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Research Article

Extension of the Jaccard distance measure for interval-valued intuitionistic fuzzy sets

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ABSTRACT

An interval-valued intuitionistic fuzzy sets (IVIFSs) theory, which is an effective tool for dealing with uncertainty, has attracted the attention of researchers and has been applied to many fields. One of the significant topics in intuitionistic fuzzy sets is the measure of distance. Many distance measures have been developed for IVIFSs in the literature over the last fifteen years. However, not all of these measures can satisfy the axioms of distance. In this study, a Jaccard distance measure and its proofs are extended under the IVIF environment. The proposed Jaccard distance measure is compared with the existing distance measures. The result of the analysis indicates that the proposed Jaccard distance measure overcomes the disadvantages of the current distance measures. Moreover, the extension of the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method based on the proposed Jaccard distance is presented to solve MCDM (Multi Criteria Decision Making) problems in the IVIF environment. Finally, a personnel selection problem is solved to illustrate the proposed MCDM method.

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INTRODUCTION

In real-life problems, crisp numbers can be insufficient to define vagueness and uncertainty properly. Hence, Zadeh [1] developed the Fuzzy Sets theory (FSs) to overcome vagueness and uncertainty. According to the FSs theory, the membership of an element to a fuzzy set is demonstrated by only a single value between zero and one. Nevertheless, in reality, the degree of non-membership of an element in a fuzzy set is not equal to 1 minus the membership degree [2,3]. In other words, there may be some hesitation about the membership of the element to the set. Therefore, Atanassov [4] extended the FSs theory to the IFSs theory, which is a generalization of the concept of FSs. The IFSs theory is characterized by membership degree, hesitation degree and non-membership degree. Atanassov and Gargov [5] then suggested the IVIFSs theory as a further generalization of the IFSs theory. Thereby, those degrees are represented by interval-valued values rather than crisp values in this theory. The IVIFSs theory provides the effective capability to cope with vague information [6].

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Due to their performance to model uncertainty in the absense of information and to handle vagueness in real-life problems, IVIFS theory has been applied in several application areas such as medical diagnosis [7,8], image segmentation [9], edge detection [10,11], decision making [12-15], pattern recognition [16], etc. One of the most interesting topics in IVIFSs theory is the some measures that compare the information carried by two IVIFSs. Many different types of measurements have been developed in the literature such as distance [6, 17-24], similarity [20, 25-29], entropy [6, 24, 25, 30-33], cross-entropy [34, 35], correlation [15, 36, 37], and divergence indices [38, 39]. Especially, distance measures between IFSs have received great attention in the last decade as they are valuable tools in several fields including decision-making, medical diagnosis, pattern recognition, and data mining.

MCDM is an approach designed for the evaluation of problems that require an efficient distance measure to make an approximate decision in an IVIF environment. Although several distance measures have been developed in the literature, not all these distance measures are effective in dealing with every MCDM problem. That is why different distance measures are needed depending on the nature of the decision-making problems. As far as the authors know, the distance measure for IVIFSs was first proposed by Xu [17]. Xu [17] defined the Euclidean distance, Hamming distance, and their versions combined with the Hausdorff metric. Then, these measures were applied to pattern recognition. Park, et al. [18] modified the distance measures defined by Xu [17] considering the amplitude margin. Xu and Yager [20] introduced a distance measure based on the Hamming distance containing the indeterminacy degree for IVIFSs. Zhao et al. [40] extended the Euclidean distance measure with the indeterminacy degree. Zhou et al. [41] developed a distance measure based on the convex combination of the difference between membership and non-membership degrees. Düğenci [19] developed a generalized distance measure used to calculate the separation measures in the IVIF extension of the TOPSIS method. Baccour and Alimi [21] and Fares, et al. [22] introduced new distance measures which were illustrated from the pattern recognition point of view. The hesitant degree was not taken into account in the above-mentioned distance measures. However, the hesitant degree plays an important role when the membership and non-membership degrees are not very different for the two IVIFSs. Thus, some authors considered the hesitant degree when developing the distance degree. Tiwari and Gupta [24] extended distance measures by considering the hesitant degree for IVIFSs. Anusha and Sireesha [23] offered Jaccard distance for IVIF sets. Here mod defined the Euclidean distance from the set to the origin. Ohlan [42], presented a distance measure under an IVIF environment using an exponential function. Gohain et al. [43] developed a distance measure that accounts for the optimistic viewpoint of the information contained in the IVIFSs and the cross-time information via the difference between the maximum and minimum cross-information factors.

The aforementioned articles have focused on popular distance measurements such as Euclidean, Manhattan, and Hamming. The Jaccard index is one of the most useful similarity measure. However, the investigation of the Jaccard distance similarity measurement of IVIF sets is limited. As far as it is known, the only study that developed the Jaccard distance for the IVIF set was done by Anusha and Sireesha [23]. But, it is seen that this measure produces unreasonable distance values for some IVIFSs. Motivated by the rising importance of decision-making methods under the IVIF environment, to fill in this gap in the literature, this study develops the Jaccard distance measures of IVIFSs by using membership and non-membership intervals of IVIFS. Then, this measure is compared to well-known distance measures for IVIFSs. The results obtained show that the proposed distance measure produces reasonable results.

PRELIMINARIES

In this section, the basic concepts of IVIFSs are introduced. Additionally, the properties of the distance and similarity measures are listed.

Definition 2.1 (IVIFSs [5]) Let X be a universe set and $E = \{x_1, x_2, ..., x_n\}$ be a subset of its elements, then an IVIFS A having the form as below with the conditions $0 \le \mu_A^U(x_i) + v_A^U(x_i) \le 1$:

$$A = \left\{ \left\langle x_i, \left(\left[\mu_A^L(x_i), \mu_A^U(x_i) \right], \left[v_A^L(x_i), v_A^U(x_i) \right] \right) \right\rangle, x_i \in E \right\}. \quad (1)$$

Definition 2.2 (Hesitancy degree [5]) For each $x_i \in E$, the hesitancy degree of any IVIFS A is described as:

$$\pi_A(x_i) = \left[\left(1 - \mu_A^U(x_i) - v_A^U(x_i)\right), \left(1 - \mu_A^L(x_i) - v_A^L(x_i)\right) \right]. \quad (2)$$

Definition 2.3 (Set operations and relations on IVIFSs [44]) For any two IVIFSs A and B:

$$A + B = \begin{pmatrix} \left[\mu_A^L(x_i) + \mu_B^L(x_i) - \mu_A^L(x_i)\mu_B^L(x_i), \mu_A^U(x_i) + \mu_B^U(x_i) - \mu_A^U(x_i)\mu_B^U(x_i) \right], \\ \left[v_A^L(x_i)v_B^L(x_i), v_A^U(x_i)v_B^U(x_i) \right] \end{pmatrix}, \quad (3)$$

$$AB = \begin{pmatrix} \left[\mu_A^L(x_i)\mu_B^L(x_i), \mu_A^U(x_i)\mu_B^U(x_i) \right], \\ \left[v_A^L(x_i) + v_B^L(x_i) - v_A^L(x_i)v_B^L(x_i), v_A^U(x_i) + v_B^U(x_i) - v_A^U(x_i)v_B^U(x_i) \right] \end{pmatrix}, \quad (4)$$

$$A \cap B = \left\{ \begin{pmatrix} x_i, \left[\min\left(\mu_A^L(x_i), \mu_B^L(x_i)\right), \min\left(\mu_A^U(x_i), \mu_B^U(x_i)\right) \right], \\ \left[\max\left(v_A^L(x_i), v_B^L(x_i)\right), \max\left(v_A^U(x_i), v_B^U(x_i)\right) \right] \end{pmatrix} \right\}, \quad (5)$$

$$A \cup B = \left\{ \left(x_i, \left[\max\left(\mu_A^L(x_i), \mu_B^L(x_i)\right), \max\left(\mu_A^U(x_i), \mu_B^U(x_i)\right) \right], \left[\min\left(v_A^L(x_i), v_B^L(x_i)\right), \min\left(v_A^U(x_i), v_B^U(x_i)\right) \right] \right) \right\}, \quad (6)$$

$A \subseteq B$, iff $\left[\mu_A^L(x_i), \mu_A^U(x_i)\right] \le \left[\mu_B^L(x_i), \mu_B^U(x_i)\right]$ and $\left[v_A^L(x_i), v_A^U(x_i)\right] \ge \left[v_B^L(x_i), v_B^U(x_i)\right]$,

$A = B$, iff $\left[\mu_A^L(x_i), \mu_A^U(x_i)\right] = \left[\mu_B^L(x_i), \mu_B^U(x_i)\right]$ and $\left[v_A^L(x_i), v_A^U(x_i)\right] = \left[v_B^L(x_i), v_B^U(x_i)\right]$.

