Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2025.00046

Research Article

Investigation of fractional order covid-19 model with q-homotopy analysis transform method

Mehmet MERDAN[1,*], Pınar AÇIKGÖZ[1]

[1]Department of Mathematical Engineering, Gümüşhane University, Gümüşhane, 29100, Türkiye

ARTICLE INFO

Article history
Received: 28 December 2023
Revised: 10 February 2024
Accepted: 24 February 2024

Keywords:
Caputo Fractional Derivative; Covid 19 Model; Laplace Transform; Laplace Transform Homotopy Perturbation Method(LT-HPM); The q-homotopy Analysis Transform Method

ABSTRACT

The primary objective of this paper is to derive approximate analytic solutions for the time fractional-order Covid-19 model by employing the q-homotopy analysis transform method (q-HATM) and the Laplace transform homotopy perturbation method (LT-HPM). The covid 19 model is a system of five-dimensional nonlinear ordinary differential equations. Moreover, this method applies even to more complex partial differential equations originating from mathematical biology, demonstrating computational efficiency and wide application. In this study, Caputo fractional derivative is used to obtain the fractional system and Laplace transformation is applied to analyze the approximate solutions of the system. Graphical results are presented and discussed quantitatively to illustrate the approximate solution.

Cite this article as: Merdan M, Açıkgöz P. Investigation of fractional order covid-19 model with q-homotopy analysis transform method. Sigma J Eng Nat Sci 2025;43(2):582−597.

INTRODUCTION

Mathematical modeling of real-life phenomena is used in many fields such as biology, chemistry, psychologcy, economy etc. For instance, financial models often contain stochastic equations whereas ordinary differential equation systems are the basic tools for modeling the spread of infectious diseases. The use of fractional analysis should be specifically mentioned,for current trend in mathematical modeling is to use different types of fractional derivatives with numerous solution methods to obtain the exact,approximate or numerical solutions of the system being analyzed.

Fractional analysis can be regarded as an extension of conventional calculus, wherein the orders of differentiation and integration must be integers. Fractional analysis can be seen as an extension of conventional calculus, whereby the sequences of differentiation and integration are restricted to integers. Fractional operators such as Caputo-Fabrizo fractional derivative or Riemann-Liouville fractional integral authorize a more efficient systematic modeling of recognized real-life problems. Therefore, this issue has received increasing attention in the last few decades, with many studies on chemistry, astrophysics and biological systems. The emergence of new definitions for fractional operators also increases the intensity of research in this field.

A viral infection caused by a newly discovered virus called coronavirus (SARS-CoV-2) causes the novel coronavirus disease. This virus is a member of the Coronaviridae virus family, which is responsible for COVID-19. The first COVID-19 outbreak occurred at the end of 2019 in the

*Corresponding author.
*E-mail address: mehmetmerdan@gmail.com

This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

Chinese city of Wuhan. Because of its rapid spread, this virus is mostly seen in the form of infectious cases, deaths, and economic losses[1-2] . The pandemic impacts not only the human population but also the economic growth of nations, resulting in significant material and moral losses. A significant number of individuals became unemployed, and several industries and enterprises ceased operations. A recent World Health Organization (WHO) estimate indicates that 10.27 million individuals tested positive, with over 1.52 million fatalities recorded by December 12, 2020. Official statistics indicate that 42.2 million individuals have entirely recovered from the virus, with the recovery rate surpassing the mortality rate. No approved pharmaceuticals or vaccines presently exist that provide protection against or cure COVID-19. Medications used to infected individuals generally serve as adjunctive therapy focused on alleviating symptoms. Most of the proposed mathematical models for COVID-19 are formulated with the help of ordinary, partial or stochastic differential equations. Recent studies on the mathematical model of the epidemic are given in [3-6]. In recent years, researchers have been trying to mathematically model various processes and phenomena in order to study the dynamics of such complex systems. Since the accuracy of fractional models in simulating the behavior of an existing system is much higher than models based on integer order, fractional order models are examined instead of integer order models[7-15].

This work analyzes a time fractional Covid-19 system, a nonlinear epidemic model, using q-HATM. Acquiring precise solutions for fractional differential equations proves to be more challenging than for their traditional integer-order counterparts.Therefore, more efficient numerical and semi-analytical techniques have been developed to find approximate solutions to such problems. Some of these techniques are the Adomian decomposition method (ADM) [16-18], Laplace decomposition method (LDM)[19-21], Homotopy analysis method (HAM)[22-25], Homotopy perturbation method (HPM)[26] and Variational iteration method[27]. Another very powerful method is the q-homotopy analysis transform method (q-HATM) [28-35].The q-homotopy analysis transform method ıs a more powerful method than the q-homotopy analysis method. Because the exıstence of $\left(\frac{1}{n}\right)^m$ term ın q-hatm gets a faster solutıon than q-ham.

The covid-19 model, which we discussed in our study and found approximate analytical solutions, is as follows.

$S(t) = \Lambda - \phi_c IS - \mu S + v_3 R$

$E(t) = \phi_c IS - (\sigma + \mu)E$

$I(t) = \sigma \mathrm{E} - (\alpha + v_2 + \delta + \mu)\mathrm{I} + v_1 R$

$Q(t) = \alpha I - (\theta + \mu)Q$

$R(t) = \theta Q - (v_1 + v_3 + \mu)R + v_2 I$

Where, susceptible individuals $S(t)$, exposed individuals $E(t)$, infected individuals $I(t)$, quarantined individuals $Q(t)$ and recovered individuals $R(t)$.

Λ represents the recruitment rate of individuals, ϕ_c denotes the contact rate of Covid-19 transmission, v_1 indicates the recovery rate from $I(t)$, θ signifies the recovery rate from $Q(t)$, μ is the natural death rate, α reflects the detection rate from $I(t)$, v_2 represents the relapse rate of Covid-19, and v_3 denotes the rate at which recovered individuals revert to the susceptible category. The initial conditions of the model are as follows.

$S(0) = 5000, E(0) = 2003, I(0) = 416, Q(0) = 404, R(0) = 115.$

This article uses the Laplace Transform Homotopy Perturbation Method (LT-HPM) and the q-homotopy analysis transform method (q-HATM) to compute the fractional order solution of the Covid 19 model. Definitions and essential theorems pertaining to fractional differentiation are provided in Chapter 2. Chapter 3 provides an overview of the core approaches of the Laplace Transform Homotopy Perturbation Method (LT-HPM) and the q-homotopy analysis transform method (q-HATM). The fractional order Covid 19 model was used to calculate the results in the last section.

MATERIALS AND METHODS

Definition 1. The Riemann-Liouville fractional derivative of order $0 < \alpha < 1$ of a $f(x) \in C^1\,[0,b]$, $b > 0$ function is defined as [36]

$$ {}_{0}^{RL}D_x^\alpha f(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} f(t)dt \qquad (1)$$

Definition 2. The Riemann-Louville fractional integral of order $0 < \alpha < 1$ of a function is defined as [36]

$$ {}_{0}^{RL}I_x^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt \qquad (2)$$

where $\Gamma(.)$ represents the Gamma function.

Definition 3. The Caputo fractional derivative of order $\alpha > 0$ of a function $\omega \in C^\mu$ $(\mu \geq 1)$ is defined in the context of Caputo as follows:

$$ {}_{0}^{C}D_t^\alpha \omega(t) = \frac{1}{\Gamma(\mathrm{m}-\alpha)} \int_0^t (t-\vartheta)^{m-\alpha-1} \omega^m(\vartheta) d\vartheta \qquad (3)$$

where $m - 1 < \alpha < m \in \mathbb{N}$. If $\alpha = m \in \mathbb{N}$, then

$$D_t^\alpha \omega(t) = \frac{d^m \omega(t)}{dt^m}.$$

Definition 4: The Laplace transform of function is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$$

for all real numbers $t \geq 0$ if the integral exists.

Definition 5. [37-39] The Laplace transform of the transform of the Caputo fractional derivative $D_t^\alpha \omega(t)$ is defined as

$$\mathcal{L}[D_t^\alpha \omega(t)] = s^\alpha \mathcal{L}[\omega(t)](s) - \sum_{k=0}^{m-1} s^{\alpha-k-1}\omega^k(0^+) \quad (4)$$

where $\alpha > 0$, $m-1<\alpha<m$ and $\mathcal{L}$ denotes the Laplace transform operator. In the literature, the Fractional Endurance operator can also be used as different operators when giving numerical solutions[43-46]. Recently, Q-Homotopy Shehu Analysis Transform Method and Improving Homotopy Analysis Method have been applied to differential equations[47-48]. In recent years, various analyzes of the SARS-CoV-2, Corona Virus and Covid 19 models have been carried out[49-57]. We know that many researchers work on fractional versions of different events in nature and biology[58-64].

The q-HATM's Fundamental Solution Procedure

The following general nonlinear time-fractional partial differential equation is used to illustrate the solution procedure of the q-HATM:

$$D_t^\alpha U(x,t) + \mathcal{R}U(x,t) + NU(x,t) = g(x,t),$$
$$(\alpha > 0, m-1<\alpha<m \in \mathbb{N})$$

$D_t^\alpha U(x,t)$ denotes the Caputo fractional derivative of an unknown function $U(x,t)$. $\mathcal{R}$ is a bounded linear partial differential operator that satisfies $\|\mathcal{R}U\| \leq \delta\|U\|$ for some δ >0. N is a nonlinear partial differential operator that adheres to the Lipschitz condition: $\|\mathcal{R}U_1 - \mathcal{R}U_2\| \leq \lambda\|U_1 - U_2\|$ for some $\lambda > 0$. Additionally, $g(x,t)$ is a nonhomogeneous term. on begin the q-HATM, we initially apply the Laplace transform on (4) and subsequently utilize the differentiation property (3) to get

$$\mathcal{L}(U(x,t)) - \frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0) + \frac{1}{s^\alpha}(\mathcal{L}[\mathcal{R}U(x,t)] + \mathcal{L}[NU(x,t)] - \mathcal{L}[g(x,t)]) = 0 \quad (5)$$

subsequent to simplification. Subsequently, we delineate the nonlinear operator.

$$N[\Phi(x,t;q)] = \mathcal{L}[\Phi(x,t;q)] - \frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}\Phi^{(k)}(x,t;q)(0^+) + \frac{1}{s^\alpha}(\mathcal{L}[\Phi U(x,t)] + \mathcal{L}[\Phi \mathrm{N} U(x,t)] - \mathcal{L}[g(x,t)])$$

where $q \in [0, \frac{1}{n}]$, $(n \geq 1)$ is the embedding parameter, and $\Phi(x,t;q)$ is an unspecified real-valued function of x, t, and q. For a nonzero auxiliary function. A homotopy $H(x,t)$ is built in the following manner:

$$(1-nq)\mathcal{L}[\Phi(x,t;q) - U_0(x,t)] = hqH(x,t)N\Phi(x,t;q) \quad (6)$$

where h is a nonzero auxiliary parameter and $U_0(x,t)$ represents the initial assumption of U. It is evident that the subsequent connection is valid.

