\ \omega_8(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

the normalizing condition is

$$\sum_{k=0}^{8} \omega_k(\infty) = 1 \quad (19)$$

Using (19) to give the explicit expressions for the steady-state availability of Subsystem 4 given in (19) is now

$$A_{T4}(\infty) = \frac{7\xi_0\xi_1\xi_2\xi_3\xi_4}{Q_4} \quad (20)$$

Where $Q_4 = 7\xi_0\xi_1\xi_2\xi_3\xi_4 + \delta_0 v_4 \xi_0\xi_1\xi_2\xi_3 + 2\delta_0 v_3 \xi_0\xi_1\xi_2\xi_4$
$2\delta_0 v_2 \xi_0\xi_1\xi_2\xi_4 + 2\delta_0 v_1 \xi_0\xi_2\xi_3\xi_4 + \delta_0 v_0 \xi_1\xi_2\xi_3\xi_4$
and, $\eta_9 = (\delta_0 v_0 + 2\delta_0 v_1 + 2\delta_0 v_2 + 2\delta_0 v_3 + \delta_0 v_0 + \delta_0 v_4)$

To evaluate the $MTTF_4$, the rows and columns of the absorbing (failure) states from the above matrix were deleted and transposed to obtain the new matrix L_4.

$$L_4 = \begin{pmatrix} -\eta_9 & 2\delta_0 v_1 & 2\delta_0 v_2 & 2\delta_0 v_3 \\ 0 & -v_1\delta_0 & 0 & 0 \\ 0 & 0 & -v_2\delta_0 & 0 \\ 0 & 0 & 0 & -v_3\delta_0 \end{pmatrix}$$

$$MTTF_4 = \frac{7}{\delta_0(2v_0 + 2v_1 + 2v_2 + 2v_3 + v_4)}$$

$$PF_4 = AT_4C_0 - Cr_1Br_4 \quad (22)$$

RESULTS AND DISCUSSIONS

The objective of this section is to express numerical experiment so as to see effect of the parameters on the performance by the use of MATLAB software. The findings of availability, MTTF and profit for all the four (4) scenarios in terms of failure rates v_o, v_1, v_2, v_3, and v_4 with repair rate ξ_o, ξ_1, ξ_2, ξ_3, and ξ_4 as follows: Tables 2, 3 and Figure [6 – 21] visually explain the detailed analysis of the availability,

Table 2. Variation of availability, MTTF and profit with respect to failure rate v_0 for the four scenarios

	Availability				**Mean time to Failure (MTTF)**				**Profit*10⁷**			
	Scenario				**Scenario**				**Scenario**			
v_0	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.979	0.963	0.988	0.993	152.1	54.07	172.2	507.2	2.449	2.409	2.470	2.484
0.1	0.794	0.392	0.670	0.928	47.94	9.151	28.88	94.85	1.986	0.980	1.675	2.320
0.2	0.668	0.246	0.507	0.870	28.45	4.999	15.76	52.31	1.670	0.615	1.267	2.175
0.3	0.576	0.179	0.407	0.819	20.23	3.438	10.84	36.11	1.441	0.448	1.019	2.048
0.4	0.506	0.141	0.340	0.774	15.69	2.620	8.260	27.58	1.267	0.352	0.852	1.935
0.5	0.452	0.116	0.292	0.733	12.82	2.117	6.672	22.30	1.130	0.290	0.732	1.833
0.6	0.408	0.098	0.256	0.697	10.83	1.775	5.596	18.72	1.020	0.247	0.642	1.742
0.7	0.372	0.086	0.228	0.663	9.383	1.529	4.819	16.13	0.930	0.215	0.571	1.659
0.8	0.341	0.076	0.206	0.633	8.274	1.342	4.231	14.17	0.854	0.190	0.514	1.584
0.9	0.316	0.068	0.187	0.606	7.399	1.197	3.771	12.63	0.790	0.170	0.468	1.516

Table 3. Variation of availability, MTTF and profit with respect to repair rate x_0 for the four scenarios

	Availability				Mean Time To Failure (MTTF)				Profit*10^7			
	Scenario				Scenario				Scenario			
x_0	1	2	3	4	1	2	3	4	1	2	3	4
0.0	0.000	0.000	0.000	0.000	56.00	20.62	60.83	185.1	0.000	0.000	0.000	0.000
0.1	0.934	0.892	0.962	0.979	56.00	20.62	60.83	185.1	2.337	2.232	2.406	2.448
0.2	0.935	0.895	0.963	0.979	56.00	20.62	60.83	185.1	2.338	2.239	2.409	2.449
0.3	0.935	0.896	0.964	0.979	56.00	20.62	60.83	185.1	2.339	2.242	2.410	2.449
0.4	0.935	0.897	0.964	0.979	56.00	20.62	60.83	185.1	2.339	2.243	2.411	2.449
0.5	0.935	0.897	0.964	0.979	56.00	20.62	60.83	185.1	2.339	2.244	2.411	2.449
0.6	0.936	0.897	0.964	0.979	56.00	20.62	60.83	185.1	2.339	2.244	2.411	2.449
0.7	0.936	0.898	0.964	0.979	56.00	20.62	60.83	185.1	2.340	2.245	2.411	2.449
0.8	0.936	0.898	0.964	0.979	56.00	20.62	60.83	185.1	2.340	2.245	2.412	2.449
0.9	0.936	0.898	0.964	0.979	56.00	20.62	60.83	185.1	2.340	2.245	2.412	2.449

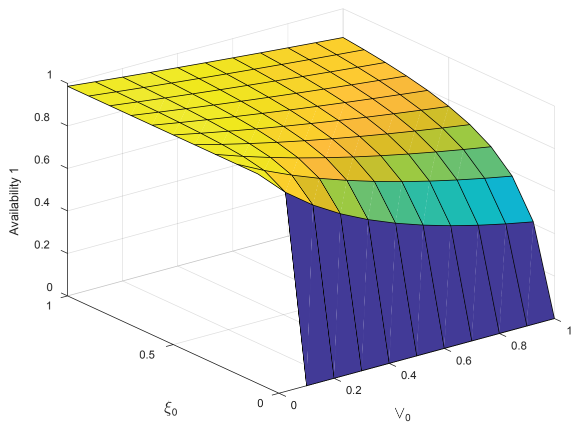


Figure 6. Availability against scenario 1 for v_0 and x_0.

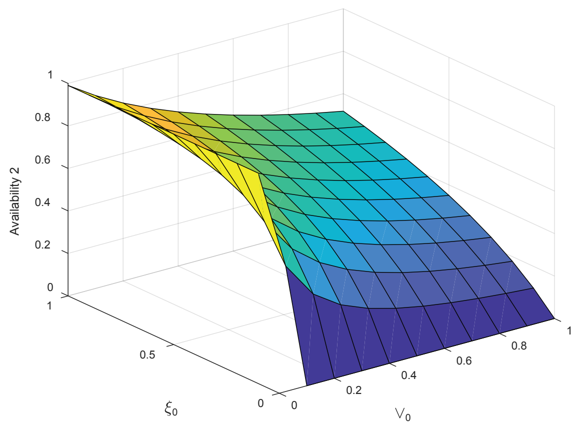


Figure 7. Availability against scenario 2 for v_0 and x_0.

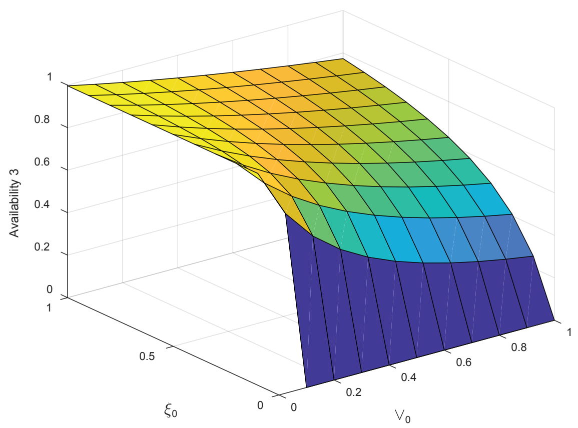


Figure 8. Availability against scenario 3 for v_0 and x_0.

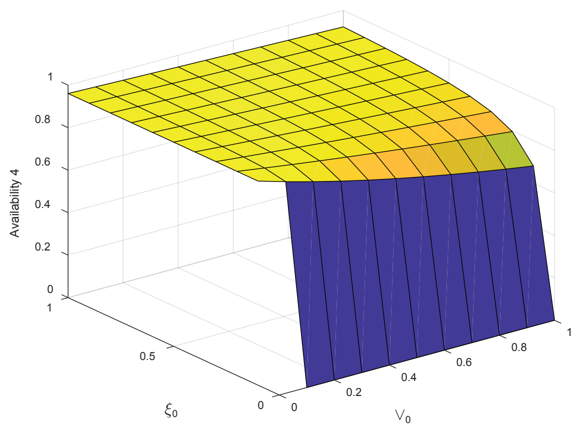


Figure 9. Availability against scenario 4 for v_0 and x_0.

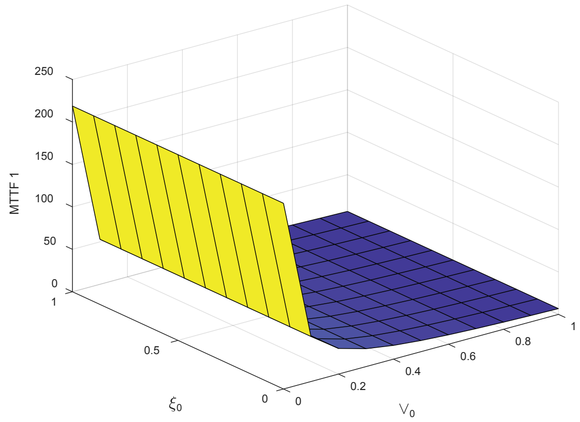


Figure 10. MTTF against scenario 1 for v_0 and x_0.

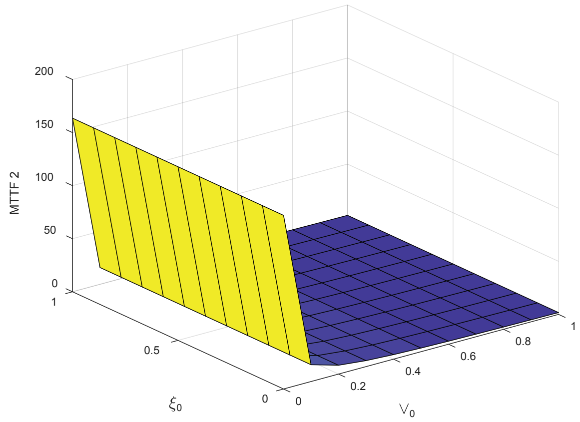


Figure 11. MTTF against scenario 2 for v_0 and x_0.

