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Research Article

Advancing logistics and resource management quality through two-stage tandem queues and single vacation techniques

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ABSTRACT

The research delves into the theoretical framework of tandem queues, elucidating their potential in optimizing resource allocation, minimizing wait times, and improving overall operational efficiency. The integration of strategic vacations aims to balance the workload, allowing logistics and resource management teams to rejuvenate during non-peak periods, ensuring heightened focus and attentiveness during critical phases. Furthermore, the study explores the impact on employee well-being, recognizing the importance of workforce satisfaction in sustaining high-quality service. The objective is to enhance the quality of service provided to customers or stakeholders within the system. This could involve reducing waiting times, ensuring timely delivery of goods or services, and maintaining consistent service levels even during periods of peak demand or resource constraints. The novelty is that optimizing warehouse operations is crucial for efficient logistics. Implementing two-stage tandem queues can help manage incoming and outgoing goods more effectively. By strategically allocating resources and employing single vacation techniques, warehouses can reduce congestion, minimize storage costs, and improve order fulfillment speed method dynamically adjusts resource utilization and server downtime to adapt to changing demand patterns, resulting in enhanced operational resilience and cost-effectiveness. The findings presented in the paper have broad applications across various industries and sectors, especially logistics and resource management, providing valuable insights and guidance for optimizing system performance, enhancing efficiency, and improving customer satisfaction and service quality. In this paper, we derive steady-state probability, Mean number of customers, and waiting time by using the Birth-death process.

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INTRODUCTION

A two-stage tandem queuing model is a specific configuration within the field of queuing theory where customers move through a system consisting of two sequential stages or servers. Each stage represents a separate queue, and customers must complete service at one stage before moving

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on to the next. The single vacation model typically refers to a system where a server or a station takes a vacation after completing service to a customer and remains idle until the vacation ends. In the contemporary landscape of global commerce, logistics, and resource management play pivotal roles in determining the efficiency and competitiveness of organizations. The increasing complexity of supply chains and resource allocation necessitates innovative approaches to enhance the quality of logistics operations. This research focuses on advancing logistics

This study explores the integration of tandem queues and single vacation techniques to provide a more adaptive and responsive framework for logistics and resource management. The rationale behind this research stems from the need to address the evolving demands and complexities inherent in contemporary logistics. Tandem queues, representing a series of linked queues, offer a dynamic model that aligns with the sequential nature of logistics processes. Additionally, the introduction of single vacation techniques, where certain resources are strategically put on temporary breaks during periods of low demand, presents an innovative solution to optimize resource utilization. The synthesis of these techniques aims to enhance the overall quality of logistics and resource management, promoting efficiency and cost-effectiveness.

Assess the impact of the proposed model on key performance indicators such as throughput, resource utilization, and response times. Provide practical insights through simulations and case studies to demonstrate the real-world applicability of the proposed logistics and resource management framework. The significance of this research lies in its potential to revolutionize how logistics and resource management systems operate by leveraging tandem queues and single vacation techniques, organizations can dynamically adapt to changing demands, minimize resource idle times, and ultimately enhance the overall quality of their logistics operations. The outcomes of this study aim to contribute valuable insights to practitioners, researchers, and decision-makers involved in optimizing supply chain processes.