Definition 2.4 (Score function [45]) Let $A = \left(\left[\mu_A^L(x_i), \mu_A^U(x_i)\right], \left[v_A^L(x_i), v_A^U(x_i)\right]\right)$ be an IVIFN, then the score function of S_{KE} where $S_{KE}(A) \in [-1,1]$ is

$$S_{KE}(A) = \frac{(\mu_A^U(x_i) - v_A^U(x_i)) + ((\mu_A^L(x_i) + v_A^L(x_i)) \times ((\mu_A^L(x_i) - v_A^U(x_i))}{2}. \quad (7)$$

Definition 2.5 (Distance measure [17]) A real function $d: IVIF(X) \times IXIF(X) \to [0,1]$ is named as the distance measure of IVIF sets on universe X, if it satisfies the following axioms for $A,B,C \in IVIF(X)$:

(A1) $0 \le d(A,B) \le 1$, if A is a crisp set

(A2) $d(A,B) = 0$, iff $A = B$

(A3) $d(A,B) = d(B,A)$,

(A4) $d(A,C) \ge d(A,B)$ and $d(A,C) \ge d(B,C)$, if $A \subseteq B \subseteq C$.

Definition 2.6 (Similarity measure [17]) A real function $s: IVIF(X) \times IXIF(X) \to [0,1]$ is named as the similarity measure of IVIF sets on universe X, if it satisfies the following axioms for $A,B,C \in IVIF(X)$:

(A1) $0 \le s(A,B) \le 1$, if A is a crisp set

(A2) $s(A,B) = 1$, iff $A = B$

(A3) $s(A,B) = s(B,A)$.

(A4) $s(A,C) \le s(A,B)$ and $s(A,C) \le s(B,C)$, if $A \subseteq B \subseteq C$.

A Novel Jaccard Distance Measure Between IVIF Sets

In this section, a new distance measure of IVIFSs is constructed based on the Jaccard index. This index, which is also known as the Jaccard similarity coefficient, is used to compare the similarity of sets. Assuming that A and B are two sample sets, the Jaccard index $J(A, B)$ is described as the size of the intersection between A and B divided by the size of the union between A and B [46, 47]. The Jaccard distance measure is complementary to the Jaccard coefficient and it measures dissimilarity between sample sets. The Jaccard distance measure [47] $d_J(A, B)$ is presented in Equation 8 [48]:

$$d_J(A,B) = \frac{\sum_{i=1}^{n} (a_i - b_i)^2}{\sum_{i=1}^{n} (a_i)^2 + \sum_{i=1}^{n} (b_i)^2 + \sum_{i=1}^{n} (a_i b_i)}. \quad (8)$$

Let $A = \left(\left[\mu_A^L(x_i), \mu_A^U(x_i)\right], \left[v_A^L(x_i), v_A^U(x_i)\right]\right)$ and $B = \left(\left[\mu_B^L(x_i), \mu_B^U(x_i)\right], \left[v_B^L(x_i), v_B^U(x_i)\right]\right)$ be two IVIFSs. The novel Jaccard distance measure extended for IVIFSs is introduced in Equation 9. The hesitant degree is also taken into account in this distance measure:

$$d_{MK_{JD}}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \left[\frac{\left(\mu_A^L(x_i) - \mu_B^L(x_i)\right)^2 + \left(\mu_A^U(x_i) - \mu_B^U(x_i)\right)^2 + \left(v_A^L(x_i) - v_B^L(x_i)\right)^2 + \left(v_A^U(x_i) - v_B^U(x_i)\right)^2 + \left(\pi_A^L(x_i) - \pi_B^L(x_i)\right)^2 + \left(\pi_A^U(x_i) - \pi_B^U(x_i)\right)^2}{\left(\begin{array}{l} \mu_A^L(x_i)^2 + \mu_A^U(x_i)^2 + v_A^L(x_i)^2 + v_A^U(x_i)^2 + \pi_A^L(x_i)^2 + \pi_A^U(x_i)^2 + \\ \mu_B^L(x_i)^2 + \mu_B^U(x_i)^2 + v_B^L(x_i)^2 + v_B^U(x_i)^2 + \pi_B^L(x_i)^2 + \pi_B^U(x_i)^2 \end{array} \right) - \left(\begin{array}{l} \mu_A^L(x_i)\mu_B^L(x_i) + \mu_A^U(x_i)\mu_B^U(x_i) + v_A^L(x_i)v_B^L(x_i) + \\ v_A^U(x_i)v_B^U(x_i) + \pi_A^L(x_i)\pi_B^L(x_i) + \pi_A^U(x_i)\pi_B^U(x_i) \end{array} \right)} \right]. \quad (9)$$

Theorem 1. $d_{MK_{JD}}(A, B)$ is the Jaccard distance between two IVIFSs A and B in X.

Proof (A1). Let A and B be two IVIFSs.

Since $0 \le \mu_A^L(x_i) \le \mu_A^U(x_i) \le 1$, $0 \le v_A^L(x_i) \le v_A^U(x_i) \le 1$, $0 \le \mu_B^L(x_i) \le \mu_B^U(x_i) \le 1$, $0 \le \pi_A^L(x_i) \le \pi_A^U(x_i) \le 1$, $0 \le \pi_B^L(x_i) \le \pi_B^U(x_i) \le 1$, $\pi_A^L(x_i) = 1 - \mu_A^U(x_i) - v_A^U(x_i)$, $\pi_A^U(x_i) = 1 - \mu_A^L(x_i) - v_A^L(x_i)$, $\pi_B^L(x_i) = 1 - \mu_B^U(x_i) - v_B^U(x_i)$, and $\pi_B^U(x_i) = 1 - \mu_B^L(x_i) - v_B^L(x_i)$ the following inequalities are obtained:

$$0 \le \left(\begin{array}{l} \left(\mu_A^L(x_i) - \mu_B^L(x_i)\right)^2 + \left(\mu_A^U(x_i) - \mu_B^U(x_i)\right)^2 + \left(v_A^L(x_i) - v_B^L(x_i)\right)^2 + \\ \left(v_A^U(x_i) - v_B^U(x_i)\right)^2 + \left(\pi_A^L(x_i) - \pi_B^L(x_i)\right)^2 + \left(\pi_A^U(x_i) - \pi_B^U(x_i)\right)^2 \end{array} \right) \le 2,$$

$$0 \le \left(\begin{array}{l} \mu_A^L(x_i)^2 + \mu_A^U(x_i)^2 + v_A^L(x_i)^2 + v_A^U(x_i)^2 + \pi_A^L(x_i)^2 + \pi_A^U(x_i)^2 + \\ \mu_B^L(x_i)^2 + \mu_B^U(x_i)^2 + v_B^L(x_i)^2 + v_B^U(x_i)^2 + \pi_B^L(x_i)^2 + \pi_B^U(x_i)^2 \end{array} \right) \le 4,$$

$$0 \le \left(\begin{array}{l} \mu_A^L(x_i)\mu_B^L(x_i) + \mu_A^U(x_i)\mu_B^U(x_i) + v_A^L(x_i)v_B^L(x_i) + \\ v_A^U(x_i)v_B^U(x_i) + \pi_A^L(x_i)\pi_B^L(x_i) + \pi_A^U(x_i)\pi_B^U(x_i) \end{array} \right) \le 2.$$