$$\begin{cases} \Phi(x,t;0) = U_0(x,t) & q=0 \\ \Phi\left(x,t;\frac{1}{n}\right) = U(x,t) & q=\frac{1}{n} \end{cases} \quad (7)$$

In other words, $\Phi(x,t;q)$ varies from the initial guess $U_0(x,t)$ to the solution $U(x,t)$ as a q varies from 0 to $\frac{1}{n}$. A Taylor's series expansion of $\Phi(x,t;q)$ about q yields

$$\Phi(x,t;q) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)q^m \quad (8)$$

where

$$U_m(x,t) = \frac{1}{m!}\frac{\partial^m \Phi(x,t;q)}{\partial q^m}\Big|_{q=0} \quad (9)$$

With suitable selections of the initial estimate $U_0(x,t)$, the auxiliary parameter h, and the auxiliary function $H(x,t)$, the series (8) converges at $q=\frac{1}{n}$ and produces a solution.

$$\Phi(x,t;q) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)\left(\frac{1}{n}\right)^m \quad (10)$$

which is at least one solution (4). Define the vectors

$$\vec{U}_m(x,t) = \{U_0(x,t), U_1(x,t), \ldots, U_m(x,t)\} \quad (11)$$

The mth-order deformation equation is obtained by taking the deriative of (6) m-times with regard to q, multiplying the result by $\frac{1}{m}$, and then setting $q=0$.

$$\mathcal{L}[U_m(x,t) - k_m U_{m-1}(x,t)] = hH(x,t)\mathcal{R}_m\left(\vec{U}_{m-1}(x,t)\right) \quad (12)$$

where

$$\mathcal{R}_m(\vec{U}_{m-1}) = \mathcal{L}U_{m-1}(x,t) - h\left(1-\frac{k_m}{n}\right)\left(\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0) + \frac{1}{s^\alpha}\mathcal{L}[g(x,t)]\right) + \frac{1}{s^\alpha}\mathcal{L}(\mathcal{R}_m U_{m-1} + H_{m-1}) \quad (13)$$

and

$$k_m = \begin{cases} 0, & m \leq 1 \\ n & m \geq 1 \end{cases} \quad (14)$$

In (13), H_m represents the homotopy polynomial, defined as

$$H_m = \frac{1}{m!}\frac{\partial^m \Phi(x,t;q)}{\partial q^m}\Big|_{q=0} \quad ve$$
$$\Phi(x,t;q) = \Phi_0 + q\Phi_1 + q^2\Phi_2 + \cdots \quad (15)$$

Applying the inverse transform (12) yields the recursive equation

$$U_m(x,t) = k_m U_m(x,t) + h\mathcal{L}^{-1}\left[H(x,t)\mathcal{R}_m\left(\vec{U}_{m-1}(x,t)\right)\right] \quad (16)$$

Thus, by substituting (13) into (16)

$$U_m(x,t) = (k_m + h)\vec{U}_{m-1}(x,t) - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1} \left[\frac{1}{s^\alpha}\sum_{k=0}^{m-1} s^{\alpha-k-1}U^{(k)}(x,0) + \frac{1}{s^\alpha}\mathcal{L}[g(x,t)]\right] + h\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}(\mathcal{R}_m U_{m-1} + H_{m-1})\right] \quad (17)$$

Lastly, the approximate analytical solution of (3) is found by truncating the subsequent series:

$$U_m(x,t) = U_0(x,t) + \sum_{m=1}^{\infty} U_m(x,t)\left(\frac{1}{n}\right)^m \quad (18)$$

The presence of the factor $\left(\frac{1}{n}\right)^m$ in the q-HATM solution (18) facilitates more rapid convergence compared to the conventional HAM. Furthermore, in the specific instance when $n = 1$, the q-HATM simplifies to the Standard Homotopy Analysis Transform Method (HATM).

q-HATM for Fractional Order Covid-19

Let's consider time-fractional Covid-19 model.

$$\begin{aligned} D_t^{\beta_1}S(t) &= \Lambda - \phi_c IS - \mu S + v_3 R \\ D_t^{\beta_2}E(t) &= \phi_c IS - (\sigma + \mu)E \\ D_t^{\beta_3}I(t) &= \sigma\mathrm{E} - (\alpha + v_2 + \delta + \mu)\mathrm{I} + v_1 R \\ D_t^{\beta_4}Q(t) &= \alpha I - (\theta + \mu)Q \\ D_t^{\beta_5}R(t) &= \theta Q - (v_1 + v_3 + \mu)R + v_2 I \end{aligned} \quad (19)$$

If the Laplace transform is taken from both sides of all equations in the systems (19), we obtained the following equations.

$$\begin{aligned} &\mathcal{L}[S(t)] - \frac{S_0}{s} + \frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c IS + \mu S - v_3 R] = 0 \\ &\mathcal{L}[E(t)] - \frac{E_0}{s} + \frac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c IS + (\sigma + \mu)E] = 0 \\ &\mathcal{L}[I(t)] - \frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma\mathrm{E} + (\alpha + v_2 + \delta + \mu)\mathrm{I} - v_1 R] = 0 \\ &\mathcal{L}[Q(t)] - \frac{Q_0}{s} + \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha I + (\theta + \mu)Q] = 0 \\ &\mathcal{L}[R(t)] - \frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q + (v_1 + v_3 + \mu)R - v_2 I] = 0 \end{aligned} \quad (20)$$

Where, $S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0$ are initial conditions and $S(0) = 5000$, $E(0) = 2003$, $I(0) = 416$, $Q(0) = 404$, $R(0) = 115$ are real valuess.

Now, we can construct the following equations (21) that we need to use to find a solution

$$\begin{aligned} N_1[\Phi(x,t;q)] &= N_1[\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5] = \mathcal{L}[\Phi_1(x,t;q)] - \frac{S_0}{s} \\ &\quad + \frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c\Phi_3(x,t;q)\Phi_1(x,t;q) + \mu\Phi_1(x,t;q) \\ &\quad - v_3\Phi_5(x,t;q)] \\ N_2[\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5] &= \mathcal{L}[\Phi_2(x,t;q)] - \frac{E_0}{s}\frac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c\Phi_3(x,t;q)\Phi_1(x,t;q) \\ &\quad - (\sigma + \mu)\Phi_2(x,t;q)] \\ N_3[\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5] &= \mathcal{L}[\Phi_3(x,t;q)] - \frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma\Phi_2(x,t;q) \\ &\quad + (\alpha + v_2 + \delta + \mu)\Phi_3(x,t;q) - v_1\Phi_5(x,t;q)] \\ N_4[\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5] &= \mathcal{L}[\Phi_4(x,t;q)] - \frac{Q_0}{s} + \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha\Phi_3(x,t;q) \\ &\quad + (\theta + \mu)\Phi_4(x,t;q)] \\ N_5[\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5] &= \mathcal{L}[\Phi_5(x,t;q)] - \frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta\Phi_4(x,t;q) \\ &\quad + (v_1 + v_3 + \mu)\Phi_5(x,t;q) - v_2\Phi_3(x,t;q)] \end{aligned} \quad (21)$$

Taking, $H(x, t) = 1$ we can find the following m-order deformation equation.

$$\begin{aligned} \mathcal{L}[S_m(t) - k_m S_{m-1}(t)] &= h\mathcal{R}_m^S[\vec{S}_{m-1}] \\ \mathcal{L}[E_m(t) - k_m E_{m-1}(t)] &= h\mathcal{R}_m^E[\vec{E}_{m-1}] \\ \mathcal{L}[I_m(t) - k_m I_{m-1}(t)] &= h\mathcal{R}_m^I[\vec{I}_{m-1}] \\ \mathcal{L}[Q_m(t) - k_m Q_{m-1}(t)] &= h\mathcal{R}_m^Q[\vec{Q}_{m-1}] \\ \mathcal{L}[R_m(t) - k_m R_{m-1}(t)] &= h\mathcal{R}_m^R[\vec{R}_{m-1}] \end{aligned} \quad (22)$$

Table 1. Parameter values used in the model[41-42]

Parameter	Definition	Value
Λ	Recovery rate of individuals	750
ϕ_c	Contact rate for Covid-19 transmission	0.0000124
σ	Exposure rate of individuals	0.000011618
μ	Rate of natural deaths	0.00324588
v_2	Relapse rate of individuals	0.001466848
δ	Death rate from Covid-19	0.00286
θ	Survival rate of individuals in quarantine	0.0766169
α	Detection rate of infectious individuals	0.010939586
v_1	Survival rate of infectious people	0.1109289
v_3	Rate of returning individuals to the susceptible class	0.0022927

Where,

$k_m = \begin{cases} 0, & m \le 1 \\ n & m \ge 1 \end{cases}$ and

$$\mathcal{R}_m^S[\vec{S}_{m-1}] = \mathcal{L}[\vec{S}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{S_0}{s} + \frac{1}{s^{\beta_1}}\mathcal{L}\left[-\Lambda + \phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i} + \mu S_{m-1} - v_3 R_{m-1}\right]$$

$$\mathcal{R}_m^E[\vec{E}_{m-1}] = \mathcal{L}[\vec{E}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{E_0}{s} + \frac{1}{s^{\beta_2}}\mathcal{L}\left[-\phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i} + (\sigma + \mu)E_{m-1}\right]$$

$$\mathcal{R}_m^I[\vec{I}_{m-1}] = \mathcal{L}[\vec{I}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{I_0}{s} + \frac{1}{s^{\beta_3}}\mathcal{L}[-\sigma E_{m-1} + (\alpha + v_2 + \delta + \mu)I_{m-1} - v_1 R_{m-1}] \tag{23}$$

$$\mathcal{R}_m^Q[\vec{Q}_{m-1}] = \mathcal{L}[\vec{Q}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{Q_0}{s} + \frac{1}{s^{\beta_4}}\mathcal{L}[-\alpha I_{m-1} + (\theta + \mu)Q_{m-1}]$$

$$\mathcal{R}_m^R[\vec{R}_{m-1}] = \mathcal{L}[\vec{R}_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right)\frac{R_0}{s} + \frac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q_{m-1} + (v_1 + v_3 + \mu)R_{m-1} - v_2 I_{m-1}]$$

