Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr

Research Article

Investigation of fractional order covid-19 model with q-homotopy analysis transform method

Mehmet MERDAN[1],* , **Pınar AÇIKGÖZ[1]**

[1]Department of Mathematical Engineering, Gümüşhane University, Gümüşhane, 29100, Türkiye

ARTICLE INFO

ABSTRACT

The primary objective of this paper is to derive approximate analytic solutions for the time fractional-order Covid-19 model by employing the q-homotopy analysis transform method (q-HATM) and the Laplace transform homotopy perturbation method (LT-HPM). The covid 19 model is a system of five-dimensional nonlinear ordinary differential equations. Moreover, this method applies even to more complex partial differential equations originating from mathematical biology, demonstrating computational efficiency and wide application. In this study, Caputo fractional derivative is used to obtain the fractional system and Laplace transformation is applied to analyze the approximate solutions of the system. Graphical results are presented and discussed quantitatively to illustrate the approximate solution.

Cite this article as: Merdan M, Açıkgöz P. Investigation of fractional order covid-19 model with q-homotopy analysis transform method. Sigma J Eng Nat Sci 2025;43(2):582–597.

INTRODUCTION

Mathematical modeling of real-life phenomena is used in many fields such as biology, chemistry, psychologcy, economy etc. For instance, financial models often contain stochastic equations whereas ordinary differential equation systems are the basic tools for modeling the spread of infectious diseases. The use of fractional analysis should be specifically mentioned,for current trend in mathematical modeling is to use different types of fractional derivatives with numerous solution methods to obtain the exact,approximate or numerical solutions of the system being analyzed.

Fractional analysis can be regarded as an extension of conventional calculus, wherein the orders of differentiation and integration must be integers. Fractional analysis can be seen as an extension of conventional calculus, whereby the sequences of differentiation and integration are restricted to integers. Fractional operators such as Caputo-Fabrizo fractional derivative or Riemann-Liouville fractional integral authorize a more efficient systematic modeling of recognized real-life problems. Therefore, this issue has received increasing attention in the last few decades, with many studies on chemistry, astrophysics and biological systems. The emergence of new definitions for fractional operators also increases the intensity of research in this field.

A viral infection caused by a newly discovered virus called coronavirus (SARS-CoV-2) causes the novel coronavirus disease. This virus is a member of the Coronaviridae virus family, which is responsible for COVID-19. The first COVID-19 outbreak occurred at the end of 2019 in the

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***Corresponding author.**
*E-mail address: mehmetmerdan@gmail.com

This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

Chinese city of Wuhan. Because of its rapid spread, this virus is mostly seen in the form of infectious cases, deaths, and economic losses[1-2] . The pandemic impacts not only the human population but also the economic growth of nations, resulting in significant material and moral losses. A significant number of individuals became unemployed, and several industries and enterprises ceased operations. A recent World Health Organization (WHO) estimate indicates that 10.27 million individuals tested positive, with over 1.52 million fatalities recorded by December 12, 2020. Official statistics indicate that 42.2 million individuals have entirely recovered from the virus, with the recovery rate surpassing the mortality rate. No approved pharmaceuticals or vaccines presently exist that provide protection against or cure COVID-19. Medications used to infected individuals generally serve as adjunctive therapy focused on alleviating symptoms. Most of the proposed mathematical models for COVID-19 are formulated with the help of ordinary, partial or stochastic differential equations. Recent studies on the mathematical model of the epidemic are given in [3-6]. In recent years, researchers have been trying to mathematically model various processes and phenomena in order to study the dynamics of such complex systems. Since the accuracy of fractional models in simulating the behavior of an existing system is much higher than models based on integer order, fractional order models are examined instead of integer order models[7-15].

This work analyzes a time fractional Covid-19 system, a nonlinear epidemic model, using q-HATM. Acquiring precise solutions for fractional differential equations proves to be more challenging than for their traditional integer-order counterparts.Therefore, more efficient numerical and semi-analytical techniques have been developed to find approximate solutions to such problems. Some of these techniques are the Adomian decomposition method (ADM) [16-18], Laplace decomposition method (LDM)[19-21], Homotopy analysis method (HAM)[22-25], Homotopy perturbation method (HPM)[26] and Variational iteration method[27]. Another very powerful method is the q-homotopy analysis transform method (q-HATM) [28-35].The q-homotopy analysis transform method ıs a more powerful method than the q-homotopy analysis method. Because the existence of $\left(\frac{1}{n}\right)^m$ term ın q-hatm gets a faster solutıon than q-ham.

The covid-19 model, which we discussed in our study and found approximate analytical solutions, is as follows.

$S(t) = \Lambda - \phi_c IS - \mu S + v_3 R$

$E(t) = \phi_c IS - (\sigma + \mu)E$

$I(t) = \sigma E - (\alpha + v_2 + \delta + \mu)I + v_1 R$

$Q(t) = \alpha I - (\theta + \mu)Q$

$R(t) = \theta Q - (v_1 + v_3 + \mu)R + v_2 I$

Where, susceptible individuals $S(t)$, exposed individuals $E(t)$, infected individuals $I(t)$, quarantined individuals $Q(t)$ and recovered individuals $R(t)$.

Λ represents the recruitment rate of individuals, ϕ_c denotes the contact rate of Covid-19 transmission, v_1 indicates the recovery rate from $I(t)$, θ signifies the recovery rate from $Q(t)$, μ is the natural death rate, α reflects the detection rate from $I(t)$, v_2 represents the relapse rate of Covid-19, and v_3 denotes the rate at which recovered individuals revert to the susceptible category. The initial conditions of the model are as follows.

$S(0) = 5000, E(0) = 2003, I(0) = 416, Q(0) = 404,$ $R(0) = 115$.

This article uses the Laplace Transform Homotopy Perturbation Method (LT-HPM) and the q-homotopy analysis transform method (q-HATM) to compute the fractional order solution of the Covid 19 model. Definitions and essential theorems pertaining to fractional differentiation are provided in Chapter 2. Chapter 3 provides an overview of the core approaches of the Laplace Transform Homotopy Perturbation Method (LT-HPM) and the q-homotopy analysis transform method (q-HATM). The fractional order Covid 19 model was used to calculate the results in the last section.

MATERIALS AND METHODS

Definition 1. The Riemann-Liouville fractional derivative of order $0 < \alpha < 1$ of a $f(x) \in C^1 [0,b]$, $b > 0$ function is defined as [36]

$$^{RL}_0 D_x^\alpha f(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} f(t)dt \qquad (1)$$

Definition 2. The Riemann-Louville fractional integral of order $0 < \alpha < 1$ of a function is defined as [36]

$$^{RL}_0 I_x^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt \qquad (2)$$

where $\Gamma(.)$ represents the Gamma function.

Definition 3. The Caputo fractional derivative of order $\alpha > 0$ of a function $\omega \in C^\mu$ ($\mu \geq 1$) is defined in the context of Caputo as follows:

$$^C_0 D_t^\alpha \omega(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\vartheta)^{m-\alpha-1} \omega^m(\vartheta)d\vartheta \qquad (3)$$

where $m - 1 < \alpha < m \in \mathbb{N}$. If $\alpha = m \in \mathbb{N}$, then

$$D_t^\alpha \omega(t) = \frac{d^m \omega(t)}{dt^m}.$$

Definition 4: The Laplace transform of function is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$$

for all real numbers $t \geq 0$ if the integral exists.

Definition 5. [37-39] The Laplace transform of the transform of the Caputo fractional derivative $D_t^\alpha \omega(t)$ is defined as

$$\mathcal{L}[D_t^\alpha \omega(t)] = s^\alpha \mathcal{L}[\omega(t)](s) - \sum_{k=0}^{m-1} s^{\alpha-k-1}\omega^k(0^+) \quad (4)$$

where $\alpha > 0$, $m - 1 < \alpha < m$ and $\mathcal{L}$ denotes the Laplace transform operator. In the literature, the Fractional Endurance operator can also be used as different operators when giving numerical solutions[43-46]. Recently, Q-Homotopy Shehu Analysis Transform Method and Improving Homotopy Analysis Method have been applied to differential equations[47-48]. In recent years, various analyzes of the SARS-CoV-2, Corona Virus and Covid 19 models have been carried out[49-57]. We know that many researchers work on fractional versions of different events in nature and biology[58-64].