Finally, the following inequality is achieved. Thus, $0 \le d_{MK_{JD}}(A, B) \le 1$.

$$0 \le \frac{\left(\mu_A^L(x_i) - \mu_B^L(x_i)\right)^2 + \left(\mu_A^U(x_i) - \mu_B^U(x_i)\right)^2 + \left(v_A^L(x_i) - v_B^L(x_i)\right)^2 + \left(v_A^U(x_i) - v_B^U(x_i)\right)^2 + \left(\pi_A^L(x_i) - \pi_B^L(x_i)\right)^2 + \left(\pi_A^U(x_i) - \pi_B^U(x_i)\right)^2}{\left(\begin{array}{l} \mu_A^L(x_i)^2 + \mu_A^U(x_i)^2 + v_A^L(x_i)^2 + v_A^U(x_i)^2 + \pi_A^L(x_i)^2 + \pi_A^U(x_i)^2 + \\ \mu_B^L(x_i)^2 + \mu_B^U(x_i)^2 + v_B^L(x_i)^2 + v_B^U(x_i)^2 + \pi_B^L(x_i)^2 + \pi_B^U(x_i)^2 \end{array} \right) - \left(\begin{array}{l} \mu_A^L(x_i)\mu_B^L(x_i) + \mu_A^U(x_i)\mu_B^U(x_i) + v_A^L(x_i)v_B^L(x_i) + \\ v_A^U(x_i)v_B^U(x_i) + \pi_A^L(x_i)\pi_B^L(x_i) + \pi_A^U(x_i)\pi_B^U(x_i) \end{array} \right)} \le 1.$$

Proof (A2). Let A and B two IVIFSs, if $A = B$ then $\mu_A^L(x_i) = \mu_B^L(x_i)$, $\mu_A^U(x_i) = \mu_B^U(x_i)$, $v_A^L(x_i) = v_B^L(x_i)$, and $v_A^U(x_i) = v_B^U(x_i)$. For this reason, the Jaccard distance $d_{MK_{JD}}(A, B)$ is equal to zero:

$$d_{MK_{JD}}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\begin{array}{c}\left(\mu_A^L(x_i)-\mu_A^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_A^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_A^L(x_i)\right)^2+\left(v_A^U(x_i)-v_A^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_A^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_A^U(x_i)\right)^2\end{array}}{\begin{array}{l}\left(\begin{array}{l}\mu_A^L(x_i)^2+\mu_A^U(x_i)^2+v_A^L(x_i)^2+v_A^U(x_i)^2+\pi_A^L(x_i)^2+\pi_A^U(x_i)^2+\\ \mu_A^L(x_i)^2+\mu_A^U(x_i)^2+v_A^L(x_i)^2+v_A^U(x_i)^2+\pi_A^L(x_i)^2+\pi_A^U(x_i)^2\end{array}\right)\\ -\left(\begin{array}{l}\mu_A^L(x_i)\mu_A^L(x_i)+\mu_A^U(x_i)\mu_A^U(x_i)+v_A^L(x_i)v_A^L(x_i)+\\ v_A^U(x_i)v_A^U(x_i)+\pi_A^L(x_i)\pi_A^L(x_i)+\pi_A^U(x_i)\pi_A^U(x_i)\end{array}\right)\end{array}}\right]=0\cdot \tag{10}$$

Proof (A3). Let A and B be two IVIFSs, The following equations can be written as:

$$\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2=\left(\mu_B^L(x_i)-\mu_A^L(x_i)\right)^2, \tag{11}$$

$$\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2=\left(\mu_B^U(x_i)-\mu_A^U(x_i)\right)^2, \tag{12}$$

$$\left(v_A^L(x_i)-v_B^L(x_i)\right)^2=\left(v_B^L(x_i)-v_A^L(x_i)\right)^2, \tag{13}$$

$$\left(v_A^U(x_i)-v_B^U(x_i)\right)^2=\left(v_B^U(x_i)-v_A^U(x_i)\right)^2, \tag{14}$$

$$\left(\pi_A^L(x_i)-\pi_B^L(x_i)\right)^2=\left(\pi_B^L(x_i)-\pi_A^L(x_i)\right)^2, \tag{15}$$

$$\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)^2=\left(\pi_B^U(x_i)-\pi_A^U(x_i)\right)^2. \tag{16}$$

Moreover, because of the commutative property of the multiplication operation (e.g. $\mu_A^L(x_i)\mu_B^L(x_i)=\mu_B^L(x_i)\mu_A^L(x_i)$ or $v_A^L(x_i)v_B^L(x_i)=v_B^L(x_i)v_A^L(x_i)$), $d_{MK_{JD}}(A, B) = d_{MK_{JD}}(B, A)$.

Proof (A4). Let A, B and C be three IVIFSs. The Jaccard distance between A and B, and A and C are the following:

$$d_{MK_{JD}}(A,C)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\begin{array}{c}\left(\mu_A^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_C^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_C^L(x_i)\right)^2+\left(v_A^U(x_i)-v_C^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_C^U(x_i)\right)^2\end{array}}{\begin{array}{l}\left(\begin{array}{l}\mu_A^L(x_i)^2+\mu_A^U(x_i)^2+v_A^L(x_i)^2+v_A^U(x_i)^2+\pi_A^L(x_i)^2+\pi_A^U(x_i)^2+\\ \mu_C^L(x_i)^2+\mu_C^U(x_i)^2+v_C^L(x_i)^2+v_C^U(x_i)^2+\pi_C^L(x_i)^2+\pi_C^U(x_i)^2\end{array}\right)\\ -\left(\begin{array}{l}\mu_A^L(x_i)\mu_C^L(x_i)+\mu_A^U(x_i)\mu_C^U(x_i)+v_A^L(x_i)v_C^L(x_i)+\\ v_A^U(x_i)v_C^U(x_i)+\pi_A^L(x_i)\pi_C^L(x_i)+\pi_A^U(x_i)\pi_C^U(x_i)\end{array}\right)\end{array}}\right]. \tag{17}$$

If $A \subseteq B \subseteq C$, then $0\le\mu_A^L(x_i)\le\mu_B^L(x_i)\le\mu_C^L(x_i)\le 1$, $0\le\mu_A^U(x_i)\le\mu_B^U(x_i)\le\mu_C^U(x_i)\le 1$, $0\le v_C^L(x_i)\le v_B^L(x_i)\le v_A^L(x_i)\le 1$, and $0\le v_C^U(x_i)\le v_B^U(x_i)\le v_A^U(x_i)\le 1$. Thus, the following inequalities are obtained. It is easy to see that $d(A,C) \ge d(A,B)$ and $d(A,C) \ge d(B,C)$;

$$\left(\begin{array}{l}\left(\mu_A^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_C^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_C^L(x_i)\right)^2+\left(v_A^U(x_i)-v_C^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_C^U(x_i)\right)^2\end{array}\right)\ge\left(\begin{array}{l}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_B^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)^2\end{array}\right),$$

$$\left(\begin{array}{l}\left(\mu_A^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_C^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_C^L(x_i)\right)^2+\left(v_A^U(x_i)-v_C^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_C^U(x_i)\right)^2\end{array}\right)\ge\left(\begin{array}{l}\left(\mu_B^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_B^U(x_i)-\mu_C^U(x_i)\right)^2+\\ \left(v_B^L(x_i)-v_C^L(x_i)\right)^2+\left(v_B^U(x_i)-v_C^U(x_i)\right)^2+\\ \left(\pi_B^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_B^U(x_i)-\pi_C^U(x_i)\right)^2\end{array}\right).$$

Theorem 2. If $d_{MK_{JD}}(A, B)$ is a Jaccard distance measure between two IVIFSs A and B in X, then $s_{MK_{JD}}(A, B) = 1 - d_{MK_{JD}}(A, B)$ is a Jaccard similarity measure between two IVIFSs A and B in X.

