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Research Article

Analysis of nonlinear partial differential equation of traveling wave solutions with an effective method

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ABSTRACT

In this paper, the exact solutions of the (2+1)-dimensional generalized Hirota–Satsuma–Ito equation are acquired via the modified exponential function method. The method facilitates the acquisition of diverse solution functions under varying conditions, enabling the investigation of the linear mathematical model's behavior from multiple perspectives. Consequently, after deriving the solution functions that characterize the behavior of the nonlinear mathematical model, the plots of these functions have been plotted using the relevant parameters.

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INTRODUCTION

One of the most important aspects of nonlinear research is the development of exact solutions for nonlinear evolution equations (NLEEs). Studies of this part can help us understand nonlinear problems like plasmas, Bose–Einstein condensation, and fluids. There are a number of wave models that illustrate these issues. The nonlinear Schrodinger and Korteweg–de Vries equations and their numerous modifications are used in these wave models. The Hirota bilinear approach is widely utilized for solution construction as a result of its ease of use and clarity, such as solitons and breathers. Also, it has been pointed out that certain types of nonlinear waves can change into other types of waves in certain situations [1-33].

In a similar fashion, many high-dimensional NLEEs include nonlinear waves, such as lump solutions and respiration waves. There are a lot of studies on breath-wave and lump wave solutions. Recently researchers have investigated skew lumps and interactions of multi-lumps within the framework of the Kadomtsev–Petviashvili equation. However, relatively few studies have been conducted on conversions in high-dimensional NLEEs [34-48].

The Hirota-Satsuma-Ito equation (HSIE), which has 2+1 dimensional, has recently attracted a lot of interest. The Jimbo-Miwa classification includes this equation, which is often utilized in the analysis of waves in relatively shallow

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water. Diverse types solutions for the (2+1)-dimensional HSIE have been derived. Furthermore, on the premise of the aforementioned research, solutions with more intricate forms is discovered. Interaction solutions have been evaluated [48-57].

In this paper, the (2+1)-dimensional generalized HSIE [39]

$$v_t + u_{xxt} + 3(uz)_x + \alpha u_x = 0, u_y = v_x, u_t = z_x \quad (1)$$

is investigated, where u is the physical field, v and z are the potentials of physical field derivatives. The aim of the study is to obtain the traveling solutions of HSIE by the modified exponential function method (MEFM).

The remainder of the work is organized as follows: In Section 2, the main idea of MEFM is presented. The application is demonstrated for (2+1)-dimensional generalized HSIE in Section 3. In Section 4, the conclusion is introduced.

MATERIALS AND METHODS

Modified Exponential Function Method

Basic information regarding MEFM are provided in this area.

Let implement the method to the following nonlinear partial differential equations (NPDEs):

$$\Lambda(\rho, \rho_x, \rho_t, \rho_{xx}, \rho_{tt}, \rho_{tx}, \dots) = 0, \quad (2)$$

where $\rho = \rho(x, y, t)$ is unknown function, Λ is a polynomial that functions as $\rho(x, y, t)$ and its partial derivatives with respect to x, y, and t.

Step 1: Assume that the traveling wave transform (TWT) is as follows:

$$\rho(x,y,t) = \Upsilon(\xi), \xi = \chi(x + y - \varpi t), \quad (3)$$

where the constants $\chi \neq 0$, $\varpi \neq 0$ and will be calculated later. By putting the derivative terms from Eq. (3) into Eq. (2), Eq. (2) is converted into a nonlinear ordinary differential equation, referred to as

$$\mathrm{T}(\Upsilon, \Upsilon', \Upsilon'', \Upsilon''', \dots.) = 0, \quad (4)$$

where T is a polynomial which has Υ and its derivatives.

Step 2: Suppose that the traveling wave solution (TWS) of Eq. (4) is stated in the form:

$$\Upsilon(\xi) = \frac{\sum_{i=0}^{N} A_i [exp(-\Phi(\xi))]^i}{\sum_{j=0}^{M} B_j [exp(-\Phi(\xi))]^j} = \frac{A_0 + A_1 exp(-\Phi) + \cdots + A_N exp(N(-\Phi))}{B_0 + B_1 exp(-\Phi) + \cdots + B_M exp(M(-\Phi))}, \quad (5)$$

where $A_N \neq 0$, $B_M \neq 0$, A_i and B_j, ($i \in [0, N]$, $j \in [0, M]$) are constants that will be calculated. $\Phi = \Phi(\xi)$ supplies the Eq. (6):

$$\Phi'(\xi) = exp(-\Phi(\xi)) + \mu exp(\Phi(\xi)) + \lambda. \quad (6)$$

By solving Eq. (6), five families of solutions are derived [18]:

Family 1: Let $\mu \neq 0$, $\lambda^2 - 4\mu > 0$. Therefore, the TWS is acquired as

$$\Phi(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \quad (7)$$

Family 2: Let $\mu \neq 0$, $\lambda^2 - 4\mu < 0$. Consequently, the TWS is obtained as

$$\Phi(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \quad (8)$$

Family 3: Let $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$. Therefore, the TWS is acquired as

$$\Phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right). \quad (9)$$

Family 4: Let $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$. Consequently, the TWS is obtained as

$$\Phi(\xi) = \ln\left(-\frac{2\lambda(\xi + E) + 4}{\lambda^2(\xi + E)}\right). \quad (10)$$

Family 5: Let $\mu = 0$, $\lambda = 0$ and $\lambda^2 - 4\mu = 0$. Therefore, the TWS is acquired as

$$\Phi(\xi) = \ln(\xi + E), \quad (11)$$

where the constants A_i ($i \in [0, N]$), B_j ($j \in [0, M]$), E, λ, μ will be determined. By using the notion of a homogeneous balance principle (BP) between the highest nonlinear terms and the highest order derivatives of Υ in Eq. (5), an association between N and M is to be established.

Step 3: Substituting Eq. (6) and the family solutions into Eq. (5) yields a polynomial of exp ($\Phi(\xi)$). The algebraic system of equations (ASEs) involving A_i ($i \in [0, N]$), B_j ($j \in [0, M]$), E, λ, and μ is derived by equating to zero the coefficients corresponding to identical powers of exp ($\Phi(\xi)$). Finally, the acquired values of coefficients substituting in equation (5), it supplies the TWSs of Eq. (2).

Application

MEFM are utilized in the part to derive the wave solutions to the (2+1)-dimensional generalized HSIE. Let us handle the TWTs:

$$\left.\begin{aligned} u(x,y,t) &= u(\xi), \xi = \chi(x + y - \varpi t), \\ v(x,y,t) &= v(\xi), \xi = \chi(x + y - \varpi t), \\ z(x,y,t) &= z(\xi), \xi = \chi(x + y - \varpi t). \end{aligned}\right\} \quad (12)$$

If the derivatives of these transformations, which should be included in Eq. (1) are taken, the following equations are found:

$$\left.\begin{aligned} V_t &= -\varpi\chi v', \\ U_{xtt} &= -\varpi\chi^3 u''', \\ U_x &= \chi u', \\ Z_x &= \chi z', \\ U_y &= \chi u', \\ V_x &= \chi v', \\ U_t &= -\varpi\chi u'. \end{aligned}\right\} \tag{13}$$

Substituting the derivative terms from Eq. (13) into Eq. (1) yields

$$\left.\begin{aligned} -\varpi\chi v' - \varpi\chi^3 u''' + 3\chi u'z + 3\chi uz' + \alpha\chi u' = 0, \\ \chi u' = \chi v', \\ -\varpi\chi u' = \chi z'. \end{aligned}\right\} \tag{14}$$

When we rearrange Eq. (14), we obtain

$$\left.\begin{aligned} -\varpi v' - \varpi\chi^2 u''' + 3(uz)' + \alpha u' = 0, \\ u' = v', \\ -\varpi u' = z'. \end{aligned}\right\} \tag{15}$$

If Eq. (15) is integrated with respect to ξ, then we find

$$\left.\begin{aligned} -\varpi v - \varpi\chi^2 u'' + 3(uz) + \alpha u = 0, \\ u = v, \\ -\varpi u = z. \end{aligned}\right\} \tag{16}$$

If we rearrange the system (16), then we have

$$-\varpi u - \varpi\chi^2 u'' - 3\varpi u^2 + \alpha u = 0. \tag{17}$$

Applying the BP to Eq. (17) yields the relationship

$$N = M + 2.$$

By selecting $M = 1$, we then determine $N = 3$. For the values of M and N, it is derived as

$$u(\xi) = \frac{A_0 + A_1 e^{-\Phi} + A_2 e^{-2\Phi} + A_3 e^{-3\Phi}}{B_0 + B_1 e^{-\Phi}}. \tag{18}$$

The ASEs with $e^{-\Phi(\xi)}$ coefficients are derived by reorganizing Eq. (18) in accordance with the requisite term in Eq. (16).