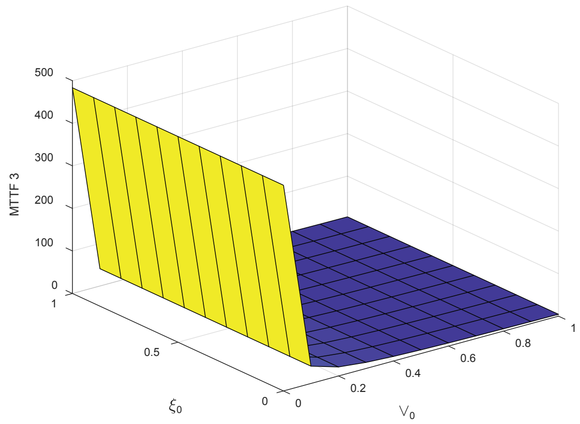


Figure 13. MTTF against scenario 3 for v_0 and x_0.

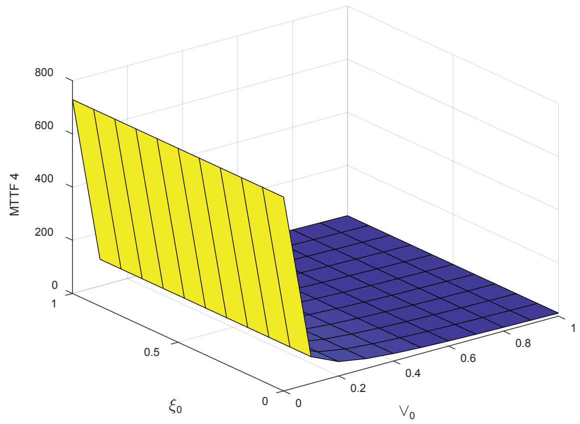


Figure 14. MTTF against scenario 4 for v_0 and x_0.

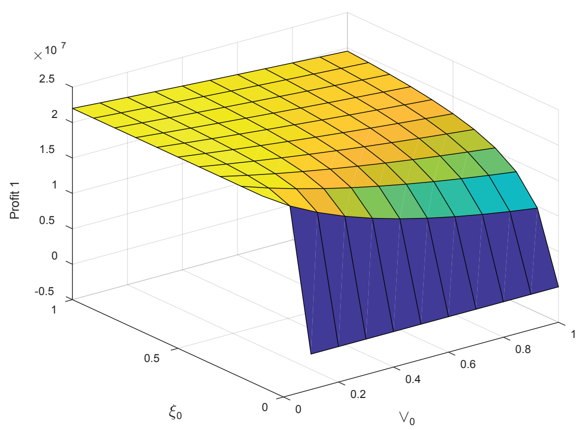


Figure 15. Profit against scenario 1 for v_0 and x_0.

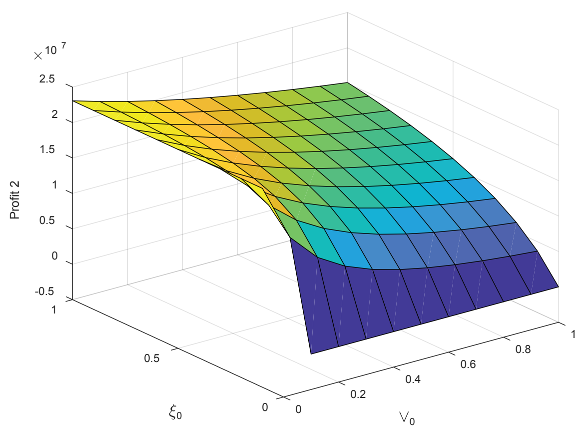


Figure 16. Profit against scenario 2 for v_0 and x_0.

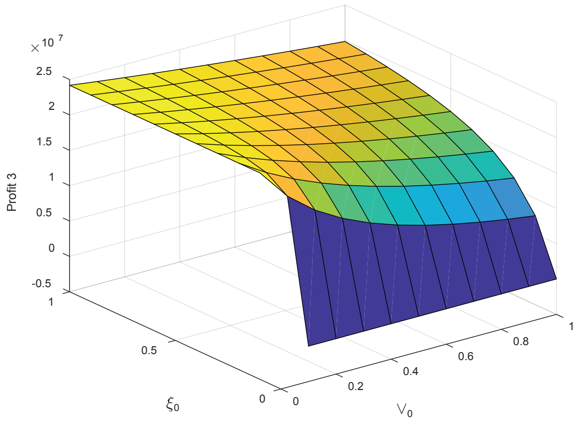


Figure 17. Profit against scenario 3 for v_0 and x_0.

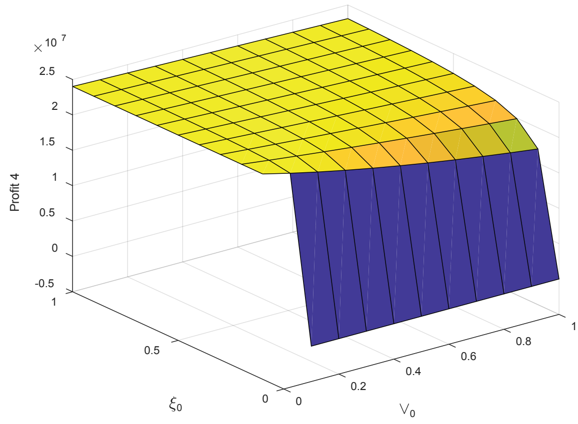


Figure 18. Profit against scenario 4 for v_0 and x_0.

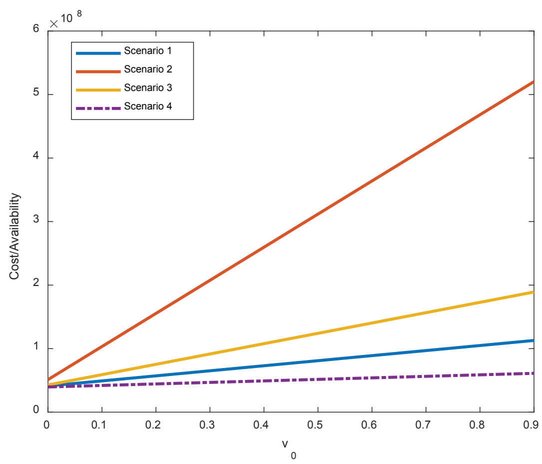
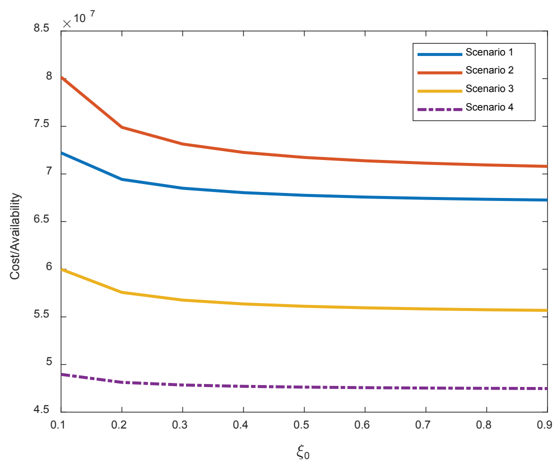


Figure 19. Cost/availability versus failure rate.



20. Cost/availability versus repair rate.

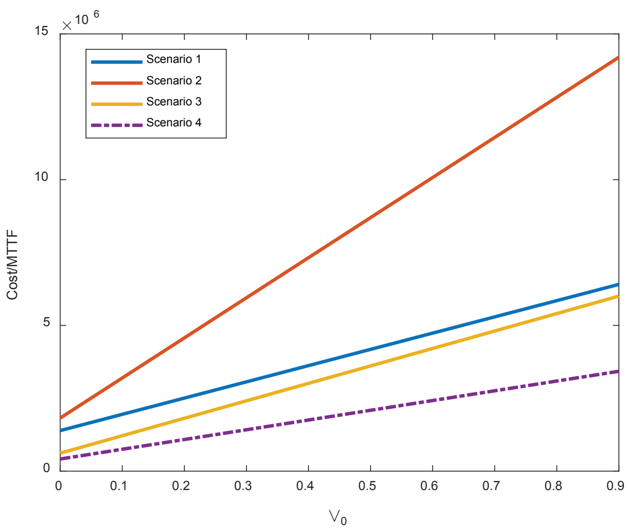


Figure 21. Cost/MTTF versus failure rate.

MTTF, profit and cost benefit respectively in terms of v_o and ξ_o. On the other hand, additional figures show an increasing pattern, highlighting the system's robustness in reaction to variations in failure and repair rates v_1 and ξ_1 as shown in Tables 4, 5 and Figure [22 – 25] in terms of availability, Figure [26 – 29] for MTTF, Figure [30 – 33] in terms of profit and Figure [34 – 36] in terms cost benefit. Tables 6, 7 and Figure [37 -51] are relevant to availability, MTTF, profit and cost benefit in terms of v_2 and ξ_2. The graphical representations encapsulated in Tables 8, 9 and figure [52 – 66] serve as a visual exploration of the intricate dynamics between failure and repair rates v_3 and ξ_3 and their consequential impact on availability, MTTF, profit and cost benefit for four different scenarios. However, availability analysis, MTTF, profit analysis and cost benefit was carried out to the same scenarios in Tables 10, 11 and figure [67 -81] it was observed that availability increases with increase in all repair rates and decreases as the failure rate increases

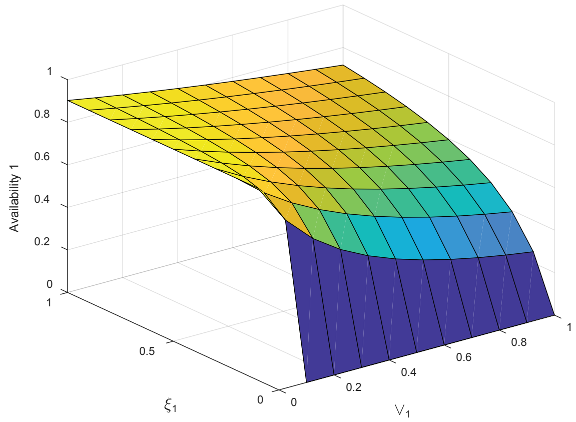


Figure 22. Availability against scenario 1 for v_1 and x_1.

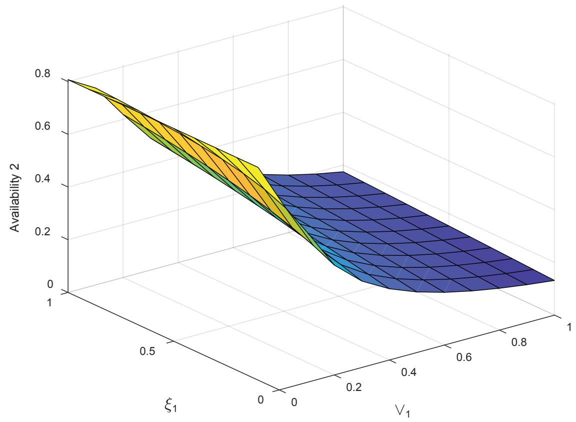


Figure 23. Availability against scenario 2 for v_1 and x_1.