$$s_{MK_{JD}}(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\begin{array}{c}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2+\\ \left(\pi_A^L(x_i)-\pi_B^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)^2\end{array}}{\begin{array}{l}\left(\begin{array}{l}\mu_A^L(x_i)^2+\mu_A^U(x_i)^2+v_A^L(x_i)^2+v_A^U(x_i)^2+\pi_A^L(x_i)^2+\pi_A^U(x_i)^2+\\ \mu_B^L(x_i)^2+\mu_B^U(x_i)^2+v_B^L(x_i)^2+v_B^U(x_i)^2+\pi_B^L(x_i)^2+\pi_B^U(x_i)^2\end{array}\right)+\\ -\left(\begin{array}{l}\mu_A^L(x_i)\mu_B^L(x_i)+\mu_A^U(x_i)\mu_B^U(x_i)+v_A^L(x_i)v_B^L(x_i)+\\ v_A^U(x_i)v_B^U(x_i)+\pi_A^L(x_i)\pi_B^L(x_i)+\pi_A^U(x_i)\pi_B^U(x_i)\end{array}\right)\end{array}}\right]. \tag{18}$$

Comparative Analysis of Distance Measures for IVIF Sets

To show the performance of the proposed Jaccard distance measure, a comparison is made between the proposed Jaccard distance measure and the existing distance measures of IVIFSs. The well-known distance measures are defined in Table 1. Table 2 provides a comprehensive comparison of the distance measures for IVIFSs with counter-intuitive examples. It can be clearly seen that the first axiom of distance measure (A1) is not provided by $d_{X_{HHD}}(A, B)$, $d_{X_{HED}}(A, B)$, $d_{P_{HED}}(A, B)$, $d_{ZXLT\ GD}(A, B)$, $d_{TG_{HD}}(A, B)$, $d_{TG_{ED}}(A, B)$, $d_{TG_{AFD}}(A, B)$, and $d_{TG_{GD}}(A, B)$ when A=([1, 1], [0, 0]) which is the max IVIF value and B=([0, 0], [1, 1]) which is min IVIF value. Another counter-intuitive case emerges when A = [(1, 1), (0, 0)], B = [(0, 0), (0, 0)], and C = [(0.5, 0.5), (0.5, 0.5)]. In this case, $d_{AS_{JD}}(A, B)$ and $d_{AS_{JD}}(C, B)$ are equal to 1.

Some distance measures are not capable of distinguishing positive differences from negative differences. For example, if A=[(0.3, 0.3), (0.3,0.3)], B=[(0.4, 0.4), (0.4, 0.4)], C=[(0.3, 0.3), (0.4, 0.4)], and D =[(0.4, 0.4), (0.3, 0.3)], then $d_{X_{HD}}(A,B)=d_{X_{HD}}(C,D)=0.1$, $d_{X_{HHD}}(A,B)=d_{X_{HHD}}(C,D)=0.05$, $d_{X_{ED}}(A,B)=d_{X_{ED}}(C,D)=0.1$, $d_{X_{HED}}(A,B)=d_{X_{HED}}(C,B)D=0.07$, $d_{P_{HD}}(A, B) = d_{P_{HD}}(C, D) = 0.1$, $d_{P_{ED}}(A, B) = d_{P_{ED}}(C, D) = 0.1$, $d_{P_{HHD}}(A,B)=d_{P_{HHD}}(C,D)=0.1$, $d_{P_{HED}}(A,B)=d_{P_{HED}}(C,D)=0.07$, $d_{ZXLT_{FD}}(A,B)=d_{ZXLT_{FD}}(C,D)=0.1$, $d_{FBA_D}(A,B)=d_{FBA_D}(C,D)=0.01$, $d_{BA_{D1}}(A,B)=d_{BA_{D1}}(C,D)=0.01$, $d_{BA_{D2}}(A,B)=d_{BA_{D2}}(C,D)=0.1$, $d_{ZXLT_{GD}}(A, B) = d_{ZXLT_{GD}}(C, D) = 0.07$.

An interesting counter-intuitive case arises when A=([0.1, 0.1], [0.6, 0.6]), B=([0.2, 0.2], [0.5, 0.5]), and C=([0.1, 0.1], [0.4, 0.4]). In this case, since these IVIF values are ranked as C>B>A according to the score function given in Definition 2.4, the degree of distance between A and C is expected to be greater than the degree of distance between A and B. However, the distance value between A and B is greater than the distance value between A and C when $d_{BA_{D1}}$, $d_{BA_{D2}}$, and d_{FBA_D}, are used. Also, the distance value between A and B is equal to the distance value between A and C when $d_{X_{HD}}$, $d_{P_{HD}}$, and $d_{X_{HHD}}$, are used, which does not seem to be reasonable. Otherwise, $d_{MK_{JD}}(A, B) = 0.147$ and $d_{MK_{JD}}(A, C) = 0.167$.

Another interesting counter-intuitive case emerges when A=([0.35, 0.4], [0.25, 0.35]), B=([0.5, 0.55], [0.3, 0.35]), and C=([0.5, 0.55], [0.25, 0.35]). In this case, since these IVIF values are ranked as C>B>A according to the

Table 1. Existing distance measures

Authors	Distance measure
Xu [17]	$d_{X_{HD}}(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\left[\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|+\left|v_A^L(x_i)-v_B^L(x_i)\right|+\left|v_A^U(x_i)-v_B^U(x_i)\right|\right]$, $d_{X_{ED}}(A,B)=\left(\frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\\\left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2\end{array}\right]\right)^{1/2}$, $d_{X_{HHD}}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\max\left[\begin{array}{l}\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|,\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|,\\\left|v_A^L(x_i)-v_B^L(x_i)\right|,\left|v_A^U(x_i)-v_B^U(x_i)\right|\end{array}\right]$, $d_{X_{HED}}(A,B)=\left(\frac{1}{2n}\sum_{i=1}^{n}\max\left[\begin{array}{l}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2,\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2,\\\left(v_A^L(x_i)-v_B^L(x_i)\right)^2,\left(v_A^U(x_i)-v_B^U(x_i)\right)^2\end{array}\right]\right)^{1/2}$.
Park, et al. [18]	$d_{P_{HD}}(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{l}\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|+\left|v_A^L(x_i)-v_B^L(x_i)\right|+\\\left|v_A^U(x_i)-v_B^U(x_i)\right|+\left|\omega_{\mu_A}(x_i)-\omega_{\mu_B}(x_i)\right|+\left|\omega_{v_A}(x_i)-\omega_{v_B}(x_i)\right|\end{array}\right]$, $d_{P_{ED}}(A,B)=\left(\frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\\\left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2+\\\left(\omega_{\mu_A}(x_i)-\omega_{\mu_B}(x_i)\right)^2+\left(\omega_{v_A}(x_i)-\omega_{v_B}(x_i)\right)^2\end{array}\right]\right)^{1/2}$, $d_{P_{HHD}}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{l}\max\left\{\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|,\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|\right\}+\\\max\left\{\left|v_A^L(x_i)-v_B^L(x_i)\right|,\left|v_A^U(x_i)-v_B^U(x_i)\right|\right\}\end{array}\right]$, $d_{P_{HED}}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\max\left\{\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|,\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|\right\}\right)^2+\\\left(\max\left\{\left|v_A^L(x_i)-v_B^L(x_i)\right|,\left|v_A^U(x_i)-v_B^U(x_i)\right|\right\}\right)^2\end{array}\right]^{1/2}$. $\omega_{\mu_A}(x)=\mu_A^U(x)-\mu_A^L(x)$, $\omega_{\mu_B}(x)=\mu_B^U(x)-\mu_B^L(x)$, $\omega_{v_A}(x)=v_A^U(x)-v_A^L(x)$, $\omega_{v_B}(x)=v_B^U(x)-v_B^L(x)$.
Xu and Yager [20]	$d_{XY_{HD}}(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{l}\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|+\left|v_A^L(x_i)-v_B^L(x_i)\right|+\\\left|v_A^U(x_i)-v_B^U(x_i)\right|+\left|\pi_A^L(x_i)-\pi_B^L(x_i)\right|+\left|\pi_A^U(x_i)-\pi_B^U(x_i)\right|\end{array}\right]$.
Zhang, et al. [6]	$d_{ZXLT_{FD}}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\max\left\{\begin{array}{l}\frac{\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|+\left|v_A^L(x_i)-v_B^L(x_i)\right|+\left|v_A^U(x_i)-v_B^U(x_i)\right|}{4}+\\\frac{\max\left(\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|,\left|v_A^L(x_i)-v_B^L(x_i)\right|+\left|v_A^U(x_i)-v_B^U(x_i)\right|\right)}{2}\end{array}\right\}$. $d_{ZXLT_{GD}}(A,B)=\left(\frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\left|\max\left(\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right),\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)\right)\right|\right)^p+\\\left(\left|\max\left(\left(v_A^L(x_i)-v_B^L(x_i)\right),\left(v_A^U(x_i)-v_B^U(x_i)\right)\right)\right|\right)^p+\end{array}\right]\right)^{1/p}$, $p\geq 2$.
Düğenci [19]	$d_{D_{GD}}(A,B)=\left(\frac{1}{4n(t+1)^p}\sum_{i=1}^{n}\left[\begin{array}{l}\left|t\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)-\left(v_A^L(x_i)-v_B^L(x_i)\right)\right|^p+\\\left|t\left(v_A^L(x_i)-v_B^L(x_i)\right)-\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)\right|^p+\\\left|t\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)-\left(v_A^U(x_i)-v_B^U(x_i)\right)\right|^p+\\\left|t\left(v_A^U(x_i)-v_B^U(x_i)\right)-\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)\right|^p\end{array}\right]\right)^{1/p}$, $t=2,3,\ldots,n$, $p=1,2,\ldots,n$.