The followings are the appropriate coefficients acquired by utilizing the Mathematica software tool.

Case-1:

$$A_0 = -\frac{1}{3}\chi^2(\lambda^2 + 2\mu)B_0,$$

$$A_1 = -\frac{1}{3}\chi^2(6\lambda B_0 + (\lambda^2 + 2\mu)B_1),$$

$$A_2 = -2\chi^2(B_0 + \lambda B_1),$$

$$A_3 = -2\chi^2 B_1,$$

$$\varpi = \frac{\alpha}{1 - \chi^2(\lambda^2 - 4\mu)}.$$

When we substitute above coefficients in Eq. (16), we acquire the solutions in the following.

Family 1: Let $\mu \neq 0$, $\lambda^2 - 4\mu > 0$. Consequently, the TWSs of Eq. (1) are found as

$$u_{1,1}(x,y,t) = \left(\frac{-\left(\left(\chi^2\Gamma^2 \mathrm{sech}\left[\frac{1}{2}\Gamma\xi\right]^2 (-4\mu + (\lambda^2 - 2\mu)\cosh[\xi\Gamma] + \lambda\Gamma\sinh[\xi\Gamma])\right)\right)}{\left(3\left(\lambda + \Gamma\tanh\left[\frac{1}{2}(\xi\Gamma)\right]\right)^2\right)}\right) - \left(\frac{\lambda\Gamma\sinh[\xi\Gamma]}{\left(3\left(\lambda + \Gamma\tanh\left[\frac{1}{2}(\xi\Gamma)\right]\right)^2\right)}\right), \tag{19}$$

$$v_{1,1}(x,y,t) = \left(\frac{(2\chi^2\Gamma^2\mu(2\mu + (\lambda^2 - 2\mu)\cosh[\xi\Gamma] - \lambda\Gamma\sinh[\xi\Gamma])}{(\lambda^2 - 2\mu + 2\mu\cosh[\xi\Gamma])^2}\right), \tag{20}$$

$$z_{1,1}(x,y,t) = -\left(\frac{(2\varpi\chi^2\Gamma^2\mu(2\mu + (\lambda^2 - 2\mu)\cosh[\xi\Gamma] - \lambda\Gamma\sinh[\xi\Gamma]))}{(\lambda^2 - 2\mu + 2\mu\cosh[\xi\Gamma])^2}\right), \tag{21}$$

where, $\xi = EE + \chi(-\varpi t + x + y)$, $\Gamma = \sqrt{\lambda^2 - 4\mu}$.

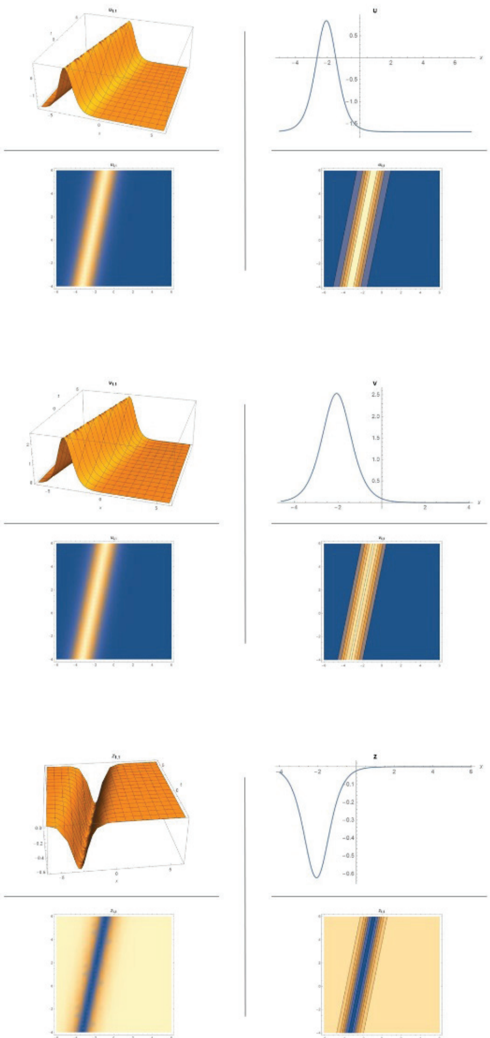

Figure 1. 2D, 3D, density, contour plots of Eqs. (19)-(21) at $\lambda = 2.5$, $\mu = 1$, $B_0 = 0.2$, $\chi = 1.5$, $\alpha = -1$, $B_1 = 1.25$, $A_0 = -1.2375$, $A_1 = -9.98438$, $A_2 = -14.9625$, $A_3 = -5.625$, $\varpi = 0.246154$, $y = 1.2$, $t = 1$, $EE = 0.75$.