The q-HATM's Fundamental Solution Procedure

The following general nonlinear time-fractional partial differential equation is used to illustrate the solution procedure of the q-HATM:

$D_t^\alpha U(x,t) + \mathcal{R}U(x,t) + NU(x,t) = g(x,t),$
$(\alpha > 0, m - 1 < \alpha < m \in \mathbb{N})$

$D_t^\alpha U(x,t)$ denotes the Caputo fractional derivative of an unknown function $U(x,t)$. $\mathcal{R}$ is a bounded linear partial differential operator that satisfies $\|\mathcal{R}U\| \le \delta\|U\|$ for some $\delta > 0$. N is a nonlinear partial differential operator that adheres to the Lipschitz condition: $\|\mathcal{R}U_1 - \mathcal{R}U_2\| \le \lambda\|U_1 - U_2\|$ for some $\lambda > 0$. Additionally, $g(x, t)$ is a nonhomogeneous term. on begin the q-HATM, we initially apply the Laplace transform on (4) and subsequently utilize the differentiation property (3) to get

$$\mathcal{L}(U(x,t)) - \frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0) + \frac{1}{s^\alpha}(\mathcal{L}[\mathcal{R}U(x,t)]$$
$$+ \mathcal{L}[NU(x,t)] - \mathcal{L}[g(x,t)]) = 0 \quad (5)$$

subsequent to simplification. Subsequently, we delineate the nonlinear operator.

$$N[\Phi(x,t;q)] = \mathcal{L}[\Phi(x,t;q)] - \frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}\Phi^{(k)}(x,t;q)(0^+)$$
$$+ \frac{1}{s^\alpha}(\mathcal{L}[\Phi U(x,t)] + \mathcal{L}[\Phi NU(x,t)] - \mathcal{L}[g(x,t)])$$

where $q \in [0, \frac{1}{n}]$, $(n \ge 1)$ is the embedding parameter, and $\Phi(x,t;q)$ is an unspecified real-valued function of x, t, and q. For a nonzero auxiliary function. A homotopy $H(x, t)$ is built in the following manner:

$$(1 - nq)\mathcal{L}[\Phi(x,t;q) - U_0(x,t)] = hqH(x,t)N\Phi(x,t;q) \quad (6)$$

where h is a nonzero auxiliary parameter and $U_0(x, t)$ represents the initial assumption of U. It is evident that the subsequent connection is valid.

$$\begin{cases} \Phi(x,t;0) = U_0(x,t) & q = 0 \\ \Phi\left(x,t;\frac{1}{n}\right) = U(x,t) & q = \frac{1}{n} \end{cases} \quad (7)$$

In other words, $\Phi(x, t; q)$ varies from the initial guess $U_0(x, t)$ to the solution $U(x, t)$ as a q varies from 0 to $\frac{1}{n}$. A Taylor's series expansion of $\Phi(x, t; q)$ about q yields

$$\Phi(x,t;q) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)q^m \quad (8)$$

where

$$U_m(x,t) = \frac{1}{m!}\frac{\partial^m \Phi(x,t;q)}{\partial q^m}\bigg|_{q=0} \quad (9)$$

With suitable selections of the initial estimate $U_0(x, t)$, the auxiliary parameter h, and the auxiliary function $H(x, t)$, the series (8) converges at $q = \frac{1}{n}$ and produces a solution.

$$\Phi(x,t;q) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)\left(\frac{1}{n}\right)^m \quad (10)$$

which is at least one solution (4). Define the vectors

$$\vec{U}_m(x,t) = \{U_0(x,t), U_1(x,t), \ldots, U_m(x,t)\} \quad (11)$$

The mth-order deformation equation is obtained by taking the deriative of (6) m-times with regard to q, multiplying the result by $\frac{1}{m}$, and then setting $q = 0$.

$$\mathcal{L}[U_m(x,t) - k_m U_{m-1}(x,t)] = hH(x,t)\mathcal{R}_m\left(\vec{U}_{m-1}(x,t)\right) \quad (12)$$

where

$$\mathcal{R}_m(\vec{U}_{m-1}) = \mathcal{L}U_{m-1}(x,t) - h\left(1 - \frac{k_m}{n}\right)\left(\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0)\right.$$
$$\left.+ \frac{1}{s^\alpha}\mathcal{L}[g(x,t)]\right) + \frac{1}{s^\alpha}\mathcal{L}(\mathcal{R}_m U_{m-1} + H_{m-1}) \quad (13)$$

and

$$k_m = \begin{cases} 0, & m \le 1 \\ n & m \ge 1 \end{cases} \quad (14)$$

In (13), H_m represents the homotopy polynomial, defined as

$$H_m = \frac{1}{m!}\frac{\partial^m \Phi(x,t;q)}{\partial q^m}\bigg|_{q=0} \quad ve$$
$$\Phi(x,t;q) = \Phi_0 + q\Phi_1 + q^2\Phi_2 + \cdots \quad (15)$$

Applying the inverse transform (12) yields the recursive equation

$$U_m(x,t) = k_m U_m(x,t) + h\mathcal{L}^{-1}\left[H(x,t)\mathcal{R}_m\left(\vec{U}_{m-1}(x,t)\right)\right] \quad (16)$$

Thus, by substituting (13) into (16)

$$U_m(x,t) = (k_m + h)\vec{U}_{m-1}(x,t) - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}$$

$$\left[\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0) + \frac{1}{s^\alpha}\mathcal{L}[g(x,t)]\right] \quad (17)$$

$$+ h\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}(\mathcal{R}_m U_{m-1} + H_{m-1})\right]$$

Lastly, the approximate analytical solution of (3) is found by truncating the subsequent series:

$$U_m(x,t) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)\left(\frac{1}{n}\right)^m \quad (18)$$

The presence of the factor $\left(\frac{1}{n}\right)^m$ in the q-HATM solution (18) facilitates more rapid convergence compared to the conventional HAM. Furthermore, in the specific instance when $n = 1$, the q-HATM simplifies to the Standard Homotopy Analysis Transform Method (HATM).

q-HATM for Fractional Order Covid-19

Let's consider time-fractional Covid-19 model.

$$\begin{aligned}
D_t^{\beta_1}S(t) &= \Lambda - \phi_c IS - \mu S + v_3 R \\
D_t^{\beta_2}E(t) &= \phi_c IS - (\sigma + \mu)E \\
D_t^{\beta_3}I(t) &= \sigma E - (\alpha + v_2 + \delta + \mu)I + v_1 R \quad (19) \\
D_t^{\beta_4}Q(t) &= \alpha I - (\theta + \mu)Q \\
D_t^{\beta_5}R(t) &= \theta Q - (v_1 + v_3 + \mu)R + v_2 I
\end{aligned}$$

If the Laplace transform is taken from both sides of all equations in the systems (19), we obtained the following equations.

$$\mathcal{L}[S(t)] - \frac{S_0}{s} + \frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c IS + \mu S - v_3 R] = 0$$

$$\mathcal{L}[E(t)] - \frac{E_0}{s} + \frac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c IS + (\sigma + \mu)E] = 0$$

$$\mathcal{L}[I(t)] - \frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma E + (\alpha + v_2 + \delta + \mu)I - v_1 R] = 0 \quad (20)$$

$$\mathcal{L}[Q(t)] - \frac{Q_0}{s} + \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha I + (\theta + \mu)Q] = 0$$

$$\mathcal{L}[R(t)] - \frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q + (v_1 + v_3 + \mu)R - v_2 I] = 0$$

Where, $S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0$ are initial conditions and $S(0) = 5000, E(0) = 2003, I(0) = 416, Q(0) = 404, R(0) = 115$ are real valuess.

Now, we can construct the following equations (21) that we need to use to find a solution

$$N_1[\Phi(x,t;q)] = N_1[\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5] = \mathcal{L}[\Phi_1(x,t;q)] - \frac{S_0}{s}$$
$$+ \frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c\Phi_3(x,t;q)\Phi_1(x,t;q) + \mu\Phi_1(x,t;q)$$
$$- v_3\Phi_5(x,t;q)]$$

$$N_2[\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5] = \mathcal{L}[\Phi_2(x,t;q)] - \frac{E_0}{s}\frac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c\Phi_3(x,t;q)\Phi_1(x,t;q)$$
$$- (\sigma + \mu)\Phi_2(x,t;q)]$$

$$N_3[\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5] = \mathcal{L}[\Phi_3(x,t;q)] - \frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma\Phi_2(x,t;q)$$
$$+ (\alpha + v_2 + \delta + \mu)\Phi_3(x,t;q) - v_1\Phi_5(x,t;q)] \quad (21)$$

$$N_4[\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5] = \mathcal{L}[\Phi_4(x,t;q)] - \frac{Q_0}{s} + \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha\Phi_3(x,t;q)$$
$$+ (\theta + \mu)\Phi_4(x,t;q)]$$

$$N_5[\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5] = \mathcal{L}[\Phi_5(x,t;q)] - \frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta\Phi_4(x,t;q)$$
$$+ (v_1 + v_3 + \mu)\Phi_5(x,t;q) - v_2\Phi_3(x,t;q)]$$

Taking, $H(x,t) = 1$ we can find the following m-order deformation equation.