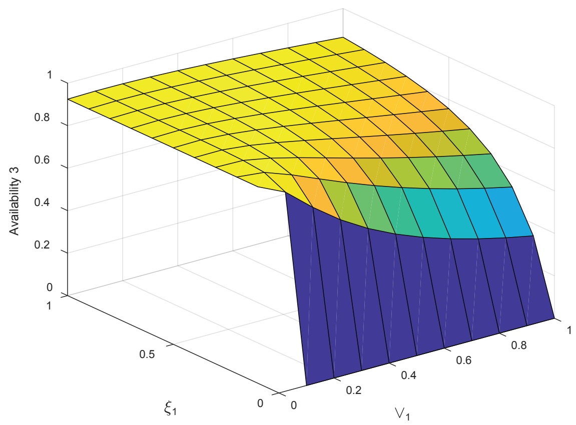


Figure 24. Availability against scenario 3 for v_1 and x_1.

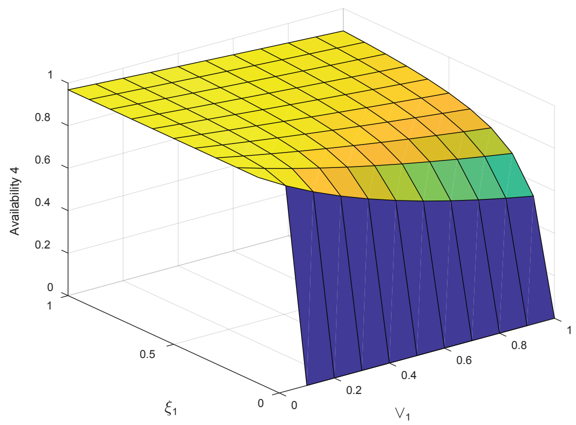


Figure 25. Availability against scenario 4 for v_1 and x_1.

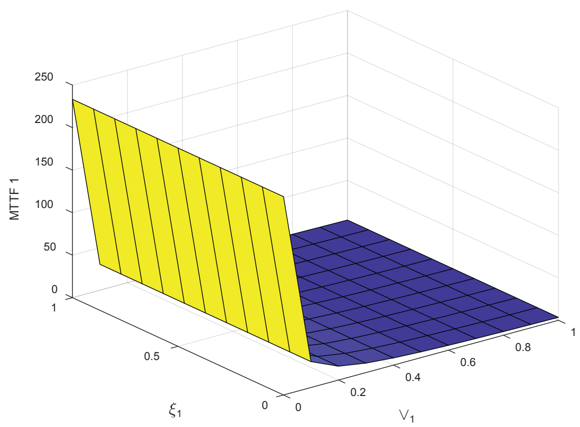


Figure 26. MTTF against scenario 1 for v_1 and x_1.

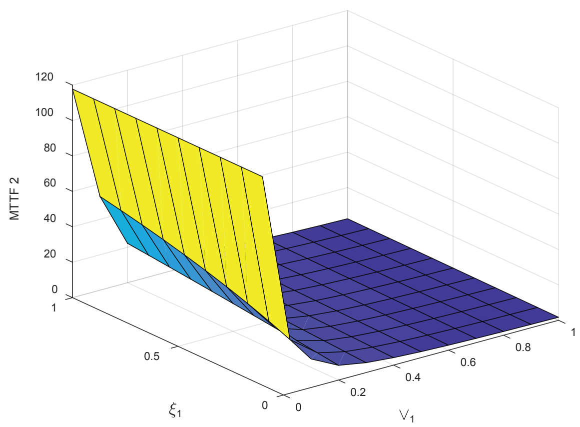


Figure 27. MTTF against scenario 2 for v_1 and x_1.

Table 4. Variation of availability, MTTF and profit with respect to failure rate v_1 for the four scenarios

	Availability				**Mean time to failure (MTTF)**				**Profit*10^7**			
	Scenario				**Scenario**				**Scenario**			
v_1	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.981	0.948	0.983	0.994	155.5	50.29	162.3	507.2	2.453	2.371	2.458	2.486
0.1	0.700	0.884	0.835	0.886	28.57	16.18	60.30	94.85	1.752	2.211	2.088	2.216
0.2	0.544	0.797	0.678	0.799	15.73	8.043	28.19	52.31	1.362	1.992	1.695	1.999
0.3	0.445	0.723	0.565	0.728	10.85	5.307	17.89	36.11	1.114	1.808	1.413	1.820
0.4	0.377	0.661	0.483	0.668	8.284	3.952	13.02	27.58	0.942	1.653	1.207	1.671
0.5	0.326	0.609	0.421	0.618	6.698	3.147	10.20	22.30	0.817	1.523	1.053	1.545
0.6	0.288	0.564	0.373	0.574	5.622	2.613	8.381	18.72	0.720	1.411	0.932	1.436
0.7	0.258	0.526	0.334	0.536	4.844	2.234	7.106	16.13	0.645	1.315	0.837	1.342
0.8	0.233	0.492	0.303	0.503	4.255	1.951	6.166	14.17	0.583	1.231	0.759	1.259
0.9	0.213	0.462	0.277	0.474	3.794	1.731	5.444	12.63	0.532	1.157	0.694	1.186

Table 5. Variation of availability, MTTF and profit with respect to repair rate x_1 for the four scenarios

	Availability				**Mean time to failure (MTTF)**				**Profit*10^7**			
	Scenario				**Scenario**				**Scenario**			
x_1	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.000	0.950	0.000	0.000	56.00	47.96	186.2	185.1	0.000	0.000	0.000	0.000
0.1	0.936	0.889	0.961	0.980	56.00	20.38	60.48	185.1	2.341	2.222	2.403	2.450
0.2	0.939	0.887	0.961	0.980	56.00	20.09	60.07	185.1	2.347	2.219	2.403	2.452
0.3	0.939	0.887	0.961	0.981	56.00	19.99	59.93	185.1	2.349	2.217	2.402	2.452
0.4	0.940	0.886	0.961	0.981	56.00	19.94	59.87	185.1	2.350	2.217	2.402	2.453
0.5	0.940	0.886	0.961	0.981	56.00	19.91	59.83	185.1	2.351	2.216	2.402	2.453
0.6	0.940	0.886	0.961	0.981	56.00	19.89	59.80	185.1	2.351	2.216	2.402	2.453
0.7	0.940	0.886	0.961	0.981	56.00	19.88	59.78	185.1	2.352	2.216	2.402	2.453
0.8	0.941	0.886	0.961	0.981	56.00	19.87	59.76	185.1	2.352	2.216	2.402	2.453
0.9	0.941	0.886	0.961	0.981	56.00	19.86	59.75	185.1	2.352	2.216	2.402	2.453

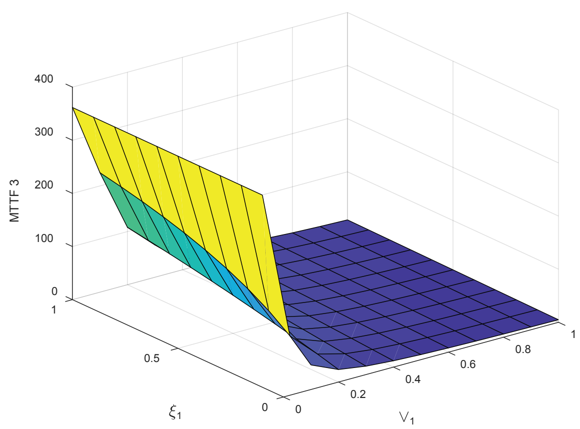
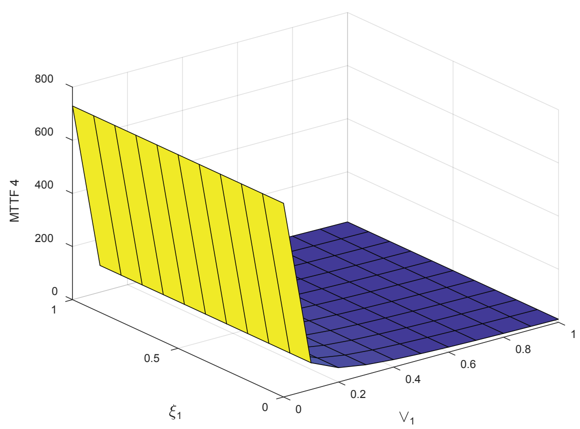


Figure 28. MTTF against scenario 1 for v_1 and x_1.



29. MTTF against scenario 4 for v_1 and x_1.

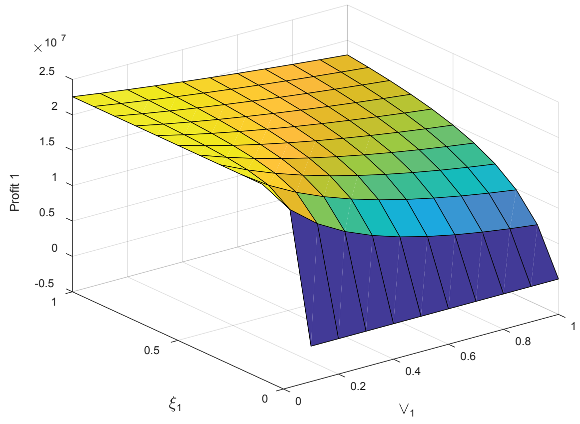


Figure 30. Profit against scenario 1 for v_1 and x_1.

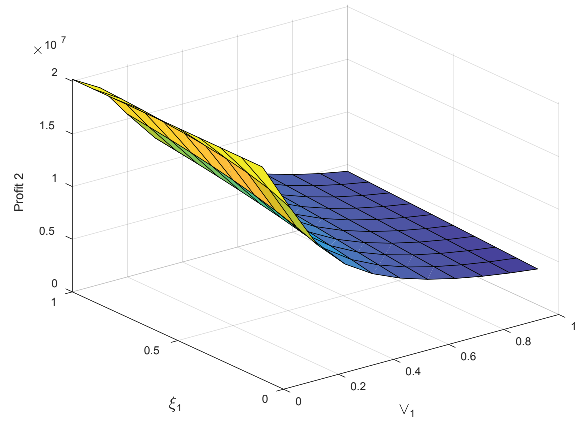


Figure 31. Profit against scenario 2 for v_1 and x_1.

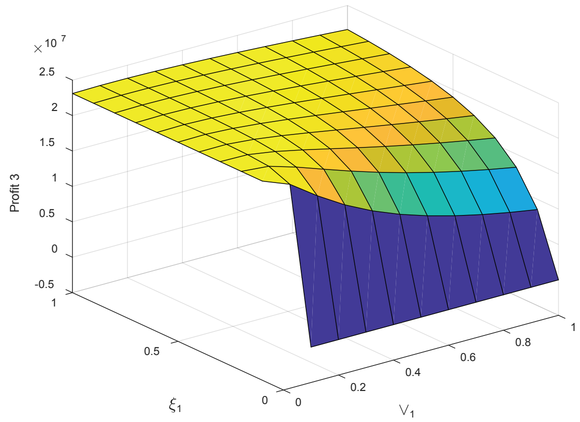


Figure 32. Profit against scenario 3 for v_1 and x_1.

