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Research Article

Comparison of NSGA-II and MODE performances by using MCDM methods for multi-response experimental data

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ABSTRACT

Multi-response experimental data, composed with more than one response variable, can be examined in three stages: modeling, optimization and decision making. In this study, these three stages were considered sequentially. Model parameters were estimated through Seemingly Unrelated Regression (SUR) method due to linear correlation between responses during the modeling stage. In the optimization stage, simultaneous optimization of predicted multiple responses were considered as a multi-objective optimization (MOO) problem. For this purpose, Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and Multi Objective Differential Evolution (MODE), were applied to obtain Pareto solution sets. In the decision making stage, compromise solution was chosen from the Pareto sets through various multi-criteria decision making (MCDM) methods. This study aims to compare performances of the NSGA-II and the MODE via various MCDM methods using three real data sets taken from different fields. The novelty of this paper is applying the MCDM methods to the Pareto solution set to choose a compromise solution by taking into account the Entropy weights of responses primarily. Afterwards, closeness of the compromise solution to the ideal solution using the mean absolute error (*MAE*) and the root mean square error (*RMSE*) metrics is calculated for decision making on the performance of the MOO methods. The results showed that compromise solution of the MODE is closer to the ideal solution than the NSGA-II according to the *MAE* and *RMSE* metrics. As a result, the MODE outperforms the NSGA-II.

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INTRODUCTION

Experimental data, which consist of input and response variables, can be produced by experimental design with measurements in many field of science and engineering. If the experimental data contains more than one response variable, it is called multi-response experimental data. Generally, the multi-response experimental data sets can

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be examined in three basic steps: (*i*) modeling, (*ii*) optimization, and (*iii*) decision making. In modeling stage, functional relationship between the responses and input variables are obtained by applying multivariate multiple linear regression analysis. Parameters of mathematical model are estimated by using Ordinary Least Squares (OLS) method to create predicted response models in polynomial type. During the modeling stage, it is necessary to define whether the responses are linearly correlated. In this case, Seemingly Unrelated Regression (SUR) method should be used to predict response functions by taking into account the linear relationship between the responses.

The SUR method was first introduced by Zellner [1] for the analysis of multi-response experimental data sets in which the responses are correlated. Liu [2] showed that the SUR estimators have smaller variance than the OLS estimators in case the experimental data size is large enough. Multi-response studies on the SUR are available in the literature [3-6].

Optimization is as important as the modeling stage during the analyses of multi-response experimental data set. For this purpose, predicted response functions are considered as objective functions and optimized simultaneously. Simultaneous optimization of the predicted response functions is achieved in multi-objective perspective, called multi-objective optimization (MOO). There are several studies for incorporating the MOO approach into the solution of multi-response problems [2,7-12]. As a result of the MOO, it is not possible to obtain a single optimal solution for all objective functions. In this case, Pareto solution set is found which has many non-dominated alternative solutions. It is possible to get the Pareto solution set through mathematical programming methods [13]. However, these methods tend to obtain elements of the Pareto solution set one at a time. In recent years, multi-objective metaheuristic (MOM) methods are frequently applied to solve the MOO problems [14]. In this study, population-based MOM methods are preferred to use since these methods can generate the Pareto solution set in a single run. Two population-based MOM methods, (*i*) NSGA-II and (*ii*) MODE, are applied for obtaining the Pareto sets. The NSGA-II and MODE, also called Artificial Intelligence Optimization Algorithms, have stochastic search mechanism with genetic operators [15,16]. There have been several studies about the NSGA-II for the optimization of multi-response problems [17-21]. Some applications of the MODE for optimization stage of multi-response problems can be seen in the studies of [22,23]. There are different studies for comparison of these two algorithms [24-28].

In decision making stage, it is necessary to decide for a compromise solution among the Pareto solutions which are non-dominated to each other. Multi-criteria decision making (MCDM) methods are preferable to choose a compromise solution from Pareto solution set. The Pareto solution set is considered as decision matrix that is a tool to evaluate and to select the best alternative considering different criteria for the MCDM. The application of the MCDM to the multi-response problems can be handled in several studies: TOPSIS and AHP [20]; TOPSIS [29]; EDAS [30]; AHP and VIKOR [31]; SAW, WASPAS, TOPSIS, EDAS, VIKOR, MOORA, COPRAS, PIV, and PSI [32]; CODAS [33]; GRA [34]; COPRAS [35,36]; TOPSIS and GRA [37]; TOPSIS, VIKOR, MOORA, and AHP [38]; WASPAS and EDAS [39]. Further, combined MCDM methods are applied to multi-response studies, e.g. PSI-TOPSIS and PSI-EDAS [8]; GRA-TOPSIS [40]; AHP-TOPSIS-VIKOR [41].

The main aim of this study is to make comparison for the optimization performances of the NSGA-II and MODE by using various MCDM methods which are TOPSIS, COPRAS, WASPAS, EDAS, CODAS, GRA and MABAC. At first, Entropy method is applied to the response variables of the experimental data sets to define the weights objectively. Then, MCDM methods are applied to the Pareto sets to define a compromise solution considering the weights. The closeness of the compromise solution and the ideal solution vectors are calculated with respect to *MAE* and *RMSE* metrics. Finally, the MOM method with the smallest MAE and RMSE metric values is considered to have the best performance.

The paper is organized as follows. In Section 2, multi-response experimenal data set and the SUR modeling are introduced briefly. In Section 3, structure of the MOO problem is defined and algorithmic steps of the NSGA-II and MODE are explained. In Section 4, considered MCDM methods are discussed for decision making. In Section 5, numerical example is carried out for three experimental data sets from different fields of engineering, chemistry, and food. The obtained results are also discussed in detailed. Finally, conclusion is given in Section 6.

MODELING OF MULTI-RESPONSE EXPERIMENTAL DATA WITH THE SUR

An experiment in which a number of responses are measured simultaneously for each setting of a group of input variables is called multi-response experiments [42]. The multi-response experimental data set with n observations, p input variables, and r response variables is shown in Table 1.

Table 1. Multi-response experimental data set

No	*Input Variables*				*Response Variables*			
	X_1	X_2	…	X_p	Y_1	Y_2	…	Y_r
1	x_{11}	x_{12}	…	x_{1p}	y_{11}	y_{12}	…	y_{1r}
2	x_{21}	x_{22}	…	x_{2p}	y_{21}	y_{22}	…	y_{2r}
⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮
n	x_{n1}	x_{n2}	…	x_{np}	y_{n1}	y_{n2}	…	y_{nr}

Correlation of the responses should be considered to perform the functional relationship between the responses and input variables. If the responses are linearly correlated, the SUR method is proper to consider the linear correlation during modeling stage. Functional relationship between the response variables and input variables is expressed by linear regression model. The response model can be written as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

In Equation (1), $\boldsymbol{Y}$ is a $rn \times 1$ vector of responses, $\boldsymbol{X}$ is a design matrix of dimension $n \times p^*$, $\boldsymbol{\beta}$ is a $p^* \times 1$ vector of regression coefficients and $\boldsymbol{\varepsilon}$ is a $rn \times 1$ vector of errors, $p^* = \sum_{i=1}^{r} p_i$. The assumptions of the response model are $E(\boldsymbol{\varepsilon}) = \mathbf{0}_s$ and $Var(\boldsymbol{\varepsilon}) = \sum \otimes I_n$.

Parameter estimation with the SUR method consists of three main steps as follows [9]:

Step 1: The predicted response model ($\hat{Y}_i$) is obtained individually by using OLS method and residuals (e_i) are calculated.

Step 2: The residual covariance matrix, $\hat{\Sigma} = [\hat{\sigma}_{il}]$, for the responses is obtained. Calculate the covariance matrix components, $\hat{\sigma}_{il} = \frac{e_i e_l}{n}$, $i,l = 1,2,...,r$.