Family 2: Let $\mu \neq 0$, $\lambda^2 - 4\mu < 0$. Therefore, the TWSs of Eq. (1) are found by

$$u_{1,2}(x,y,t) = \left(\frac{\left(-\chi^2\Gamma^2\sec\left[\frac{1}{2}\theta\xi\right]^2\left(4\mu - (\lambda^2 - 2\mu)\cos[\xi\theta] + \lambda\Gamma\sin[\xi\theta]\right)\right)}{\left(3\left(\lambda - \Gamma\tanh\left[\frac{1}{2}(\xi\theta)\right]\right)^2\right)}\right), \quad (22)$$

$$v_{1,2}(x,y,t) = \left(\frac{-2\chi^2\Gamma^2\mu(2\mu + (\lambda^2 - 2\mu)\cos[\xi\Theta] + \lambda\Gamma\sin[\xi\Theta])}{(\lambda^2 - 2\mu + 2\mu\cos[\xi\Theta])^2}\right), \quad (23)$$

$$z_{1,2}(x,y,t) = -\left(\frac{-2\varpi\chi^2\Gamma^2\mu(2\mu + (\lambda^2 - 2\mu)\cos[\xi\Theta] + \lambda\Gamma\sin[\xi\Theta])}{(\lambda^2 - 2\mu + 2\mu\cos[\xi\Theta])^2}\right), \quad (24)$$

where, $\xi = EE + \chi(-\varpi t + x + y)$, $\Theta = \sqrt{-\lambda^2 + 4\mu}$.

Family 3: Let $\mu = 0$, $\lambda \neq 0$, $\lambda^2 - 4\mu < 0$. Therefore, the TWSs of Eq. (1) are acquired as

$$u_{1,3}(x,y,t) = \left(-\frac{1}{6}\chi^2\lambda^2\left(2 + 3\operatorname{csch}\left[\frac{1}{2}\xi\lambda\right]^2\right)\right), \quad (25)$$

$$v_{1,3}(x,y,t) = \left(-\frac{\chi^2\lambda^2}{-1 + \cosh\left[\frac{1}{2}\xi\lambda\right]}\right), \quad (26)$$

$$z_{1,3}(x,y,t) = \left(-\frac{\chi^2\lambda^2}{-1 + \cosh\left[\frac{1}{2}\xi\lambda\right]}\right), \quad (27)$$

where, $\xi = EE + \chi(-\varpi t + x + y)$.

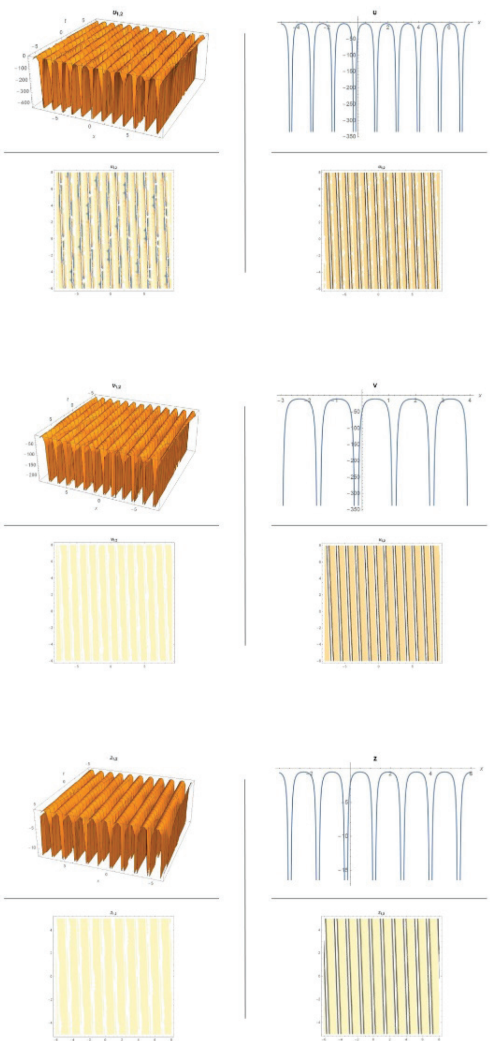

Figure 2. 2D, 3D, density, contour plots of Eqs. (22)-(24) at $\lambda = 1$, $\mu = 2.5$, $B_0 = 0.2$, $\chi = 1.5$, $\alpha = -1$, $B_1 = 1.25$, $A_0 = -0.9$, $A_1 = -6.525$, $A_2 = -6.525$, $A_3 = -5.625$, $\varpi = 0.0470588$, $y = 1.2$, $t = 1$, $EE = 0.75$.