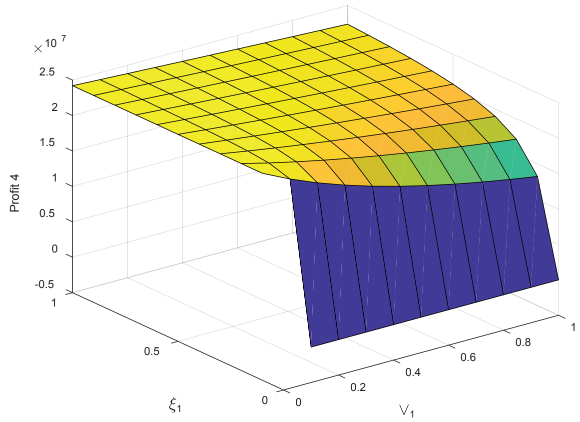


Figure 33. Profit against scenario 4 for v_1 and x_1.

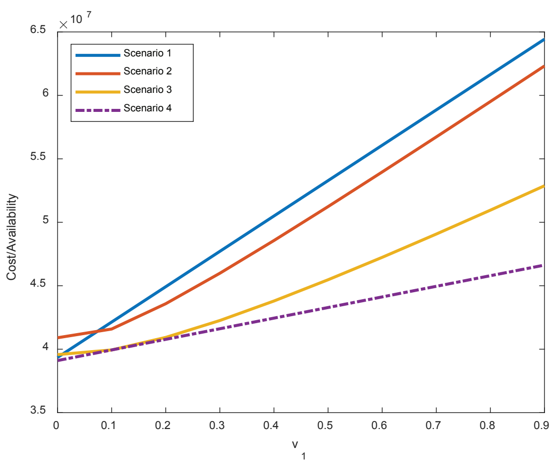


Figure 34. Cost/Availability versus failure rate.

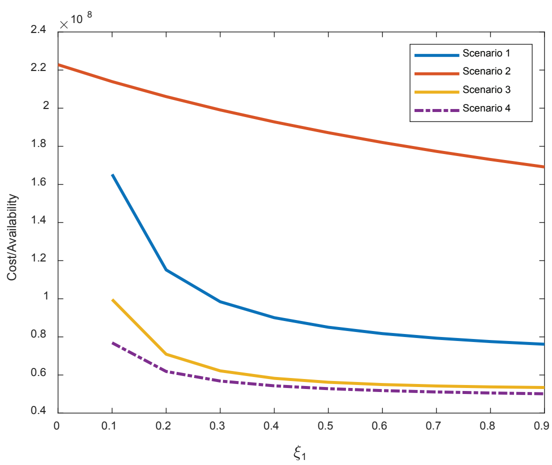


Figure 35. Cost/Availability versus repair rate.

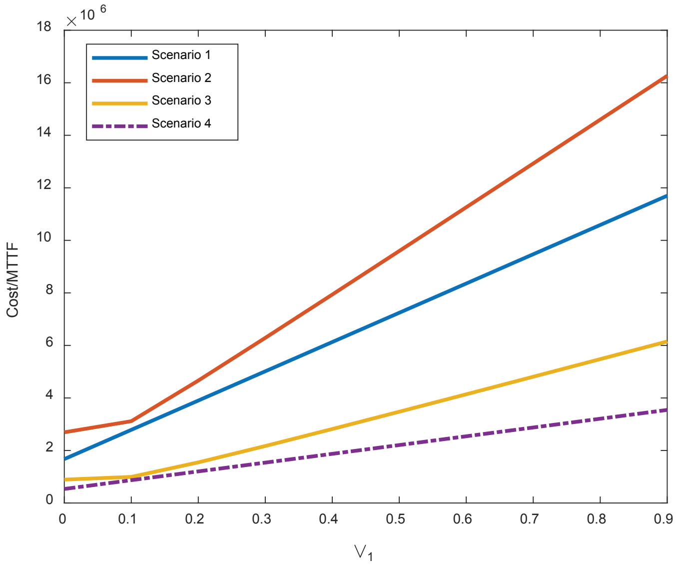


Figure 36. Cost/MTTF versus failure rate.

Table 6. Variation of Availability, MTTF and Profit with respect to failure rate v_2 for the four Scenarios

	Availability				**Mean Time To Failure (MTTF)**				**Profit*10^7**			
	Scenario				**Scenario**				**Scenario**			
v_2	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.980	0.949	0.983	0.994	155.5	50.57	162.7	507.2	2.451	2.374	2.459	2.485
0.1	0.726	0.602	0.857	0.898	28.57	16.54	62.49	94.85	1.815	1.506	2.142	2.246
0.2	0.576	0.408	0.725	0.819	15.73	8.170	29.03	52.31	1.441	1.021	1.814	2.048
0.3	0.478	0.307	0.626	0.753	10.85	5.371	18.32	36.11	1.195	0.769	1.565	1.883
0.4	0.408	0.246	0.550	0.697	8.284	3.992	13.27	27.58	1.021	0.616	1.374	1.742
0.5	0.356	0.056	0.489	0.648	6.698	3.174	10.37	22.30	0.891	0.514	1.224	1.621
0.6	0.316	0.176	0.441	0.606	5.622	2.633	8.500	18.72	0.790	0.440	1.104	1.516
0.7	0.284	0.154	0.401	0.569	4.844	2.250	7.195	16.13	0.710	0.385	1.004	1.423
0.8	0.257	0.137	0.368	0.536	4.255	1.963	6.235	14.17	0.644	0.342	0.921	1.341
0.9	0.236	0.123	0.340	0.507	3.794	1.742	5.500	12.63	0.590	0.308	0.851	1.268

Table 7. Variation of Availability, MTTF and Profit with respect to repair rate x_2 for the four Scenarios

	Availability				**Mean Time To Failure (MTTF)**				**Profit*10^7**			
	Scenario				**Scenario**				**Scenario**			
x_2	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.000	0.000	0.000	0.000	56.00	37.74	140.2	185.1	0.000	0.000	2.405	2.454
0.1	0.941	0.892	0.962	0.981	56.00	20.02	59.86	185.1	2.354	2.231	2.406	2.464
0.2	0.954	0.894	0.962	0.985	56.00	18.92	58.17	185.1	2.386	2.236	2.406	2.468
0.3	0.958	0.894	0.962	0.987	56.00	18.52	57.59	185.1	2.396	2.234	2.406	2.469
0.4	0.961	0.893	0.962	0.988	56.00	18.32	57.30	185.1	2.402	2.233	2.406	2.471
0.5	0.962	0.892	0.962	0.988	56.00	18.19	57.12	185.1	2.405	2.232	2.406	2.471
0.6	0.963	0.892	0.962	0.988	56.00	18.11	57.00	185.1	2.407	2.231	2.406	2.472
0.7	0.963	0.892	0.962	0.988	56.00	18.05	56.92	185.1	2.409	2.230	2.406	2.472
0.8	0.964	0.891	0.962	0.989	56.00	18.00	56.85	185.1	2.410	2.229	2.406	2.472
0.9	0.964	0.891	0.962	0.989	56.00	17.97	56.80	185.1	2.411	2.229	2.406	2.472

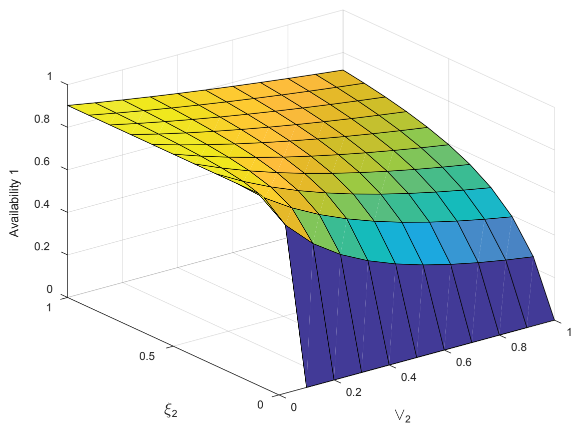


Figure 37. Availability against scenario 1 for v_2 and x_2.

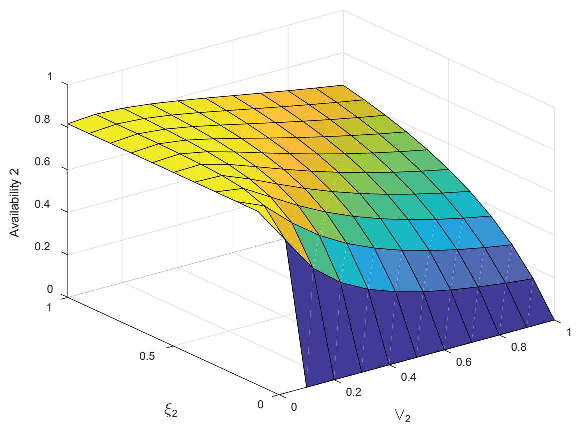


Figure 38. Availability against scenario 2 for v_2 and x_2.

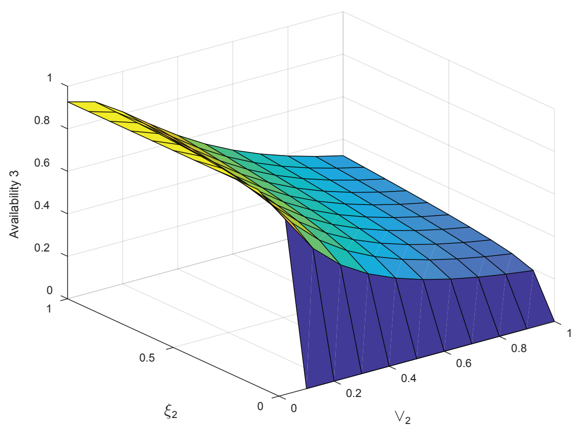


Figure 39. Availability against scenario 3 for v_2 and x_2.

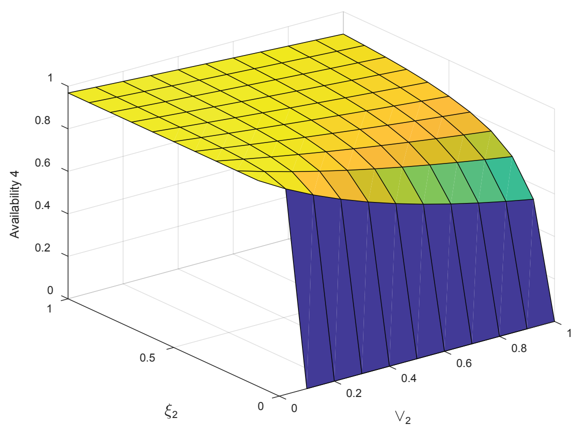


Figure 40. Availability against scenario 4 for v_2 and x_2.

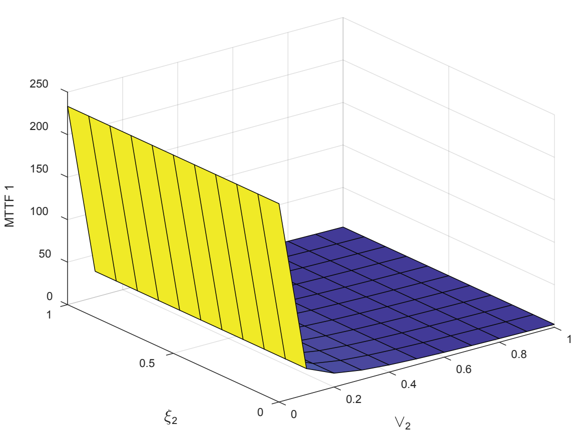


Figure 41. MTTF against scenario 1 for v_2 and x_2.