Step 3: The estimator vector of $\boldsymbol{\beta}$ for the SUR is calculated as

$$\hat{\boldsymbol{\beta}}^{SUR} = \left(\boldsymbol{X}^T (\hat{\sum}^{-1} \otimes \boldsymbol{I}_n) \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T (\hat{\sum}^{-1} \otimes \boldsymbol{I}_n) \boldsymbol{Y}. \tag{2}$$

The predicted response model can be written in polynomial form according to parameter estimates as

$$\hat{Y}_i = \hat{\beta}_{i0} + \sum_{j=1}^{p} \hat{\beta}_{ij} x_{ij} + \sum_{j<k} \sum \hat{\beta}_{ijk} x_{ij} x_{ik} + \sum_{j=1}^{p} \hat{\beta}_{ijj} x_{ij}^2 \ , \ i = 1,2,...,r. \tag{3}$$

MULTI-OBJECTIVE OPTIMIZATION FOR MULTI-RESPONSE PROBLEM

Optimization of multi-response problem can be considered as a multi-objective optimization (MOO) problem.

Mathematical formulation of the MOO problem can be written as

$$\begin{aligned} &min/max\, f_1 = \hat{Y}_1(\boldsymbol{X}) \\ &min/max\, f_2 = \hat{Y}_2(\boldsymbol{X}) \\ &\quad\vdots \\ &min/max\, f_r = \hat{Y}_r(\boldsymbol{X}) \\ &\boldsymbol{X} \in S \end{aligned} \tag{4}$$

In Equations (4), $\boldsymbol{X} = [X_1\ X_2 ... X_p]$ is input variable vector and S is constraint set. Each predicted response model $\hat{Y}_i(\boldsymbol{X})$, i=1,2,...,r, is considered as an objective function, f_i, $i = 1,2,...,r$. Solution set of the MOO problem, is called Pareto solution set. The Pareto solution set is obtained by using two population-based MOM methods, NSGA-II and MODE, which are quite successful to find Pareto solutions in a single run without getting stuck local solutions.

Fast non-dominanted sorting and crowding distance approaches are two main principles for obtaining Pareto solution set by the MOM methods. The former ensures that solutions are sorted by separating them into surfaces according to their dominance as Rank-1, Rank-2, and etc. as shown in Figure 1, while the latter provides the measurement of the distances between the solutions on each surface and each other as illustrated in Figure 2.

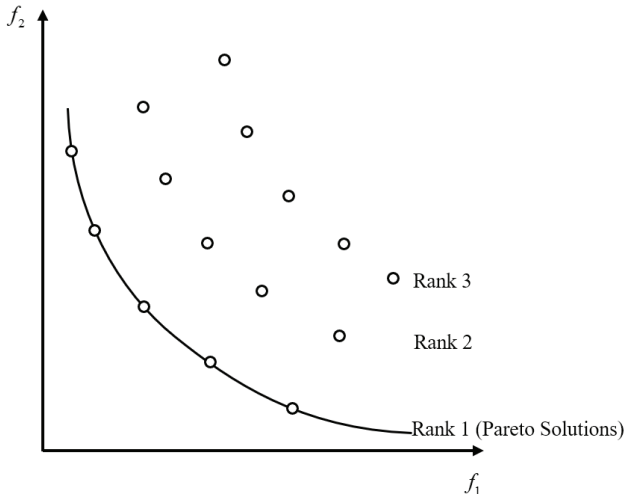

Figure 1. Surfaces determined by the fast non-dominated sorting.

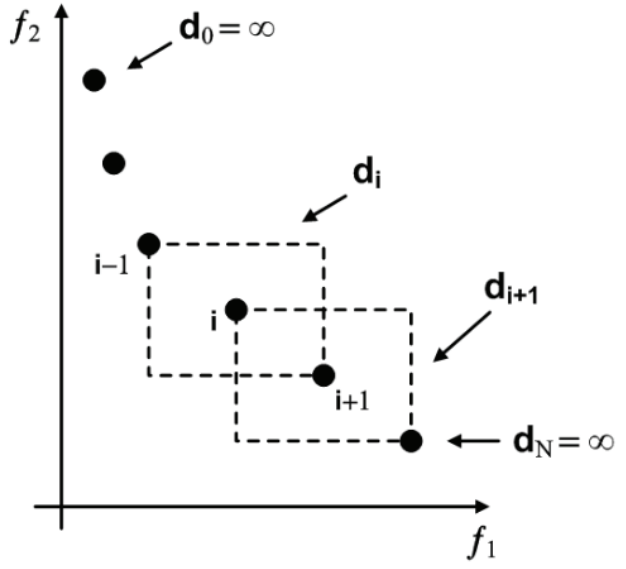

Figure 2. Crowding distances on the surfaces.

Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

The NSGA-II introduced by Deb et al. [43], is based on genetic operators. Simulations involving difficult problems have also shown that solutions are well spread and convergence is strong.

Step 1: Determine the tuning parameters, population size (n_{pop}), maximum generation number ($maxn_{gen}$), genetic operators which are selection, crossover and mutation. Generate initial population (P) with size n_{pop} randomly and $n_{gen} = 0$.

Step 2: Create a new generation population (Q) with size n_{pop} by applying genetic operators to P.

Step 3: Combine parent and new generation population, $R = P \cup Q$.

Step 4: Obtain and sort non-dominated surface.

Step 5: Calculate crowding distances of the solutions on each surface. Sort surfaces in ascending order and crowding distances within the surface in descending order.

Step 6: Carry out crossover and mutation operators. Create new generation solutions (Q) according to crossover and mutation rates. Update number of generation, $n_{gen} = n_{gen} + 1$.

Step 7: If number of generations is not reached, Return to Step 2, otherwise algorithm is terminated.

Multi Objective Differential Evolution (MODE)

The MODE proposed by Souza et al. [23], is an extension of the Differential Evolution (DE) algorithm developed by Storn and Price [44], for the single objective algorithm to multi-objective optimization. The MODE is a preferred method for difficult problems due to its ease of application.

Step 1: Determine the tuning parameters, number of variables (D), population size ($n_{pop} \geq 4$), maximum generation number ($maxn_{gen}$), scaling vector (F) and crossover ratio (CR).

Step 2: Determine initial population.

$\forall_i \leq n_{pop} \wedge \forall_j \leq D : x_{j,i,G=0} = lower(x_j) + rand_j[0,1].(upper(x_j) - lower(x_j))$

$lower(x_j), upper(x_j)$: The lower and upper bounds of the variables

r_1, r_2, r_3: Randomly selected chromosomes from the population ($r_1 \neq r_2 \neq r_3 \neq i$)

$x_{j,i,G}$: jth gene of ith chromosome in G generation

Step 3: Compute mutation operator.

$\forall_j \leq D : n_{j,i,G+1} = x_{j,r_3,G} + F \times (x_{j,r_1,G} - x_{j,r_2,G})$

$n_{j,i,G+1}$: Mutated intermediate chromosome

Step 4: Carry out crossover operator.

$$\forall_j \leq D, x_{j,u,G+1} = \begin{cases} n_{j,i,G+1} & if\ (rand_j[0,1]) < CR \vee j = j_{rand} \\ x_{j,i,G} & other \end{cases}$$

$x_{j,u,G+1}$: Chromosome formed after crossover

Step 5: Perform selection.

$$\forall_i \leq n_{pop} : x_{i,G+1} = \begin{cases} x_{u,G+1} & if f(x_{u,G+1}) \leq f(x_{i,G}) \\ x_{i,G} & other \end{cases}$$

Step 6: Repeat Step 3–Step 5 until the $maxn_{gen}$ is reached.

Step 7: Apply non-dominated sorting.

Step 8: Calculate crowding distance.

Step 9: Sort surfaces in ascending order, crowding distances within the surface in descending order [16,22].

MULTI-CRITERIA DECISION MAKING METHODS FOR PARETO SOLUTION SET

Multi-Criteria Decision Making (MCDM) methods can be classified into three categories as sum-based, max/min value-based and range-based normalization techniques. In this study, TOPSIS and COPRAS for sum-based category; WASPAS, EDAS and CODAS for max/min value-based category; GRA and MABAC for range-based category are considered. These MCDM methods are applied to Pareto solution set to choose a compromise solution. The obtained Pareto solution set is considered as a decision matrix, $F = [f_{ij}]_{m \times r}$, $i = 1,2,...,m$, $j = 1,2,...,r$. Each row of the decision matrix indicates non-dominated solution, called alternative and each column of the decision matrix indicates objective function value, called criteria.