The logistics and resource management landscape faces challenges arising from the dynamic nature of global supply chains, fluctuating demand, and the need for effective resource allocation. Tandem queuing models play a crucial role in optimizing system performance across various domains, including communication networks, cloud computing, and industrial engineering. Kim and Dudin [1] introduced a priority-based tandem queuing model with admission control, which effectively manages traffic flow by prioritizing service requests. Reddy et al. [2] investigated tandem queues with three multi-server units and bulk service, emphasizing the efficiency of handling large-scale service demands. Gomez-Corral [3] analyzed a tandem queue with blocking under a Markovian Arrival Process, providing insights into the impact of arrival patterns on system congestion. Keerthika et al. [4] conducted a comprehensive survey on tandem queuing models, outlining their scope and applications, while their subsequent work [5] introduced a two-stage retrial tandem queuing model incorporating working vacation techniques to enhance QoS in internet and web services. Additionally, Keerthika et al. [6] applied artificial intelligence to improve queue efficiency by modeling reneging behavior in tandem queuing systems. Nazarov et al. [7] examined the scaling limits of tandem queues with infinite orbits, offering a theoretical perspective on queue dynamics. Sun et al. [8] explored the performance of mobile cloud computing under bursty demand conditions, demonstrating the role of queuing models in managing fluctuating workloads. Larionov et al. [9] utilized NS-3 simulations to calibrate a tandem queue with PH service time, providing practical insights into multi-hop wireless networks. Amuthan et al. [10] proposed a congestion-adaptive routing mechanism using multi-stage tandem queuing models to optimize MANET performance. Wu et al. [11] focused on finite buffer capacity in tandem queues, addressing real-world constraints on system resources. Earlier foundational studies by Kim and Dudin [12], Reddy et al. [13], and Gomez-Corral [14] have significantly contributed to the understanding of tandem queuing models by examining priority-based control, multi-server bulk service, and blocking mechanisms. These studies collectively advance the theoretical and practical applications of tandem queuing models in optimizing system performance and service quality.

Tandem queuing models and their applications have been extensively explored across various domains, including production systems, queuing networks, and logistics. Jia and Zhang [15] analyzed serial production lines with geometric machines and finite production runs, providing insights into performance optimization and system-theoretic properties. Lee et al. [16] investigated Bernoulli production lines with waiting time constraints, emphasizing the importance of queue control in manufacturing systems. Kalyanaraman and Sundaramoorthy [17] introduced a Markovian working vacation queue with server state-dependent arrival rates and partial breakdowns, contributing to queue stability and efficiency. Baumann and Sandmann [18] examined a multi-server tandem queue with a Markovian arrival process, phase-type service times, and finite buffers, demonstrating the impact of arrival and service processes on system performance. Avrachenkov and Yechiali [19] studied tandem blocking queues with a common retrial queue, offering a novel perspective on congestion management and retrial mechanisms. Kim et al. [20] analyzed a tandem retrial queuing system with correlated arrival flow, where the operation of the second station is described by a Markov chain, enhancing the understanding of retrial queuing behavior. Valentina and Dudina [21] explored a retrial tandem queue with a controllable strategy for repeated attempts, focusing on dynamic control policies to optimize queue performance. Andrew and Grindlay [22] examined tandem queues with dynamic priorities, highlighting the role of priority-based service mechanisms.

Kamoun [23] conducted a performance analysis of two priority queuing systems in tandem, further extending the research on priority-based service disciplines. Dudin et al. [24] investigated a tandem queue with multi-server stages and group service at the second stage, while their subsequent study with Chakravarthy [25] examined blocking and group service mechanisms in a tandem queuing system, contributing to the analysis of multi-stage service systems. Niranjan et al. [26] developed a multiple-control policy for an unreliable two-phase bulk queuing system incorporating active Bernoulli feedback and vacation, offering valuable insights into system reliability and performance optimization. Lastly, Chatterjee and Mohanty [27] studied inbound logistics in an automobile manufacturing supply chain, demonstrating the practical relevance of tandem queuing models in optimizing industrial operations. Collectively, these studies contribute to the theoretical and practical advancements of tandem queuing models, providing insights into production line optimization, priority queuing, retrial mechanisms, and congestion management across various sectors.

Literature reveals a limited exploration of the integration of tandem queues and single vacation techniques in the logistics and resource management domains. The synergy of these two approaches offers a promising avenue for addressing the challenges of resource allocation in dynamic supply chain environments. This study intends to fill that gap by providing a thorough understanding of the combined impact on logistics quality. Prior studies often employed various performance metrics, such as throughput, response times, and resource utilization, to evaluate the effectiveness of logistics and resource management models.