$$\begin{aligned}
\mathcal{L}[S_m(t) - k_m S_{m-1}(t)] &= h\mathcal{R}_m^S[\vec{S}_{m-1}] \\
\mathcal{L}[E_m(t) - k_m E_{m-1}(t)] &= h\mathcal{R}_m^E[\vec{E}_{m-1}] \\
\mathcal{L}[I_m(t) - k_m I_{m-1}(t)] &= h\mathcal{R}_m^I[\vec{I}_{m-1}] \quad (22) \\
\mathcal{L}[Q_m(t) - k_m Q_{m-1}(t)] &= h\mathcal{R}_m^Q[\vec{Q}_{m-1}] \\
\mathcal{L}[R_m(t) - k_m R_{m-1}(t)] &= h\mathcal{R}_m^R[\vec{R}_{m-1}]
\end{aligned}$$

Table 1. Parameter values used in the model[41-42]

Parameter	Definition	Value
Λ	Recovery rate of individuals	750
ϕ_c	Contact rate for Covid-19 transmission	0.0000124
σ	Exposure rate of individuals	0.000011618
μ	Rate of natural deaths	0.00324588
v_2	Relapse rate of individuals	0.001466848
δ	Death rate from Covid-19	0.00286
θ	Survival rate of individuals in quarantine	0.0766169
α	Detection rate of infectious individuals	0.010939586
v_1	Survival rate of infectious people	0.1109289
v_3	Rate of returning individuals to the susceptible class	0.0022927

Where,

$$k_m = \begin{cases} 0, & m \leq 1 \\ n & m \geq 1 \end{cases} \text{ and}$$

$$\mathcal{R}_m^S[\vec{S}_{m-1}] = \mathcal{L}[\vec{S}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{S_0}{s}$$
$$+ \frac{1}{s^{\beta_1}}\mathcal{L}\left[-\Lambda + \phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i} + \mu S_{m-1} - v_3 R_{m-1}\right]$$

$$\mathcal{R}_m^E[\vec{E}_{m-1}] = \mathcal{L}[\vec{E}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{E_0}{s}$$
$$+ \frac{1}{s^{\beta_2}}\mathcal{L}\left[-\phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i} + (\sigma + \mu)E_{m-1}\right]$$

$$\mathcal{R}_m^I[\vec{I}_{m-1}] = \mathcal{L}[\vec{I}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma E_{m-1} \qquad (23)$$
$$+ (\alpha + v_2 + \delta + \mu)I_{m-1} - v_1 R_{m-1}]$$

$$\mathcal{R}_m^Q[\vec{Q}_{m-1}] = \mathcal{L}[\vec{Q}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{Q_0}{s}$$
$$+ \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha I_{m-1} + (\theta + \mu)Q_{m-1}]$$

$$\mathcal{R}_m^R[\vec{R}_{m-1}] = \mathcal{L}[\vec{R}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q_{m-1}$$
$$+ (v_1 + v_3 + \mu)R_{m-1} - v_2 I_{m-1}]$$

If the expressions (23) are substituted in (22) and taken in reverse Laplace, we get to following equations.

$$S_m - k_m S_{m-1} = h S_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i}\right.$$
$$\left. + \mu S_{m-1} - v_3 R_{m-1}]\right\}$$
$$\qquad\qquad (23.1)$$
$$S_m = (k_m + h)S_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i}\right.$$
$$\left. + \mu S_{m-1} - v_3 R_{m-1}]\right\}$$

$$E_m - k_m E_{m-1} = h E_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{E_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_2}}\mathcal{L}\left[-\phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i}\right.\right.$$
$$\left.\left. + (\sigma + \mu)E_{m-1}\right]\right\}$$
$$\qquad\qquad (23.2)$$

$$E_m = (k_m + h)E_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{E_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i}\right.$$
$$\left. + (\sigma + \mu)E_{m-1}]\right\}$$

$$I_m = (k_m + h)I_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{I_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma E_{m-1} + (\alpha + v_2 + \delta + \mu)I_{m-1} \quad (23.3)\right.$$
$$\left. - v_1 R_{m-1}]\right\}$$

$$Q_m = (k_m + h)Q_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{Q_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_4}}\mathcal{L}\{-\alpha I_{m-1} + (\theta + \mu)Q_{m-1}\}\right\}$$
$$\qquad\qquad (23.4)$$

$$R_m = (k_m + h)R_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{R_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q_{m-1} + (v_1 + v_3 + \mu)R_{m-1} \quad (23.5)\right.$$
$$\left. - v_2 I_{m-1}]\right\}$$

From here, general solution is found as in (24).

$$S(t) = S_0 + \sum_{m=1}^{\infty} S_m(t)\left(\frac{1}{n}\right)^m$$
$$E(t) = E_0 + \sum_{m=1}^{\infty} E_m(t)\left(\frac{1}{n}\right)^m$$
$$I(t) = I_0 + \sum_{m=1}^{\infty} I_m(t)\left(\frac{1}{n}\right)^m \qquad (24)$$
$$Q(t) = Q_0 + \sum_{m=1}^{\infty} Q_m(t)\left(\frac{1}{n}\right)^m$$
$$R(t) = S_0 + \sum_{m=1}^{\infty} R_m(t)\left(\frac{1}{n}\right)^m$$

(23.1), (23.2), (23.3), (23.4), (23.5) for $i = 0, m = 1$ for (23.1) equation, when using the (14) feature and performing the necessary actions, the following results are obtained.

$$S_1 = (k_1 + h)S_0 - h\left(1 - \frac{k_1}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0]\right\}$$
$$S_1 = (0 + h)S_0 - h\left(1 - \frac{0}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0]\right\} \qquad (25)$$
$$S_1 = h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}\left[(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0).\frac{1}{s}\right]\right\}$$
$$S_1 = h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1+1)}$$

For $i = 1, m = 1$ E_1, I_1, Q_1 and R_1 are found as follows, similar to S_1 operations in the equations (23.2),(23.3),(23.4) and (23.5) and again (14) feature.

$$E_1 = h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2+1)} \qquad (26)$$

$$I_1 = h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3+1)} \quad (27)$$

$$Q_1 = h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4+1)} \qquad (28)$$

$$R_1 = h(\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5+1)} \quad (29)$$

From equation (23.1) and property (14) for $m = 2$, S_2 is obtained.

$$S_2 = (k_2 + h)S_1 - h\left(1 - \frac{k_2}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right]$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c(S_0 I_1 + S_1 I_0 + \mu S_1 - v_3 R_1]\right\}$$

(25), (27), (29) expressions are written instead; S_2 is found as

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}\left[A\frac{t^{\beta_3}}{\Gamma(\beta_3 + 1)} + B\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}\right.\right.$$
$$\left.\left. + C\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + D\frac{t^{\beta_5}}{\Gamma(\beta_5 + 1)}\right]\right\}$$

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}$$
$$+ h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\left[A\frac{1}{s^{\beta_3+1}} + B\frac{1}{s^{\beta_1+1}} + C\frac{1}{s^{\beta_1+1}} + \frac{1}{s^{\beta_5+1}}\right]\right\}$$

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}$$
$$+ h\left(A\frac{t^{\beta_3+\beta_1}}{\Gamma(\beta_3 + \beta_1 + 1)} + B\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)}\right.$$
$$\left. + C\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + D\frac{t^{\beta_1+\beta_5}}{\Gamma(\beta_1 + \beta_5 + 1)}\right)$$

where

$$A = -\Lambda + \phi_c S_0 h\left(\sigma E_0 + ((\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\right)$$
$$B = \left((\phi_c I_0 h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0))\right)$$
$$C = \left(\mu h((-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0))\right)$$
$$D = -v_3 h\left((\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)\right)$$

Again, similar to the operations that find E_2, I_2, Q_2 and R_2 are found as follows:

$$E_2 = (n + h)h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2 + 1)}$$
$$+ h\left(E_*\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3 + \beta_2 + 1)} + F\frac{t^{\beta_2+\beta_1}}{\Gamma(\beta_2 + \beta_1 + 1)}\right.$$
$$\left. + G\frac{t^{2\beta_2}}{\Gamma(2\beta_2 + 1)}\right)$$

where

$$E_* = -\phi_c S_0 h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$
$$F = -\phi_c I_0 h((-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0))$$
$$G = (\sigma + \mu)h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)$$

Similarly,

$$I_2 = (n + h)h\left((-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0\right.$$
$$\left. - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3 + 1)}\right) + h\left(H\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3 + \beta_2 + 1)}\right.$$
$$\left. + I_1\frac{t^{2\beta_3}}{\Gamma(2\beta_3 + 1)} + K\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3 + \beta_5 + 1)} + L\frac{t^{2\beta_3}}{\Gamma(2\beta_3 + 1)}\right)$$

where,

$$H = -\sigma h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)$$
$$I_1 = (\alpha + v_2 + \delta + \mu)h(-\sigma E_0 + (\alpha + v_2$$
$$+ \delta + \mu)I_0 - v_1 R_0)$$
$$K = -v_1 h(\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)$$