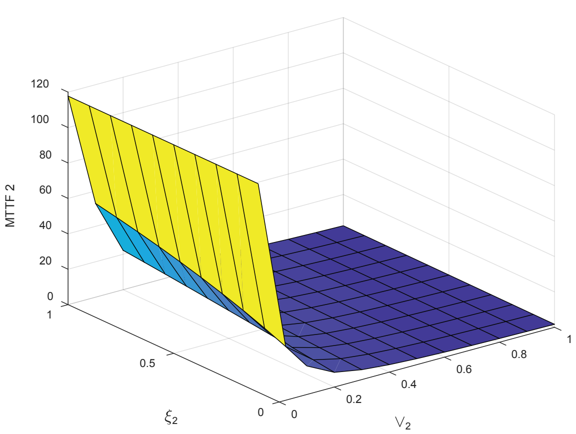


Figure 42. Availability against scenario 2 for v_2 and x_2.

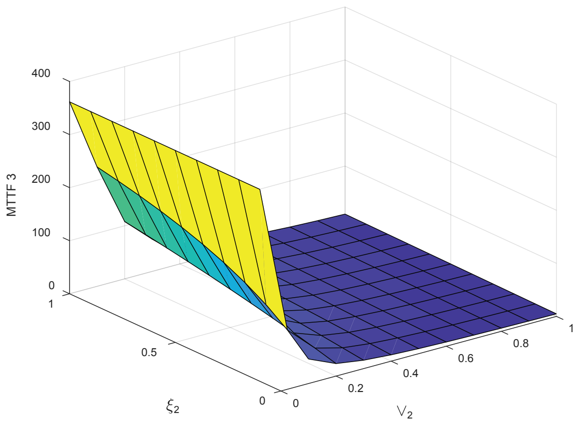


Figure 43. MTTF against scenario 3 for v_2 and x_2.

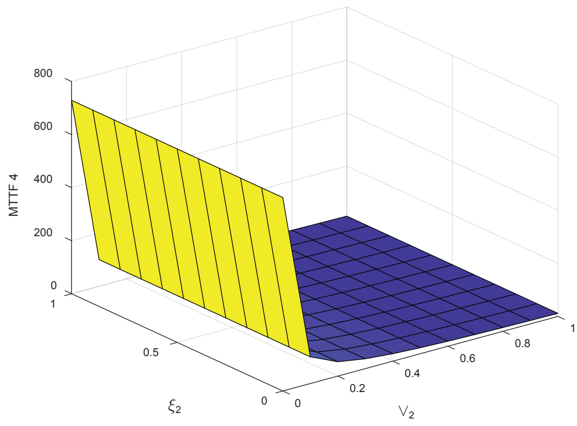


Figure 44. MTTF against scenario 4 for v_2 and x_2.

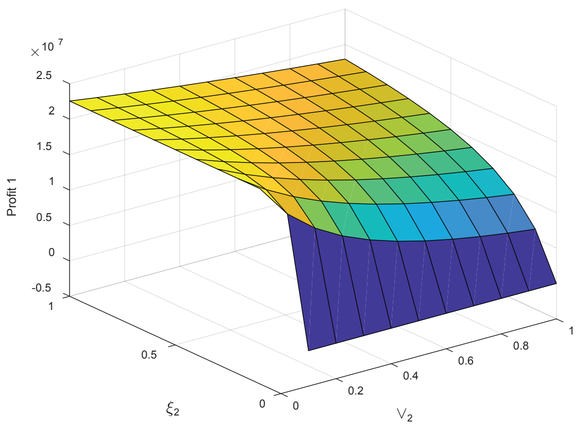


Figure 45. Profit against scenario 1 for v_2 and x_2.

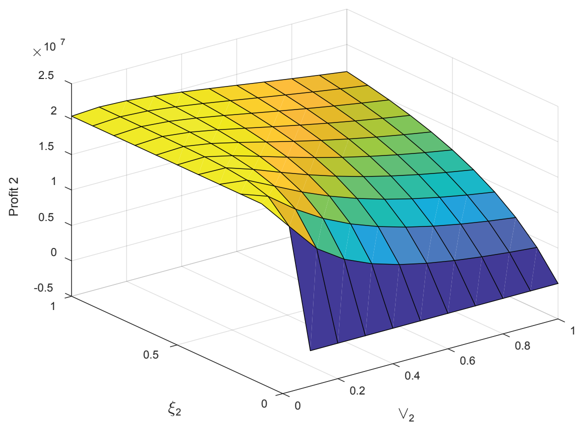


Figure 46. Profit against scenario 2 for v_2 and x_2.

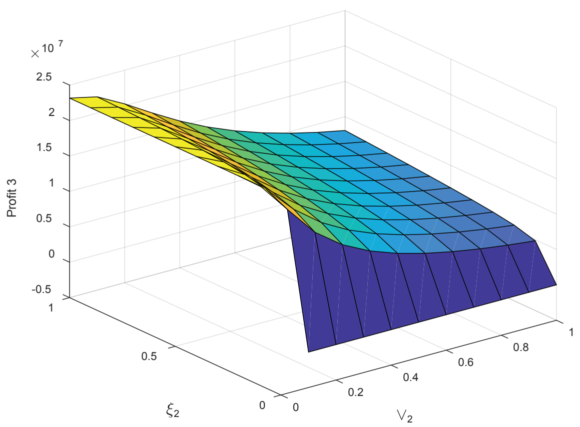


Figure 47. Profit against scenario 3 for v_2 and x_2.

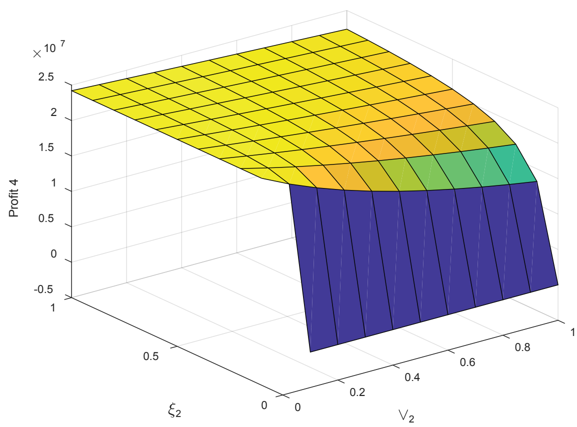


Figure 48. Profit against scenario 4 for v_2 and x_2.

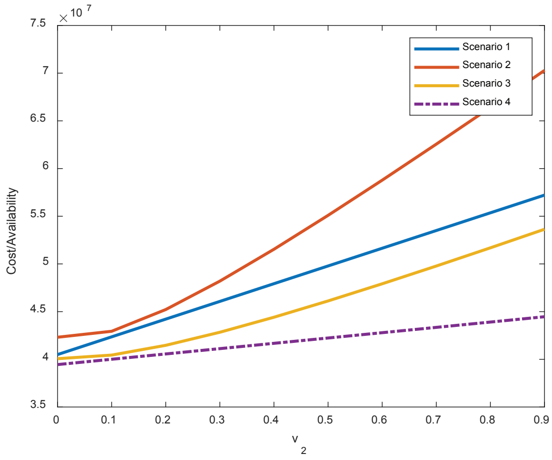


Figure 49. Cost/Availability versus failure rate.

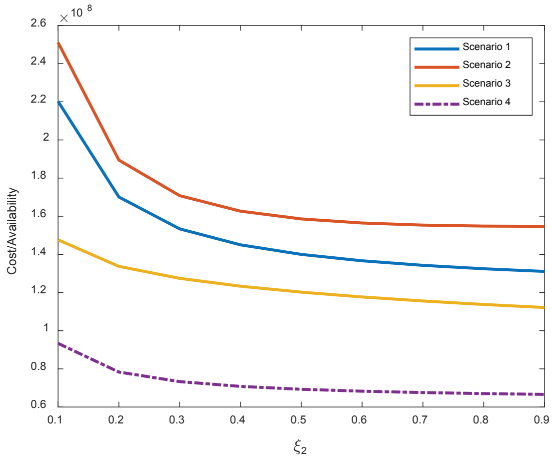


Figure 50. Cost/Availability versus repair rate.

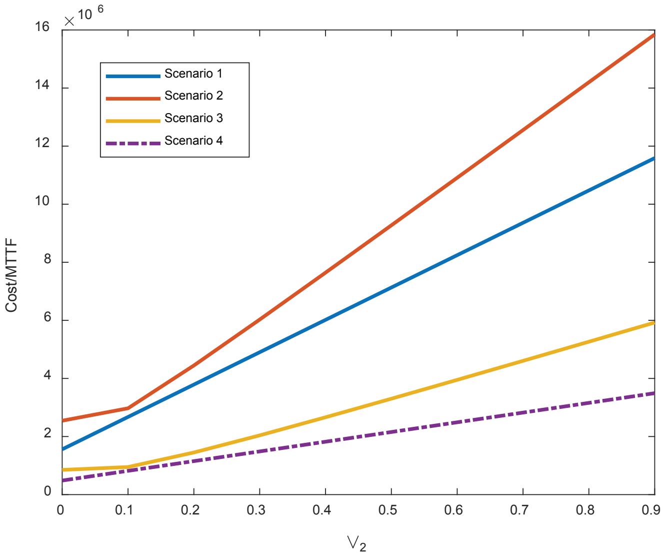


Figure 51. Cost/MTTF versus failure rate.

Table 8. Variation of Availability, MTTF and Profit with respect to failure rate v_3 for the four Scenarios

	Availability				Mean Time To Failure (MTTF)				Profit*10^7			
	Scenario				Scenario				Scenario			
v_3	1	2	3	4	1	2	3	4	1	2	3	4
0.0	0.950	0.937	0.980	0.984	155.5	54.76	169.4	507.2	2.376	2.344	2.452	2.461
0.1	0.919	0.930	0.979	0.974	28.57	32.92	128.0	94.85	2.298	2.325	2.448	2.435
0.2	0.890	0.900	0.973	0.964	15.73	16.23	73.46	52.31	2.225	2.252	2.433	2.410
0.3	0.862	0.863	0.963	0.954	10.85	9.751	44.98	36.11	2.156	2.157	2.409	2.386
0.4	0.836	0.823	0.951	0.944	8.284	6.705	30.34	27.58	2.092	2.057	2.379	2.361
0.5	0.812	0.783	0.938	0.935	6.698	5.017	22.07	22.30	2.031	1.958	2.345	2.338
0.6	0.789	0.745	0.923	0.926	5.622	3.971	16.97	18.72	1.974	1.864	2.309	2.315
0.7	0.768	0.710	0.908	0.916	4.844	3.267	13.60	16.13	1.920	1.776	2.271	2.292
0.8	0.747	0.677	0.893	0.908	4.255	2.766	11.24	14.17	1.868	1.693	2.233	2.270
0.9	0.728	0.646	0.877	0.899	3.794	2.393	9.517	12.63	1.820	1.617	2.194	2.248

Table 9. Variation of Availability, MTTF and Profit with respect to repair rate x_3 for the four Scenarios

