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Research Article

A consensus reaching process with fuzzy matrix of interpersonal influences

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ABSTRACT

Consensus reaching process is an influential feature of social influence network theory and group decision making. The influence model undertakes a pivotal role where group decision making is based on opinion dynamics. Interpersonal influences of experts have an exceptional part of the influence model which is a dominant addition in social influence networks. Influence based model is a simple and satisfactory mathematical representation of the change of opinions due to the experts' influences. Uncertainty is presupposed in almost every direction of decision making and opinion dynamics so should also be included in the influence model. In this paper, the influence model is refined by utilizing the triangular fuzzy numbers in place of crisp numbers, where not only the initial opinions but also the interpersonal influences are represented as fuzzy numbers. This extends the influence model from ordinary numbers to fuzzy numbers. A fuzzy inverse matrix is computed by using a system of linear equations where coefficients and constants are fuzzy numbers. These equations are called fuzzy linear equations.

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INTRODUCTION

Decision-making is an imperative element of our routine life whether we are at home or work and have become complicated due to the development of society. Organizations arrange group of members for their decision-making processes, which is known as group decision making (GDM). GDM models are usually concerned with two processes: the consensus reaching process (CRP) [1-3] and the selection process [4,5]. CRP is an iterative process leading to the final solution after discussions and interpersonal influences of experts. Yao and Gu [6] propose a consensus model based on an influence network for large-scale GDM. After the influence model was introduced by Friedkin and Johnsen [7], there have been presented its related concepts and applications by different authors [8-10]. Influence models contribute particularly in GDM, where initial opinions of n experts are revised due to their interpersonal and social influences [5,11]. These models are based on an iterative process where the matri $W = [w_{ij}]_{n\times n}$, ($w_{ij} \in [0,1]$), of interpersonal influences is a basic component. Weights w_{ij} are assumed to be satisfy the normalization property i.e. $\sum_{j=1}^{n} w_{ij} = 1$.

Fuzzy set theory [12] is suggested to deal with uncertainty which is an important aspect of almost all

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decision-making processes. Fuzzy set have many generalizations with a large number of applications in optimization and other decision making [13-15]. The mathematical theory of interpersonal influences leads us to construct the influence models. Fuzzy preference relations (FPRs) also play an important role in such problems. FPRs are utilized to form the initial opinions of experts and through a mathematical model final opinions are obtained [16-18].

Fuzzy numbers have been introduced by Jain [19] and Dubois and Prade [20] to model the uncertain information. Ranking fuzzy numbers is a crucial step in investigating fuzzy information in decision making process. A lot of contibutions have been made in ranking of fuzzy numbers [21-24]. A fuzzy number gets its extension from the real number and can be considered as a function whose domain is a specified set. A fuzzy set satisfying some conditions, is called a fuzzy number. Fuzzy numbers and the extension principle are the basis for fuzzy arithmetic [25]. Lee [26] has also discussed fuzzy numbers with their basic operations $\oplus$, $\ominus$, $\otimes$ and $\oslash$, where these operations are conducted for triangular fuzzy numbers (TFNs), with the help of α-cuts. Such operations are not fuzzy, the numbers on which the operations are performed are fuzzy, so the results of these operations are also fuzzy. TFN (a_l, a_0, a_r) assigns an interval, called α-cut, for each value in the interval [0,1]. Fuzzy numbers have a wide utilization in decision sciences and engineering applications [27,28].

A fuzzy matrix has two different meanings in the literature: firstly, $A = (a_{ij})_{m\times n}$ is called a fuzzy matrix if $a_{ij} \in [0,1]$, $(i = 1,2, \dots, m; j = 1,2, \dots, n)$, on the other hand a matrix with entries of fuzzy numbers is called a fuzzy matrix too. Fuzzy matrices of second class is considered in this article. There is a strong connection between interval matrix and fuzzy matrix because each α-cut of a fuzzy matrix is an interval matrix that contains interval numbers. The interval matrix, its operations and singularity were discussed by Rohn [29,30]. Clustering with covariance matrix and mediative fuzzy relations which contain a large number of uncertainties, also took advantage of fuzzy matrices [31-34]. An approximate inverse of an uncertain matrix was introduced in Ghaoui [35]. A square matrix A of order n represents the co-efficient matrix in a system $Ax = b$ of linear equations whose solution is uniquely determined by finding the inverse of A. In the case of a fuzzy matrix, the computation of its inverse is quite complicated. Dehghan et al. [36] presented some conditions for the invertibility of fuzzy matrices in terms of interval matrices. Farahani et.al. [37] presented a method to find the inverse of a fuzzy matrix by using eigen value method. Dequan and Guo [38] investigated a class of fuzzy linear matrix equation by using the embedding approach. Some recent developments are observed in neutrosophic fuzzy matrices [39,40]. Basaran [41] has suggested a method to calculate fuzzy inverse matrix by using a fuzzy linear equation system. This method of obtaining the inverse fuzzy matrix is utilized in this paper for the matrix of triangular fuzzy numbers and to present the influence model where the matrix of interpersonal influences is a fuzzy matrix with triangular fuzzy numbers as its entries.

The concepts of social influence network theory and opinion dynamics are not mathematical in nature, but formation of the matrices of opinions and influences prepares the fundamentals for mathematical procedures. This research aims to distend the insufficiency of mathematical techniques in opinion dynamics and social influences. Crisp numbers are replaced by fuzzy numbers in the CRP [7]. TFNs are utilized which are suitable for ranking and comparison of the influences of experts. TFNs give comprehensive results while inverse of the matrix is computed.