The novelty lies in creating a comprehensive framework that addresses both the sequential nature of processes within logistics and resource management and the dynamic scheduling of resources to optimize efficiency. This integrated approach allows for more precise modeling, analysis, and optimization of complex systems, leading to improved operational outcomes and resource utilization. The paper contributes to filling a research gap in the field of logistics and resource management by proposing innovative solutions to existing challenges. By exploring the application of two-stage tandem queues and single vacation techniques in this context, the paper advances the theoretical understanding and practical implementation of queuing theory in logistics operations.

ADVANTAGE OF THE MODEL

- **Improved Efficiency:** The model can lead to enhanced efficiency in logistics and resource management processes by optimizing queuing systems and resource utilization. This can result in reduced waiting times, faster throughput, and increased productivity.
- **Resource Optimization:** By employing two-stage tandem queues and single vacation techniques, the model facilitates better utilization of resources such as warehouse space, vehicles, and manpower. This leads to reduced idle time and improved overall resource efficiency.
- **Enhanced Service Quality:** Optimizing queuing processes and resource management can lead to better service quality in terms of faster order processing, timely delivery, and improved customer satisfaction. This can help organizations gain a competitive edge in the marketplace.
- **Cost Savings:** The model can help organizations reduce costs associated with resource downtime, inventory holding, and inefficient operations. By minimizing idle time and optimizing resource utilization, organizations can achieve significant cost savings in their logistics operations

Figure 1 describes the following components: The input stage is the initial stage where incoming tasks, orders, or requests are received by the system. These inputs could be customer orders, production requests, service inquiries, or any other form of demand placed on the system. Processing Stages the system comprises two processing stages organized in tandem, where tasks move sequentially from one stage to the next. Each processing stage involves the allocation of resources, such as servers, machinery, or personnel, to perform specific tasks or services. The output Stage is the final stage where completed tasks or processed items are delivered or made available to customers or downstream processes. Outputs from the system could include finished products, completed service requests, or processed goods. The graphical depiction in Figure 1 illustrates the operational efficiency and throughput of a tandem queue system incorporating logistical considerations.

System Description and Assumptions

The logistics and resource management system under study is a complex network involving multiple stages of processing and resource allocation. It serves to streamline the flow of goods, services, or tasks through a series of interconnected stages, each contributing to the overall fulfillment of customer demands or operational objectives. We take into account a tandem queuing system with a single server vacation with the following assumptions:

- Customers are expected to arrive in the batch queue using Poisson processes with a mean arrival rate. Customers will be served in the order that they come. The service is hosted on two servers S_1 and S_2. The first service server provides batch service to all customers in the queue. When the batch service is finished, the server quickly switches to the second server to serve all of the customers in the single service. Batch service times are assumed to follow an exponential distribution μ_1, and single service times are assumed to be exponentially distributed μ_2.
- The single vacation model typically refers to a system where a server or a station takes a vacation after