	Availability				**Mean Time To Failure (MTTF)**				**Profit*10^7**			
	Scenario				**Scenario**				**Scenario**			
x_3	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
0.0	0.000	0.000	0.000	0.000	56.00	30.32	107.6	185.1	0.000	0.000	0.000	0.000
0.1	0.919	0.874	0.959	0.974	56.00	20.68	61.34	185.1	2.299	2.186	2.399	2.436
0.2	0.944	0.891	0.962	0.982	56.00	19.31	58.88	185.1	2.361	2.228	2.406	2.456
0.3	0.953	0.893	0.962	0.985	56.00	18.77	58.00	185.1	2.383	2.233	2.407	2.463
0.4	0.957	0.893	0.962	0.986	56.00	18.47	57.55	185.1	2.394	2.233	2.407	2.467
0.5	0.960	0.893	0.962	0.987	56.00	18.29	57.27	185.1	2.400	2.232	2.407	2.469
0.6	0.962	0.892	0.962	0.988	56.00	18.16	57.09	185.1	2.405	2.231	2.406	2.470
0.7	0.963	0.892	0.962	0.988	56.00	18.07	56.95	185.1	2.408	2.230	2.406	2.471
0.8	0.964	0.891	0.962	0.989	56.00	18.00	56.85	185.1	2.410	2.229	2.406	2.472
0.9	0.965	0.891	0.962	0.989	56.00	17.95	56.78	185.1	2.412	2.229	2.406	2.473

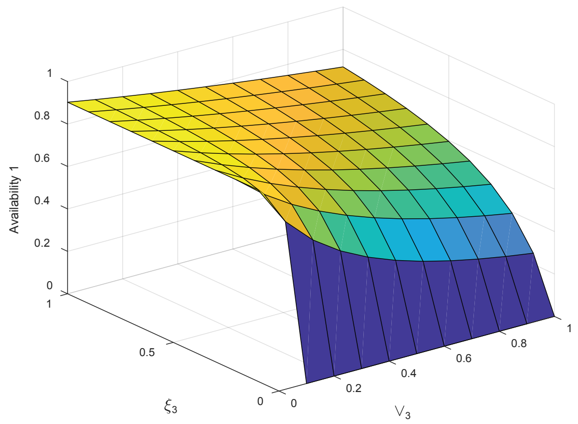


Figure 52. Availability against scenario 1 for v_3 and x_3.

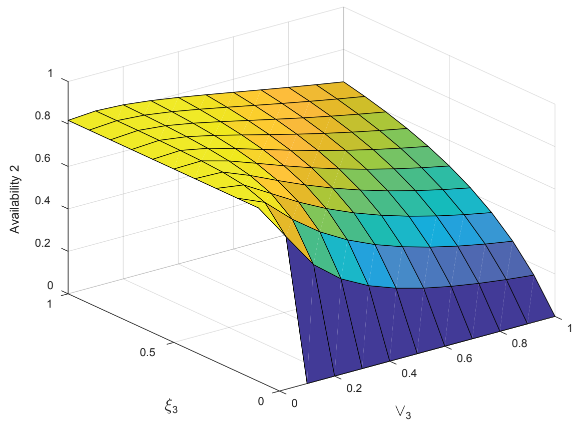


Figure 53. Availability against scenario 2 for v_3 and x_3.

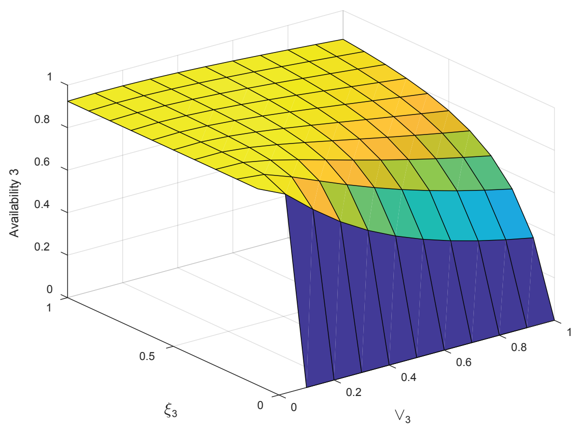


Figure 54. Availability against scenario 3 for v_3 and x_3.

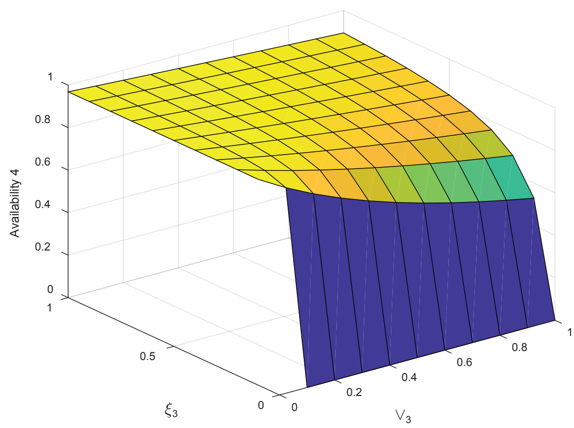


Figure 55. Availability against scenario 4 for v_3 and x_3.

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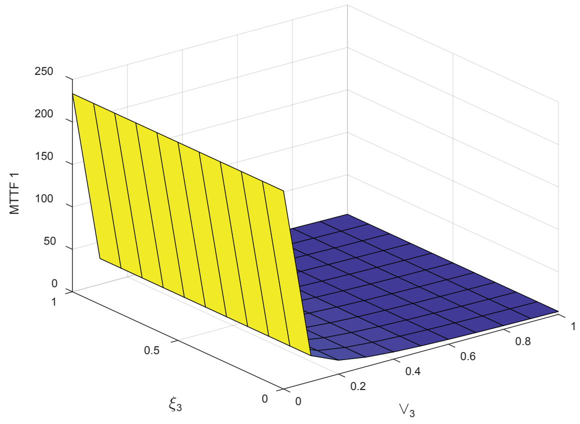

Figure 56. MTTF against scenario 1 for v_3 and x_3

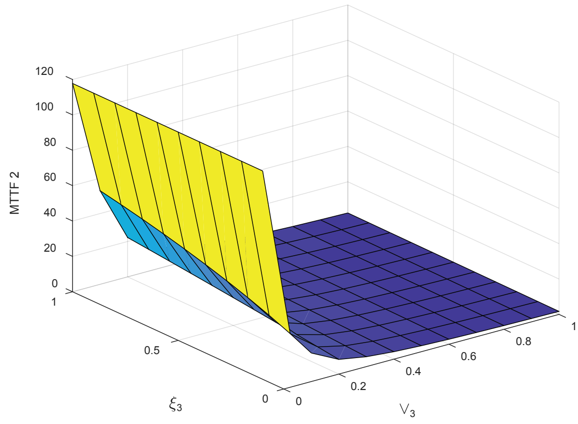

Figure 57. MTTF against scenario 2 for v_3 and x_3.

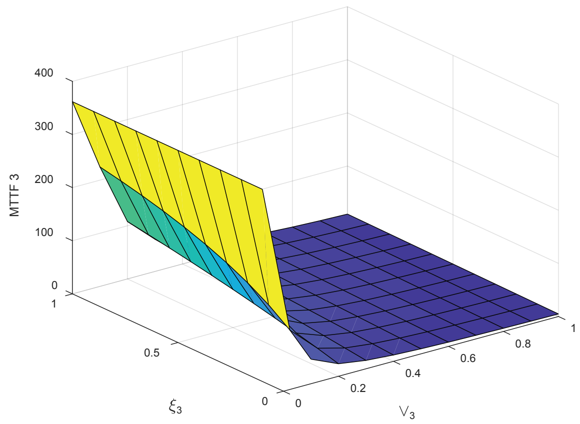

Figure 58. MTTF against scenario 3 for v_3 and x_3.

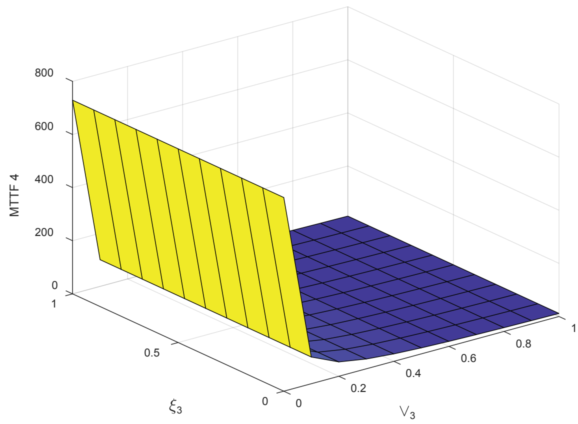

Figure 59. MTTF against scenario 4 for v_3 and x_3.

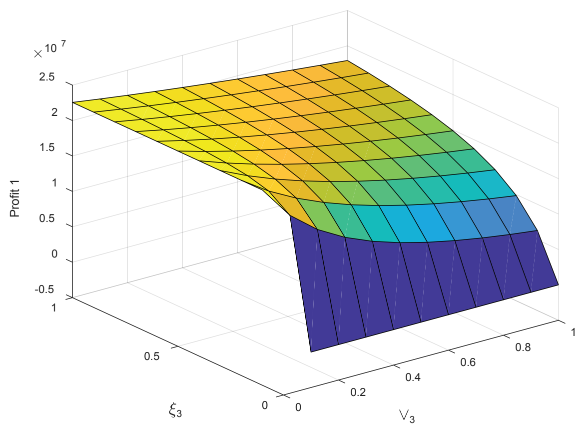

Figure 60. Profit against scenario 1 for v_3 and x_3.

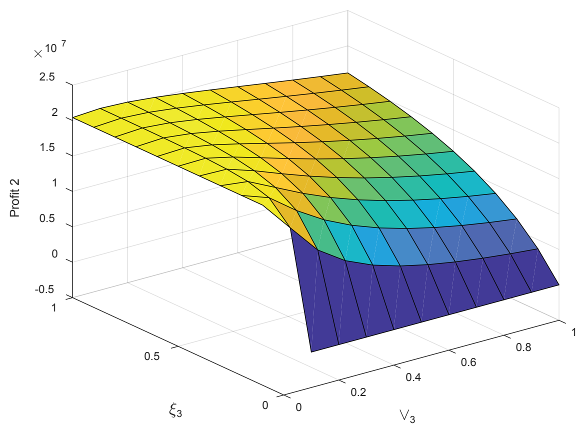

Figure 61. Profit against scenario 2 for v_3 and x_3.

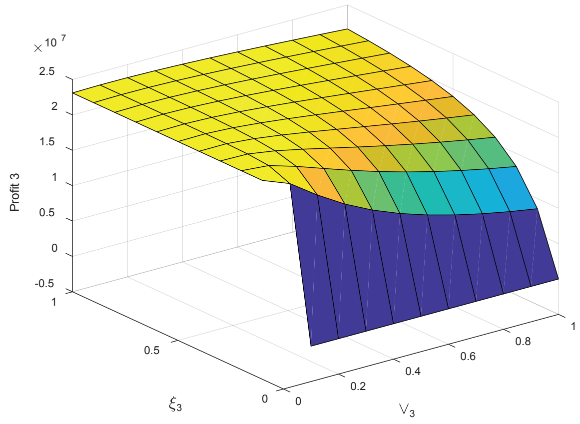

Figure 62. Profit against scenario 3 for v_3 and x_3.