If the expressions (23) are substituted in (22) and taken in reverse Laplace, we get to following equations.

$$\begin{aligned} S_m - k_m S_{m-1} &= hS_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{S_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c \textstyle\sum_{i=0}^{m-1} S_i I_{m-1-i} + \mu S_{m-1} - v_3 R_{m-1}]\right\} \\ S_m &= (k_m + h)S_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{S_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c \textstyle\sum_{i=0}^{m-1} S_i I_{m-1-i} + \mu S_{m-1} - v_3 R_{m-1}]\right\} \end{aligned} \tag{23.1}$$

$$E_m - k_m E_{m-1} = hE_{m-1} - h\left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left[\frac{E_0}{s}\right] + h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_2}}\mathcal{L}\left[-\phi_c \sum_{i=0}^{m-1} S_i I_{m-1-i} + (\sigma + \mu)E_{m-1}\right]\right\} \tag{23.2}$$

$$E_m = (k_m + h)E_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{E_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_2}}\mathcal{L}[-\phi_c \textstyle\sum_{i=0}^{m-1} S_i I_{m-1-i} + (\sigma + \mu)E_{m-1}]\right\}$$

$$I_m = (k_m + h)I_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{I_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_3}}\mathcal{L}[-\sigma E_{m-1} + (\alpha + v_2 + \delta + \mu)I_{m-1} - v_1 R_{m-1}]\right\} \tag{23.3}$$

$$Q_m = (k_m + h)Q_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{Q_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_4}}\mathcal{L}\{-\alpha I_{m-1} + (\theta + \mu)Q_{m-1}\}\right\} \tag{23.4}$$

$$R_m = (k_m + h)R_{m-1} - h\left(1 - \tfrac{k_m}{n}\right)\mathcal{L}^{-1}\left[\tfrac{R_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_5}}\mathcal{L}[-\theta Q_{m-1} + (v_1 + v_3 + \mu)R_{m-1} - v_2 I_{m-1}]\right\} \tag{23.5}$$

From here, general solution is found as in (24).

$$\begin{aligned} S(t) &= S_0 + \sum_{m=1}^{\infty} S_m(t)\left(\frac{1}{n}\right)^m \\ E(t) &= E_0 + \sum_{m=1}^{\infty} E_m(t)\left(\frac{1}{n}\right)^m \\ I(t) &= I_0 + \sum_{m=1}^{\infty} I_m(t)\left(\frac{1}{n}\right)^m \\ Q(t) &= Q_0 + \sum_{m=1}^{\infty} Q_m(t)\left(\frac{1}{n}\right)^m \\ R(t) &= S_0 + \sum_{m=1}^{\infty} R_m(t)\left(\frac{1}{n}\right)^m \end{aligned} \tag{24}$$

(23.1), (23.2), (23.3), (23.4), (23.5) for $i = 0, m = 1$ for (23.1) equation, when using the (14) feature and performing the necessary actions, the following results are obtained.

$$\begin{aligned} S_1 &= (k_1 + h)S_0 - h\left(1 - \tfrac{k_1}{n}\right)\mathcal{L}^{-1}\left[\tfrac{S_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0]\right\} \\ S_1 &= (0 + h)S_0 - h\left(1 - \tfrac{0}{n}\right)\mathcal{L}^{-1}\left[\tfrac{S_0}{s}\right] + h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0]\right\} \\ S_1 &= h\mathcal{L}^{-1}\left\{\tfrac{1}{s^{\beta_1}}\mathcal{L}\left[(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0).\tfrac{1}{s}\right]\right\} \\ S_1 &= h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} \end{aligned} \tag{25}$$

For $i = 1, m = 1$ E_1, I_1, Q_1 and R_1 are found as follows, similar to S_1 operations in the equations (23.2),(23.3),(23.4) and (23.5) and again (14) feature.

$$E_1 = h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2 + 1)} \tag{26}$$

$$I_1 = h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3 + 1)} \tag{27}$$

$$Q_1 = h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4 + 1)} \tag{28}$$

$$R_1 = h(\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5 + 1)} \tag{29}$$

From equation (23.1) and property (14) for $m = 2$, S_2 is obtained.

$$S_2 = (k_2 + h)S_1 - h\left(1 - \frac{k_2}{n}\right)\mathcal{L}^{-1}\left[\frac{S_0}{s}\right] + h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}[-\Lambda + \phi_c(S_0 I_1 + S_1 I_0 + \mu S_1 - v_3 R_1]\right\}$$

(25), (27), (29) expressions are written instead; S_2 is found as

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\mathcal{L}\left[A\frac{t^{\beta_3}}{\Gamma(\beta_3 + 1)} + B\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + C\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + D\frac{t^{\beta_5}}{\Gamma(\beta_5 + 1)}\right]\right\}$$

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + h\mathcal{L}^{-1}\left\{\frac{1}{s^{\beta_1}}\left[A\frac{1}{s^{\beta_3+1}} + B\frac{1}{s^{\beta_1+1}} + C\frac{1}{s^{\beta_1+1}} + \frac{1}{s^{\beta_5+1}}\right]\right\}$$

$$S_2 = (n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + h\left(A\frac{t^{\beta_3+\beta_1}}{\Gamma(\beta_3 + \beta_1 + 1)} + B\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + C\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + D\frac{t^{\beta_1+\beta_5}}{\Gamma(\beta_1 + \beta_5 + 1)}\right)$$

where

$$A = -\Lambda + \phi_c S_0 h\left(\sigma E_0 + \left((\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0\right)\right)$$
$$B = \left(\left(\phi_c I_0 h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\right)\right)$$
$$C = \left(\mu h\left((-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\right)\right)$$
$$D = -v_3 h\left((\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)\right)$$

Again , similar to the operations that find E_2, I_2, Q_2 and R_2 are found as follows:

$$E_2 = (n + h)h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2 + 1)} + h\left(E_*\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3 + \beta_2 + 1)} + F\frac{t^{\beta_2+\beta_1}}{\Gamma(\beta_2 + \beta_1 + 1)} + G\frac{t^{2\beta_2}}{\Gamma(2\beta_2 + 1)}\right)$$

where

$$E_* = -\phi_c S_0 h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$
$$F = -\phi_c I_0 h\left((-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\right)$$
$$G = (\sigma + \mu)h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)$$

Similarly,

$$I_2 = (n + h)h\left((-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3 + 1)}\right) + h\left(H\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3 + \beta_2 + 1)} + I_1\frac{t^{2\beta_3}}{\Gamma(2\beta_3 + 1)} + K\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3 + \beta_5 + 1)} + L\frac{t^{2\beta_3}}{\Gamma(2\beta_3 + 1)}\right)$$

where,

$$H = -\sigma h(-\phi_c S_0 I_0 + (\sigma + \mu)E_0)$$
$$I_1 = (\alpha + v_2 + \delta + \mu)h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$
$$K = -v_1 h(\theta Q_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)$$

Similarly,

$$Q_2 = (n + h)h(-\alpha I_0 + (\theta + \mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4 + 1)} + h\left(M\frac{t^{\beta_3+\beta_4}}{\Gamma(\beta_3 + \beta_4 + 1)} + N\frac{t^{2\beta_4}}{\Gamma(2\beta_4 + 1)}\right)$$

where

$$M = -\alpha h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$
$$N = (\theta + \mu)h(-\alpha I_0 + (\theta + \mu)Q_0)$$

Similarly,

$$R_2 = (n + h)h(\theta Q_0 + (v_1 + v_3 + \mu)R_0 - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5 + 1)} + h\left(O\frac{t^{\beta_4+\beta_5}}{\Gamma(\beta_4 + \beta_5 + 1)} + \ddot{O}\frac{t^{2\beta_5}}{\Gamma(2\beta_5 + 1)} + P\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3 + \beta_5 + 1)}\right)$$

where

$$O = \left(\theta h(-\alpha I_0 + (\theta + \mu)Q_0)\right)$$
$$\ddot{O} = (v_1 + v_3 + \mu)h(\theta R_0 - (v_1 + v_3 + \mu)R_0 - v_2 I_0)$$
$$P = -v_2 h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)$$

The solution using equations is,

$$S(t) = S_0 + S_1(t)\left(\frac{1}{n}\right) + S_2(t)\left(\frac{1}{n}\right)^2$$

$$S(t) = S_0 + \left(h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)}\right)\left(\frac{1}{n}\right) + \left[(n + h)h(-\Lambda + \phi_c S_0 I_0 + \mu S_0 - v_3 R_0)\frac{t^{\beta_1}}{\Gamma(\beta_1 + 1)} + h\left(A\frac{t^{\beta_3+\beta_1}}{\Gamma(\beta_3 + \beta_1 + 1)} + B\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + C\frac{t^{2\beta_1}}{\Gamma(2\beta_1 + 1)} + D\frac{t^{\beta_1+\beta_5}}{\Gamma(\beta_1 + \beta_5 + 1)}\right)\right]\left(\frac{1}{n}\right)^2$$

$$E(t) = E_0 + E_1(t)\left(\frac{1}{n}\right) + E_2(t)\left(\frac{1}{n}\right)^2$$

$$\begin{aligned} E(t) = E_0 &+ \left(h(-\phi_c S_0 I_0 + (\sigma+\mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2+1)}\right)\left(\frac{1}{n}\right) \\ &+ \left[(n+h)h(-\phi_c S_0 I_0 + (\sigma+\mu)E_0)\frac{t^{\beta_2}}{\Gamma(\beta_2+1)} \right. \\ &+ h\left(E_*\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3+\beta_2+1)} + F\frac{t^{\beta_2+\beta_1}}{\Gamma(\beta_2+\beta_1+1)} \right. \\ &\left.\left. + G\frac{t^{2\beta_2}}{\Gamma(2\beta_2+1)}\right)\right]\left(\frac{1}{n}\right)^2 \end{aligned}$$

$$I(t) = I_0 + I_1(t)\left(\frac{1}{n}\right) + I_2(t)\left(\frac{1}{n}\right)^2$$

$$\begin{aligned} I(t) = I_0 &+ \left(h(-\sigma E_0 + (\alpha + v_2 + \delta + \mu)I_0 \right. \\ &\left. - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3+1)}\right)\left(\frac{1}{n}\right) + \left[(n+h)h\left((-\sigma E_0 \right.\right. \\ &\left. + (\alpha + v_2 + \delta + \mu)I_0 - v_1 R_0)\frac{t^{\beta_3}}{\Gamma(\beta_3+1)}\right) \\ &+ h\left(H\frac{t^{\beta_3+\beta_2}}{\Gamma(\beta_3+\beta_2+1)} + I_1\frac{t^{2\beta_3}}{\Gamma(2\beta_3+1)} \right. \\ &\left.\left. + K\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3+\beta_5+1)} + L\frac{t^{2\beta_3}}{\Gamma(2\beta_3+1)}\right)\right]\left(\frac{1}{n}\right)^2 \end{aligned}$$