Table 1. Existing distance measures (continue)

Authors	Distance measure
Tiwari and Gupta [24]	$d_{TG_{HD}}(A,B)=\frac{1}{8n}\sum_{i=1}^{n}\left[\begin{array}{l}\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert+\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert+\lvert v_A^L(x_i)-v_B^L(x_i)\rvert+\\\lvert v_A^U(x_i)-v_B^U(x_i)\rvert+\lvert\pi_A^L(x_i)-\pi_B^L(x_i)\rvert+\lvert\pi_A^U(x_i)-\pi_B^U(x_i)\rvert\end{array}\right]$, $d_{TG_{ED}}(A,B)=\left(\frac{1}{12n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\\\left(v_A^U(x_i)-v_B^U(x_i)\right)^2+\left(\pi_A^L(x_i)-\pi_B^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)^2\end{array}\right]\right)^{1/2}$, $d_{TG_{HHD}}(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\max\left\{\begin{array}{l}\frac{\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert+\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert}{2},\\\frac{\lvert v_A^L(x_i)-v_B^L(x_i)\rvert+\lvert v_A^U(x_i)-v_B^U(x_i)\rvert}{2},\\\frac{\lvert\pi_A^L(x_i)-\pi_B^L(x_i)\rvert+\lvert\pi_A^U(x_i)-\pi_B^U(x_i)\rvert}{2},\end{array}\right\}$, $d_{TG_{AFD}}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\max\left\{\begin{array}{l}\frac{\begin{array}{l}\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert+\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert+\lvert v_A^L(x_i)-v_B^L(x_i)\rvert+\\\lvert v_A^U(x_i)-v_B^U(x_i)\rvert+\lvert\pi_A^L(x_i)-\pi_B^L(x_i)\rvert+\lvert\pi_A^U(x_i)-\pi_B^U(x_i)\rvert\end{array}}{8}+\\\frac{\max\left(\begin{array}{l}\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert+\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert,\\\lvert v_A^L(x_i)-v_B^L(x_i)\rvert+\lvert v_A^U(x_i)-v_B^U(x_i)\rvert,\\\lvert\pi_A^L(x_i)-\pi_B^L(x_i)\rvert+\lvert\pi_A^U(x_i)-\pi_B^U(x_i)\rvert\end{array}\right)}{4}\end{array}\right\}$, $d_{TG_{GD}}(A,B)=\left(\frac{1}{12n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\left\lvert\max\left(\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right),\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)\right)\right\rvert\right)^p+\\\left(\left\lvert\max\left(\left(v_A^L(x_i)-v_B^L(x_i)\right),\left(v_A^U(x_i)-v_B^U(x_i)\right)\right)\right\rvert\right)^p+\\\left(\left\lvert\max\left(\left(\pi_A^L(x_i)-\pi_B^L(x_i)\right),\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)\right)\right\rvert\right)^p\end{array}\right]\right)^{1/p}$, $p\geq 2$.
Baccour and Alimi [21]	$d_{BA_{D1}}(A,B)=\left(\frac{1}{4n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\sqrt{\mu_A^L(x_i)}-\sqrt{\mu_B^L(x_i)}\right)^2+\left(\sqrt{\mu_A^U(x_i)}-\sqrt{\mu_B^U(x_i)}\right)^2+\\\left(\sqrt{v_A^L(x_i)}-\sqrt{v_B^L(x_i)}\right)^2+\left(\sqrt{v_A^U(x_i)}-\sqrt{v_B^U(x_i)}\right)^2\end{array}\right]\right)$, $d_{BA_{D2}}(A,B)=\frac{1}{8n}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\sqrt{\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert}+\sqrt{\lvert v_A^L(x_i)-v_B^L(x_i)\rvert}\right)^2+\\\left(\sqrt{\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert}+\sqrt{\lvert v_A^U(x_i)-v_B^U(x_i)\rvert}\right)^2\end{array}\right]$.
Fares, et al. [22]	$d_{FBA_D}(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\lvert\mu_A^L(x_i)-\mu_B^L(x_i)\rvert^p}{\mu_A^L(x_i)+\mu_B^L(x_i)}+\frac{\lvert\mu_A^U(x_i)-\mu_B^U(x_i)\rvert^p}{\mu_A^U(x_i)+\mu_B^U(x_i)}+\\\frac{\lvert v_A^L(x_i)-v_B^L(x_i)\rvert^p}{2-v_A^L(x_i)-v_B^L(x_i)}+\frac{\lvert v_A^U(x_i)-v_B^U(x_i)\rvert^p}{2-v_A^U(x_i)-v_B^U(x_i)}\end{array}\right)$.
Anusha and Sireesha [23]	$d_{AS_{JD}}(A,B)=1-\frac{\begin{array}{c}\{\lvert[\min(\mu_A^L(x_i),\mu_B^L(x_i)),\min(\mu_A^U(x_i),\mu_B^U(x_i))],\\ [\max(v_A^L(x_i),v_B^L(x_i)),\max(v_A^U(x_i),v_B^U(x_i))],\\ [\min(\pi_A^L(x_i),\pi_B^L(x_i)),\max(\pi_A^U(x_i),\pi_B^U(x_i))]\rvert\}\end{array}}{\begin{array}{c}\{\lvert[\max(\mu_A^L(x_i),\mu_B^L(x_i)),\max(\mu_A^U(x_i),\mu_B^U(x_i))],\\ [\min(v_A^L(x_i),v_B^L(x_i)),\min(v_A^U(x_i),v_B^U(x_i))],\\ [\min(\pi_A^L(x_i),\pi_B^L(x_i)),\max(\pi_A^U(x_i),\pi_B^U(x_i))]\rvert\}\end{array}}$.