The criteria may not be of equal importance. The importance of the criteria is expressed by weights. In this study, the weight of the criteria w_j, $j = 1,2,...,r$, is calculated by using Entropy method which is one of the MCDM methods. The Entropy method is applied to response values which is presented as in Table 1. The weight vector of the criteria is represented as, $\boldsymbol{w} = [w_j]_{1xr}$, $j = 1,2,...,r$, for each MCDM method. Entropy method is widely used for determining weights for MCDM problems. Mathematical theory of the Entropy was proposed by Shannon [45]. Subjective, objective, and integral weighting methods are the three main weighting methods. Objective entropy weighting method is applied to evaluate weights for MCDM. Advantage of the Entropy method is that it avoids subjective evaluations of decision makers in finding criterion weights, therefore increasing the objectivity of comprehensive evaluation results which are reliable and effective [21,46,47].

The algorithmic steps of the MCDM, called TOPSIS, COPRAS, WASPAS, EDAS, CODAS, GRA and MABAC, can be seen below, respectively.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

The TOPSIS method developed by Hwang and Yoon [48] defines a closeness coefficient to determine ranking order of all alternatives by calculating distances to both the positive ideal solution and negative ideal solution simultaneously.

Algorithm of the TOPSIS method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m \times r}$, $i = 1,2,...,m$, $j = 1,2,...,r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = [w_j]_{1xr}$, $j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = [n_{ij}]_{mxr}$ by using

$$n_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^{m} f_{ij}^2}}, j \in B, j \in C \quad (5)$$

In Equation (5), B and C denote benefit criteria and cost criteria, respectively.

Step 4: Construct weighted normalized decision matrix values of $V = [v_{ij}]_{mxr}$ as follows:

$$v_{ij} = w_j \times n_{ij} \quad (6)$$

Step 5: Determine positive ideal solution $A^+ = [v_j^+]_{1\times r}$ and negative ideal solution $A^- = [v_j^-]_{1\times m}$ as follows:

$$v_j^+ = \begin{cases} \max\limits_i v_{ij}, j \in B \\ \min\limits_i v_{ij}, j \in C \end{cases} \quad (7)$$

$$v_j^- = \begin{cases} \min\limits_i v_{ij}, j \in B \\ \max\limits_i v_{ij}, j \in C \end{cases} \quad (8)$$

Step 6: Compute distances S_i^+ and S_i^- from A^+ and A^- of each alternative by following,

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \quad (9)$$

$$S_i^- = \sqrt{\sum_{j=1}^{r} (v_{ij} - v_j^-)^2} \quad (10)$$

Step 7: Calculate closeness coefficients of each alternatives as follows:

$$CC_i = \frac{S_i^-}{S_i^- + S_i^+} \quad (11)$$

Step 8: Rank the alternatives according to descending order of CC_i, $i = 1,2,...,m$ [49,50].

Complex Proportional Assessment (COPRAS)

The COPRAS method, developed by Zavadskas et al. [51], has noticeable advantages among the other methods. Calculation time is very short and it can be easily implemented to any program source code.

Algorithm of the COPRAS method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m\times r}$, $i = 1,2,...,m$, $j = 1,2,...,r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = [w_j]_{1xr}$, $j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = [n_{ij}]_{mxr}$, $i = 1,2,...,m$, $j = 1,2,...,r$, by using

$$n_{ij} = \frac{f_{ij}}{\sum_{i=1}^{m} f_{ij}}, j \in B, j \in C \quad (12)$$

Step 4: Construct weighted normalized decision matrix values of $V = [v_{ij}]_{mxr}$ as follows:

$$v_{ij} = w_j \times n_{ij} \quad (13)$$

Step 5: Compute S_i^+ and S_i^- values of each alternative as follows:

$$S_i^- = \sum_{j=1}^{k} v_{ij}, \ j \in C \quad (14)$$

$$S_i^+ = \sum_{j=1}^{k} v_{ij}, \ j \in B \quad (15)$$

Step 6: Compute relative importance values Q_i of each alternative by using:

$$Q_i = S_i^+ + \frac{\sum_{i=1}^{m} S_i^-}{\left(S_i^-\right)\left(\sum_{i=1}^{m} \frac{1}{S_i^-}\right)} \quad (16)$$

Step 7: Calculate performance index P_i of each alternative by using as follows:

Step 8: Rank the alternatives according to descending order of Q_i *or* P_i, $i = 1,2,...,m$ [51].

Weighted Aggregated Sum Product Assessment (WASPAS)

The WASPAS method combines weighted sum model (WSM) and weighted product model (WPM). Due to its mathematical simplicity and capability, WASPAS provides more accurate results as compared to WSM and WPM methods.

Algorithm of the WASPAS method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m\times r}$, $i = 1,2,...,m$, $j = 1,2,...,r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = [w_j]_{1xr}$, $j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = [n_{ij}]_{mxr}$, $i = 1,2,...,m$, $j = 1,2,...,r$, by using

$$n_{ij} = \begin{cases} \dfrac{f_{ij}}{\max\limits_i f_{ij}}, j \in B \\ \dfrac{\min\limits_i f_{ij}}{f_{ij}}, j \in C \end{cases} \quad (17)$$

Step 4: Calculate total relative importance of each alternative based on WSM method, as follows:

$$Q_i^{(1)} = \sum_{j=1}^{r} w_j \times n_{ij} \quad (18)$$

Step 5: Calculate total relative importance of each alternative based on WPM method, as follows:

$$Q_i^{(2)} = \prod_{j=1}^{r} (n_{ij})^{w_j} \quad (19)$$

Step 6: Compute joint generalized criterion of weighted aggregation of additive and multiplicative methods for each alternative by:

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)} \quad (20)$$

Step 7: Rank the alternatives according to decreasing values of Q_i, $i = 1, 2, ..., m$ [52].

Evaluation Based on Distance from Average Solution (EDAS)

The EDAS method introduced by Keshavarz-Ghorabaee et al. [53] uses average solution for appraising the alternatives. Two measures which called positive distance from average (PDA) and negative distance from average(NDA) are considered for the appraisal which are calculated according to type of criteria, benefit or cost.

Algorithm of the EDAS method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m \times r}$, $i = 1, 2, ..., m$, $j = 1, 2, ..., r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = [w_j]_{1xr}$, $j = 1, 2, ..., r$, by Entropy method.

Step 3: Determine average values of the criteria, $AV = [AV_j]_{1xr}$ by,

$$AV_j = \frac{1}{m} \sum_{i=1}^{m} f_{ij} \quad (21)$$

Step 4: Compute positive distance from average (PDA) and negative distance from average (NDA) of each alternative with respect to each criteria as,

$$PDA_{ij} = \begin{cases} \dfrac{\max\left(0, (f_{ij} - AV_j)\right)}{AV_j}, j \in B \\ \dfrac{\max\left(0, (AV_j - f_{ij})\right)}{AV_j}, j \in C \end{cases} \quad (22)$$

$$NDA_{ij} = \begin{cases} \dfrac{\max\left(0, (AV_j - f_{ij})\right)}{AV_j}, j \in B \\ \dfrac{\max\left(0, (f_{ij} - AV_j)\right)}{AV_j}, j \in C \end{cases} \quad (23)$$

Step 5: Calculate SP_i and SN_i values of each alternative as follows,

$$SP_i = \sum_{j=1}^{r} w_j PDA_{ij} \quad (24)$$

$$SN_i = \sum_{j=1}^{r} w_j NDA_{ij} \quad (25)$$

Step 6: Calculate normalized NSP_i and NSN_i values of each alternative as follows, respectively:

$$NSP_i = \frac{SP_i}{\max\limits_i SP_i} \quad (26)$$

$$NSN_i = 1 - \frac{SN_i}{\max\limits_i SN_i} \quad (27)$$

Step 7: Calculate the assessment score AS_i of each alternative as follows:

$$AS_i = \frac{1}{2}\left(NSP_i + NSN_i\right) \quad (28)$$

Step 8: Rank the alternatives according to decreasing values of AS_i, $i = 1, 2, ..., m$ [53].

Combinative Distance-based Assessment (CODAS)

The CODAS method was introduced by Keshavarz-Ghorabaee et al. [54] as an efficient method to solve MCDM problems. It is used to obtain the desirability of attributes by a ranking which is estimated by two measures, Euclidean distance and Taxicab distance, from a negative ideal solution.