$$Q(t) = Q_0 + Q_1(t)\left(\frac{1}{n}\right) + Q_2(t)\left(\frac{1}{n}\right)^2$$

$$\begin{aligned} Q(t) = Q_0 &+ \left(h(-\alpha I_0 + (\theta+\mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4+1)}\right)\left(\frac{1}{n}\right) \\ &+ \left[(n+h)h(-\alpha I_0 + (\theta+\mu)Q_0)\frac{t^{\beta_4}}{\Gamma(\beta_4+1)} \right. \\ &\left. + h\left(M\frac{t^{\beta_3+\beta_4}}{\Gamma(\beta_3+\beta_4+1)} + N\frac{t^{2\beta_4}}{\Gamma(2\beta_4+1)}\right)\right]\left(\frac{1}{n}\right)^2 \end{aligned}$$

$$R(t) = R_0 + R_1(t)\left(\frac{1}{n}\right) + R_2(t)\left(\frac{1}{n}\right)^2$$

$$\begin{aligned} R(t) = R_0 &+ \left(h(\theta Q_0 - (v_1 + v_3 + \mu)R_0 \right. \\ &\left. - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5+1)}\right)\left(\frac{1}{n}\right) + \left[(n+h)h(\theta Q_0 \right. \\ &+ (v_1 + v_3 + \mu)R_0 - v_2 I_0)\frac{t^{\beta_5}}{\Gamma(\beta_5+1)} \\ &+ h\left(O\frac{t^{\beta_4+\beta_5}}{\Gamma(\beta_4+\beta_5+1)} + \ddot{O}\frac{t^{2\beta_5}}{\Gamma(2\beta_5+1)} \right. \\ &\left.\left. + P\frac{t^{\beta_3+\beta_5}}{\Gamma(\beta_3+\beta_5+1)}\right)\right]\left(\frac{1}{n}\right)^2 \end{aligned}$$

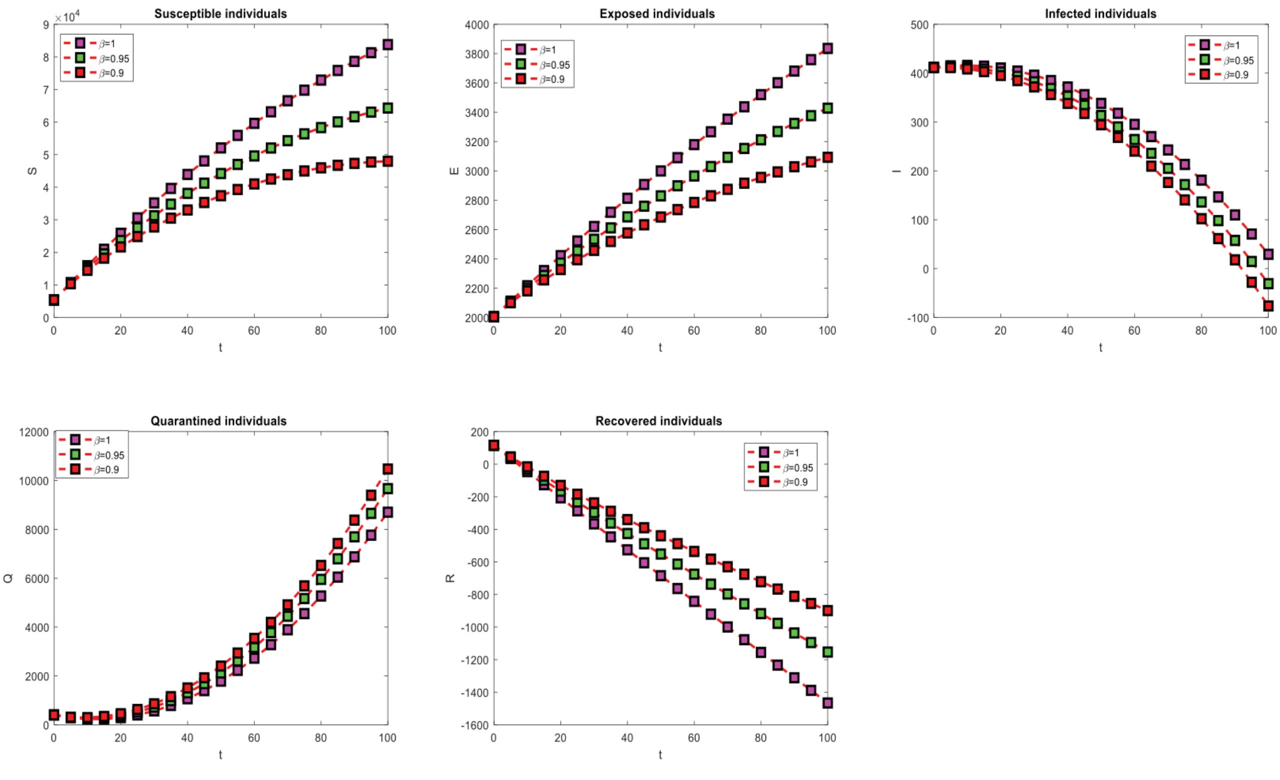

Figure 1. Variation of S, E, I, Q and R when $\beta = 1$, $\beta = 0.95$, $\beta = 0.9$.

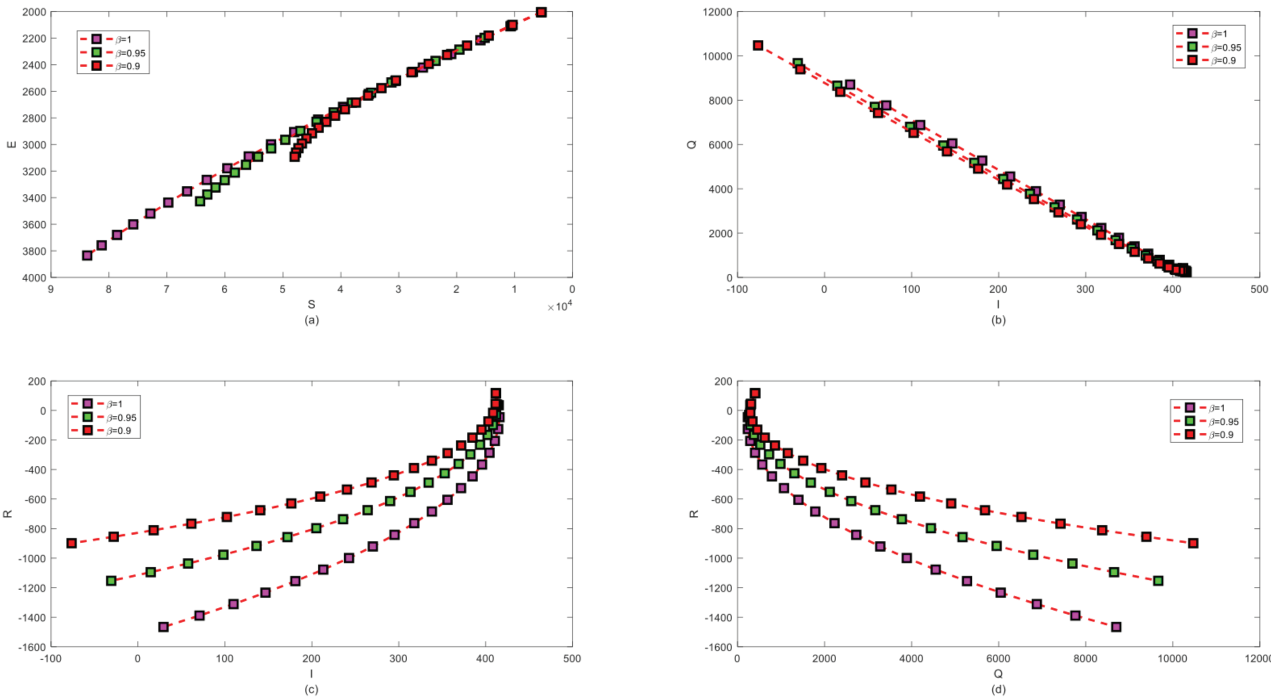

Figure 2. Phase diagram of fractional Covid-19 Model : (a) $S(t){-}E(t)$, (b) $I(t){-}Q(t)$, (c) $I(t){-}R(t)$, (d) $Q(t){-}R(t)$.

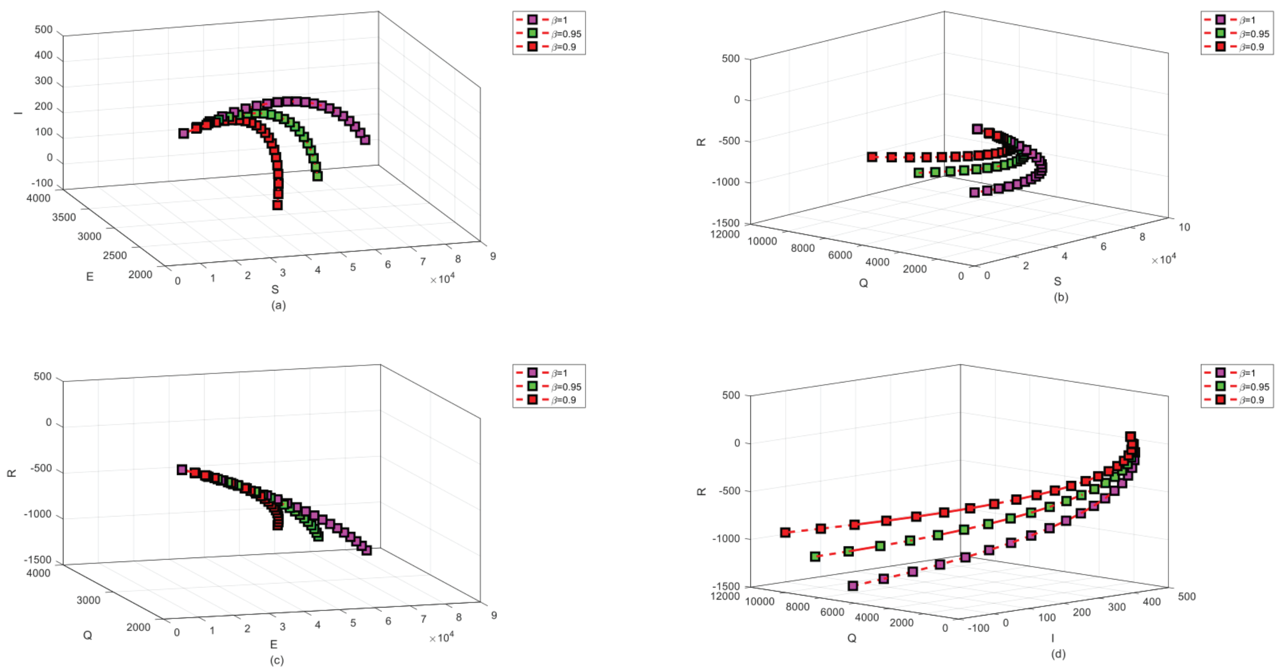

Figure 3. Phase portrait of fractional Covid-19 model.