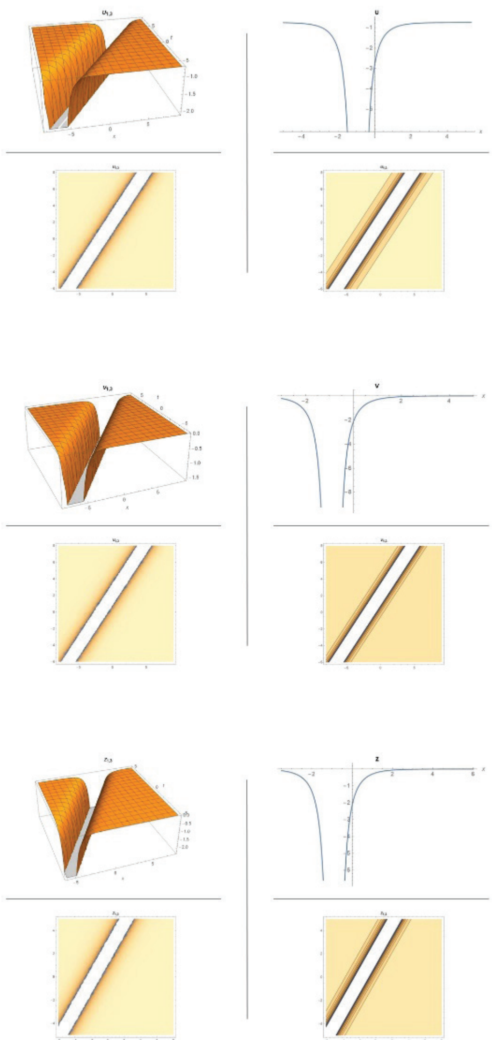

Figure 3. 2D, 3D, density, contour plots of Eqs. (25)-(27) at $\lambda = 1$, $\mu = 0$, $B_0 = 0.2$, $\chi = 1.5$, $\alpha = -1$, $B_1 = 1.25$, $A_0 = -0.15$, $A_1 = -1.8375$, $A_2 = -6.525$, $A_3 = -5.625$, $\varpi = 0.8$, $y = 1.2$, $t = 1$, $EE = 0.75$.

Family 4: Let $\mu \neq 0, \lambda \neq 0, \lambda^2 - 4\mu = 0$. Consequently, the TWSs of Eq. (1) are found as

$$u_{1,4}(x, y, t) = \left(\frac{1}{6}\chi^2\left(\frac{\lambda^2(-8 + \varsigma(4 + \varsigma))}{(2 + \varsigma)^2} - 4\mu\right)\right), \tag{28}$$

$$v_{1,4}(x, y, t) = \left(-\frac{2\chi^2\lambda^2}{(2 + \varsigma)^2}\right), \tag{29}$$

$$z_{1,4}(x, y, t) = \left(\frac{2\varpi\chi^2\lambda^2}{(2 + \varsigma)^2}\right), \tag{30}$$

where, $\xi = \text{EE} + \chi(-\varpi t + x + y)$, $\varsigma = \xi\lambda$.

Family 5: Let $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$. Consequently, the TWSs of Eq. (1) are obtained as

$$u_{1,5}(x, y, t) = \left(-\frac{2\chi^2}{\xi^2}\right), \tag{31}$$

$$v_{1,5}(x, y, t) = \left(-\frac{2\chi^2}{\xi^2}\right), \tag{32}$$

$$z_{1,5}(x, y, t) = \left(\frac{2\varpi\chi^2}{\xi^2}\right), \tag{33}$$

where, $\xi = \text{EE} + \chi(-\varpi t + x + y)$.