Algorithm of the CODAS method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m \times r}$, $i = 1, 2, ..., m$, $j = 1, 2, ..., r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = \left[w_j\right]_{1xr}, j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = \left[n_{ij}\right]_{mxr}, i = 1,2,...,m,\ j = 1,2,...,r$, by using

$$n_{ij} = \begin{cases} \dfrac{f_{ij}}{\max\limits_i f_{ij}}, j \in B \\ \dfrac{\min\limits_i f_{ij}}{f_{ij}}, j \in C \end{cases} \quad (29)$$

Step 4: Construct weighted normalized decision matrix values of $V = \left[v_{ij}\right]_{mxr}$ as follows:

$$v_{ij} = w_j \times n_{ij} \quad (30)$$

Step 5: Determine negative ideal solution $NS = \left[ns_j\right]_{1\times r}$ as follows:

$$ns_j = \min_i v_{ij} \quad (31)$$

Step 6: Compute Euclidean distance (E_i) and Taxicab distance (T_i) of each alternative from the negative ideal solution by using,

$$E_i = \sqrt{\sum_{j=1}^{r} (v_{ij} - ns_j)^2} \quad (32)$$

$$T_i = \sum_{j=1}^{r} \left| v_{ij} - ns_j \right| \quad (33)$$

Step 7: Define relative assessment matrix, $RA = \left[h_{ik}\right]_{m\times r}$ as follows:

$$h_{ik} = (E_i - T_k) + \left(\psi(E_i - E_k) \times (T_i - T_k)\right),\ k = 1,2,...,r \quad (34)$$

In Equation (34), ψ indicates a threshold function according to the threshold parameter τ set by decision maker, defined as

$$\psi(x) = \begin{cases} 1, & if\ |x| \geq \tau \\ 0, & if\ |x| < \tau \end{cases} \quad (35)$$

Step 8: Calculate assessment score AS_i of each alternative as follows:

$$AS_i = \sum_{k=1}^{r} h_{ik} \quad (36)$$

Step 9: Rank the alternatives according to decreasing values of AS_i, $i = 1,2,...,m$. The alternative with the highest assessment score is the most desirable alternative [54].

Grey Relational Analysis (GRA)

Grey system theory developed by Deng [55] deals with poor, incomplete, and uncertain information. The GRA method based on grey system theory is appropriate for solving MCDM problems with complicated interrelationships between multiple factors and variables.

Algorithm of the GRA method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m\times r}$, $i = 1,2,...,m,\ j = 1,2,...,r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = \left[w_j\right]_{1xr}, j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = \left[n_{ij}\right]_{mxr}$, by using

$$n_{ij} = \begin{cases} \dfrac{f_{ij} - \min\limits_i f_{ij}}{\max\limits_i f_{ij} - \min\limits_i f_{ij}}, j \in B \\ \dfrac{f_{ij} - \max\limits_i f_{ij}}{\min\limits_i f_{ij} - \max\limits_i f_{ij}}, j \in C \end{cases} \quad (37)$$

Step 4: Construct absolute value matrix of $\Delta = \left[\Delta_{ij}\right]_{mxr}$ as follows:

$$\Delta_{ij} = \left| n_{0j}^* - n_{ij}^* \right| \quad (38)$$

In Equation (38), $n_{0j}^* = \begin{cases} \max\limits_i n_{ij}, j \in B \\ \min\limits_i n_{ij}, j \in C \end{cases}$

Step 5: Compute grey relational coefficient matrix as, $\gamma = \left[\gamma_{ij}\right]_{mxr}$ by following:

$$\gamma_{ij} = \frac{\Delta_{\min} + \eta\Delta_{\max}}{\Delta_{ij} + \eta\Delta_{\max}} \quad (39)$$

In Equation (39), $0 < \eta < 1$ defines distinguishing coefficient, $\Delta_{\min} = \min\limits_i \min\limits_j \left(\Delta_{ij}\right)$, and $\Delta_{\max} = \max\limits_i \max\limits_j \left(\Delta_{ij}\right)$

Step 6: Calculate grey relational degree Γ_i of each alternative by using:

$$\Gamma_i = \sum_{j=1}^{n} w_j \gamma_{ij} \quad (40)$$

Step 7: Rank the alternatives according to descending order of Γ_i, $i = 1,2,...,m$ [42,55,56].

Multi-Attributive Border Approximation Area Comparison(MABAC)

The MABAC method, developed by Pamucar and Ćirović [57], is based on distance of the criterion function of each alternative from the border approximation area.

Algorithm of the MABAC method for Pareto solution set of multi-response problems are given as follows:

Step 1: Construct decision matrix as, $F = [f_{ij}]_{m\times r}$, $i = 1,2,...,m,\ j = 1,2,...,r$.

Step 2: Obtain weight vector of the criteria as, $\boldsymbol{w} = [w_j]_{1xr}, j = 1,2,...,r$, by Entropy method.

Step 3: Determine normalized decision matrix, $N = [n_{ij}]_{mxr}$, by using

$$n_{ij} = \begin{cases} \dfrac{f_{ij} - \min\limits_i f_{ij}}{\max\limits_i f_{ij} - \min\limits_i f_{ij}}, j \in B \\ \dfrac{f_{ij} - \max\limits_i f_{ij}}{\min\limits_i f_{ij} - \max\limits_i f_{ij}}, j \in C \end{cases} \quad (41)$$

Step 4: Construct weighted normalized decision matrix values of $V = [v_{ij}]_{mxr}$ as follows:

$$v_{ij} = w_j \times (n_{ij} + 1) \quad (42)$$

Step 5: Compute Border Approximation Area for each criterion as, $G = [g_j]_{mxr}$ by:

$$g_j = \left(\prod_{i=1}^{m} v_{ij} \right)^{\frac{1}{m}} \quad (43)$$

Step 6: Calculate distance of the alternative from border approximation area for matrix elements (Q) as follows:

$$Q = V - G \quad (44)$$

Step 7: Determine belonging of each alternative A_i to approximation area (G^+, G or G^-) as follows:

$$A_i = \begin{cases} G^+, \, if \; q_{ij} > 0 \\ G, \, if \; q_{ij} = 0 \\ G^-, \, if \; q_{ij} < 0 \end{cases} \quad (45)$$

In Equation (45), G indicates border approximation area, G^+ indicates upper approximation area which contains the ideal alternative (A^+) while G^- indicates lower approximation area which contains the anti-ideal alternative(A^-).

Step 8: Compute final values of the criterion function S_i for each alternative which is based on the sum of the distance of the alternatives from the border approximation areas as:

$$S_i = \sum_{j=1}^{r} q_{ij} \quad (46)$$

Step 9: Rank the alternatives according to decreasing values of S_i, $i = 1,2,...,m$. The alternative with the highest assessment score is the most desirable alternative [57].

NUMERICAL EXAMPLES

Three real data sets related to multiple-response experiments are given to illustrate for the performance comparison of the NSGA-II and MODE. These are (*i*) engineering data set, (*ii*) chemistry data set, and (*iii*) food data set. Modeling stage was achieved by using the SUR method since responses are correlated for each data set. Pareto solution sets are obtained by using the NSGA-II and MODE in the optimization stage. In decision making stage, compromise solutions are determined via seven MCDM methods. The *MAE* and *RMSE* metrics are calculated to evaluate these compromise solutions as follows:

$$MAE = \frac{1}{r} \sum_{i=1}^{r} | Y_i - \hat{Y}_i | \quad (47)$$

$$RMSE = \sqrt{\frac{1}{r} \sum_{i=1}^{r} (Y_i - \hat{Y}_i)^2} \quad (48)$$

R 3.6.1 and MATLAB R2021b were used in all steps and computations.

Example 1 (Engineering data set)

Engineering data set which consist of three input and two response variables is shown in Table 2 [5]. The input variables are number of wings (X_1), wing height (X_2) and wing thickness (X_3). The response variables are friction coefficient multiplied by the Nusselt number (Y_1) and the Reynolds number (Y_2), wanted to be maximized and minimized, respectively. The experimental data set is composed with Box-Behnken design. It should be noted that input variables are given by coded values in Table 2. The coded levels of the input variables and corresponding real values are presented in [5].