Similarly,

$$Q_2 = (n + h)h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4 + 1)}$$
$$+ h\left(M\frac{t^{\beta_3+\beta_4}}{\Gamma(\beta_3 + \beta_4 + 1)} + N\frac{t^{2\beta_4}}{\Gamma(2\beta_4 + 1)}\right)$$

where

$$M = -\alpha h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$
$$N = (\theta + \mu)h(-\alpha I_0 + (\theta + \mu)Q_0)$$

Similarly,

$$R_2 = (n + h)h(\theta Q_0 + (v_1 + v_3 + \mu)R_0$$
$$- v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5 + 1)} + h\left(O\frac{t^{\beta_4+\beta_5}}{\Gamma(\beta_4 + \beta_5 + 1)}\right.$$
$$\left. + \ddot{O}\frac{t^{2\beta_5}}{\Gamma(2\beta_5 + 1)} + P\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3 + \beta_5 + 1)}\right)$$

where

$$O = \left(\theta h(-\alpha I_0 + (\theta + \mu)Q_0)\right)$$
$$\ddot{O} = (v_1 + v_3 + \mu)h(\theta R_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)$$
$$P = -v_2 h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$

The solution using equations is,

$$S(t) = S_0 + S_1(t)\left(\frac{1}{n}\right) + S_2(t)\left(\frac{1}{n}\right)^2$$
$$S(t) = S_0 + \left(h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}\right)\left(\frac{1}{n}\right)$$
$$+ \left[(n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}\right.$$
$$+ h\left(A\frac{t^{\beta_3+\beta_1}}{\Gamma(\beta_3 + \beta_1 + 1)} + B\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + C\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)}\right.$$
$$\left.\left. + D\frac{t^{\beta_1+\beta_5}}{\Gamma(\beta_1 + \beta_5 + 1)}\right)\right]\left(\frac{1}{n}\right)^2$$

$$E(t) = E_0 + E_1(t)\left(\frac{1}{n}\right) + E_2(t)\left(\frac{1}{n}\right)^2$$

$$E(t) = E_0 + \left(h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2+1)}\right)\left(\frac{1}{n}\right)$$

$$+ \Bigg[(n+h)h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2+1)}$$

$$+ h\left(E_*\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3+\beta_2+1)} + F\frac{t^{\beta_2+\beta_1}}{\Gamma(\beta_2+\beta_1+1)}\right.$$

$$\left.+ G\frac{t^{2\beta_2}}{\Gamma(2\beta_2+1)}\right)\Bigg]\left(\frac{1}{n}\right)^2$$

$$I(t) = I_0 + I_1(t)\left(\frac{1}{n}\right) + I_2(t)\left(\frac{1}{n}\right)^2$$

$$I(t) = I_0 + \left(h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0\right.$$

$$\left.- v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3+1)}\right)\left(\frac{1}{n}\right) + \Bigg[(n+h)h\left((-\sigma E_0\right.$$

$$+ (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3+1)}\right)$$

$$+ h\left(H\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3+\beta_2+1)} + I_1\frac{t^{2\beta_3}}{\Gamma(2\beta_3+1)}\right.$$

$$\left.+ K\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3+\beta_5+1)} + L\frac{t^{2\beta_3}}{\Gamma(2\beta_3+1)}\right)\Bigg]\left(\frac{1}{n}\right)^2$$

$$Q(t) = Q_0 + Q_1(t)\left(\frac{1}{n}\right) + Q_2(t)\left(\frac{1}{n}\right)^2$$

$$Q(t) = Q_0 + \left(h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4+1)}\right)\left(\frac{1}{n}\right)$$

$$+ \Bigg[(n+h)h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4+1)}$$

$$+ h\left(M\frac{t^{\beta_3+\beta_4}}{\Gamma(\beta_3+\beta_4+1)} + N\frac{t^{2\beta_4}}{\Gamma(2\beta_4+1)}\right)\Bigg]\left(\frac{1}{n}\right)^2$$

$$R(t) = R_0 + R_1(t)\left(\frac{1}{n}\right) + R_2(t)\left(\frac{1}{n}\right)^2$$

$$R(t) = R_0 + \left(h(\theta Q_0 - (v_1 + v_3 + \mu)R_0\right.$$

$$\left.- v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5+1)}\right)\left(\frac{1}{n}\right) + \Bigg[(n+h)h(\theta Q_0$$

$$+ (v_1 + v_3 + \mu)R_0 - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5+1)}$$

$$+ h\left(O\frac{t^{\beta_4+\beta_5}}{\Gamma(\beta_4+\beta_5+1)} + \ddot{O}\frac{t^{2\beta_5}}{\Gamma(2\beta_5+1)}\right.$$

$$\left.+ P\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3+\beta_5+1)}\right)\Bigg]\left(\frac{1}{n}\right)^2$$
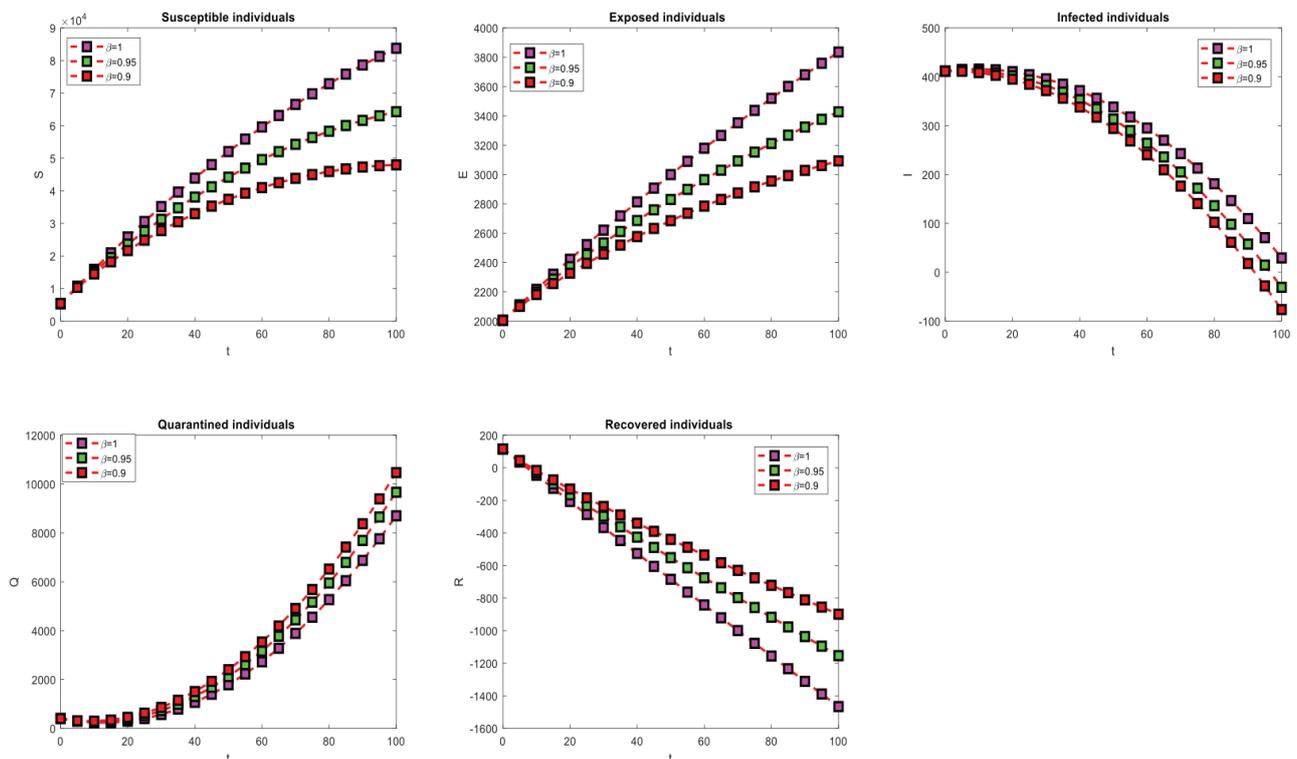