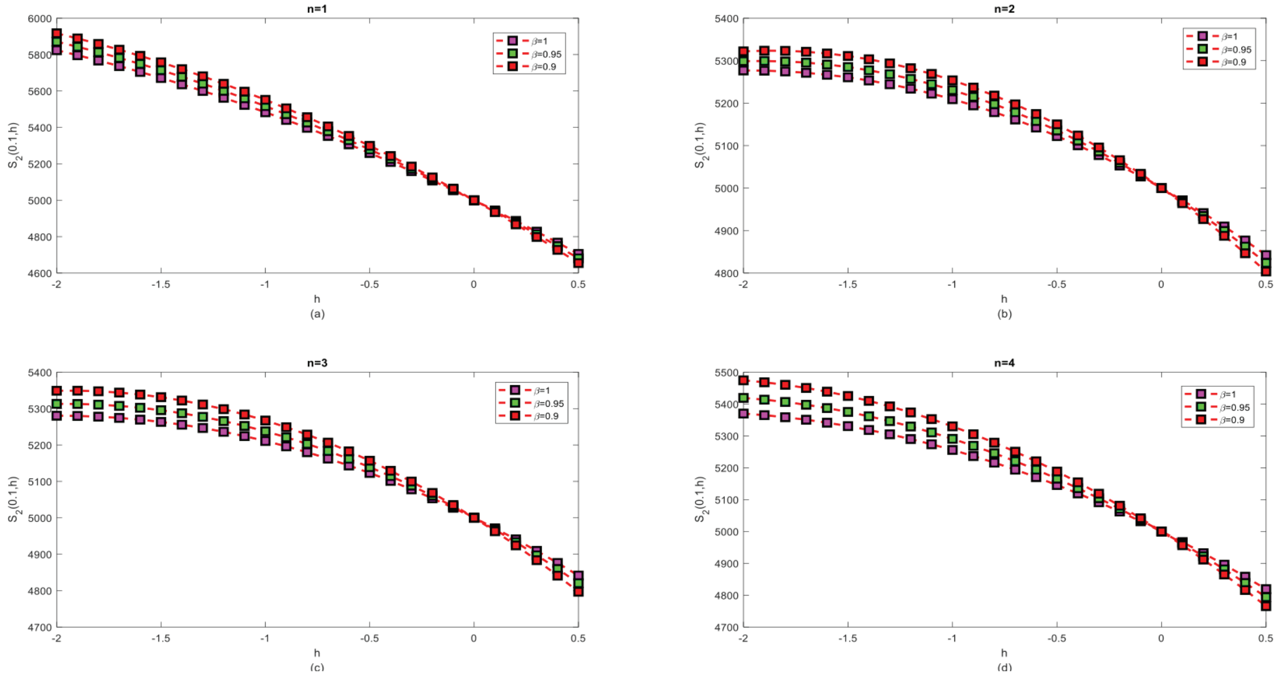

Figure 4. h-curves of $S(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.

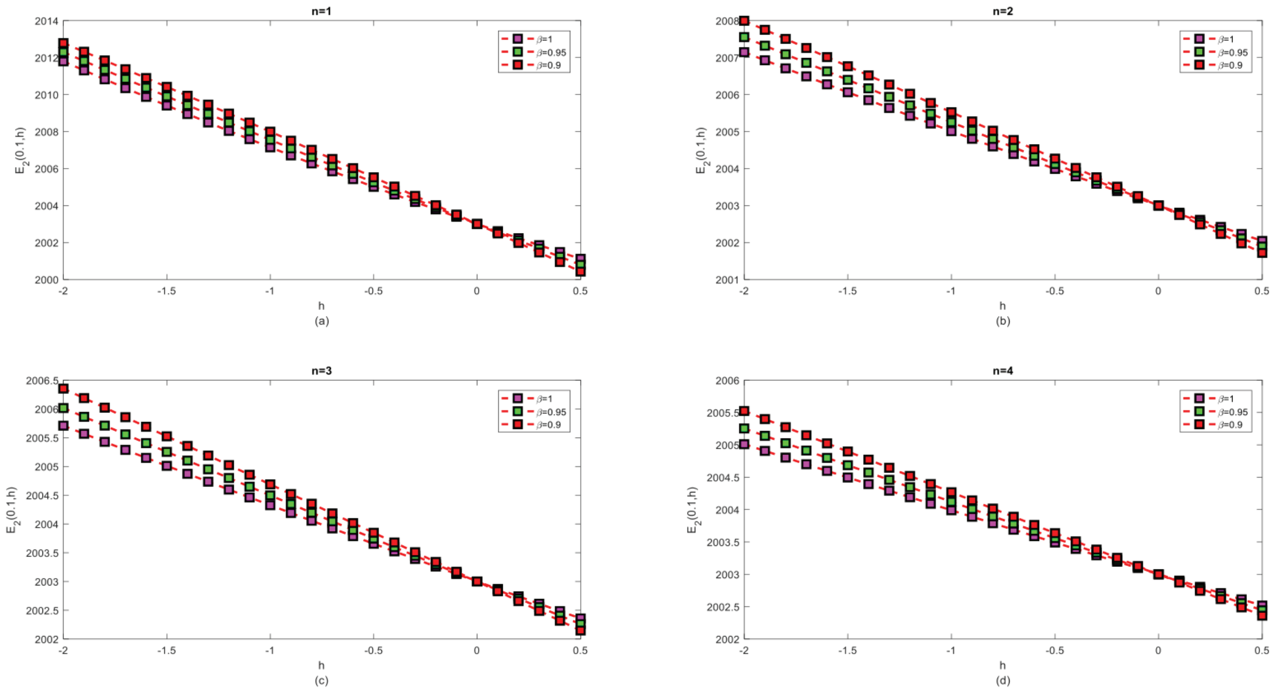

Figure 5. h-curves of $E(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.

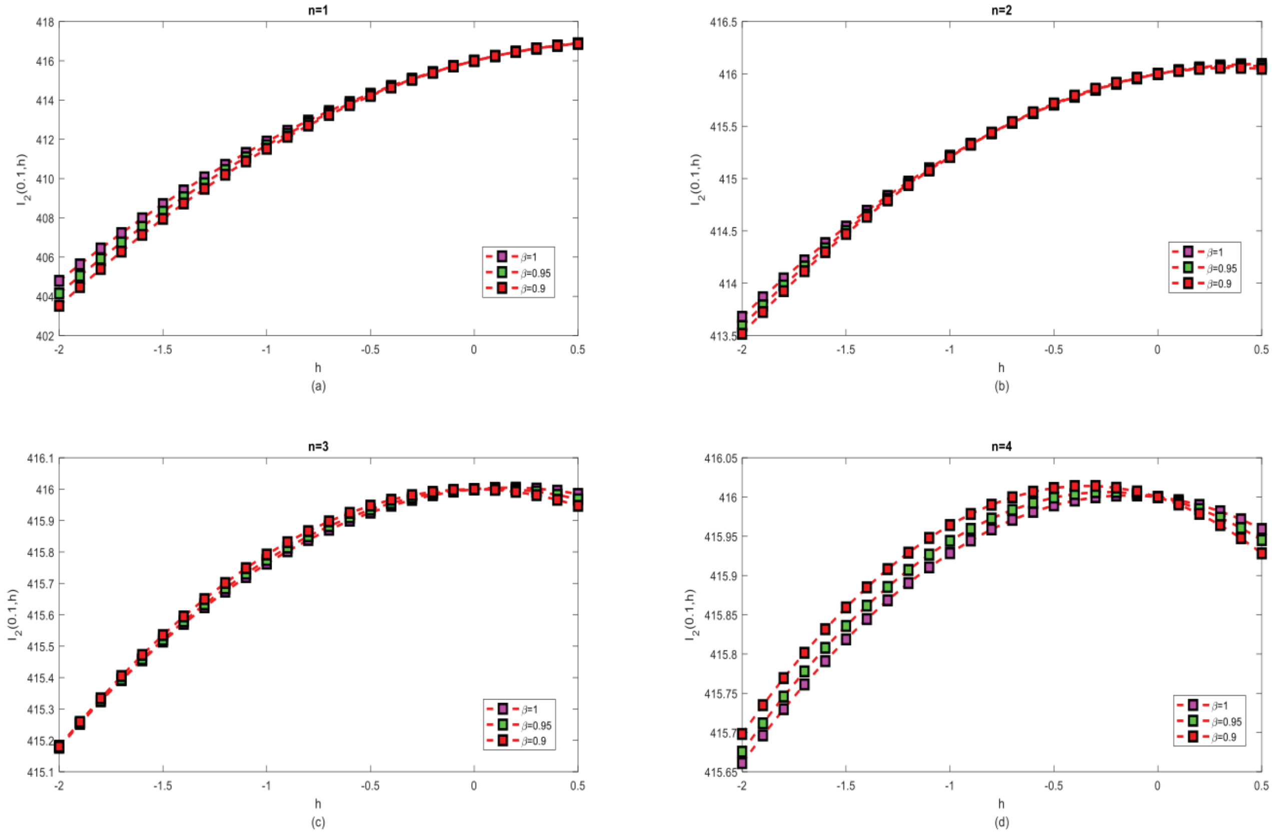

Figure 6. h-curves of $I(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.