Table 2. Engineering data set

	Input variables			*Response variables*	
No	X_1	X_2	X_3	Y_1	Y_2
1	-1	-1	0	7	15
2	+1	-1	0	4	11
3	-1	+1	0	18	26
4	+1	+1	0	11	18.5
5	-1	0	-1	12.6	19
6	+1	0	-1	8	14.8
7	-1	0	+1	12.6	19.7
8	+1	0	+1	8.5	15
9	0	-1	-1	6.5	13.7
10	0	1	-1	15.8	22.7
11	0	-1	1	8.45	15.7
12	0	1	1	16.6	23.8
13	0	0	0	12.2	19
14	0	0	0	12.4	18.4
15	0	0	0	12.8	18.9
16	0	0	0	12.3	18.1
17	0	0	0	12.5	18.6

The normality assumption for the responses was checked. It was seen that the responses were normally distributed (p-value of Y_1= 0.52 >0.05, p-value of Y_2= 0.57>0.05). Pearson correlation coefficient between the responses is calculated as 0.98 which is statistically significant. Due to relationship between the responses, predicted response models were obtained by SUR method. In order to obtain predicted response model with the SUR, variance-covariance matrix estimation is calculated from residuals of OLS response estimations. Predicted response models are written as

$$\hat{Y}_1 = 12.440 - 2.338X_1 + 4.431X_2 + 0.406X_3 - 1.926X_1^2 - 0.514X_2^2 - 1.000X_1X_2,\ R^2 = 0.9931 \quad (49)$$

$$\hat{Y}_2 = 18.600 - 2.550X_1 + 4.450X_2 + 0.500X_3 - 1.413X_1^2 + 0.438X_2^2 - 0.875X_1X_2,\ R^2 = 0.9889 \quad (50)$$

According to ANOVA results belonging to $\hat{Y}_1$ (p-value = 0.0001<0.05) and $\hat{Y}_2$ (p-value = 0.0001<0.05), models obtained by the SUR method were found to be statistically significant. Predicted response models are considered as objective functions, f_1 and f_2. Ideal solution vector is calculated as $\boldsymbol{f}^* = \left[f_1^* \ \ f_2^*\right] = [18.2093\ 9.25]$ by optimizing objective functions f_1 and f_2, individually. The MOO problem can be given as follows:

$$\begin{aligned} &Max\, f_1 = \hat{Y}_1(\boldsymbol{X}) \\ &Min\, f_2 = \hat{Y}_2(\boldsymbol{X}) \\ &\mathbf{-1} \le \boldsymbol{X} \le \mathbf{1} \end{aligned} \quad (51)$$

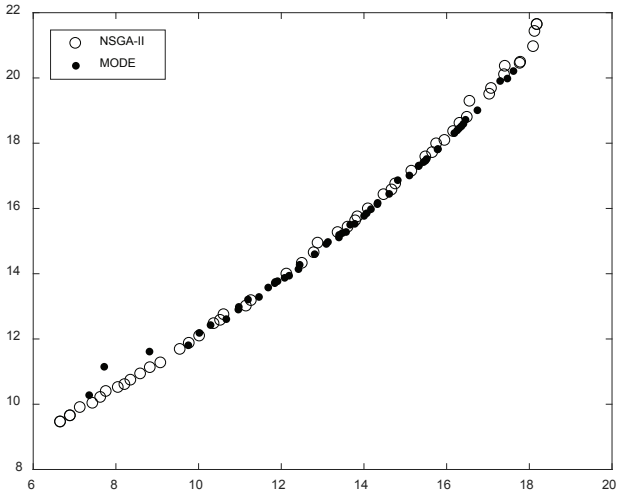

Figure 3. Pareto solutions obtained with the NSGA-II and the MODE for engineering data set.

The MOO is achieved through the NSGA-II and the MODE. The tuning parameters of the NSGA-II are n_{pop} = 100, $maxn_{gen}$ = 600 and the tuning parameters of the MODE are n_{pop} = 75, $maxn_{gen}$ = 50, CR = 0.8, F = 0.6. Pareto solutions are obtained by the NSGA-II and the MODE as shown in Figure 3. Size of Pareto solution set is 53 which is number of Rank-1 non-dominated solutions.

Entropy method was applied to obtain weights of the response variables Y_1 and Y_2 through experimental response values given in Table 2. The weight vector is calculated as $\boldsymbol{w} = [w_1\ w_2] = [0.7312\ 0.2688]$. Each MCDM method was applied to Pareto solutions to determine compromise solutions. The closeness of the compromise solution to the

Table 3. Compromise solutions of NSGA-II and MODE with metrics for engineering data set

	MCDM	*X*			*f*		*Metrics*	
		x_1	x_2	x_3	f_1	f_2	*MAE*	*RMSE*
	TOPSIS	0.982	0.941	0.652	17.781	20.443	5.8106	5.6006
	COPRAS	0.962	0.991	0.881	18.105	20.955	5.9047	5.8527
	WASPAS	0.962	0.991	0.881	18.105	20.955	5.9047	5.8527
NSGA-II	EDAS	0.962	0.991	0.881	18.105	20.955	5.9047	5.8527
	CODAS	0.824	1	0.970	18.194	21.632	6.1985	6.1910
	GREY	0.824	1	0.970	18.194	21.632	6.1985	6.1910
	MABAC	0.962	0.991	0.881	18.105	20.955	5.9047	5.8527
	TOPSIS	**1**	**0.931**	**0.015**	**17.469**	**19.982**	**5.7361**	**5.3788**
	COPRAS	1	1	-0.378	17.616	20.211	5.7772	5.4885
	WASPAS	1	1	-0.378	17.616	20.211	5.7772	5.4885
MODE	EDAS	1	1	-0.378	17.616	20.211	5.7772	5.4885
	CODAS	1	1	-0.378	17.616	20.211	5.7772	5.4885
	GREY	1	1	-0.378	17.616	20.211	5.7772	5.4885
	MABAC	1	1	-0.378	17.616	20.211	5.7772	5.4885

Table 4. Chemistry data set

	Input variables			*Response variables*	
No	X_1	X_2	X_3	Y_1	Y_2
1	+1	+1	0	0.206	2.122
2	0	-1	+1	0.107	1.888
3	-1	0	-1	0.138	1.519
4	0	-1	-1	0.159	1.750
5	-1	+1	0	0.123	1.468
6	0	+1	+1	0.196	2.473
7	+1	-1	0	0.185	1.612
8	+1	0	-1	0.167	1.956
9	0	0	0	0.244	2.113
10	0	0	0	0.238	1.987
11	-1	-1	0	0.058	1.348
12	-1	0	+1	0.014	1.485
13	+1	0	+1	0.098	2.286
14	0	+1	-1	0.251	1.992
15	0	0	0	0.226	2.031

ideal solution is calculated by the *MAE* and *RMSE* metrics as given in Table 3. It was seen that compromise solution with the smallest *MAE* and *RMSE* values belongs to MODE. This compromise solution was chosen by TOPSIS method. These results show that the MODE can find a better compromise solution closer to ideal solution than NSGA-II.

Example 2 (Chemistry data set)

Chemistry data set which consist of three input and two response variables is shown in Table 4 [12]. The input variables are temperature (X_1), pressure (X_2) and co-solvent (X_3). The response variables are extraction efficiency of carotenoids (Y_1) and the extraction efficiency of chlorophylls (Y_2) which are wanted to be maximized. The experimental data set is composed with the Box-Behnken design. Coded values and real values for chemistry data set are shown in [13].

The normality assumption for the responses was checked. It was seen that the responses were normally distributed (p-value of Y_1= 0.55 > 0.05, p-value of Y_2= 0.67> 0.05). Pearson correlation coefficient between the response variables is determined as 0.56 and it is found to be statistically significant. Due to relationship between the responses, predicted response models were used by SUR method. In order to achieve predicted response model with the SUR, variance-covariance matrix estimation is calculated from the residuals of the OLS response estimations.