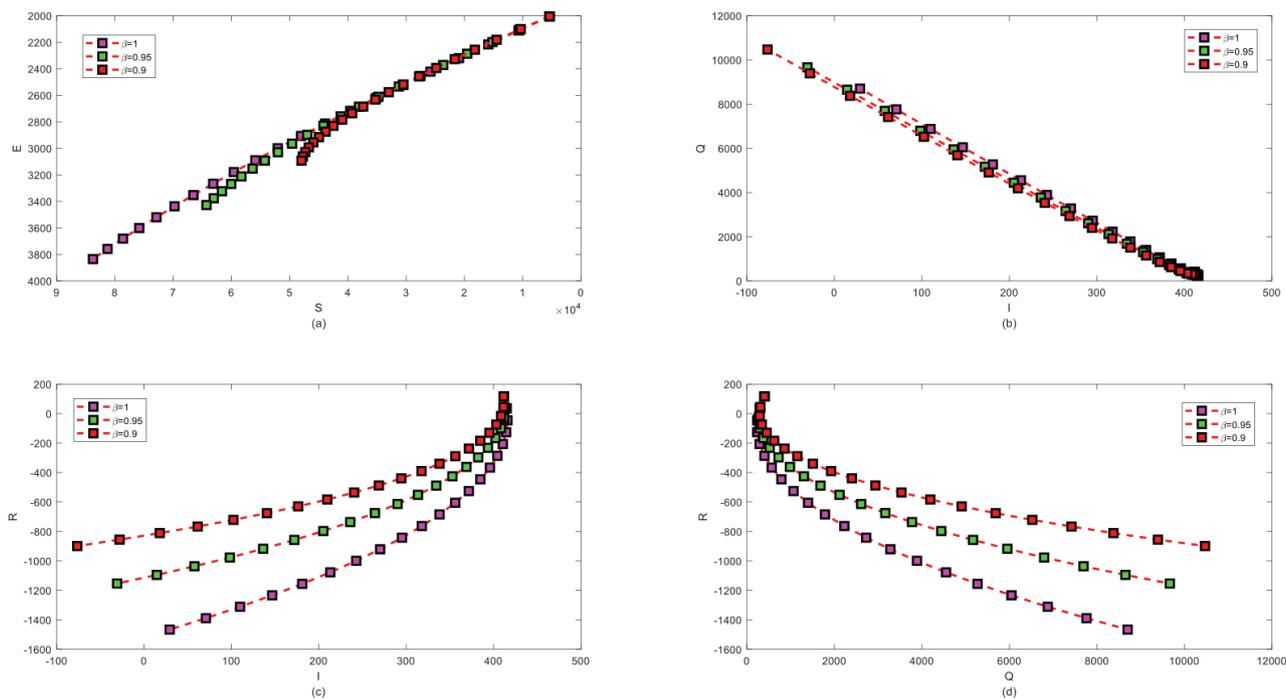**Figure 1.** Variation of *S, E, I, Q* and R when $\beta = 1$, $\beta = 0.95$, $\beta = 0.9$.

Figure 2. Phase diagram of fractional Covid-19 Model : (a) $S(t)-E(t)$, (b) $I(t)-Q(t)$, (c) $I(t)-R(t)$, (d) $Q(t)-R(t)$.
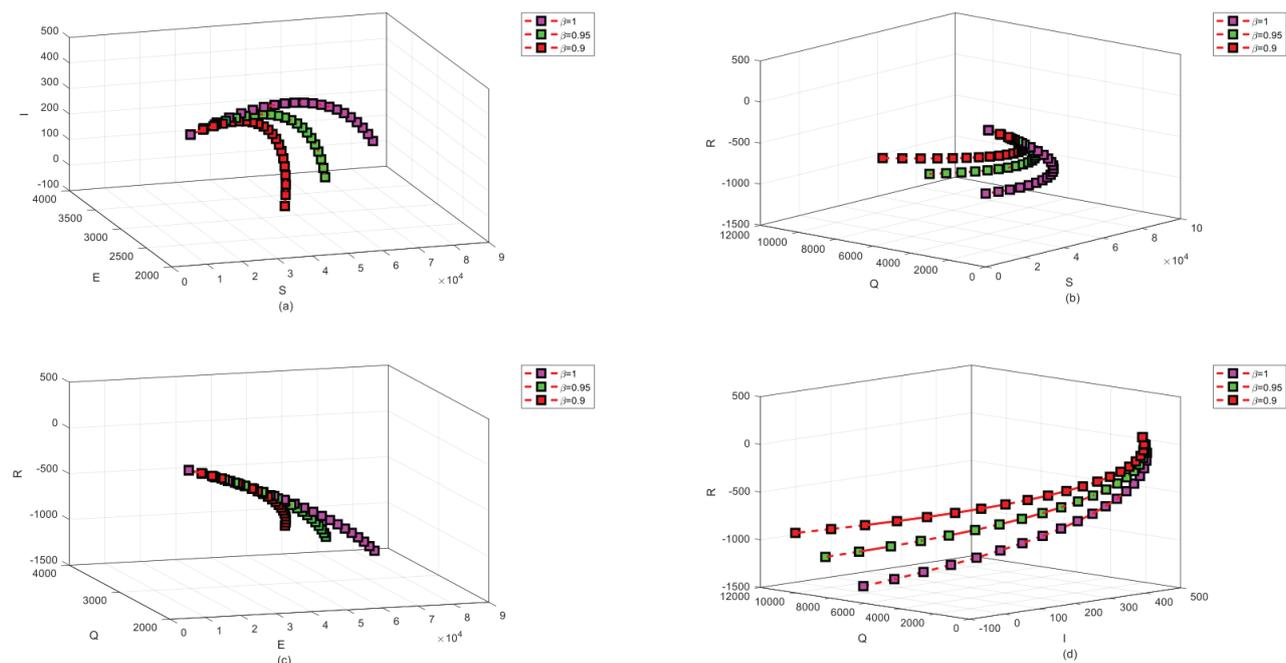
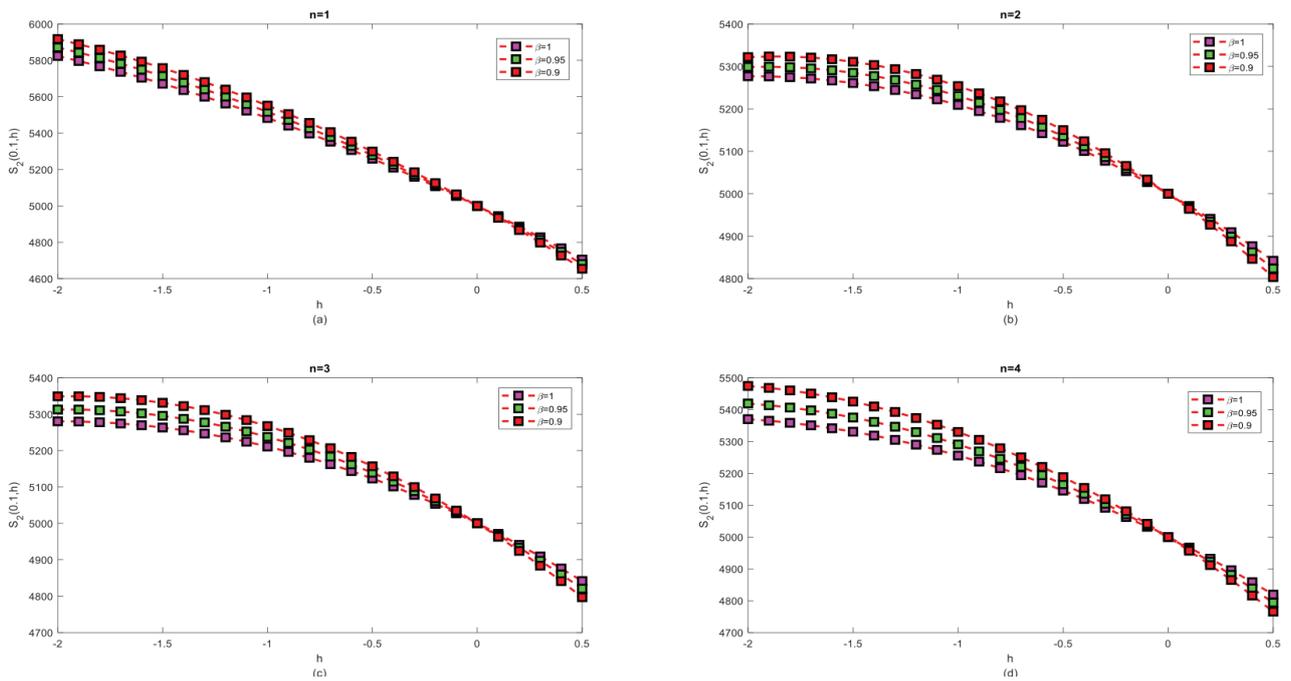

Figure 3. Phase portrait of fractional Covid-19 model.