Table 2. The comparison results of distance measures ($p = 2$, $t = 2$, $t_1 = 2$, $t_2 = 2$)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
A	([1, 1], [0, 0])	([0.5, 0.5], [0.5, 0.5])	([0.3, 0.3], [0.3, 0.3])	([0.3, 0.3], [0.4, 0.4])	([0.1, 0.1], [0.6, 0.6])	([0.1, 0.1], [0.6, 0.6])	([0.35, 0.4], [0.25, 0.35])	([0.35, 0.4], [0.25, 0.35])
B	([0, 0], [1, 1])	([0, 0], [1, 1])	([0.4, 0.4], [0.4, 0.4])	([0.4, 0.4], [0.3, 0.3])	([0.2, 0.2], [0.5, 0.5])	([0.1, 0.1], [0.4, 0.4])	([0.5, 0.55], [0.3, 0.35])	([0.5, 0.55], [0.25, 0.35])
d_{XHD} [17]	1.000	0.500	**0.100**	**0.100**	**0.100**	**0.100**	0.088	0.075
d_{XED} [17]	1.000	0.500	**0.100**	**0.100**	0.100	0.141	0.109	0.106
d_{XHHD} [17]	**0.500**	0.250	**0.050**	**0.050**	0.050	0.100	**0.075**	**0.075**
d_{XHED} [17]	**0.707**	0.354	**0.071**	**0.071**	0.071	0.141	**0.106**	**0.106**
d_{PHD}[18]	1.000	0.500	**0.100**	**0.100**	**0.100**	**0.100**	0.100	0.075
d_{PED}[18]	1.000	0.500	**0.100**	**0.100**	0.100	0.141	0.112	0.106
d_{PHHD}[18]	1.000	0.500	**0.100**	**0.100**	**0.100**	**0.100**	0.100	0.075
d_{PHED}[18]	**0.707**	0.354	**0.071**	**0.071**	0.071	0.100	0.079	0.075
d_{XYHD} [20]	1.000	0.500	0.200	0.100	0.100	0.200	0.175	0.150
d_{ZXLTFD}[6]	1.000	0.500	**0.100**	**0.100**	0.100	0.150	0.119	0.113
d_{ZXLTGD}[6]	**0.710**	0.350	**0.071**	**0.071**	0.071	0.100	**0.075**	**0.075**
d_{DGD} [19]	1.000	0.500	0.033	0.100	0.100	0.105	**0.070**	**0.079**
d_{TGHD}[24]	**0.500**	0.250	0.100	0.050	0.050	0.100	0.088	0.075
d_{TGED}[24]	**0.577**	0.289	0.100	0.058	0.058	0.115	0.096	0.087
d_{TGHHD}[24]	1.000	0.500	0.200	0.100	0.100	0.200	0.175	0.150
d_{TGAFD}[24]	**0.500**	0.250	0.100	0.050	0.050	0.100	0.088	0.075
d_{TGGD}[24]	**0.118**	0.059	0.020	0.012	0.012	0.024	0.021	0.018
d_{BAD21}[21]	1.000	0.293	**0.007**	**0.007**	**0.011**	**0.010**	0.007	0.006
d_{BAD2} [21]	1.000	0.500	**0.100**	**0.100**	**0.100**	**0.050**	0.065	0.038
d_{FBAD} [22]	1.000	0.500	**0.011**	**0.011**	**0.022**	**0.020**	**0.013**	**0.013**
d_{ASJD} [23]	**1.000**	**1.000**	0.107	0.146	0.173	0.259	0.073	0.055
d_{MKJD}	1.000	0.500	0.160	0.060	0.047	0.167	0.136	0.113

score function given in Definition 2.4, the degree of distance between A and C is expected to be greater than the degree of distance between A and B. But, the distance value between A and B is equal to the distance value between A and C when $d_{X_{HHD}}$, $d_{X_{HED}}$, $d_{ZXLT_{GD}}$ and d_{FBA_D} is used. Furthermore, the distance value between A and B is greater than the distance value between A and C when $d_{D_{GD}}$ are preferred. Otherwise, $d_{MK_{JD}}(A, B) = 0.113$ and $d_{MK_{JD}}(A, C) = 0.136$. Thus, the proposed Jaccard distance measure is compatible with the score function in Equation (7). The comparison of distance measures is presented in Table 2. Counter-intuitive cases are marked as bold type. It is seen from Table 2 that the proposed Jaccard distance measure has no counterintuitive cases.

The MCDM Method Usıng the Jaccard Distance Measure

This section presents the extension of the TOPSIS method to an IVIF environment for solving multi-criteria group decision-making problems. In this method, the proposed Jaccard distance measure is used to obtain separation measures. Let $A = \{A_1, A_2, ..., A_m\}$ be a set of alternatives, $C = \{C_1, C_2, ..., C_n\}$ be a set of criteria, $W = \{w_1, w_2, ..., w_n\}$ be a set of weights of criteria, and $\lambda = \{\lambda_1, \lambda_2, ..., \lambda_k\}$ be a set of weights of experts the steps of the IVIF-TOPSIS method are given as follows:

Step 1. Construct the IVIF decision matrices $\tilde{D}_k$ based on the evaluations of alternatives obtained by experts. The IVIF decision matrices $\tilde{D}_k$ can be defined as in Equation (19):

$$\tilde{D}_k = \begin{bmatrix} & A_1 & A_2 & \cdots & A_m \\ C_1 & \left[\mu_{11k}^L, \mu_{11k}^U\right], \left[v_{11k}^L, v_{11k}^U\right] & \left[\mu_{12k}^L, \mu_{12k}^U\right], \left[v_{12k}^L, v_{12k}^U\right] & \cdots & \left[\mu_{1mk}^L, \mu_{1mk}^U\right], \left[v_{1mk}^L, v_{1mk}^U\right] \\ C_2 & \left[\mu_{21k}^L, \mu_{21k}^U\right], \left[v_{21k}^L, v_{21k}^U\right] & \left[\mu_{22k}^L, \mu_{22k}^U\right], \left[v_{22k}^L, v_{22k}^U\right] & \cdots & \left[\mu_{2mk}^L, \mu_{2mk}^U\right], \left[v_{2mk}^L, v_{2mk}^U\right] \\ & \vdots & \vdots & \ddots & \vdots \\ C_n & \left[\mu_{n1k}^L, \mu_{n1k}^U\right], \left[v_{n1k}^L, v_{n1k}^U\right] & \left[\mu_{n2k}^L, \mu_{n2k}^U\right], \left[v_{n2k}^L, v_{n2k}^U\right] & \cdots & \left[\mu_{nmk}^L, \mu_{nmk}^U\right], \left[v_{nmk}^L, v_{nmk}^U\right] \end{bmatrix}. \quad (19)$$

Step 2. Determine the IVIF positive ideal solution (PIS) and IVIF negative ideal solution (NIS) for *kth* expert by using Equations (20) and (21). Here $\left(\left[\mu_{1*k}^L, \mu_{1*k}^U\right], \left[v_{1*k}^L, v_{1*k}^U\right]\right)$ and $\left(\left[\mu_{1-k}^L, \mu_{1-k}^U\right], \left[v_{1-k}^L, v_{1-k}^U\right]\right)$ represent maximum and minimum IVIF values, respectively, among the values of alternatives for *ith* criterion:

$$I\tilde{S}_k^+ = \left(\left(\left[\mu_{1*k}^L, \mu_{1*k}^U\right], \left[v_{1*k}^L, v_{1*k}^U\right]\right), \left(\left[\mu_{2*k}^L, \mu_{2*k}^U\right], \left[v_{2*k}^L, v_{2*k}^U\right]\right), \cdots\cdots, \left(\left[\mu_{n*k}^L, \mu_{n*k}^U\right], \left[v_{n*k}^L, v_{n*k}^U\right]\right)\right), \quad (20)$$

$$I\tilde{S}_k^- = \left(\left(\left[\mu_{1-k}^L, \mu_{1-k}^U\right], \left[v_{1-k}^L, v_{1-k}^U\right]\right), \left(\left[\mu_{2-k}^L, \mu_{2-k}^U\right], \left[v_{2-k}^L, v_{2-k}^U\right]\right), \cdots\cdots, \left(\left[\mu_{n-k}^L, \mu_{n-k}^U\right], \left[v_{n-k}^L, v_{n-k}^U\right]\right)\right). \quad (21)$$