The predicted response models can be written as

$$\hat{Y}_1 = 0.236 + 0.040X_1 + 0.033X_2 - 0.038X_3 - 0.084X_1^2 - 0.048X_3^2,\ R^2 = 0.9237 \quad (52)$$

$$\hat{Y}_2 = 2.044 + 0.270X_1 + 0.182X_2 + 0.114X_3 - 0.310X_1^2 - 0.096X_2^2 + 0.078X_3^2 + 0.098X_1X_2 + 0.091X_1X_3 + 0.086X_2X_3,\ R^2 = 0.9742 \quad (53)$$

According to ANOVA results belonging to $\hat{Y}_1$ (p-value = 0.009 < 0.05) and $\hat{Y}_2$ (p-value = 0.002 < 0.05), models obtained by the SUR method were found to be statistically significant. Ideal solution vector is calculated as $\boldsymbol{f}^* = \left[f_1^* \ \ f_2^*\right] = [0.3987\ \ 2.7699]$. MOO problem is presented in Equation (54) as follows:

$$\begin{aligned} &max\, f_1 = \hat{Y}_1\,(\boldsymbol{X}) \\ &max\, f_2 = \hat{Y}_2\,(\boldsymbol{X}) \\ &\mathbf{-1} \le \boldsymbol{X} \le \mathbf{1} \end{aligned} \quad (54)$$

The MOO is achieved through the NSGA-II and the MODE. The tuning parameters of the NSGA-II are $n_{pop} = 100$, $maxn_{gen} = 600$ and the tuning parameters of the MODE are $n_{pop} = 75$, $maxn_{gen} = 50$, $CR = 0.8$, $F = 0.6$. Pareto solutions obtained with the NSGA-II and MODE for chemistry data set are illustrated in Figure 4.

Entropy method was applied to obtain weights of the response variables Y_1 and Y_2 through experimental response values given in Table 4. The weight vector is calculated as $\boldsymbol{w} = [w_1\ \ w_2] = [0.8840\ \ 0.1160]$. Each MCDM method was applied to Pareto solutions to determine compromise solutions. The closeness of the compromise solution to the ideal solution is calculated by the *MAE* and *RMSE* metrics as given in Table 5. It was seen that compromise solution with the smallest *MAE* and *RMSE* values belongs to the MODE. This compromise solution was chosen by COPRAS method. These results show that the MODE can find a better compromise solution closer to ideal solution than the NSGA-II.

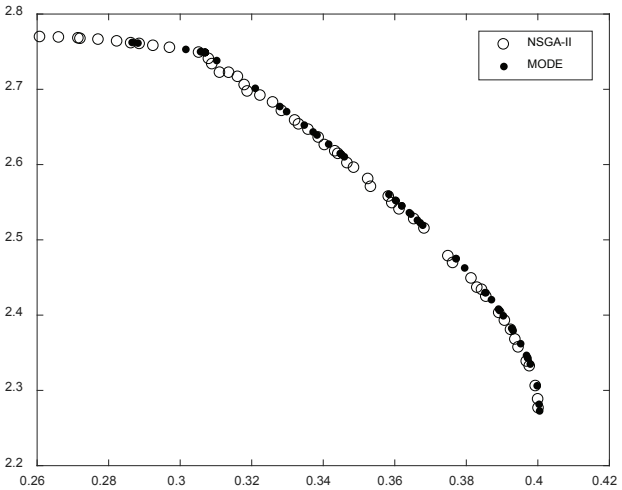

Figure 4. Pareto solutions obtained with the NSGA-II and MODE for chemistry data set.

Table 5. Compromise solutions of NSGA-II and MODE with metrics for chemistry data set

	MCDM	*X*			*f*		*Metrics*	
		x_1	x_2	x_3	f_1	f_2	*MAE*	*RMSE*
NSGA-II	TOPSIS	0.998	0.999	-0.173	0.398	2.332	0.2193	0.2190
	COPRAS	0.999	0.999	-0.399	0.400	2.276	0.2476	0.2470
	WASPAS	0.999	0.999	-0.348	0.400	2.288	0.2416	0.2410
	EDAS	0.999	0.999	-0.348	0.400	2.288	0.2416	0.2410
	CODAS	0.999	0.999	-0.348	0.400	2.288	0.2416	0.2410
	GREY	0.999	0.999	-0.348	0.400	2.288	0.2416	0.2410
	MABAC	0.999	0.999	-0.348	0.400	2.288	0.2416	0.2410
MODE	TOPSIS	1	1	-0.274	0.4000	2.306	0.2326	0.2320
	COPRAS	**0.897**	**1**	**1**	**0.2870**	**2.762**	**0.0598**	**0.0560**
	WASPAS	1	1	-0.415	0.4010	2.273	0.2496	0.2485
	EDAS	1	1	-0.415	0.4010	2.273	0.2496	0.2485
	CODAS	1	1	-0.415	0.4010	2.273	0.2496	0.2485
	GREY	1	1	-0.415	0.4010	2.273	0.2496	0.2485
	MABAC	1	1	-0.274	0.4010	2.306	0.2331	0.2320

Example 3 (Food data set)

Food data set which consist of three input variables and two response variables for 17 observations is shown in Table 6 [58]. The input variables are temperature (X_1), washing time (X_2) and washing rate (X_3). The response variables are thiobarbituric acid number (Y_1) and percent cooking loss (Y_2) which are wanted to be maximized. The experimental data set is composed with Central Composite Design (CCD). Real values and coded values of input variables are shown in [58].

Table 6. Food data set

	Input variables			*Response variables*	
No	X_1	X_2	X_3	Y_1	Y_2
1	-1	-1	-1	29.31	29.50
2	+1	-1	-1	39.32	19.40
3	-1	+1	-1	25.16	25.70
4	+1	+1	-1	40.81	27.10
5	-1	-1	+1	29.82	21.40
6	+1	-1	+1	32.20	24.00
7	-1	+1	+1	22.01	19.60
8	+1	+1	+1	40.02	25.10
9	-1.682	0	0	33.00	24.20
10	+1.682	0	0	51.59	30.60
11	0	-1.682	0	20.35	20.90
12	0	+1.682	0	20.53	18.90
13	0	0	-1.682	23.85	23.00
14	0	0	+1.682	20.16	21.20
15	0	0	0	21.72	18.50
16	0	0	0	21.21	18.60
17	0	0	0	21.55	16.80

The normality assumption for the responses was checked. It was seen that one of the responses was normally distributed (p-value of Y_1= 0.01 < 0.05, p-value of Y_2= 0.41> 0.05). However, it was assumed that the Y_1 has normally distributed. Then, Pearson correlation between the response variables is determined as 0.68 and it is found to be statistically significant. The predicted response models obtained by the SUR method can be written as

$$\hat{Y}_1 = 21.257 + 5.661X_1 - 1.227X_3 + 8.1685X_1^2 + 2.6587X_1X_2,\ R^2 = 0.9302 \quad (55)$$

$$\hat{Y}_2 = 17.933 - 1.071X_3 + 3.452X_1^2 + 1.579X_3^2 + 1.800X_1X_2 + 2.100X_1X_3,\ R^2 = 0.7799 \quad (56)$$

According to ANOVA results for $\hat{Y}_1$ (p-value = 0.001< 0.05) and $\hat{Y}_2$ (p-value = 0.001< 0.05), models obtained by the SUR method were found to be statistically significant. Ideal solution vector is calculated as $\boldsymbol{f}^* = \left[f_1^* \quad f_2^* \right] = [16.05\ 16.48]$. MOO problem is presented in Equation (57) as follows:

$$\begin{aligned} &min\, f_1 = \hat{Y}_1(\boldsymbol{X}) \\ &min\, f_2 = \hat{Y}_2(\boldsymbol{X}) \\ &\textbf{-1.682} \le \boldsymbol{X} \le \textbf{1.682} \end{aligned} \quad (57)$$

The MOO is achieved through the NSGA-II and the MODE. The tuning parameters of the NSGA-II are $n_{pop} = 100$, $maxn_{gen} = 600$ and the tuning parameters of the MODE are $n_{pop} = 75$, $maxn_{gen} = 50$, $CR = 0.8$, $F = 0.6$. Pareto

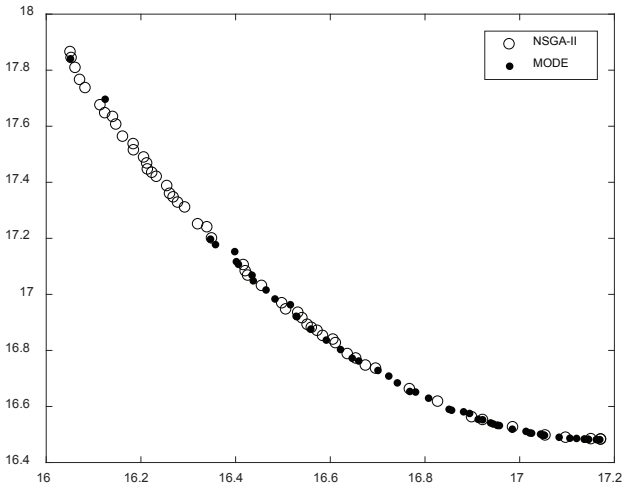

Figure 5. Pareto solutions obtained with the NSGA-II and MODE for food data set.

results show that the MODE can find a better compromise solution closer to ideal solution than the NSGA-II.