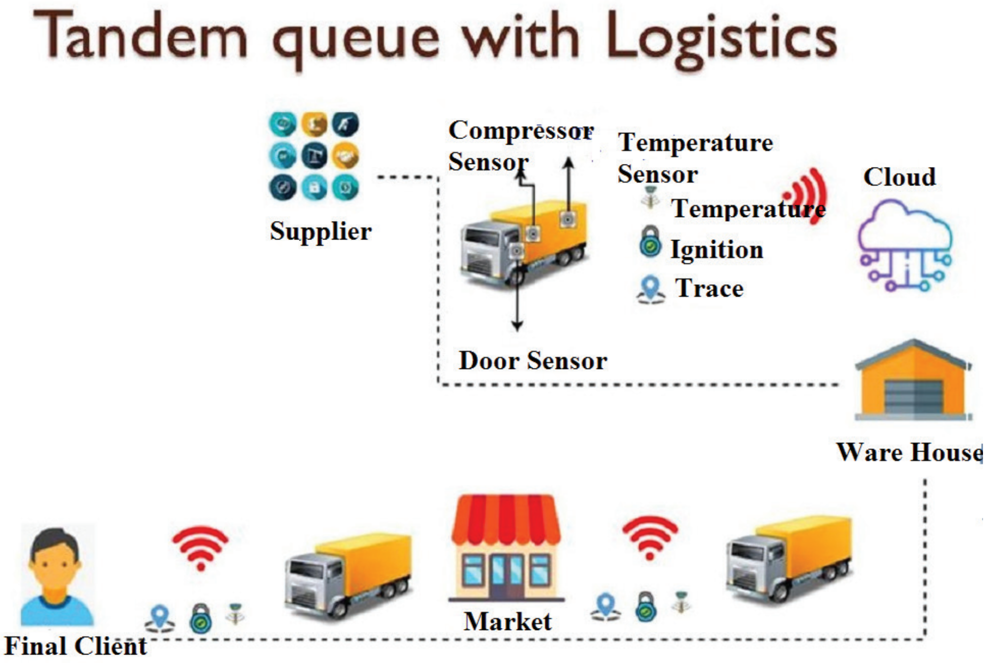

Figure 1. Tandem queue with logistics.

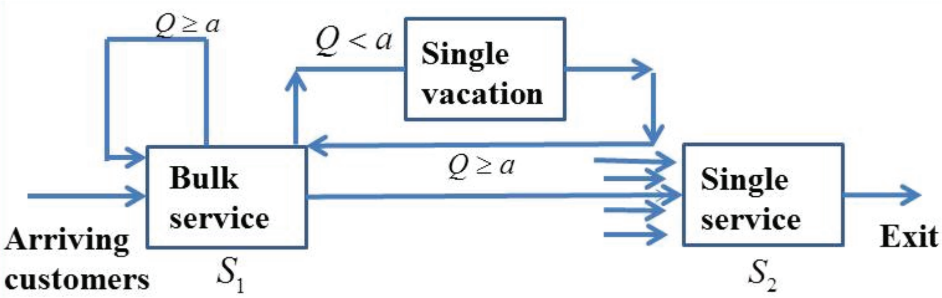

Figure 2. Schematic representation of tandem queue with single server vacation.

completing service to a customer and remains idle until the vacation ends. The vacation rate is denoted by υ.

- We adopt the following notations used in the queuing equations which are as follows
- χ-arrival rate at server S_1
- μ_1-service rate at server S_1
- x_1-number of customers at service at S_1
- Z-total number of customers at S_1
- $W_q^{(1)}$- A customer's queue waits time at S_1
- b- Maximum batch size being served at S_1
- μ_2-service rate at server S_2
- y_1-number of customers at service at S_2
- $W_q^{(2)}$-A customer's queue waits time at S_2
- W- A customer's total waiting time in the system.
- υ- Single vacation rate.
- The proposed model is represented schematically in Figure 2. Where Q-Queue length, a - minimum capacity

Steady–State Probabilities

For this model, the possible states are given below in the table

$Q_{j,l,m}$=Probability that the system is in the state j, with l customers waiting at S_1 and m customers waiting at S_2. (Since the arrivals to the system S_1 are Poisson with

Table 1. State description for the Tandem queuing model with vacation

State	Description
0	The system is empty, no service at both servers
1	Customers in service at S_1 only
2	Customers in service at S_2 only
3	Customers are in process at both S_1 and S_2
4	The server is on vacation

parameter χ and service rate is exponential with parameters μ_1 and μ_2 respectively), we can write the difference equation

$$Q_{0,0,0}(t+h) = Q_{0,0,0}(t)(1-\chi h) + Q_{1,0,0}(t)\mu_1 Q_0 h$$

$$Q_{0,0,0}(t+h) - Q_{0,0,0}(t) = -Q_{0,0,0}(t)\ \chi h + Q_{1,0,0}(t)\mu_1 Q_o h$$

$$Lim_{h\to 0}\frac{Q_{0,0,0}(t+h) - Q_{0,0,0}(t)}{h} = -\chi Q_{0,0,0}(t) + \mu_1 Q_o Q_{1,0,0}(t)$$