Step 3. Calculate the positive separation measures D_j^{k+} and negative separation measures D_j^{k-} between the alternatives and IVIFPIS and IVIFNIS for each expert using Equations (22) and (23), respectively:

$$D_j^{*k} = \frac{1}{n}\sum_{i=1}^{n} w_i^T \left[\frac{\left(\mu_{1jk}^L - \mu_{1*k}^L\right)^2 + (\mu_{1jk}^U - \mu_{1*k}^U)^2 + (v_{1jk}^L - v_{1*k}^L)^2 + (v_{1jk}^U - v_{1*k}^U)^2 + (\pi_{1jk}^L - \pi_{1*k}^L)^2 + (\pi_{1jk}^U - \pi_{1*k}^U)^2}{\left(\begin{matrix}\mu_{1jk}^{L\,2} + \mu_{1jk}^{U\,2} + v_{1jk}^{L\,2} + v_{1jk}^{U\,2} + \pi_{1jk}^{L\,2} + \pi_{1jk}^{U\,2} + \\ \mu_{1*k}^{L\,2} + \mu_{1*k}^{U\,2} + v_{1*k}^{L\,2} + v_{1*k}^{U\,2} + \pi_{1*k}^{L\,2} + \pi_{1*k}^{U\,2}\end{matrix}\right) - \left(\begin{matrix}\mu_{1jk}^L \mu_{1*k}^L + \mu_{1jk}^U \mu_{1*k}^U + v_{1jk}^L v_{1*k}^L + \\ v_{1jk}^U v_{1*k}^U + \pi_{1jk}^L \pi_{1*k}^L + \pi_{1jk}^U \pi_{1*k}^U\end{matrix}\right)}\right], \quad (22)$$

$$D_j^{-k} = \frac{1}{n}\sum_{i=1}^{n} w_i^T \left[\frac{\left(\mu_{1jk}^L - \mu_{1-k}^L\right)^2 + (\mu_{1jk}^U - \mu_{1-k}^U)^2 + (v_{1jk}^L - v_{1-k}^L)^2 + (v_{1jk}^U - v_{1-k}^U)^2 + (\pi_{1jk}^L - \pi_{1-k}^L)^2 + (\pi_{1jk}^U - \pi_{1-k}^U)^2}{\left(\begin{matrix}\mu_{1jk}^{L\,2} + \mu_{1jk}^{U\,2} + v_{1jk}^{L\,2} + v_{1jk}^{U\,2} + \pi_{1jk}^{L\,2} + \pi_{1jk}^{U\,2} + \\ \mu_{1-k}^{L\,2} + \mu_{1-k}^{U\,2} + v_{1-k}^{L\,2} + v_{1-k}^{U\,2} + \pi_{1-k}^{L\,2} + \pi_{1-k}^{U\,2}\end{matrix}\right) - \left(\begin{matrix}\mu_{1jk}^L \mu_{1-k}^L + \mu_{1jk}^U \mu_{1-k}^U + v_{1jk}^L v_{1-k}^L + \\ v_{1jk}^U v_{1-k}^U + \pi_{1jk}^L \pi_{1-k}^L + \pi_{1jk}^U \pi_{1-k}^U\end{matrix}\right)}\right]. \quad (23)$$

Step 4. Aggregate the separation measures for the experts using Equation (24) and Equation (25) where λ_k is the weight of expert k. Then, obtain the closeness coefficient of each alternative by using Equation (26):

$$D_j^+ = \sum_{k=1}^{K} \lambda_k D_j^{k+}, \quad (24)$$

$$D_j^- = \sum_{k=1}^{K} \lambda_k D_j^{k-}, \quad (25)$$

$$U_j = \frac{D_j^{k-}}{(D_j^{k-} + D_j^{k+})}. \quad (26)$$

Step 5. Rank the alternatives in descending order of relative closeness and select the alternative.

NUMERICAL EXAMPLE

A personnel selection problem is considered to illustrate the proposed extended TOPSIS method for group decision-making with IVIF numbers. Then, the proposed IVIF-TOPSIS method is compared to the IVIF-TOPSIS methods introduced by Ye [3], Izadikhah [49], and Bai [50].

Personnel Evaluation Problem

A retail company plans to employ a process analyst. Some of the candidates applying for the job positions are evaluated as part of the preliminary consideration and some candidates are eliminated. Five candidates are listed for consideration by the expert committee. This committee consists of three human resources experts. In the evaluation process, five criteria are determined: technical skills (C1), communication skills (C2), self-confidence (C3), past experience (C4), and foreign language (C5). The experts should balance these five criteria simultaneously and select the best candidate for the job. With the assumption that $w_i^T = \{0.15, 0.30, 0.10, 0.20, 0.25\}$ is the weight vector of criteria and $\lambda_k^T = \{0.3, 0.5, 0.2\}$ is the weight vector of experts, the evaluations corresponding to five candidates provided by experts are presented in Table 3.

By using Equations (20) and (21), the IVIFPIS and IVIFNIS are determined for each expert, and these values are given in Table 3. Then, using Equation (22), the positive separation measure (D_j^{k+}) between each candidate and the Public Interest Score (PIS) for each expert is calculated, and the positive separation measure is given in Table 4. Similarly, the negative separation measure (D_j^{k-}) between each candidate and the network and Information Security (NIS) for each expert is obtained by using Equation (23) and also given in Table 4.

Table 3. The IVIF decision matrix for each expert

Experts		A_1	A_2	A_3	A_4	A_5
E#1	C_1	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.5,0.8],[0.1,0.2],[0,0.4])	([0.4,0.6],[0.1,0.3],[0.1,0.5])
	C_2	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.4,0.6],[0.2,0.3],[0.1,0.4])	([0.5,0.7],[0.2,0.3],[0,0.3])
	C_3	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.5,0.7],[0.1,0.3],[0,0.4])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.6,0.7],[0.1,0.2],[0.1,0.3])
	C_4	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.8,0.9],[0,0.1],[0,0.2])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.7,0.8],[0.1,0.2],[0,0.2])
	C_5	([0.6,0.8],[0.1,0.2],[0,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.5,0.7],[0.1,0.2],[0.1,0.4])
E#2	C_1	([0.4,0.5],[0.3,0.4],[0.1,0.3])	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.5,0.6],[0.3,0.4],[0,0.2])
	C_2	([0.7,0.9],[0,0.1],[0,0.3])	([0.6,0.7],[0.1,0.2],[0.1,0.3])	([0.5,0.7],[0.2,0.3],[0,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.5,0.7],[0.2,0.3],[0,0.3])
	C_3	([0.6,0.8],[0,0.1],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.3,0.4],[0.4,0.5],[0.1,0.3])	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.6,0.7],[0.2,0.3],[0,0.2])
	C_4	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.4,0.5],[0.3,0.4],[0.1,0.3])	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.7,0.9],[0,0.1],[0,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])
	C_5	([0.5,0.7],[0.1,0.2],[0.1,0.4])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.6,0.8],[0,0.1],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.4,0.6],[0.3,0.4],[0,0.3])
E#3	C_1	([0.5,0.6],[0.3,0.4],[0,0.2])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.6,0.8],[0.1,0.2],[0,0.3])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.6,0.8],[0.1,0.2],[0,0.3])
	C_2	([0.5,0.7],[0.2,0.3],[0,0.3])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.5,0.7],[0.1,0.2],[0.1,0.4])
	C_3	([0.6,0.7],[0,0.2],[0.1,0.4])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.4,0.6],[0.2,0.3],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])
	C_4	([0.7,0.8],[0,0.1],[0.1,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.7,0.9],[0,0.1],[0,0.3])	([0.5,0.6],[0.2,0.3],[0.1,0.3])
	C_5	([0.4,0.7],[0.1,0.2],[0.1,0.5])	([0.4,0.6],[0.3,0.4],[0,0.3])	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.5,0.7],[0.1,0.2],[0.1,0.4])