CONCLUSION

In this study, analysis of multi-response experimental data was achieved in three basic stages, modeling, optimization and decision making. The SUR method was used for modeling of the responses since the responses were linearly correlated. The predicted response models were considered as objective functions. Ideal solution vector is provided by optimizing objective functions individually. Multi-response problem was handled as a MOO problem considering the objective functions simultaneously. The NSGA-II and the MODE, two commonly used MOM methods, were applied to obtain Pareto solution sets which were the solution of the MOO problem.

Table 7. Compromise solutions for NSGA-II and MODE with metrics for food data set

	MCDM	*X*			*f*		*Metrics*	
		x_1	x_2	x_3	f_1	f_2	*MAE*	*RMSE*
	TOPSIS	-0.653	1.681	1.599	16.162	17.562	0.5970	0.5439
	COPRAS	-0.647	1.682	1.681	16.072	17.765	0.6535	0.6426
	WASPAS	-0.647	1.682	1.681	16.072	17.765	0.6535	0.6426
NSGA-II	EDAS	-0.624	1.682	1.680	16.054	17.843	0.6835	0.6815
	CODAS	-0.617	1.682	1.681	16.052	17.864	0.6930	0.6920
	GREY	-0.617	1.682	1.681	16.052	17.864	0.6930	0.6920
	MABAC	-0.647	1.682	1.681	16.072	17.765	0.6535	0.6426
	TOPSIS	-0.616	1.682	1.622	16.1240	17.6960	0.6450	0.6091
	COPRAS	-0.628	1.682	1.682	16.0510	17.8410	0.6810	0.6805
	WASPAS	-0.628	1.682	1.682	16.0510	17.8410	0.6810	0.6805
MODE	**EDAS**	**-0.667**	**1.682**	**1.381**	**16.4370**	**17.0480**	**0.4775**	**0.3437**
	CODAS	-0.628	1.682	1.682	16.0510	17.8410	0.6810	0.6805
	GREY	-0.628	1.682	1.682	16.0510	17.8410	0.6810	0.6805
	MABAC	-0.628	1.682	1.682	16.0510	17.8410	0.6810	0.6805

solutions obtained with the NSGA-II and the MODE for chemistry data set are illustrated in Figure 5.

Entropy method was applied to obtain weights of the response variables Y_1 and Y_2 through experimental response values given in Table 4. The weight vector is calculated as $\boldsymbol{w} = [w_1 \ w_2] = [0.7579 \ 0.2421]$. Each MCDM method was applied to Pareto solutions to determine compromise solutions. The closeness of the compromise solution to the ideal solution is calculated by *MAE* and *RMSE* metrics as given in Table 7. It was seen that compromise solution with the smallest *MAE* and *RMSE* values belongs to MODE. This compromise solution was chosen by EDAS method. These

In multi response studies each response may have different importance. Thus, in this study, Entropy method was applied to obtain weights of the response variables objectively over experimental results for each dataset. Each MCDM method, considering weights, was applied to the Pareto solutions to determine a compromise solution. The main aim of this study is to compare the performance of the NSGA-II and MODE. For this purpose, firstly, the MCDM methods applied to the Pareto sets to define a compromise solution. Secondly, closeness of the compromise solution and the ideal solution vectors were calculated with respect to *MAE* and *RMSE* metrics. Three data sets related to engineering, chemistry and food fields were considered for

application. Calculation results show that the compromise solution of the MODE is closer to the ideal solution than the compromise solution of the NSGA-II. The MODE has the smallest *MAE* and *RMSE* values according to the at least one of the MCDM methods, for instance TOPSIS for engineering data set, COPRAS for chemistry data set, EDAS for food data set. Accordingly, the MODE performs better than NSGA-II to find Pareto solution set that contains a compromise solution closer to the ideal solution.

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REFERENCES

[1] Zellner A. An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias. J Am Stat Assoc 1962;57:348–368. [CrossRef]

[2] Liu A. Efficient estimation of two seemingly unrelated regression equations. J Multivar Anal 2002;82:445–456. [CrossRef]

[3] Bera S, Barman G, Mukherjee I. Effect of seemingly unrelated regression-based modeling approach on solution quality for correlated multiple response optimization problems. Proceedings of the 2011 IEEE IEEM. 2011. p. 1490–1494. [CrossRef]

[4] Costa N, Lourenço J, Pereira ZL. Responses modeling and optimization criteria impact on the optimization of multiple quaility characteristics. Comput Ind Eng 2012;62:927–935. [CrossRef]

[5] Rout SK, Hussein AK, Mohanti CP. Multi-objective optimization of a three-dimensional internally finned tube based on response surface methodology(RSM). Therm Eng 2015;1:131–145. [CrossRef]

[6] Shah HK, Montgomery DC, Carlyle WM. Response surface modeling and optimization in multiresponse experiments using seemingly unrelated regression. Qual Eng 2004;16:387–397. [CrossRef]

[7] Akçay H, Anagün AS. Multiresponse optimization application on a manufacturing factory. Math Comput Appl 2013;18:531–538. [CrossRef]

[8] Shivakoti I, Kalita K, Kibria G, Sharma A, Pradhan BB, Ghadai RK. Parametric analysis and multi response optimization of laser surface texturing of titanium super alloy. J Braz Soc Mech Sci Eng 2021;43:400. [CrossRef]

[9] Tunçel S. Çok yanıtlı deneysel verilerin görünüşte ilişkisiz regresyon analizi ile modellenmesi ve optimal değişken değerlerinin belirlenmesi (Yüksek lisans tezi). Ankara: Ankara Üniversitesi; 2022.

[10] Türkşen Ö. Optimization and decision making stages for multiple responses: an application of NSGA-II and FCM clustering algorithm. Gazi Univ J Sci 2015;28:321–330.

[11] Türkşen Ö, Vural N. Analyses of replicated spectophotometric data by using soft computing methods. J Iran Chem Soc 2020;17:2651–2661. [CrossRef]

[12] Yıldırım A, Güneş F, Belen MA. Differential evolution optimization applied to the performance analysis of a microwave transistor. Sigma J Eng Nat Sci 2017;8:135–144.

[13] Lu J, Feng X, Han Y, Xue C. Optimization of subcritical fluid extraction of carotenoids and chlorophyll a from Laminaria japonica aresh by response surface methodology. J Sci Food Agric 2014;94:139–145. [CrossRef]

[14] Deb K. Multi objective optimization using evolutionary algorithms. New York: John-Wiley and Sons; 2001. p. 497.