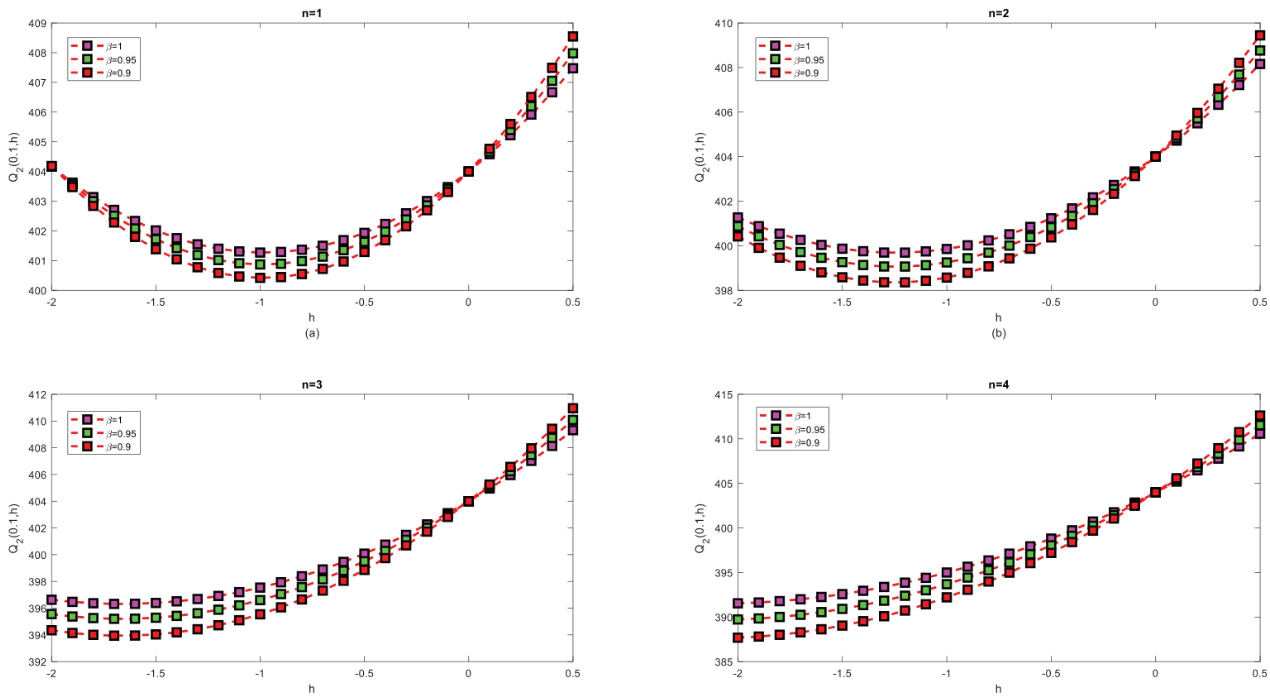

Figure 7. h-curves of $Q(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.

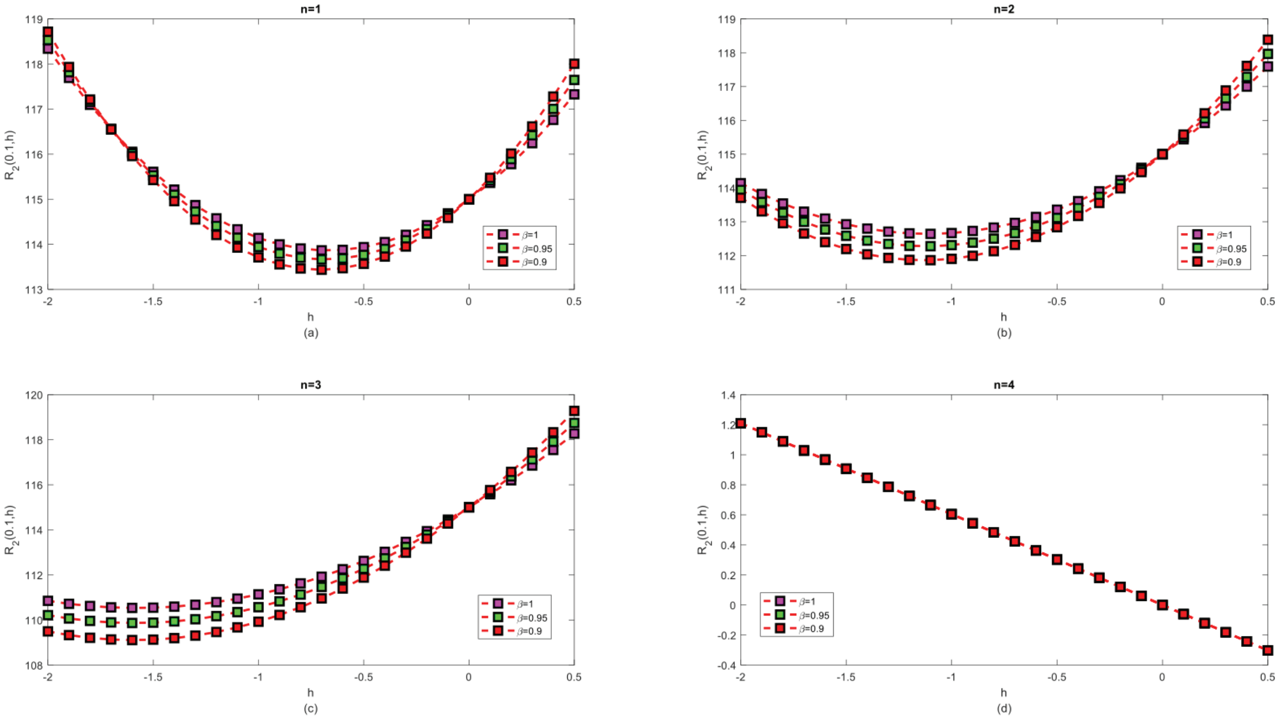

Figure 8. h-curves of $R(t)$ for different values of β when $t = 0.1$: (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$.

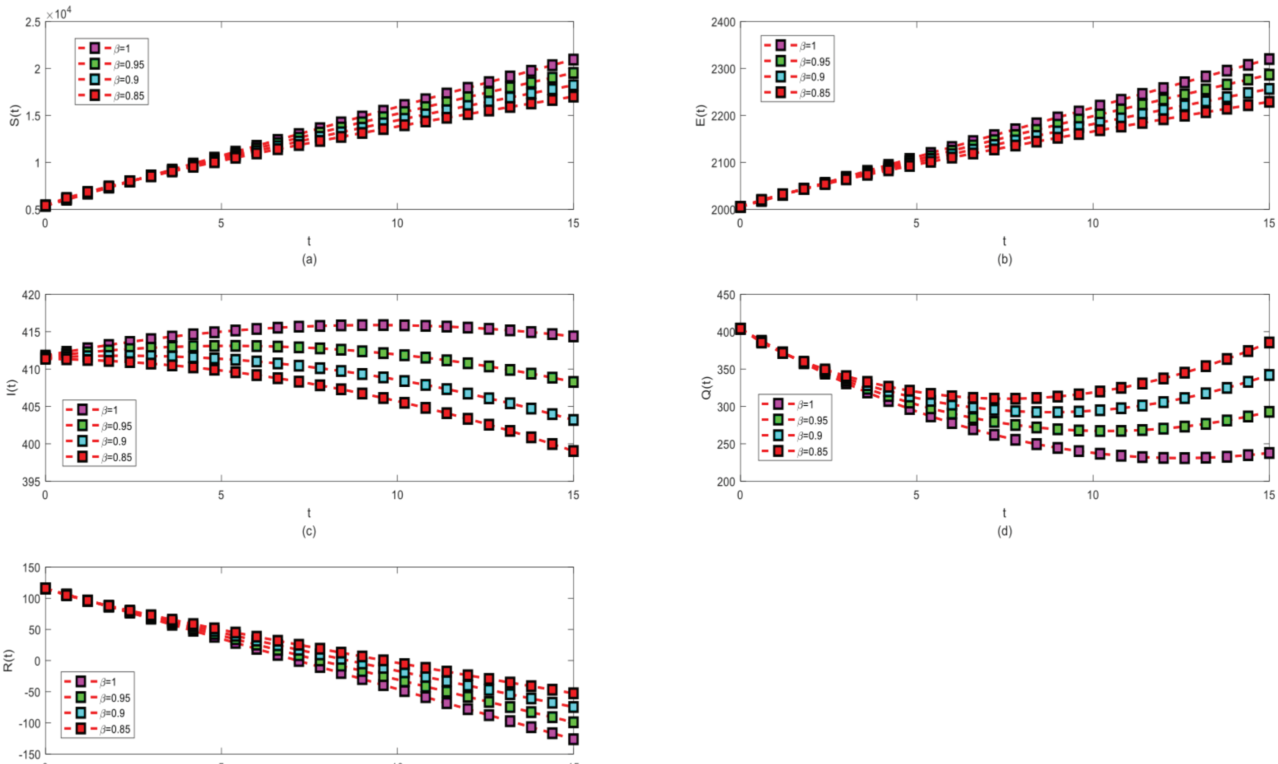

Figure 9. Numerical solutions of the Covid-19 model for differant values of β when h=−1 and n=1: (a)Susceptible individuals–$S(t)$, (b) exposed individuals –$E(t)$,(c) infected individuals –$I(t)$,(d)quarantined individuals –$Q(t)$ and (e) recovered individuals –$R(t)$.

Laplace Transform Homotopy Perturbation Method(LT-HPM)

To demonstrate the functionality of (LH-HPM), assume a general nonlinear fractional differential equation system represented as follows:

$$D_*^{\beta} x(t) = f(t) - Rx(t) - Nx(t); \tag{30}$$

where $n-1<\alpha<n$, $n \in N$, $t>0$, with the following boundary conditions,

$B(x; \frac{dx}{dt})$,

where R is a linear operator, N is a nonlinear operator, $f(t)$ is a well-known analytical function, and D_*^{β} represents the fractional derivative in the caputo sense. It is presumed that the solution $x(t)$ is a well-known function, and B is the boundary operator.

Initially, we shall formulate a homotopy as follows:

$$(1-p)\left[D_*^{\beta} x(t) - D_*^{\beta} x_0(t)\right] + p\left[D_*^{\beta} x_0(t) - f(t) + Rx(t) + Nx(t)\right] \tag{31}$$

or

$$D_*^{\beta} x(t) = D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - Rx(t) - Nx(t)\right] \tag{32}$$

Let p be a homotopy parameter, where $p \in [0,1]$, and x_0 represents the initial approximation for the solution of (30) that fulfills the boundary condition.