Figure 4. *h*-curves of $S(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.
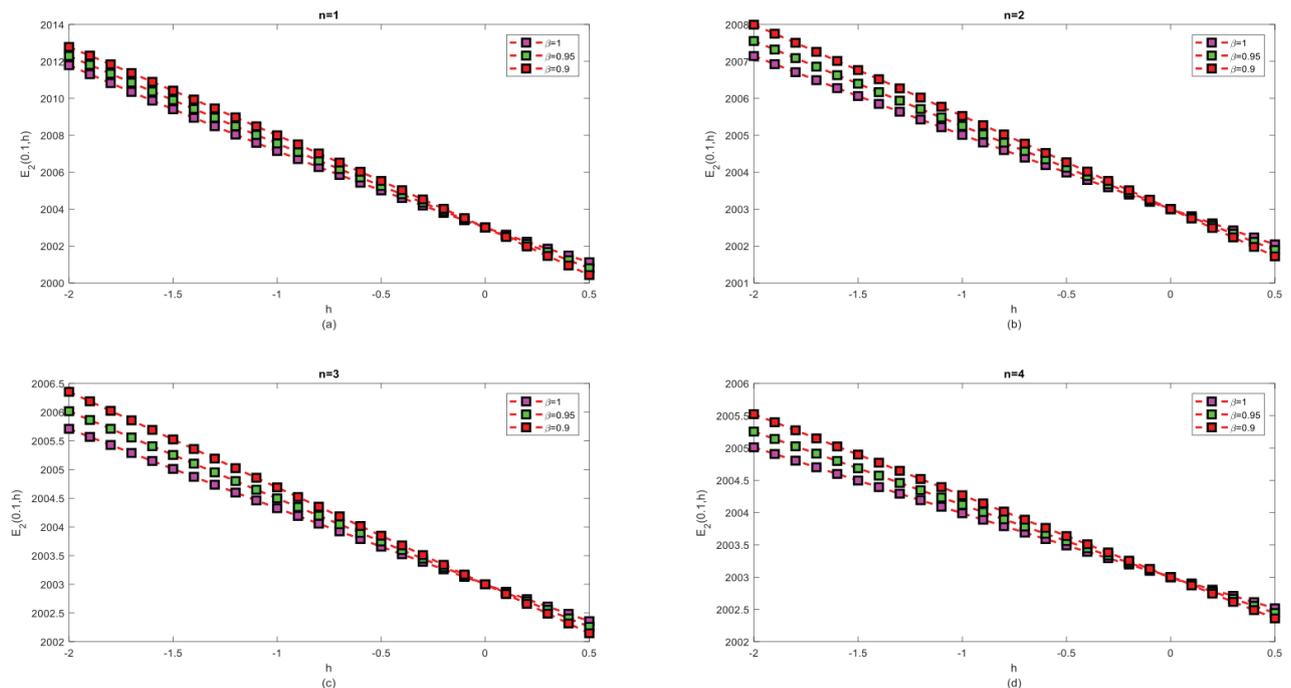

Figure 5. *h*-curves of $E(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.
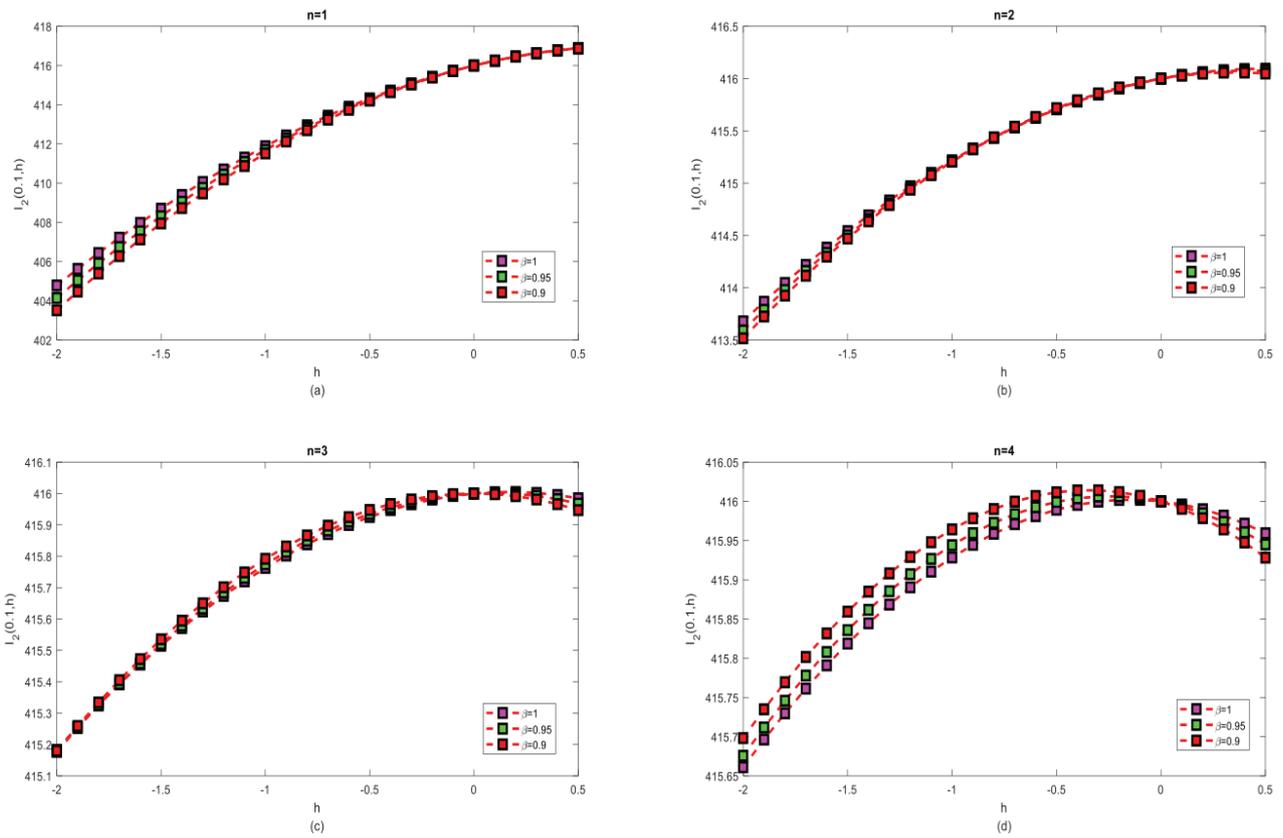
Figure 6. *h*-curves of $I(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.
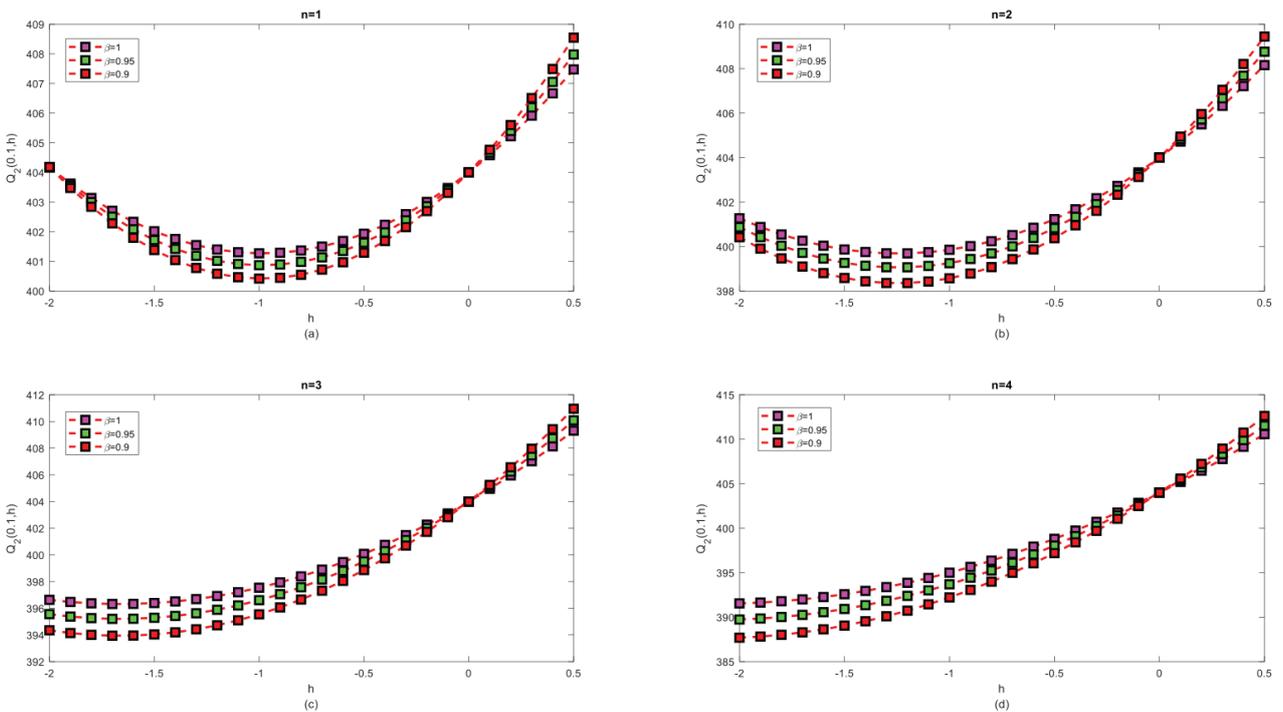