Assuming steady state, the above equation reduces to

$$0 = -\chi Q_{0,0,0} + \mu_1 Q_0 Q_{1,0,0} \tag{1}$$

Similarly the other steady state equations can be written as

$$(\chi+\mu_1)Q_{1,0,0} = \chi Q_{0,0,0} + \mu_2 Q_0 Q_{1,1,0} + \mu_2 Q_{2,0,0} \tag{2}$$

$$(\chi+\mu_1)Q_{1,l,0} = \chi Q_{1,l-1,0} + \mu_1 Q_0 Q_{1,l+1,0} + \mu_2 Q_{2,l-1,0},\quad 1\le l\le M-1 \tag{3}$$

$$(\chi+\mu_1)Q_{1,l,0} = \chi Q_{1,l-1,0} + \mu_1 Q_{3,l-1,0} \tag{4}$$

$$(\chi+\mu_2)Q_{2,0,0} = \mu_1 Q_{1,0,0} + \mu_1 Q_0 Q_{3,0,0} \tag{5}$$

$$(\chi+\mu_2)Q_{2,0,m} = \mu_1 Q_{3,0,m-1} + \mu_1 Q_0 Q_{3,0,m}, 1\le m\le N \tag{6}$$

$$\begin{aligned}(\chi+\mu_1+\mu_2)Q_{3,0,0} = {} & \mu_1 Q_0 Q_{3,1,0} + \mu_1 Q_1 Q_{1,1,0} + \mu_2 Q_{2,0,1}\\ & + \chi Q_{2,0,0}\end{aligned} \tag{7}$$

$$\begin{aligned}(\chi+\mu_1+\mu_2)Q_{3,l,0} = {} & \chi Q_{3,l-1,0} + \mu_2 Q_{3,l-1,1} + \mu_1 Q_0 Q_{3,l-1,1}\\ & + \mu_1 Q_1 Q_{3,l+1,0} + \mu_1 Q_1 Q_{1,l+1,0}\end{aligned} \tag{8}$$

$$(\mu_1+\mu_2)Q_{3,N-1,0} = \chi Q_{3,N-2,0} + \mu_2 Q_{2,N-1,1} + \mu_1 Q_1 Q_{1,N,0} \tag{9}$$

$$\begin{aligned}(\chi+\mu_1+\mu_2)Q_{3,0,l} = {} & \mu_1 Q_0 Q_{3,1,l} + \mu_1 Q_1 Q_{3,1,l-1} + \mu_1 Q_{2,0,l+1}\\ & + \chi Q_{2,0,l},\ 1\le l\le M\end{aligned} \tag{10}$$

$$\begin{aligned}(\chi+\mu_1+\mu_2)Q_{3,l,m} = {} & \chi Q_{3,l-1,m} + \mu_1 Q_0 Q_{3,l+1,m} + \mu_1 Q_1 Q_{3,l+1,m-1}\\ & + \mu_2 Q_{3,l-1,m+1}, 1\le l,m\le N-1\end{aligned} \tag{11}$$

$$(\chi+\mu_1)Q_{4,0,N} = \mu_0 Q_{3,0,N} \tag{12}$$

$$(\chi+\mu_1)Q_{4,l,N} = \chi Q_{4,l-1,N} + \mu_1 Q_{3,l,N}, 1\le l\le N \tag{13}$$