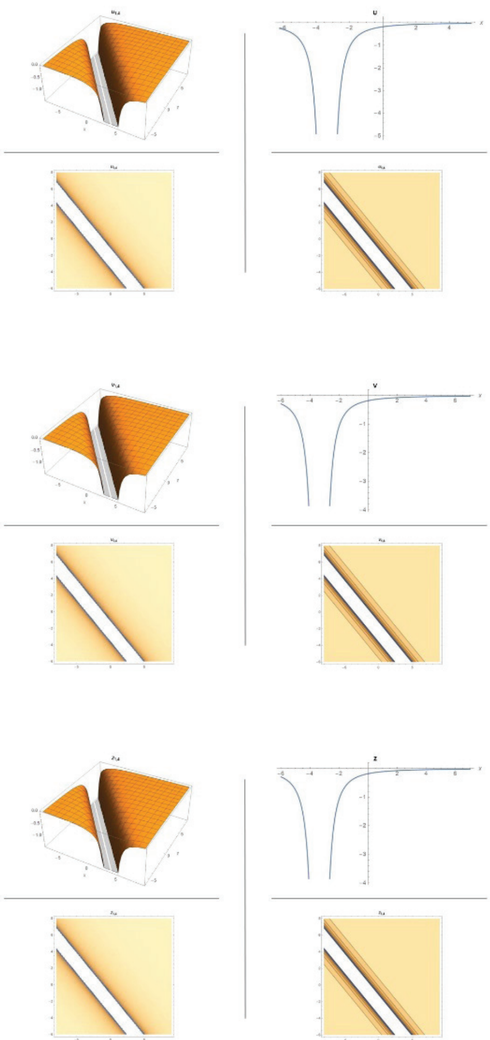

Figure 4. 2D, 3D, density, contour plots of Eqs. (28)-(30) at $\lambda = 2$, $\mu = 1$, $B_0 = 0.2$, $\chi = 1.5$, $\alpha = -1$, $B_1 = 1.25$, $A_0 = -0.9$, $A_1 = -7.425$, $A_2 = -12.15$, $A_3 = -5.625$, $\varpi = -1$, $y = 1.2$, $t = 1$, $EE = 0.75$.

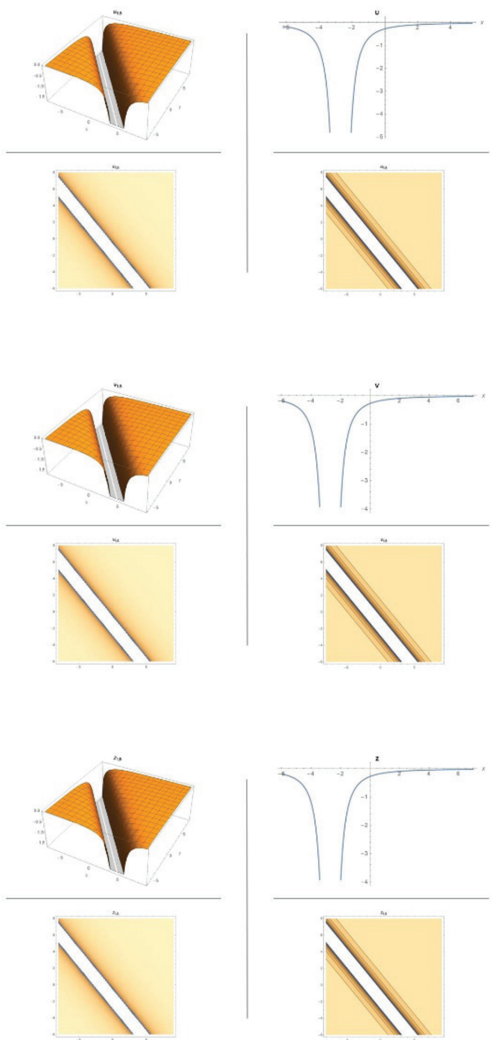

Figure 5. 2D, 3D, density, contour plots of Eqs. (31)-(33) at $\lambda = 0$, $\mu = 0$, $B_0 = 0.2$, $\chi = 1.5$, $\alpha = -1$, $B_1 = 1.25$, $A_0 = 0$, $A_1 = 0$, $A_2 = -0.9$, $A_3 = -5.625$, $\varpi = -1$, $y = 1.2$, $t = 1$, $EE = 0.75$.

CONCLUSION

In the paper, the TWSs of (2+1)-dimensional generalized HSIE by utilizing the MEFM are acquired. In Mathematica software, we obtain the TWSs of (2+1)-dimensional generalized HSIE. Two dimensional, three dimensional, density and contour plots of the TWSs by choosing the suitable parameters have been plotted in Mathematica software. The proposed method is predicted to be an exceedingly efficient way for acquiring exact solutions of such NPDEs. The derived solutions are anticipated to be beneficial in elucidating the behavior of frequency waves within the realm of physics.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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