Table 4. The positive ideal solution and negative ideal solution for each expert

Experts		C_1	C_2	C_3	C_4	C_5
E#1	IVIFNIS	([0.4,0.6],[0.1,0.3],[0.1,0.5])	([0.4,0.6],[0.2,0.3],[0.1,0.4])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.5,0.7],[0.1,0.2],[0.1,0.4])
	IVIFPIS	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.6,0.7],[0.1,0.2],[0.1,0.3])	([0.8,0.9],[0,0.1],[0,0.2])	([0.7,0.8],[0.1,0.2],[0,0.2])
E#2	IVIFNIS	([0.4,0.5],[0.3,0.4],[0.1,0.3])	([0.5,0.7],[0.2,0.3],[0,0.3])	([0.3,0.4],[0.4,0.5],[0.1,0.3])	([0.4,0.5],[0.3,0.4],[0.1,0.3])	([0.4,0.6],[0.3,0.4],[0,0.3])
	IVIFPIS	([0.6,0.7],[0.2,0.3],[0,0.2])	([0.7,0.9],[0,0.1],[0,0.3])	([0.6,0.8],[0,0.1],[0.1,0.4])	([0.7,0.9],[0,0.1],[0,0.3])	([0.6,0.8],[0,0.1],[0.1,0.4])
E#3	IVIFNIS	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.4,0.5],[0.2,0.4],[0.1,0.4])	([0.4,0.6],[0.2,0.3],[0.1,0.4])	([0.5,0.6],[0.2,0.3],[0.1,0.3])	([0.4,0.6],[0.3,0.4],[0,0.3])
	IVIFPIS	([0.6,0.8],[0.1,0.2],[0,0.3])	([0.7,0.8],[0.1,0.2],[0,0.2])	([0.6,0.7],[0,0.2],[0.1,0.4])	([0.7,0.9],[0,0.1],[0,0.3])	([0.5,0.7],[0.1,0.2],[0.1,0.4])

Table 5. Separation measures for each expert

Alternative	Expert #1		Expert #2		Expert #3	
A_1	$D_1^{1-}=0.006$	$D_1^{1+}=0.004$	$D_1^{2-}=0.011$	$D_1^{2+}=0.004$	$D_1^{3-}=0.008$	$D_1^{3+}=0.003$
A_2	$D_2^{1-}=0.014$	$D_2^{1+}=0.001$	$D_2^{2-}=0.008$	$D_2^{2+}=0.010$	$D_2^{3-}=0.003$	$D_2^{3+}=0.010$
A_3	$D_3^{1-}=0.007$	$D_3^{1+}=0.005$	$D_3^{2-}=0.008$	$D_3^{2+}=0.009$	$D_3^{3-}=0.012$	$D_3^{3+}=0.002$
A_4	$D_4^{1-}=0.001$	$D_4^{1+}=0.011$	$D_4^{2-}=0.012$	$D_4^{2+}=0.003$	$D_4^{3-}=0.009$	$D_4^{3+}=0.004$
A_5	$D_5^{1-}=0.006$	$D_5^{1+}=0.006$	$D_5^{2-}=0.007$	$D_5^{2+}=0.009$	$D_5^{3-}=0.007$	$D_5^{3+}=0.005$

Table 6. The aggregated separation measures and closeness coefficients

	A_1	A_2	A_3	A_4	A_5
D_j^-	0.009	0.008	0.009	0.008	0.007
D_j^+	0.004	0.008	0.006	0.005	0.007
U_j	0.709	0.490	0.604	0.613	0.487

Table 7. Comparison of the alternative priorities

References	A_1	A_2	A_3	A_4	A_5	Rank	Best Alternative
Ye [3]	0.585	0.499	0.551	0.539	0.475	$A_1 > A_3 > A_4 > A_2 > A_5$	A_1
Izadikhah [49]	0.734	0.477	0.395	0.475	0.313	$A_1 > A_2 > A_4 > A_3 > A_5$	A_1
Bai [50]	0.755	0.721	0.724	0.730	0.682	$A_1 > A_4 > A_3 > A_2 > A_5$	A_1
This study	0.709	0.490	0.604	0.613	0.487	$A_1 > A_4 > A_3 > A_2 > A_5$	A_1

Finally, the separation measures are aggregated using Equations (24) and (25). Furthermore, the closeness coefficients of all alternatives are obtained using Equation (26) and shown in Table 6. The candidates are ranked as $A_1 > A_4 > A_3 > A_2 > A_5$ according to descending order of closeness coefficients of candidates where the symbol ">" means superior to. Thus, A_1 is the best candidate for a process analyst for the company.

Comparison with Other Methods

The proposed IVIF-TOPSIS method is compared with several other IVIF-TOPSIS methods. While the IVIF-TOPSIS method proposed by Ye [3] uses the Euclidean distance in the calculation phase of the separation measure, the IVIF-TOPSIS method proposed by Izadikhah [49] uses the Hamming distance. On the other hand, the separation measures of each alternative from the positive and negative ideal solutions are calculated using the score function in the IVIF-TOPSIS method introduced by Bai [50]. The calculated results are shown in Table 7, which present the overall priorities of the alternatives. The ranking order obtained using our proposed methodology is as $A_1 > A_4 > A_3 > A_2 > A_5$. This order indicates that A1 is the appropriate alternative for selection in line with the results of Bai [50]. In other words, IVIF-TOPSIS methods proposed by Bai [50] and developed in this study give the same ranking, although they use different closeness coefficients. The method proposed by Ye [3] and Izadikhah [49] also showed A_1 as the most suitable alternative. Hence, the method based on the novel Jaccard distance is a suitable solution for MCDM problems owing to its ability to deal with the imprecise information.

CONCLUSION

This study introduces the Jaccard distance measure as a new distance measure for IVIFSs. This measure is then compared with well-known distance measures for IVIFSs using a few counter-intuitive cases. As the result of the comparative analysis the proposed Jaccard distance measure does not have counterintuitive cases. Moreover, this study presents the extension of TOPSIS to the IVIF environment to solve multi-criteria group decision-making problems. The proposed Jaccard distance measure is used to obtain separation measures in the IVIF-TOPSIS method. In the evaluation process, the ratings of each alternative are represented as IVIFVs ın IVIF-TOPSIS method. After calculating IVIF-PIS and IVIF-NIS, separation measures are obtained based on the proposed Jaccard distance measure.

Then, the closeness coefficients of each alternative are determined and ranked. The personnel selection problem is illustrated to show the application of the proposed Jaccard distance measure to the IVIF-TOPSIS method. This illustrative example is also used to demonstrate the differences between the proposed IVIF-TOPSIS method and several IVIF-TOPSIS methods. A comparison of the alternative priorities shows that the most suitable alternative produced by the new IVIF-TOPSIS is the same as the most suitable alternative obtained by other IVIF-TOPSIS methods.

For further research, many extensions of fuzzy sets, such as hesitant fuzzy sets, spherical fuzzy sets, or Neutrosophic fuzzy sets are suggested for solving the same problem.

LIST OF ABBREVIATIONS

FSs	Fuzzy Sets
IFSs	Intuitionistic Fuzzy Sets
IVIFS	Interval-Valued Intuitionistic Fuzzy Set
MCDM	Multi-Criteria Decision-Making
TOPSIS	Technique for Order Preference by Similarity to Ideal Solutions
HD	Hamming Distance
ED	Euclidean Distance
HED	Hausdorff Euclidean Distance
HHD	Hausdorff Hamming Distance
FD	Fifth Distance
GD	Generalized Distance
JD	Jaccard Distance

CONFLICT OF INTEREST

There are no conflicts of interest/competing interests.

ETHICS APPROVAL

The author approves that the research presented in this paper is conducted following the principles of ethical and professional conduct.

CONSENT FOR PUBLICATION

Not applicable, the author used publicly available data only and provide the corresponding references.

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