[15] Bektaş G, Niğdeli SM, Yücel M, Kayabekir AE. Yapay Zeka Optimizasyon Algoritmaları ve Mühendislik Uygulamaları. Ankara: Seçkin Yayıncılık; 2021. [Turkish]

[16] Karaboğa D. Yapay Zeka Optimizasyon Algoritmaları. Ankara: Nobel Yayınları; 2020. [Turkish]

[17] Esfe MH, Hajmohammad H, Moradi R, Arani AAA. Multi-objective optimization of cost and thermal performance of double walled carbon nanotubes/water nanofluids by NSGA-II using response surface method. Appl Therm Eng 2017;112:1648–1657. [CrossRef]

[18] Joshi M, Ghadai RK, Madhu S, Kalita K, Gao X. Comparison of NSGA-II,MAOLA and MODA for multi-objective optimization of micro-machining processes. Materials (Basel) 2021;14:5109. [CrossRef]

[19] Ravichandran S, Kumudinidevi RP, Bharathidasan SG, Jeba E. Cordinated controller design of grid connected DFIG based wind turbine using response surface methodology and NSGA II. Sustain Energy Technol Assess 2014;8:120–130. [CrossRef]

[20] Thirumalai R, Seenivasan M, Panneerselvam K. Experimental investigation and multi response optimization of turning process parameters for inconel 718 using TOPSIS approach. Mater Today Proc 2021;45:467–472. [CrossRef]

[21] Torabi SHR, Alibabaei S, Bonab BBM, Sadeghi H, Faraji G. Design and optimization of turbine blade preform forging using RSM and NSGA II. J Intell Manuf 2017;28:1409–1419. [CrossRef]

[22] Babu B V, Anbarasu B. Multi-objective differential evolution (MODE): an evolutionary algorithm for multi-objective optimization problems (moops). Proceedings of International Symposium and 58th Annual Session of IIChE, 2005.

[23] Souza MN, Silva MA, Machado AR, Lobato FS. Treatment of multi-response surface applied to machinability of stainess steel using multi-objective optimization differential evolution. 21. Brazilian Congress of Mechanical Engineering, 2011.

[24] Basu M.: Economic environmental dispatch using multi-objective differential evolution. Appl Soft Comput 2011;11:2845–2853. [CrossRef]
[25] Gaitonde VN, Manjaiah M, Maradi S, Karnik SR, Petkar PM, Davim JP. Multiresponse optimization in wire electric discharge machining (WEDM) of HCHCR steel by integrating response surface methodology (RSM) with differential evolution(DE). Comput Methods Eng 2017;199–221. [CrossRef]
[26] Monsef H, Naghashzadegen M, Jamali A, Farmani R. Comparison of evolutionary multi objective optimization algorithms in optimum design of water distribution network. Ain Shams Eng 2019;10:103–111. [CrossRef]
[27] Singh GK, Yadava V, Kumar R. Multiresponse optimization of electro-discharge diamond face grinding process using robust design of experiments. Mater Manuf Process 2013;25:851–856. [CrossRef]
[28] Zhao W, Ma A, Ji J, Chen X, Yao T. Multiobjective optimization of a double-side linear vernier PM motor using response surface method and differential evolution. IEEE Trans Ind 2020;67:80–90. [CrossRef]
[29] Reddy VV. Turning process parameters optimization of al7075 hybrid mmc's composite using TOPSIS method. Sigma J Eng Nat Sci 2020;38:2043–2055.
[30] Yalçın N, Uncu N. Applying EDAS as an applicable mcdm method for industrial robot selection. Sigma J Eng Nat Sci 2019;37:779–796.
[31] Bölükbaş U, Güneri AF. A fuzzy multi-criteria decision approach for measuring technology competency performance of smes. Sigma J Eng Nat Sci 2017;8:31–40.
[32] Trung DD. A combination method for multi-criteria decision making problem in turning process. Manuf Rev. 2021;8:1–17. [CrossRef]
[33] Pandiyan GK, Prabaharan T, James DJD, Sivalingam V. Machinability Analysis and Optimization of Electrical Discharge Machining in AA6061-T6/15wt.% SiC Composite by the Multi-criteria Decision-Making Approach. J Mater Eng Perform 2022;31:3741–3752. [CrossRef]
[34] Khan MM, Dey A. Hybrid MCDM approach for examining the high-stress abrasive wear behaviour of in situ ZA-27/TiCp MMCs. Mater Chem Phys 2022;277:125319. [CrossRef]
[35] Patil SB, Patole TA, Jadhav RS, Suryawanshi SS, Raykar SJ. Complex proportional assessment (COPRAS) based multiple-criteria decision making (MCDM) paradigm for hard turning process parameters. Mater Today Proc 2022;59:835–840. [CrossRef]
[36] Rahman Z, Siddiquee AN, Khan AZ, Ahmad S. Multi-response optimization of FSP parameters on mechanical properties of surface composite, Mater Today Proc 2022;62:5–8. [CrossRef]
[37] Somasundaram M, Kumar JP. Multi response optimization of EDM process parameters for biodegradable AZ31 magnesium alloy using TOPSIS and grey relational analysis. Sādhanā 2022;47:136. [CrossRef]
[38] Divya C, Raju LS, Singaravel B. Application of mcdm methods for process parameter optimization in turning process-a review. Recent Trends in Mechanical Engineering. Singapore: Springer; 2021. p. 199–207. [CrossRef]
[39] Bagal DK, Giri A, Pattanaik AK, Jeet S, Barua A, Panda SN. MCDM optimization of characteristics in resistance spot welding for dissimilar meterials utilizing advanced hybrid taguchi method-coupled CoCoSo, EDAS and WASPAS method. Next Generation Meterials and Processing Technologies. Singapore: Springer; 2020. [CrossRef]
[40] Abhilash P, Chakradhar D. Multi-response optimization of wire EDM of inconel 718 using a hybrid entropy weighted GRA-TOPSIS method. Process Integr Optim Sustain 2022;6:61-72. [CrossRef]
[41] Singh A, Prapoorna Biligiri K, Venkatesh Sampath P. Development of framework for ranking pervious concrete pavement mixtures: application of multi-criteria decision-making methods. Int J Pavement Eng 2022;24:2021406. [CrossRef]
[42] Kuo Y, Yang T, Huang GW. The use of grey relational analysis in solving multiple attribute decision-making problems, Comput Ind Eng 2008;55:80–93. [CrossRef]
[43] Deb K, Pratab A, Agarwal S, Meyarivan T. Fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans Evol Comput 2002;6:182–197. [CrossRef]
[44] Storn R, Price K. Differential evolution- a simple and efficient adaptive scheme for global optimization over continuous spaces. J Glob Optim 1997;11:341–359. [CrossRef]
[45] Shannon CE. A mathematical theory of communication. Bell Syst Tech J 1948;27:379–423. [CrossRef]
[46] Zavadskas EK, Podvezko V. Integrated determination of objective criteria weights in MCDM. Int J Inf Technol Decis Mak 2016;15:267–283. [CrossRef]
[47] Gül Ş, Fırat M. Determination of priority regions for rehabilitation in water networks by multiple criteria decision making methods. Sigma J Eng Nat Sci 2020;38:1481–1494.
[48] Hwang CL, Yoon K. Multiple Attribute Decision Making: Methods and Applications. New York: Springer- Verlag; 1981. [CrossRef]
[49] Chen C. Extensions of the TOPSIS for group decision-making under fuzzy environment. Fuzzy Sets Syst. 2000;114:1–9. [CrossRef]
[50] Opricovic S, Tzeng G. Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. Eur J Oper Res 2004;156:445–455. [CrossRef]

[51] Zavadskas EK, Kaklauskas A, Sarka V. The new method of multi-criteria Complex Proportional Assessment (COPRAS) of Projects. Technol Econ Dev Econ 1994;1:131-139.
[52] Chakraborty S, Zavadskas EK, Antucheviciene J. Applications of WASPAS method as a multi-criteria decision-making tool. Econ Comput Econ Cyber Studi Res 2015;49:5–22.
[53] Keshavarz-Ghorabaee M, Zavadskas EK, Olfat L, Turkis Z. Multi-criteria ınventory classification using a new method of evaluation based on distance from average solution (EDAS). Informatica 2015;26:435–451. [CrossRef]
[54] Keshavarz-Ghorabaee M, Zavadskas Turkis Z. Antucheviciene, J. A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making. Econ Comp and Econ Cyb Stud Res 2016;50:25–44.
[55] Deng J. Control problems of grey systems. Syst Control Lett 1982;5:288-294. [CrossRef]
[56] Lin CL. Use of the Taguchi Method and grey relational analysis to optimize turning operations with multiple performance characteristics. Mater Manuf Process 2004;19:209–220. [CrossRef]
[57] Pamučar D, Ćirović G. The selection of transport and handling resources in logistics centers using Multi-Attributive Border Approximation area Comparison (MABAC). Expert Syst Appl 2015;42:3016–3028. [CrossRef]
[58] Khuri AI, Cornell JA. Response Surfaces: Designs and Analyses. New York: Marcel Dekker; 1996. p. 510.