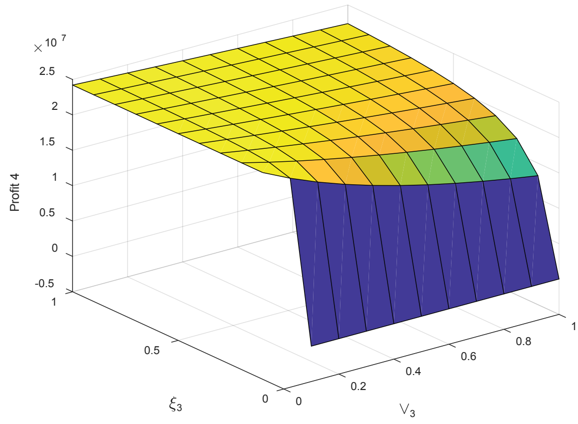

Figure 63. Profit against scenario 4 for v_3 and x_3.

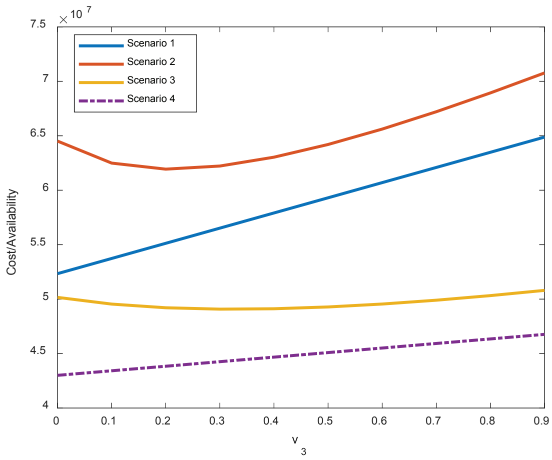

Figure 64. Cost/Availability versus failure rate.

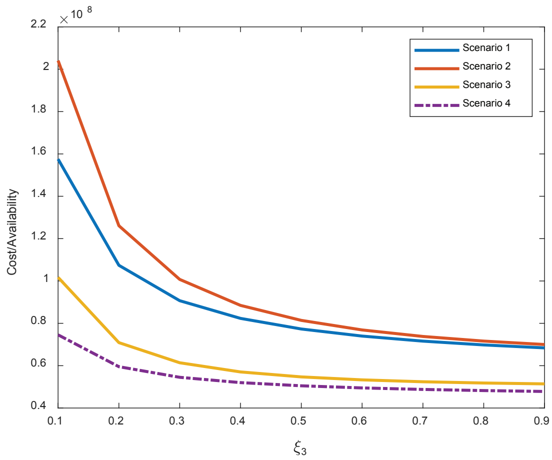

Figure 65. Cost/Availability versus repair rate.

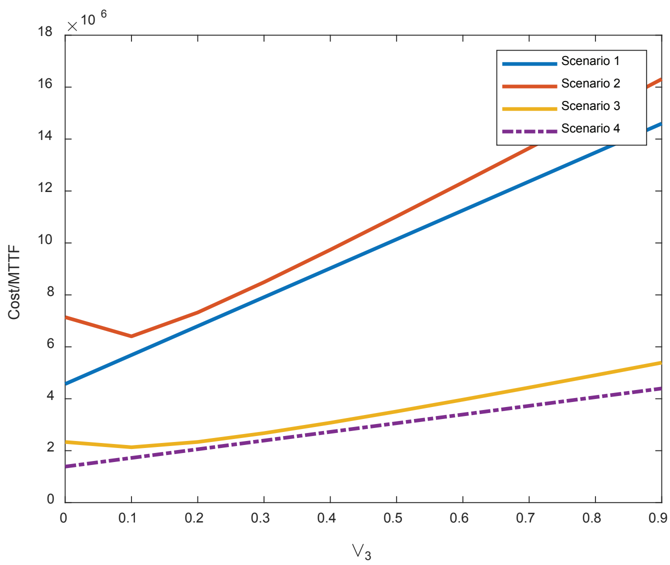

Figure 66. Cost/MTTF versus failure rate.

Table 11. Variation of Availability, MTTF and Profit with respect to repair rate x_4 for the four Scenarios

	Availability				Mean Time To Failure (MTTF)				Profit*10^7			
	Scenario				Scenario				Scenario			
x_4	1	2	3	4	1	2	3	4	1	2	3	4
0.0	0.000	0.000	0.000	0.000	56.00	18.29	57.27	185.1	0.000	0.000	0.000	0.000
0.1	0.901	0.650	0.851	0.968	56.00	18.29	57.27	185.1	2.252	1.625	2.128	2.420
0.2	0.937	0.783	0.917	0.980	56.00	18.29	57.27	185.1	2.342	1.958	2.294	2.450
0.3	0.949	0.840	0.942	0.984	56.00	18.29	57.27	185.1	2.374	2.101	2.355	2.461
0.4	0.956	0.872	0.955	0.986	56.00	18.29	57.27	185.1	2.390	2.181	2.387	2.466
0.5	0.960	0.893	0.962	0.987	56.00	18.29	57.27	185.1	2.400	2.232	2.407	2.469
0.6	0.963	0.907	0.968	0.988	56.00	18.29	57.27	185.1	2.407	2.267	2.420	2.471
0.7	0.964	0.917	0.971	0.989	56.00	18.29	57.27	185.1	2.412	2.267	2.429	2.473
0.8	0.966	0.925	0.974	0.989	56.00	18.29	57.27	185.1	2.415	2.313	2.437	2.474
0.9	0.966	0.931	0.977	0.990	56.00	18.29	57.27	185.1	2.418	2.329	2.442	2.475

Table 10. Variation of Availability, MTTF and Profit with respect to failure rate v_4 for the four Scenarios

	Availability				Mean Time To Failure (MTTF)				Profit*10^7			
	Scenario				Scenario				Scenario			
v_4	1	2	3	4	1	2	3	4	1	2	3	4
0.0	0.949	0.966	0.991	0.984	107.6	307.8	1577	353.5	2.373	2.416	2.478	2.460
0.1	0.924	0.843	0.941	0.976	42.42	12.71	37.10	140.5	2.310	2.109	2.354	2.440
0.2	0.900	0.748	0.897	0.967	26.41	6.492	18.80	87.71	2.251	1.871	2.242	2.419
0.3	0.877	0.672	0.856	0.959	19.17	4.359	12.60	63.75	2.194	1.681	2.140	2.399
0.4	0.856	0.610	0.819	0.952	15.05	3.281	9.400	50.07	2.141	1.526	2.047	2.380
0.5	0.835	0.559	0.785	0.944	12.38	2.630	7.600	41.22	2.089	1.397	1.962	2.361
0.6	0.816	0.515	0.753	0.936	10.52	2.195	6.300	35.03	2.041	1.289	1.884	2.342
0.7	0.797	0.478	0.724	0.929	9.150	1.883	5.400	30.46	1.994	1.196	1.811	2.323
0.8	0.780	0.446	0.697	0.922	8.092	1.649	4.700	26.94	1.950	1.115	1.744	2.305
0.9	0.763	0.418	0.672	0.914	7.253	1.466	4.200	24.15	1.907	1.045	1.682	2.286

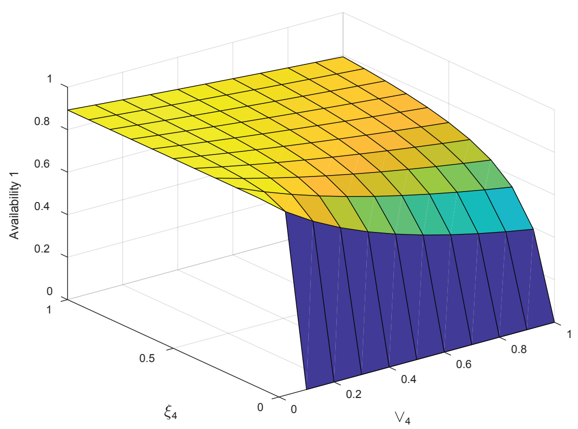

Figure 67. Availability against scenario 1 for v_4 and x_4.

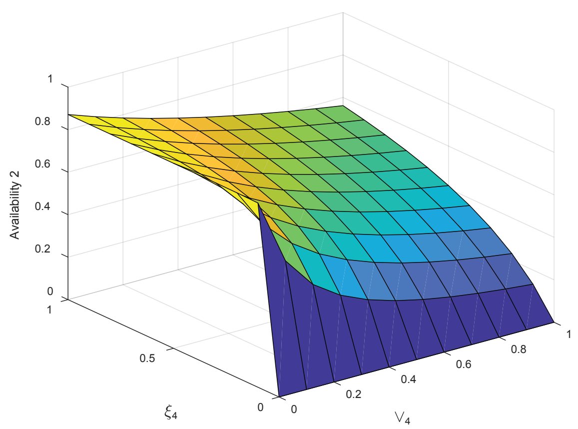

Figure 68. Availability against scenario 2 for v_4 and x_4.

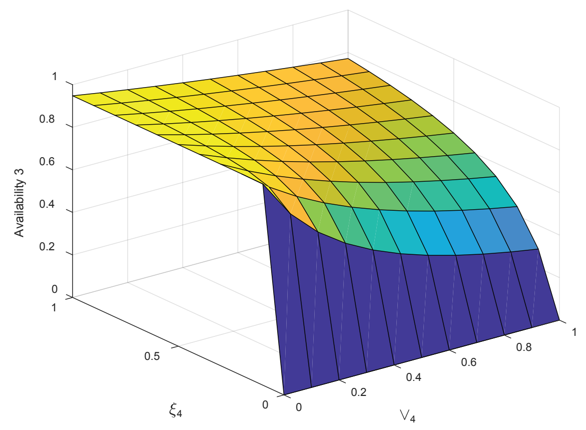

Figure 69. Availability against scenario 3 for v_4 and x_4.

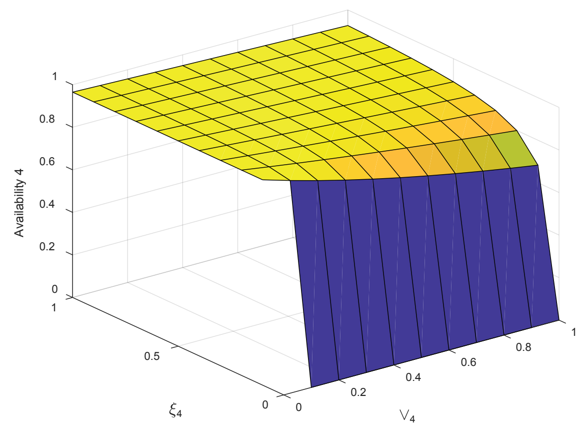

Figure 70. Availability against scenario 4 for v_4 and x_4.