We will now apply the Laplace transform to both sides of equation (32), yielding:

$$\mathfrak{L}\left(D_*^{\beta} x(t)\right) = \mathfrak{L}\left[D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - Rx(t) - Nx(t)\right]\right] \tag{33}$$

Applying the formula for the Laplace transform, we get:

$$s^{\beta}\mathfrak{L}[x(t)] - \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) = \mathfrak{L}\left[D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - Rx(t) - Nx(t)\right]\right] \tag{34}$$

where m-1 < β < m, or

$$\mathfrak{L}[x(t)] = \frac{1}{s^{\beta}} \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^{\beta}}\, \mathfrak{L}\left[D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - Rx(t) - Nx(t)\right]\right] \tag{35}$$

Using the inverse Laplace transform on both sides of (9), we get:

$$x(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^{\beta}} \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^{\beta}}\, \mathfrak{L}\left[D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - Rx(t) - Nx(t)\right]\right]\right] \tag{36}$$

The solution of (36) may be expressed as a power series in terms of p as follows:

$$x(t) = \sum_{n=0}^{\infty} p^n x_n \tag{37}$$

Next, substituting (37) into (36), we obtain:

$$\sum_{n=0}^{\infty} p^n x_n = \mathfrak{L}^{-1}\left[\frac{1}{s^{\beta}} \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0) + \frac{1}{s^{\beta}}\, \mathfrak{L}\left[D_*^{\beta} x_0(t) + p\left[-D_*^{\beta} x_0(t) + f(t) - R\left[\sum_{n=0}^{\infty} p^n x_n\right] - N\left[\sum_{n=0}^{\infty} p^n x_n\right]\right]\right]\right] \tag{38}$$

By equating the terms with equivalent powers of p, we obtain:

$$\begin{aligned}
p^0&: x_0(t) = \mathfrak{L}^{-1}\left[\left[\frac{1}{s^{\beta}} \sum_{k=0}^{m-1} s^{\beta-k-1} f^{(k)}(0)\right] + \frac{1}{s^{\beta}}\, \mathfrak{L}\left[D_*^{\beta} x_0(t)\right]\right] \\
p^1&: x_1(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^{\beta}}\, \mathfrak{L}\left[-D_*^{\beta} x_0(t) + f(t) - R(x_0) - N(x_0)\right]\right] \\
p^2&: x_2(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^{\beta}}\, \mathfrak{L}\left[-R(x_0, x_1) - N(x_0, x_1))\right]\right] \\
p^3&: x_3(t) = \mathfrak{L}^{-1}\left[\frac{1}{s^{\beta}}\, \mathfrak{L}\left[-R(x_0, x_1, x_2) - N(x_0, x_1, x_2))\right]\right] \\
&\vdots
\end{aligned} \tag{39}$$

Assuming the initial approximation is expressed as $x(0) = \gamma_0, x'(0) = \gamma_1, \ldots, x^{(m-1)}(0) = \gamma_{m-1}$

Consequently, the precise answer may be derived as follows:

$$x = \lim_{p \to 1} \sum_{n=0}^{\infty} p^n x_n = x_0 + x_1 + x_2 + \cdots.$$

Initially, we will address (19) using the LT-HPM by replacing (38) and selecting $s_0(t) = 5000$, $e_0(t) = 2003$, $\iota_0(t) = 416$, $q_0(t) = 404$, $r_0(t) = 115$,

we obtain:

$$\sum_{n=0}^{\infty} p^n s_n = s_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\, \mathfrak{L}\left[\Lambda - \phi_c \sum_{n=0}^{\infty} p^n \iota_n \sum_{n=0}^{\infty} p^n s_n - \mu \sum_{n=0}^{\infty} p^n s_n + v_3 \sum_{n=0}^{\infty} p^n r_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n e_n = e_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_2}}\, \mathfrak{L}\left[\phi_c \sum_{n=0}^{\infty} p^n \iota_n \sum_{n=0}^{\infty} p^n s_n - (\sigma + \mu) \sum_{n=0}^{\infty} p^n e_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n \iota_n = \iota_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_3}}\, \mathfrak{L}\left[\sigma \sum_{n=0}^{\infty} p^n e_n - (\alpha + v_2 + \delta + \mu) \sum_{n=0}^{\infty} p^n \iota_n + v_1 \sum_{n=0}^{\infty} p^n r_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n q_n = q_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\, \mathfrak{L}\left[\alpha \sum_{n=0}^{\infty} p^n \iota_n - (\theta + \mu) \sum_{n=0}^{\infty} p^n q_n\right]\right]$$

$$\sum_{n=0}^{\infty} p^n r_n = r_0 + \mathfrak{L}^{-1}\left[\frac{p}{s^{\beta_1}}\, \mathfrak{L}\left[\theta \sum_{n=0}^{\infty} p^n q_n - (v_1 + v_3 + \mu) \sum_{n=0}^{\infty} p^n r_n + v_2 \sum_{n=0}^{\infty} p^n \iota_n\right]\right]$$

$$\begin{aligned}
S(t) &= 5000 + 1516.213419 t^{\frac{9}{10}} - 0.01938357757 t^{\frac{27}{10}} - 1.346390957 t^{\frac{9}{5}} \\
E(t) &= 2003 + 20.03318362 t^{\frac{9}{10}} + 0.01938357757 t^{\frac{27}{10}} - 0.03743710883 t^{\frac{9}{5}} \\
I(t) &= 416 + 5.280834026 t^{\frac{9}{10}} + 1.146277267 t^{\frac{9}{5}} \\
Q(t) &= 404 - 28.81526129 t^{\frac{9}{10}} + 1.353320950 t^{\frac{9}{5}}
\end{aligned}$$

Comparison

In order to compare accuracy of the obtained results by q-HATM and LT-HPM, Table 2,3 are presented.

Table 2. Table for numerical solutions of the Covid-19 model for $\beta = 0.9$ with LT-HPM

t	$S(t)$	$E(t)$	$I(t)$	$Q(t)$	$R(t)$
0.0	5000.	2003.	416.	404.	115.
0.1	5190.858582	2005.521474	416.6829849	400.3938222	117.3383382
0.2	5356.120044	2007.704456	417.3038556	397.3052983	119.2988352
0.3	5512.906629	2009.775362	417.9181995	394.4043519	121.1036538
0.4	5664.422246	2011.776651	418.5353209	391.6279520	122.7967567
0.5	5812.129021	2013.727749	419.1591106	388.9469232	124.3989815
0.6	5956.864980	2015.639824	419.7916089	386.3443237	125.9226853
0.7	6099.172362	2017.520127	420.4340179	383.8090547	127.3761218
0.8	6239.430012	2019.373748	421.0870970	381.3332687	128.7652329
0.9	6377.917112	2021.204447	421.7513479	378.9111153	130.0945239
1.0	6514.847644	2023.015131	422.4271113	376.5380596	131.3675435

Table 3. Table for numerical solutions of the Covid-19 model for $\beta = 0.9$ when h = −1 and $n = 1$ with q-HATM

t	$S(t)$	$E(t)$	$I(t)$	$Q(t)$	$R(t)$
0.0	5000	2003	416	404	115
0.1	5099.7381	2005.5589	416.5845	400.3848	117.3928
0.2	5197.4811	2007.8348	416.9608	397.2800	119.4884
0.3	5299.9824	2010.0456	417.2065	394.3601	121.4969
0.4	5408.0820	2012.2299	417.3408	391.5643	123.4568
0.5	5522.0261	2014.4045	417.3742	388.8648	125.3853
0.6	5641.8827	2016.5787	417.3133	386.2457	127.2921
0.7	5767.6499	2018.7583	417.1632	383.6965	129.1835
0.8	5899.2955	2020.9472	416.9276	381.2102	131.0637
0.9	6036.7735	2023.1480	416.6095	378.7814	132.9357
1.0	6180.0314	2025.3628	416.2115	376.4060	134.8021

Figure 1 and Table 2 show that similar results were obtained for Susceptible individuals–$S(t)$, exposed individuals $-E(t)$, infected individuals $-I(t)$,quarantined individuals $-Q(t)$ and recovered individuals $-R(t)$. Figure 9 and Table 3 show that similar results were obtained for Susceptible individuals–$S(t)$, exposed individuals $-E(t)$, infected individuals $-I(t)$, quarantined individuals $-Q(t)$ and recovered individuals $-R(t)$.

Once more, it has been noted that the Laplace Transform Homotopy Perturbation Method yields consistent results when contrasted with the q-homotopy analysis transform method.

CONCLUSION

In the present paper, approximate solutions for a five-dimensional Covid-19 model is investigated by the q-HATM for five sets of initial data. Time-fractional derivatives are taken in the Caputo sense. q-HAM with parameter $q \in \left[0, \frac{1}{n}\right], (n \geq 1)$ and the usual The Laplace transform method does not involve any linearization, discretization or restrictive assumptions. The schema also includes a helper parameter h that allows us to manipulate and control the serial solution to achieve fast convergence. The graphical representations provide an understanding of the behavior of the series solution in comparison to a varying fractional, parameterized, or auxiliary parameter h. The graphs illustrate the continuous reliance of the model solutions on the fractional degree parameter and the chosen system parameters. In addition to the Covid 19 model, q-HAM method, a solution has also been obtained with the Laplace Transform Homotopy Perturbation Method. The solutions obtained from both methods are shown in tables. In conclusion, we state that, in the light of the present work, q-HATM is only efficient and highly efficient systems of nonlinear fractional differential equations in a wide class, explaining various biological phenomena and other emerging systems in different fields of science and engineering.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Khan MA, Ullah S, Kumar S. A robust study on 2019-nCOV outbreaks through non-singular derivative. Eur Phys J Plus 2021;136:168. [CrossRef]

[2] Ortiz-Ospina E, Roser M, Ritchie H, Hasell J. Coronavirus pandemic (COVID-19). Our World in Data. 2020.