The characteristic equation for the difference equation is

$$\mu_1 z^{l+1} - (\chi+\mu_2)z + \chi = 0 \tag{14}$$

To find the $Q_{0,0,0}$ using the normalizing condition is

$$\sum_{j=0}^{5}\sum_{l=0}^{M}\sum_{m=0}^{N} Q_{j,l,m} = 1$$

From (1)

$$\frac{\chi}{\mu_1}Q_{0,0,0} = \sum_{j=1}^{5} Q_{j,0,0}$$

$$Q_{0,0,0}\left[\frac{\chi}{\mu_1} - \sum_{m=1}^{b}\left(\frac{\chi}{\chi+\mu_1}\right)^m\right] = \upsilon\frac{\mu_1}{\chi}\frac{1}{1-\frac{\chi+\mu_2}{\chi}\upsilon}\sum_{m=1}^{b}\left(\frac{\chi}{\chi+\mu_1}\right)^m [1-(\frac{\chi+\mu_2}{\chi}\upsilon)^m \tag{15}$$

L.H.S of the above equation is

$$= Q_{0,0,0}[\frac{\chi}{\mu_1} - \frac{\chi}{\chi+\mu_2}[1+\frac{\chi}{\chi+\mu_2} + \ldots\ldots.. + \left(\frac{\chi}{\chi+\mu_2}\right)^{m-1}$$

$$\begin{aligned} &= Q_{0,0,0}\left\{\frac{\chi}{\mu_1} - \frac{\chi}{\chi+\mu_2}\frac{1-\left[\frac{\chi}{\chi+\mu_1}\right]^m}{1-\frac{\chi}{\chi+\mu_1}}\right\}\\ &= Q_{0,0,0}\left\{\frac{\chi}{\mu_1} - \frac{\chi}{\mu_2}[1-\left(\frac{\chi}{\chi+\mu_1}\right)^m\right\}\\ &= Q_{0,0,0}\left\{\frac{\chi}{\mu_1}[\frac{\chi}{\chi+\mu_1}]^m\right\}\end{aligned} \tag{16}$$

RHS of the equation is (15) is

$$\begin{aligned} &= \frac{\mu_1\upsilon}{\chi}\frac{1}{[1-\frac{[\chi+\mu_1]\upsilon}{\chi}}\left\{\frac{-\chi}{\mu_1}\left[\frac{\chi}{\chi+\mu_2}\right]^m\right\}\\ &= \upsilon\frac{1}{\frac{\chi+\mu_1}{\chi}\upsilon - 1}\left[\frac{\chi}{\chi+\mu_2}\right]\end{aligned} \tag{17}$$

Thus we get,

$$Q_{0,0,0}\frac{\chi}{\mu_1}\left[\frac{\chi}{\chi+\mu_1}\right]^m = \upsilon\frac{1}{\frac{\chi+\mu_1}{\chi}}\left[\frac{\chi}{\chi+\mu_1}\right]^m$$

Hence

$$Q_{0,0,0} = \frac{\mu_1}{\chi}\frac{1}{\left[\frac{\chi+\mu_2}{\chi}\right]} \quad \text{Where } \rho = \frac{\chi}{\chi+\upsilon} \tag{18}$$

Mean Number of Customers

The average number of units waiting in the queue at server 1 is given by

$$E(x_1) = \sum_{l=1}^{b} lQ_{j,l,0}$$

$$E(x_1) = Q_{0,0,0}\sum_{l=1}^{b} l\left[\frac{\chi}{\chi+\mu_1}\right]^l + \frac{\mu_1\upsilon}{\chi}[-\frac{\chi}{\mu_1 v^{l+1}}]\sum_{l=1}^{b} l[(\frac{\chi}{\chi+\mu_1})^b$$

$$= \frac{\mu_1}{\chi}\frac{1}{\left[\frac{\chi+\mu_2}{\chi}\right]}\sum_{l=1}^{b} l\left[\frac{\chi}{\chi+\mu_1}\right]^l + \frac{\mu_1\upsilon}{\chi}[-\frac{\chi}{\mu_1 v^{l+1}}]\sum_{l=1}^{b} p[(\frac{\chi}{\chi+\mu_1})^b \tag{19}$$

$$E(x_1) = \frac{\mu_1}{\chi}\left[\frac{1}{\theta}\sum_{l=1}^{b} l\theta^l + \upsilon^l\sum_{l=1}^{b} l\theta^b\right]$$