Figure 7. *h*-curves of $Q(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.
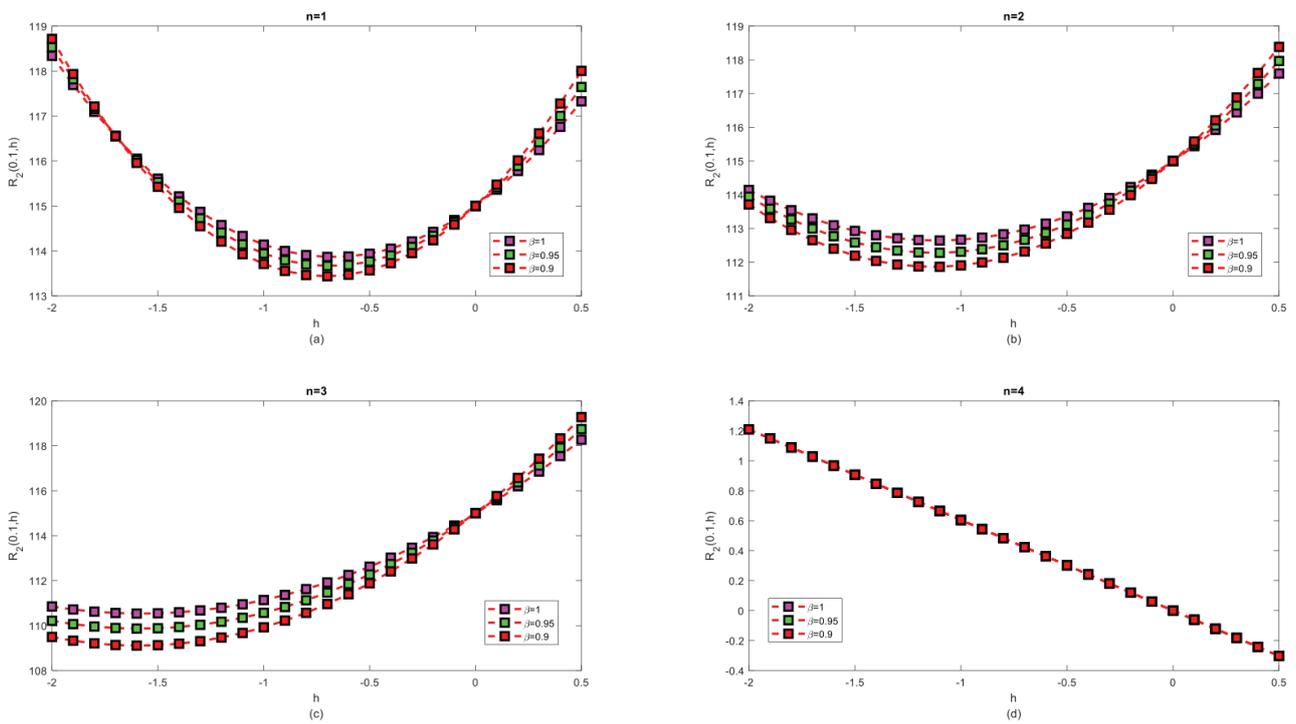
Figure 8. h-curves of $R(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.
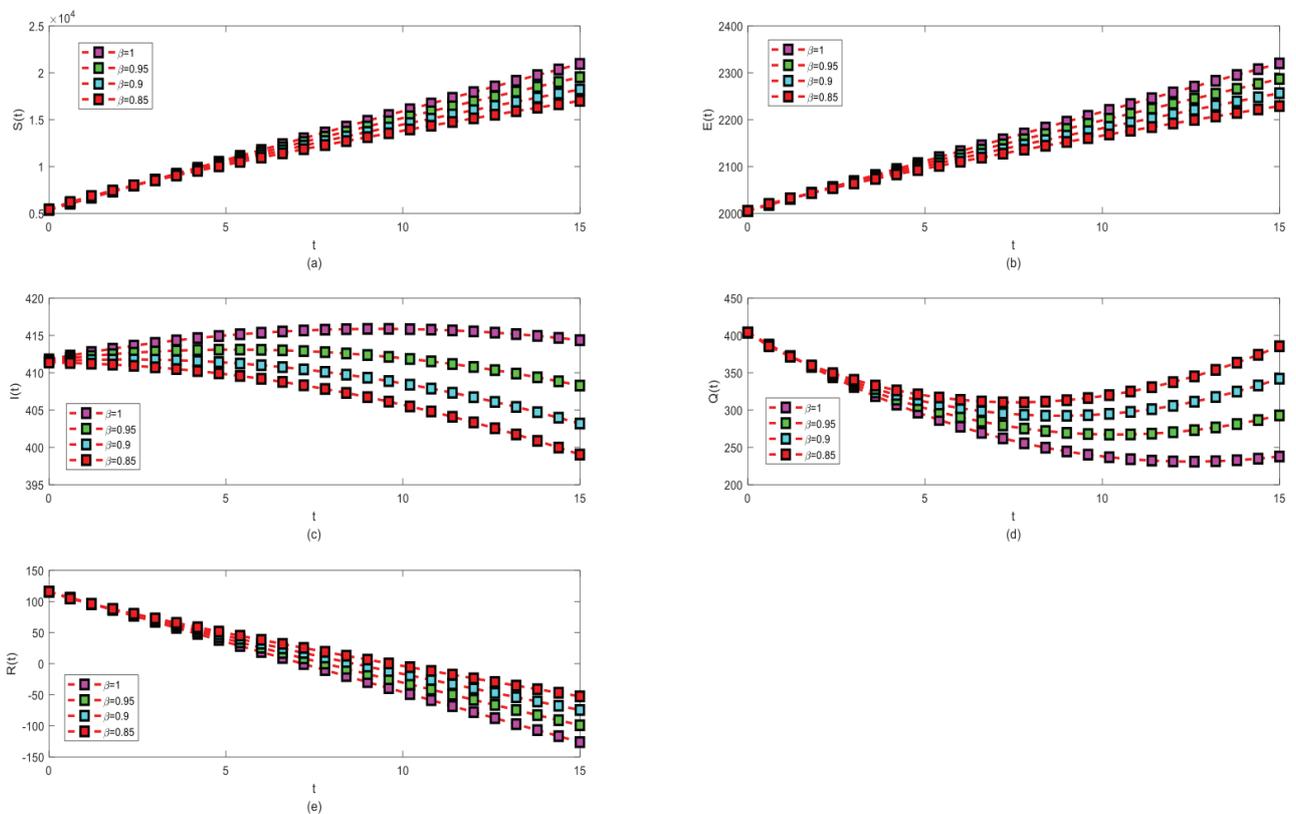

Figure 9. Numerical solutions of the Covid-19 model for differant values of β when $h=-1$ and $n=1$: (a)Susceptible individuals–$S(t)$, (b) exposed individuals –$E(t)$,(c) infected individuals –$I(t)$,(d)quarantined individuals –$Q(t)$ and (e) recovered individuals –$R(t)$.

Laplace Transform Homotopy Perturbation Method(LT-HPM)

To demonstrate the functionality of (LH-HPM), assume a general nonlinear fractional differential equation system represented as follows:

$$D_*^\beta x(t) = f(t) - Rx(t) - Nx(t);$$ (30)

where $n - 1 < \alpha < n, n \in N, t > 0$, with the following boundary conditions,

$$B(x; \frac{dx}{dt}),$$

where R is a linear operator, N is a nonlinear operator, $f(t)$ is a well-known analytical function, and D_*^β represents the fractional derivative in the caputo sense. It is presumed that the solution $x(t)$ is a well-known function, and B is the boundary operator.

Initially, we shall formulate a homotopy as follows:

$$(1-p)[D_*^\beta x(t) - D_*^\beta x_0(t)] + p[D_*^\beta x_0(t) - f(t) + Rx(t) + Nx(t)]$$ (31)

or

$$D_*^\beta x(t) = D_*^\beta x_0(t) + p[-D_*^\beta x_0(t) + f(t) - Rx(t) - Nx(t)]$$ (32)

Let p be a homotopy parameter, where $p \in [0,1]$, and x_0 represents the initial approximation for the solution of (30) that fulfills the boundary condition.