[3] Aba Oud MA, Ali A, Alrabaiah H. A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation, and environmental viral load. Adv Differ Equ 2021;106. [CrossRef]

[4] Yousif R, Jeribi A, Al-Azzawi S. Fractional-Order SEIRD model for global COVID-19 outbreak. Mathematics 2023;11:1036. [CrossRef]

[5] Kumar S, Chauhan RP, Momani S, Hadid S. Numerical investigations on COVID-19 model through singular and non-singular fractional operators. Numer Methods Partial Differ Equ 2020;1–27. [CrossRef]

[6] Kumar S, Kumar R, Osman MS, Samet B. A wavelet-based numerical scheme for fractional order SEIR epidemic of measles by using Genocchi polynomials. Numer Methods Partial Differ Equ 2021;37:1250–1268. [CrossRef]

[7] Alaroud M, Tahat N, Al-Omari S, Suthar D, Ozyurt SG. An attractive approach associated with transform functions for solving certain fractional Swift-Hohenberg equation. J Funct Spaces 2021;2021:3230272. [CrossRef]

[8] Aljarrah H, Alaroud M, Ishak A, Darus M. Adaptation of residual-error series algorithm to handle fractional system of partial differential equations. Mathematics 2021;9:2868. [CrossRef]

[9] Aljarrah H, Alaroud M, Ishak A, Darus M. Approximate solution of nonlinear time-fractional PDEs by Laplace residual power series method. Mathematics 2022;10:1980. [CrossRef]

[10] Kumar S, Kumar A, Samet B, Dutta H. A study on fractional host-parasitoid population dynamical model to describe insect species. Numer Methods Partial Differ Equ 2021;37:1673–1692. [CrossRef]

[11] Enyi CD, Nwaeze ER, Omaba MSE. Numerical and analytical solution to a conformable fractional Fornberg-Whitham equation. Math Sci 2022;2022:1–14. [CrossRef]

[12] Enyi CD. Efficacious analytical technique applied to fractional Fornberg-Whitham model and two-dimensional fractional population model. Symmetry 2020;12:1976. [CrossRef]

[13] Mohammad H, Kumar S, Rezapour S, Etemad S. A theoretical study of the Caputo-Fabrizio fractional modeling for hearing loss due to Mumps virus with optimal control. Chaos Solitons Fractals 2021;144:110668. [CrossRef]

[14] Alaroud M, Al-Smadi M, Arqub OA, Ahmad RR, Hammad MA, Momani S. Numerical solutions of linear time-fractional Klein-Gordon equation by using power series approach. Proc Int Conf Fract Differ Appl (ICFDA). 2018. [CrossRef]

[15] Liaqat MI, Okyere E. Comparative analysis of the time-fractional Black-Scholes option pricing equations (BSOPE) by the Laplace residual power series method (LRPSM). J Math. 2023;2023:6092283. [CrossRef]

[16] Wazwaz A. A reliable modification of Adomian decomposition method. Appl Math Comput 1999;102:77–86. [CrossRef]

[17] Momani S. An explicit and numerical solutions of the fractional KdV equation. Math Comput Simul 2005;70:110–118. [CrossRef]

[18] Momani S, Odibat Z. Analytical solution of a time-fractional Navier-Stokes equation by Adomian decomposition method. Appl Math Comput 2006;177:488–494. [CrossRef]

[19] Jafari H, Khalique CM, Nazari M. Application of Laplace decomposition method for solving linear and nonlinear fractional diffusion-wave equations. Appl Math Lett 2011;24:1799–1805. [CrossRef]

[20] Khan M, Hussain M. Application of Laplace decomposition method on semi-infinite domain. Numer Algorithms 2011;56:211–218. [CrossRef]

[21] Khan Y, Austin F. Application of the Laplace decomposition method to nonlinear homogeneous and non-homogeneous advection equations. Z Naturforsch A 2010;65:849–853. [CrossRef]

[22] Abbasdandy S. Homotopy analysis method for heat radiation equations. Int Commun Heat Mass Transf 2007;34:380–387. [CrossRef]

[23] Liao SJ. Beyond perturbation: introduction to the homotopy analysis method. Boca Raton: Chapman & Hall/CRC Press; 2003.

[24] Liao SJ. The proposed homotopy analysis technique for the solution of nonlinear problems [Dissertation]. Shanghai Jiao Tong Univ; 1992.

[25] Zurigat M, Momani S, Odibat Z, Alawneh A. The homotopy analysis method for handling systems of fractional differential equations. Appl Math Model 2010;34:24–35. [CrossRef]

[26] Momani S, Abdulaziz O, Hashim I. Application of homotopy-perturbation method to fractional IVPs. J Comput Appl Math 2008;216:574–584. [CrossRef]

[27] Jafari H, Kadem A, Baleanu D, Yilmaz T. Solution of the fractional Davey-Stewartson equations with variational iteration method. Rom Rep Phys 2012;64:337–346.

[28] Cheng X, Wang L, Hou J. Solving time fractional Keller-Segel type diffusion equations with symmetry analysis, power series method, invariant subspace method and q-homotopy analysis method. Chin J Phys 2022;77:1639–1653. [CrossRef]

[29] Kumar D, Singh J, Baleanu D. A new analysis for fractional model of regularized long-wave equations arising in ion acoustic plasma waves. Math Methods Appl Sci 2017;40:5642–5653. [CrossRef]

[30] Okposo NI, Jonathan AM. Application of the q-homotopy analysis transform method to the solution of a fractional attraction Keller-Segel chemotaxis model. Sci World J 2020;15:1–12.

[31] Prakasha A, Kaur H. Numerical solution for fractional model of Fokker-Planck equation by using q-HATM. Chaos Solitons Fractals 2017;105:99–110. [CrossRef]

[32] Singh J, Secer A, Swroop R, Kumar D. A reliable analytical approach for a fractional model of advection-dispersion equation. Nonlinear Eng 2019;8:107–116. [CrossRef]

[33] Karim Jbr R, AL-Rammahi A. q-homotopy analysis method for solving nonlinear Fredholm integral equation of the second kind. Int J Nonlinear Anal Appl 2021;12:2145–2152.

[34] El-Tawil MA, Huseen SN. The q-homotopy analysis method (q-HAM). Int J Appl Math Mech 2012;8:51–75. [CrossRef]

[35] El-Tawil MA, Huseen SN. On convergence of the q-homotopy analysis method. Int J Contemp Math Sci 2013;8:481–497. [CrossRef]

[36] Podlubny I. Fractional differential equations. San Diego: Academic Press; 1999.

[37] Caputo M, Fabrizio MA. New definition of fractional derivative without singular kernel. Prog Fract Differ Appl 2015;1:73–85.

[38] Caputo M. Elasticità e dissipazione. Bologna: Zanichelli; 1969.

[39] Miller KS, Ross B. An introduction to the fractional calculus and fractional differential equations. USA: Wiley-Interscience; 1993.

[40] Kilbas AA, Srivastava HM, Trujillo JJ. Theory and applications of fractional differential equations. USA: Elsevier Science; 2006.

[41] Krishna MV, Prakash J. Mathematical modelling on phase-based transmissibility of coronavirus. Infect Dis Model 2020;5:375–385. [CrossRef]

[42] Abioye AI, Peter OJ, Ogunseye HA, Oguntolu FA, Oshinubi K, Ibrahim AA, Khan I. Mathematical model of COVID-19 in Nigeria with optimal control. Results Phys 2021;28:104598. [CrossRef]

[43] Agarwal RP, Bohner M, Peterson A. Inequalities on time scales: A survey. Math Inequal Appl 2001;4:535–555. [CrossRef]

[44] Akın L, Zeren Y. On innovations of the multivariable fractional Hardy-type inequalities on time scales. Sigma J Eng Nat Sci 2023;41:415–422. [CrossRef]

[45] Akın L. On generalized weighted dynamic inequalities for diamond integral on time scales calculus. Indian J Pure Appl Math 2024;55:363–376. [CrossRef]

[46] Akın L. A new approach for the fractional integral operator in time scales with variable exponent Lebesgue spaces. Fractal Fract 2021;5:7. [CrossRef]

[47] Alkan A. Improving homotopy analysis method with an optimal parameter for time-fractional Burgers equation. Karamanoglu Mehmetbey Univ J Eng Nat Sci 2022;4:117–134. [CrossRef]

[48] Bektaş U, Anaç H. Q-Homotopy Shehu analysis transform method of time-fractional coupled Burgers equations. Eskisehir Tech Univ J Sci Technol A Appl Sci Eng 2023;24:177–191. [CrossRef]

[49] Mishra VN, Khatri K, Mishra LN, Deepmala. Inverse result in simultaneous approximation by Baskakov-Durrmeyer-Stancu operators. J Inequal Appl 2013;2013:586. [CrossRef]

[50] Mishra VN, Khatri K, Mishra LN. Some approximation properties of q-Baskakov-Beta-Stancu type operators. J Calc Var 2013;2013:814824. [CrossRef]

[51] Mishra VN, Khatri K, Mishra LN. Statistical approximation by Kantorovich type discrete q-Beta operators. Adv Differ Equ 2013;2013:345. [CrossRef]

[52] Sharma MK, Dhiman N, Vandana, Mishra VN. Mediative fuzzy logic mathematical model: A contradictory management prediction in COVID-19 pandemic. Appl Soft Comput 2021;105:107285. [CrossRef]

[53] Verma H, Mishra VN, Mathur P. Effectiveness of lockdown to curtail the spread of coronavirus: A mathematical model. ISA Trans 2022;124:124–134. [CrossRef]

[54] Nath BJ, Dehingia K, Mishra VN, Chu YM, Sarmah HK. Mathematical analysis of a within-host model of SARS-CoV-2. Adv Differ Equ 2021;113. [CrossRef]

[55] Nandal A, Blagojevic M, Milosevic D, Dhaka A, Mishra LN. Fuzzy enhancement and deep hash layer-based neural network to detect COVID-19. J Intell Fuzzy Syst. 2021;41:1341−1351. [CrossRef]
[56] Sharma MK, Dhiman N, Mishra LN, Mishra VN, Sahani SK. Mediative fuzzy extension technique and its consistent measurement in the decision-making of medical application. Math Probl Eng 2021;2021:5530681. [CrossRef]
[57] Sharma MK, Chaudhary S, Rathour L, Mishra VN. Modified genetic algorithm with novel crossover and mutation operator in travelling salesman problem. Sigma J Eng Nat Sci 2024;42:1876−1883. [CrossRef]
[58] Same ME, Zanganeh H, Aydogan SM. Investigation of a class of the singular fractional integro-differential quantum equations with multi-step methods. J Math Ext 2021;15:1−54.
[59] Baleanu D, Ghafarnezhad K, Rezapour S, Shabibi M. On the existence of solutions of a three steps crisis integro-differential equation. Adv Differ Equ 2018;2018:135. [CrossRef]
[60] Rezapour S, Imran A, Hussain A, Martínez F, Etemad S, Kaabar MKA. Condensing functions and approximate endpoint criterion for the existence analysis of quantum integro-difference FBVPs. Symmetry 2021;13:469. [CrossRef]
[61] Aydogan SM. On a K-dimensional system of hybrid fractional differential equations with multi-point boundary conditions. J Math Ext 2021;15:10−18.
[62] Baleanu D, Aydogan SM, Mohammadi H, Rezapour S. On modelling of epidemic childhood diseases with the Caputo-Fabrizio derivative by using the Laplace Adomian decomposition method. Alex Eng J 2020;59:3029−3039. [CrossRef]
[63] Baleanu D, Jajarmi A, Mohammadi H, Rezapour S. A new study on the mathematical modelling of human liver with Caputo-Fabrizio fractional derivative. Chaos Solitons Fractals 2020;134:109705. [CrossRef]
[64] Baleanu D, Etemad S, Mohammadi H, Rezapour S. A novel modeling of boundary value problems on the glucose graph. Commun Nonlinear Sci Numer Simul 2021;100:105844. [CrossRef]