The average number of queued units at server 2 is provided by

$$E(y_1) = \sum_{l=1}^{b} lQ_{j,0,m}$$

The average number of queued units at server 2 is calculated as

$E(y_1) = G'(1)$ as given in M. L. Chaudhary and J.G. Templeton [1983]

$$G(z) = \prod_1^b \frac{1-h_p}{1-h_p z}$$

Taking log

$$\log G(z) = \sum_{p=1}^{b} \log(1-h_p) - \sum_{p=1}^{b} \log(1-h_p z)$$

Differentiating $\frac{G'(z)}{G(z)} = \sum_{m=1}^{b} \frac{h_p}{1-h_p z}$

Putting z=1 we get $\sum_{p=1}^{b} \frac{h_p}{1-h_p}$

Thus Mean number of customers in the system is given by

$$E(z) = E(x_1) + E(y_1)$$

$$\text{E (z)} = = \frac{\mu_1}{\chi}\left[\frac{1}{\theta}\sum_{l=1}^{b} l\theta^l + \upsilon^l \sum_{l=1}^{b} l\theta^b\right] + \sum_{p=1}^{b} \frac{h_p}{1-h_p} \tag{20}$$

Waiting Time Distribution

The p. d. f of $W_q^{(1)}$ is given by

$$\omega_1(t) = Q_{o,o,o} + \sum_1^{b-1} Q_{0,l,0} + \sum_1^{b-1} Q_{0,0,m}$$

The average waiting time in the queue at S_1 is given by

$$E(w_q^{(1)}) = \int_0^\infty t\omega_1(t)dt = \frac{\chi}{\mu_1(1-\frac{\chi}{\mu_1})} v \tag{21}$$

The average waiting time in the queue at S_2 is given by

$$E(w_q^{(2)}) = \left[\sum_{k=1}^{b} (1-h_m)^{-1} - b + \frac{1}{2}(\frac{m_2}{\bar{a}} - 1)\right] / \mu_2 \tag{22}$$

Where $m_2 = \sum_1^b j^2 q_j$ and $\bar{a} = \sum_1^b jq_j$

The mean waiting time in the system is

$$E(w) = E(w_q^{(1)}) + E(w_q^{(2)})$$

Numerical Analysis

An effect of arrival rate on average size and waiting time is given in Table 1 and Figure 3; we can see that,when the arrival rate increases, it means that more customers are entering the system per unit of time. This leads to a higher average number of customers in the system because arrivals are outpacing the rate at which customers are being served. Consequently, the waiting time also increases because more customers are competing for service, resulting in longer queues.

From Fig.4 and Table 2, we can observe that, When the service rate increases, it means that more customers are being served per unit of time. This reduces the time each customer spends in the system, leading to a decrease in waiting time. However, as the service rate increases, more customers can be served in the same amount of time, which can lead to an increase in the average number of customers in the system. This is because even though customers are being served faster, there are more of them entering the system due to shorter waiting times, leading to a higher overall customer load.

Table 2. Arrival rate vs performance measure

χ	ρ	$E(z)$	$E(w)$
2.00	0.0823	10.0276	1.8278
2.50	0.1977	11.0282	2.3486
3.00	0.1231	17.0081	3.0426
3.50	0.1375	24.4203	3.6089
4.00	0.1528	33.3902	4.4006
4.50	0.1682	43.0055	5.1323
5.00	0.1791	55.4542	5.8115
5.50	0.1846	71.9373	6.7568
6.00	0.2000	107.125	7.5518

Table 3. Service rate vs performance measure

μ_1	ρ	$E(z)$	$E(w)$
4.00	1.1447	78.2610	7.8261
4.10	1.1433	77.0575	7.6256
4.20	1.1419	76.7096	7.4724
4.30	1.1406	75.2629	7.4148
4.40	1.1395	74.7240	7.3992
4.50	1.1354	74.3297	7.3702
4.60	1.1321	74.1532	7.3102
4.70	1.1306	74.1279	7.3006
4.80	1.2906	73.9260	7.2060

Figure 3 represents the expected queue length and waiting time when $\mu_1 = 4$, $\mu_2 = 5$, $\upsilon = 3$.