We will now apply the Laplace transform to both sides of equation (32), yielding:

$$\mathfrak{L}\left(D_*^\beta x(t)\right) = \mathfrak{L}\left[D_*^\beta x_0(t) + p[-D_*^\beta x_0(t) + f(t) - Rx(t) - Nx(t)]\right]$$ (33)

Applying the formula for the Laplace transform, we get:

$$s^\beta \mathfrak{L}[x(t)] - \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) = \mathfrak{L}\left[D_*^\beta x_0(t) + p[-D_*^\beta x_0(t) + f(t) - Rx(t) - Nx(t)]\right]$$ (34)

where $m-1 < \beta < m$, or

$$\mathfrak{L}[x(t)] = \frac{1}{s^\beta}\sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^\beta}\mathfrak{L}\left[D_*^\beta x_0(t) + p[-D_*^\beta x_0(t) + f(t) - Rx(t) - Nx(t)]\right]$$ (35)

Using the inverse Laplace transform on both sides of (9), we get:

$$x(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^\beta}\sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^\beta}\mathfrak{L}\left[D_*^\beta x_0(t) + p[-D_*^\beta x_0(t) + f(t) - Rx(t) - Nx(t)]\right]\right]$$ (36)

The solution of (36) may be expressed as a power series in terms of p as follows:

$$x(t) = \sum_{n=0}^{\infty} p^n x_n$$ (37)

Next, substituting (37) into (36), we obtain:

$$\sum_{n=0}^{\infty} p^n x_n = \mathfrak{L}^{-1}\left[\frac{1}{s^\beta}\sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^\beta}\mathfrak{L}\left[D_*^\beta x_0(t) + p\left[-D_*^\beta x_0(t) + f(t) - R[\sum_{n=0}^{\infty} p^n x_n] - N[\sum_{n=0}^{\infty} p^n x_n]\right]\right]\right]$$ (38)

By equating the terms with equivalent powers of p, we obtain:

$$p^0 : x_0(t) = \mathfrak{L}^{-1}\left[\left[\frac{1}{s^\beta}\sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0)\right] + \frac{1}{s^\beta}\mathfrak{L}[D_*^\beta x_0(t)]\right]$$

$$p^1 : x_1(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^\beta}\mathfrak{L}[-D_*^\beta x_0(t) + f(t) - R(x_0) - N(x_0)]\right]$$

$$p^2 : x_2(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^\beta}\mathfrak{L}[-R(x_0, x_1) - N(x_0, x_1))]\right]$$

$$p^3 : x_3(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^\beta}\mathfrak{L}[-R(x_0, x_1, x_2) - N(x_0, x_1, x_2))]\right]$$

$$\vdots$$ (39)

Assuming the initial approximation is expressed as $x(0) = \gamma_0, x'(0) = \gamma_1, \ldots, x^{(m-1)}(0) = \gamma_{m-1}$

Consequently, the precise answer may be derived as follows:

$$x = \lim_{p \to 1}\sum_{n=0}^{\infty} p^n x_n = x_0 + x_1 + x_2 + \cdots.$$

Initially, we will address (19) using the LT-HPM by replacing (38) and selecting $s_0(t) = 5000, e_0(t) = 2003, \iota_0(t) = 416, q_0(t) = 404, r_0(t) = 115,$

we obtain:

$$\sum_{n=0}^{\infty} p^n s_n = s_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\mathfrak{L}\left[\Lambda - \phi_c \sum_{n=0}^{\infty} p^n \iota_n \sum_{n=0}^{\infty} p^n s_n - \mu \sum_{n=0}^{\infty} p^n s_n + v_3 \sum_{n=0}^{\infty} p^n r_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n e_n = e_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_2}}\mathfrak{L}\left[\phi_c \sum_{n=0}^{\infty} p^n \iota_n \sum_{n=0}^{\infty} p^n s_n - (\sigma + \mu)\sum_{n=0}^{\infty} p^n e_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n \iota_n = \iota_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_3}}\mathfrak{L}\left[\sigma \sum_{n=0}^{\infty} p^n e_n - (\alpha + v_2 + \delta + \mu)\sum_{n=0}^{\infty} p^n \iota_n + v_1 \sum_{n=0}^{\infty} p^n r_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n q_n = q_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\mathfrak{L}\left[\alpha \sum_{n=0}^{\infty} p^n \iota_n - (\theta + \mu)\sum_{n=0}^{\infty} p^n q_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n r_n = r_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\mathfrak{L}\left[\theta \sum_{n=0}^{\infty} p^n q_n - (v_1 + v_3 + \mu)\sum_{n=0}^{\infty} p^n r_n + v_2 \sum_{n=0}^{\infty} p^n \iota_n\right]\right]$$

$$S(t) = 5000 + 1516.213419 t^{\frac{9}{10}} - 0.01938357757 t^{\frac{27}{10}} - 1.346390957 t^{\frac{9}{5}}$$
$$E(t) = 2003 + 20.03318362 t^{\frac{9}{10}} + 0.01938357757 t^{\frac{27}{10}} - 0.03743710883 t^{\frac{9}{5}}$$
$$I(t) = 416 + 5.280834026 t^{\frac{9}{10}} + 1.146277267 t^{\frac{9}{5}}$$
$$Q(t) = 404 - 28.81526129 t^{\frac{9}{10}} + 1.353320950 t^{\frac{9}{5}}$$

Comparison

In order to compare accuracy of the obtained results by q-HATM and LT-HPM, Table 2,3 are presented.

Table 2. Table for numerical solutions of the Covid-19 model for β = 0.9 with LT-HPM

t	S(t)	E(t)	I(t)	Q(t)	R(t)
0.0	5000.	2003.	416.	404.	115.
0.1	5190.858582	2005.521474	416.6829849	400.3938222	117.3383382
0.2	5356.120044	2007.704456	417.3038556	397.3052983	119.2988352
0.3	5512.906629	2009.775362	417.9181995	394.4043519	121.1036538
0.4	5664.422246	2011.776651	418.5353209	391.6279520	122.7967567
0.5	5812.129021	2013.727749	419.1591106	388.9469232	124.3989815
0.6	5956.864980	2015.639824	419.7916089	386.3443237	125.9226853
0.7	6099.172362	2017.520127	420.4340179	383.8090547	127.3761218
0.8	6239.430012	2019.373748	421.0870970	381.3332687	128.7652329
0.9	6377.917112	2021.204447	421.7513479	378.9111153	130.0945239
1.0	6514.847644	2023.015131	422.4271113	376.5380596	131.3675435

Table 3. Table for numerical solutions of the Covid-19 model for β = 0.9 when h = −1 and n = 1 with q-HATM

t	S(t)	E(t)	I(t)	Q(t)	R(t)
0.0	5000	2003	416	404	115
0.1	5099.7381	2005.5589	416.5845	400.3848	117.3928
0.2	5197.4811	2007.8348	416.9608	397.2800	119.4884
0.3	5299.9824	2010.0456	417.2065	394.3601	121.4969
0.4	5408.0820	2012.2299	417.3408	391.5643	123.4568
0.5	5522.0261	2014.4045	417.3742	388.8648	125.3853
0.6	5641.8827	2016.5787	417.3133	386.2457	127.2921
0.7	5767.6499	2018.7583	417.1632	383.6965	129.1835
0.8	5899.2955	2020.9472	416.9276	381.2102	131.0637
0.9	6036.7735	2023.1480	416.6095	378.7814	132.9357
1.0	6180.0314	2025.3628	416.2115	376.4060	134.8021

Figure 1 and Table 2 show that similar results were obtained for Susceptible individuals–$S(t)$, exposed individuals –$E(t)$, infected individuals –$I(t)$,quarantined individuals –$Q(t)$ and recovered individuals –$R(t)$. Figure 9 and Table 3 show that similar results were obtained for Susceptible individuals–$S(t)$, exposed individuals –$E(t)$, infected individuals –$I(t)$, quarantined individuals –$Q(t)$ and recovered individuals –$R(t)$.

Once more, it has been noted that the Laplace Transform Homotopy Perturbation Method yields consistent results when contrasted with the q-homotopy analysis transform method.

CONCLUSION

In the present paper, approximate solutions for a five-dimensional Covid-19 model is investigated by the q-HATM for five sets of initial data. Time-fractional derivatives are taken in the Caputo sense. q-HAM with parameter $q \in \left[0, \frac{1}{n}\right], (n \geq 1)$ and the usual The Laplace transform method does not involve any linearization, discretization or restrictive assumptions. The schema also includes a helper parameter h that allows us to manipulate and control the serial solution to achieve fast convergence. The graphical representations provide an understanding of the behavior of the series solution in comparison to a varying fractional, parameterized, or auxiliary parameter h. The graphs illustrate the continuous reliance of the model solutions on the fractional degree parameter and the chosen system parameters. In addition to the Covid 19 model, q-HAM method, a solution has also been obtained with the Laplace Transform Homotopy Perturbation Method. The solutions obtained from both methods are shown in tables. In conclusion, we state that, in the light of the present work, q-HATM is only efficient and highly efficient systems of nonlinear fractional differential equations in a wide class, explaining various biological phenomena and other emerging systems in different fields of science and engineering.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

ETHICS

There are no ethical issues with the publication of this manuscript.

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