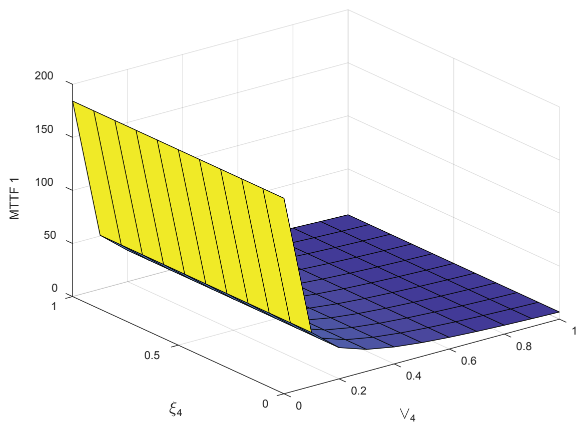

Figure 71. MTTF against scenario 1 for v_4 and x_4.

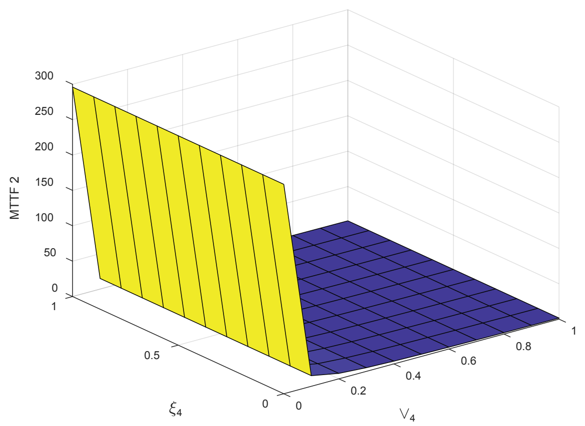

Figure 72. MTTF against scenario 2 for v_4 and x_4.

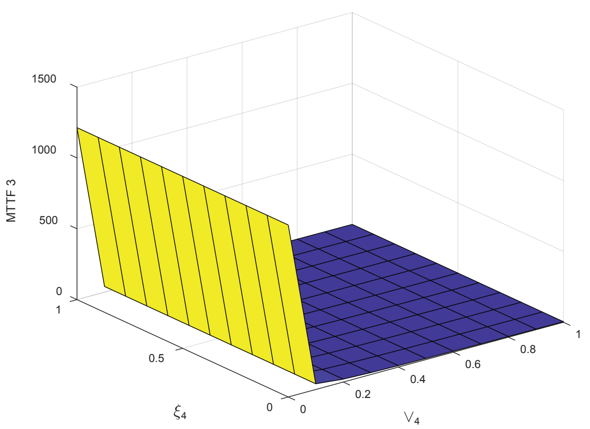

Figure 73. MTTF against scenario 3 for v_4 and x_4.

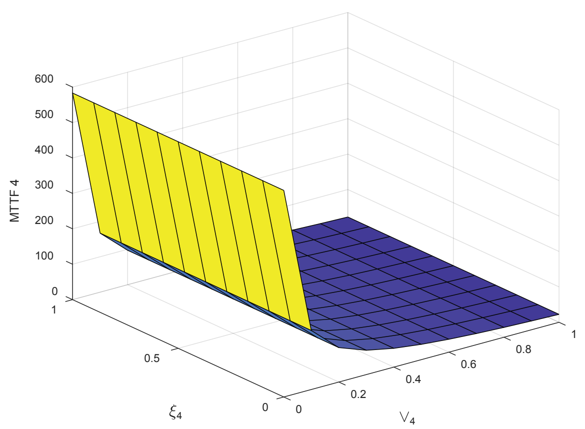

Figure 74. MTTF against scenario 5 for v_4 and x_4.

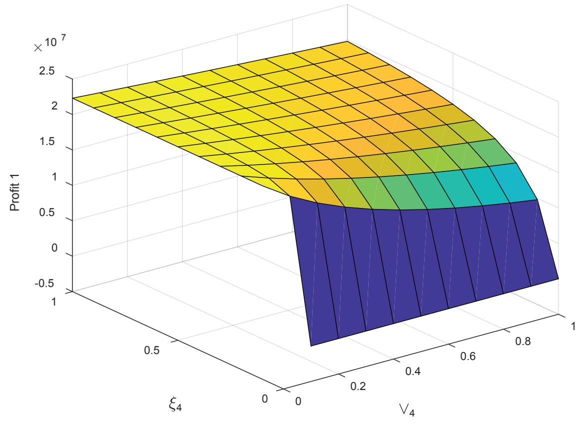

Figure 75. Profit against scenario 1 for v_4 and x_4.

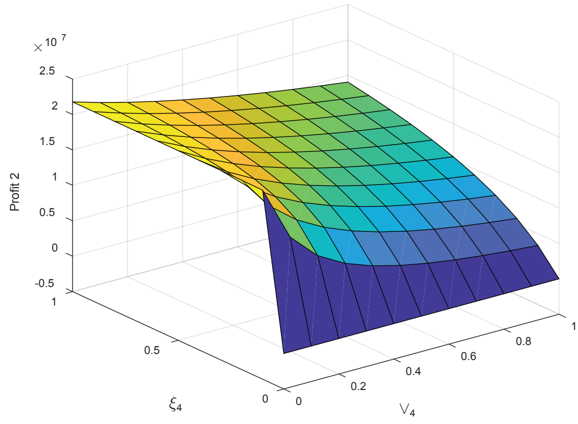

Figure 76. Profit against scenario 2 for v_4 and x_4.

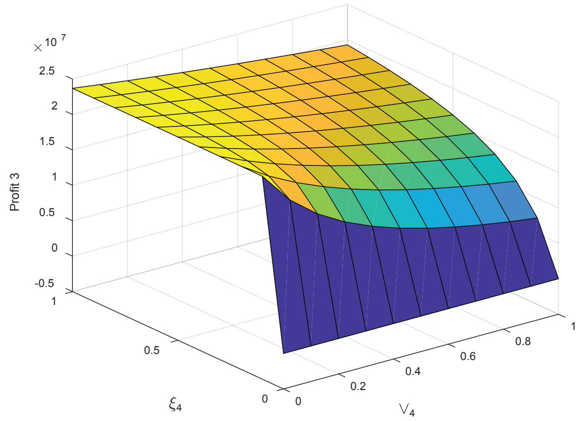

Figure 77. Profit against scenario 3 for v_4 and x_4.

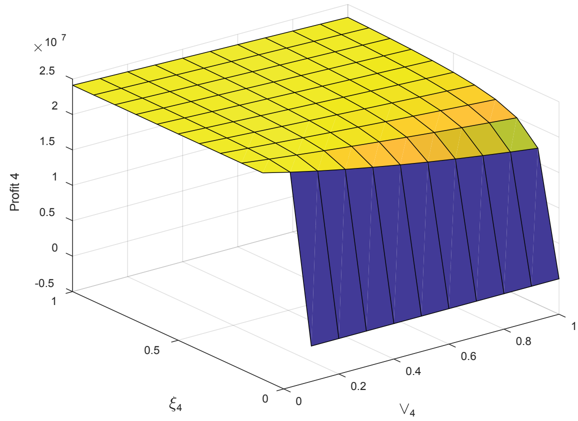

Figure 78. Profit against scenario 4 for v_4 and x_4.

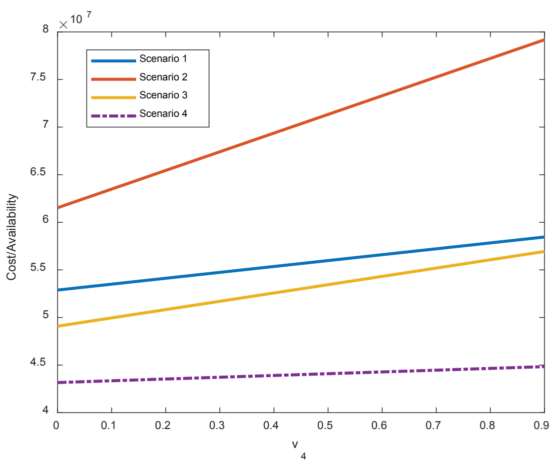

Figure 79. Cost/Availability versus failure rate.

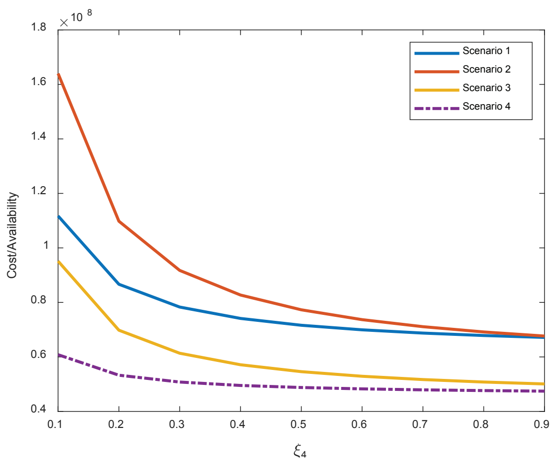

Figure 80. Cost/Availability versus repair rate.

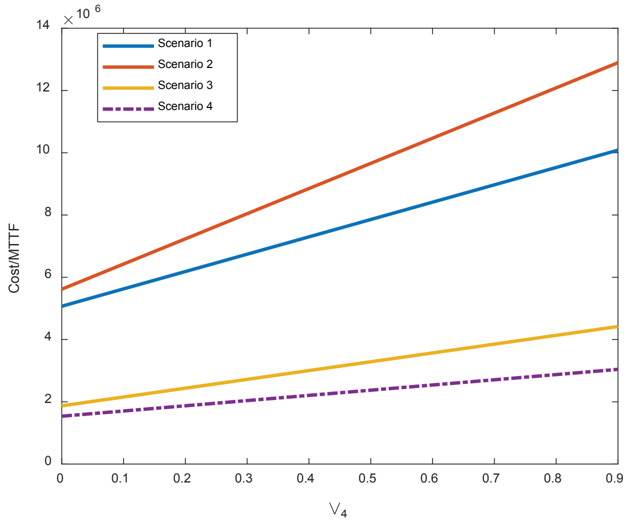

Figure 81. Cost/MTTF versus failure rate.

and at the same time MTTF and Cost analysis increases as the repair rate increases. Lastly, profit analysis was carried out in all the four (4) scenarios and throughout the findings it observed that profit in terms of failure rate decreases as the failure rate increase and also profit increases as the repair rate increases. To this fact, the clients require optimal maintenance action in order to avoid huge downfall and adequate the life span of the network.

CONCLUSION

In this research, computer network system consisting 2-clients, load balancers, 2-fog nodes and 2-cloud server arranged in series-parallel was considered. Thorough differential equations and detailed mathematical expressions of availability, sensitivity, MTTF and profit were derived. It is true from the figures that optimal performance level was obtained. The presented numerical results have shown the implication of repair and failure rates on the network availability. From numerical results, it is enough to show that availability or reliability and MTTF can be increase with adequate maintenance to truncate network downfall through fault tolerant units/subsystem like introducing more load balancers and more number of cloud servers. The presented research work will help plant management to shun away on an erroneous performance assessment caused by poor system design. Failure occurrence and monitoring of condition can be extended and incorporated to allow management in approving the optimal replacement/ maintenance time.

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AUTHORSHIP CONTRIBUTIONS

Muhammad Salihu Isa initiate the model and do all the writing and mathematical analysis while Jinbiao Wu helps in editing and supervision.

CONFLICT OF INTEREST

Authors have declared that there is no conflict of interest with regard to this research

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