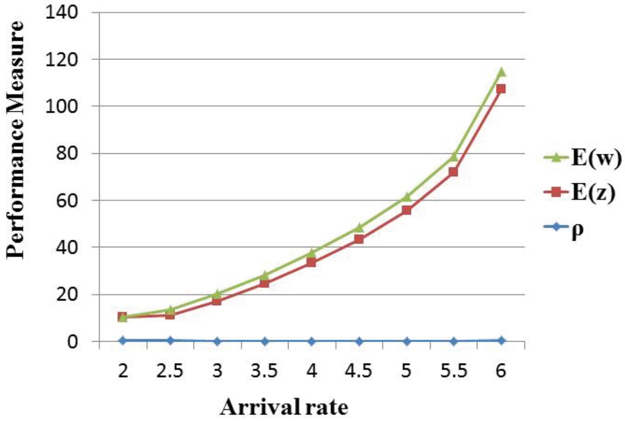

Figure 3. Arrival rate vs performance measures.

Figure 4 represents the Expected Queue length and Waiting time when $\chi = 5$, $\mu_2 = 3$, $\upsilon = 6$.

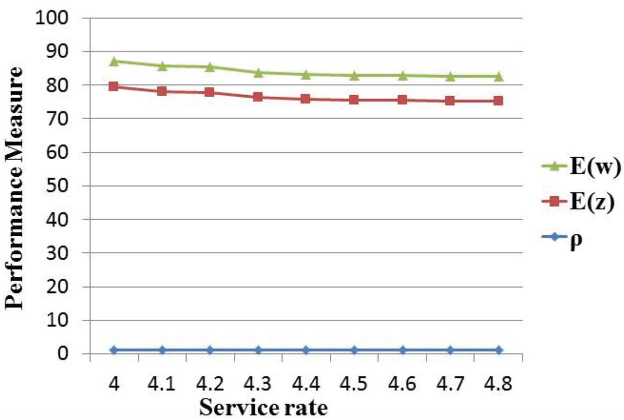

Figure 4. Service rate vs performance measure.

CONCLUSION

This study has delved into the application of two-stage tandem queues and single vacation techniques to advance the quality of logistics and resource management. Through a thorough analysis of the proposed model, several key insights have emerged. The results of this study highlight the significance of adopting advanced queuing techniques in logistics and resource management. The findings can serve as a valuable guide for practitioners and decision-makers in optimizing their operations, improving resource utilization, and ultimately enhancing the quality of service delivery. In this article, we have analyzed the Mean number of customers and waiting time distribution by using the Birth-death process. Numerical illustrations are given for some of the performance measures. We can observe that higher arrival rates lead to more customers entering the system, increasing the average number of customers and prolonging waiting times due to heightened competition for service, and Elevated service rates decrease customer wait times by speeding up service delivery. However, this can also elevate the average number of customers in the system as faster service attracts more customers, potentially increasing the overall system load.

With growing concerns about environmental sustainability and resource conservation, future research could focus on integrating sustainability criteria into queueing models and optimization algorithms. This could include incorporating energy consumption metrics, carbon footprint reduction goals, and other sustainability indicators into the decision-making process. By optimizing operations with sustainability in mind, organizations can reduce their environmental impact while also improving operational efficiency and cost-effectiveness.

AUTHORSHIP CONTRIBUTION

Authors equally contributed to this work

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the findings of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflict of interest with respect to the research, authorship, a and/or publication of this article

ETHICS

There are no ethical issues with the publication of this manuscript.

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