PRELIMINARIES

Triangular Fuzzy Numbers

Definition 1

[27] A fuzzy number $A = (a_l, a_0, a_r)$ where $a_l, a_0, a_r \in R$ and $a_l \le a_0 \le a_r$, is a triangular fuzzy number if it is represented as the following membership function:

$$\mu_{(A)}(x) = \begin{cases} 0, & x < a_l \\ \dfrac{x - a_l}{a_0 - a_l} & a_l \le x \le a_0 \\ \dfrac{a_r - x}{a_r - a_0} & a_0 \le x \le a_r \\ 0 & x > a_r \end{cases}.$$

Operations on fuzzy numbers

The arithmetic operations on fuzzy numbers can be defined by the extension principle.

Definition 2

[43] Let A and B be two fuzzy numbers and $\times$ be an operation on R, such as $+, -, *, \div, \cdots$. By extension principle, the extended operation $\otimes$ on fuzzy numbers can be defined by:

$$\mu_{A\otimes B}(z) = \sup_{x,y:\, z = x\times y} \min(A(x), B(y))$$

Suppose $A = (a_l, a_0, a_r)$ and B= (b_l, b_0, b_r) are two triangular fuzzy numbers. The formulae for the extended addition, subtraction, multiplication and division operations become [17,23]:

1. Addition

$$A \oplus B = (a_l, a_0, a_r) \oplus (b_l, b, b_r) = (a_l + b_l, a_0 + b_0, a_r + b_r).$$

2. Subtraction

$$A \ominus B = (a_l, a_0, a_r) \ominus (b_l, b, b_r) = (a_l - b_l, a_0 - b_0, a_r - b_r).$$

3. Multiplication

$$A \otimes B \cong (a_l, a_0, a_r) \otimes (b_l, b, b_r)$$
$$\cong (a_l b_l, a_0 b_0, a_r b_r).$$

4. Division

$$A \oslash B \cong (a_l, a_0, a_r) \oslash (b_l, b, b_r)$$
$$\cong \left(\frac{a_l}{b_r}, \frac{a_0}{b_0}, \frac{a_r}{b_l}\right).$$

Ordering of triangular fuzzy numbers

[43] The set of all fuzzy numbers is denoted by N_F. A total ordering $\preccurlyeq$ on N_T (the set of all triangular fuzzy numbers) may be defined as: $(a_l, a_0, a_r) \preccurlyeq (b_l, b, b_r)$ if and only if

1. $a_l < b_l$, or
2. $a_l = b_l$ but $a_0 < b_0$, or
3. $a_l = b_l$, $a_0 = b_0$ but $a_r < b_r$.

Fuzzy Inverse Matrix

Definition 3

[44] A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number.

let $A = (a_l, a_0, a_r)$ and $B = (b_l, b_0, b_r)$ be two $m \times n$ and $n \times p$ fuzzy matrices. The size of the product of two fuzzy matrices is $m \times p$ and is written as follows: $\tilde{A} \times \tilde{B} = \tilde{C} = (\tilde{c}_{ij})$, where $\tilde{c}_{ij} = \oplus(\tilde{a}_{ik} \otimes \tilde{b}_{kj})$ where $\otimes$ is the approximated multiplication.

Before defining fuzzy inverse matrix, it is necessary to define fuzzy zero number and fuzzy one number to develop fuzzy identity matrix.

Definition 4

[41] If the center value of a fuzzy number is 0 and the left and right spread values are α and β where $0 < \alpha < \beta < 1$, this fuzzy number is called fuzzy zero number and is denoted as $\tilde{0} = (-\alpha, 0, \beta)$.

Definition 5

If the center value of a fuzzy number is $\tilde{1}$ and the left and right spread values are δ and λ where $0 < \delta < \lambda < 1$, this fuzzy number is called fuzzy one number and is denoted as $\tilde{1} = (1 - \delta, 1, 1 + \lambda)$.

Definition 6

If the diagonal elements of a fuzzy matrix are fuzzy one numbers and the off-diagonal elements are fuzzy zero numbers, then this fuzzy matrix is called fuzzy identity matrix and is denoted by $\tilde{I}$.

$$\tilde{I} = \begin{bmatrix} \tilde{1} & \tilde{0} & \cdots & \tilde{0} \\ \tilde{0} & \tilde{1} & \cdots & \tilde{0} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{0} & \tilde{0} & \cdots & \tilde{1} \end{bmatrix}.$$

Social Influence Network Theory

Let $W = w_{ij}$ be the matrix of interpersonal influences among n experts with $w_{ij} \in [0,1]$ and $\sum_{j=1}^{n} w_{ij} = 1$. The diagonal matrix $A = diag(a_{11}, a_{22}, \ldots, a_{nn})$ is obtained from the matrix W computing $a_{ii} = 1 - w_{ii}$, $i = 12, \ldots, n$. The matrix A is clarified as the susceptibility of all experts to interpersonal influence [8]. With the first opinion $g^{(1)}$, the following iterative plan is suggested to find the revised and final opinion:

$$g^{(t)} = AWg^{(t-1)} + (I - A)g^{(1)} \quad (1)$$

If $I - AW$ is non-singular, then this process reaches the following

$$g^{(\infty)} = (I - AW)^{-1}(I - A)g^{(1)}$$

Figure 1 describes the general consensus reaching process with interpersonal influences of experts.

INFLUENCE MODEL WITH FUZZY MATRICES

Fuzzy numbers generalize real numbers and are very useful to represent data corresponding to uncertain situations. In this section, an influence model is presented which is similar to the model presented in section 2.3 but the ordinary numbers are replaced by fuzzy numbers. TFNs consider only 3 data points and two linear functions so suitable to represent the influences and opinions. Final opinions are

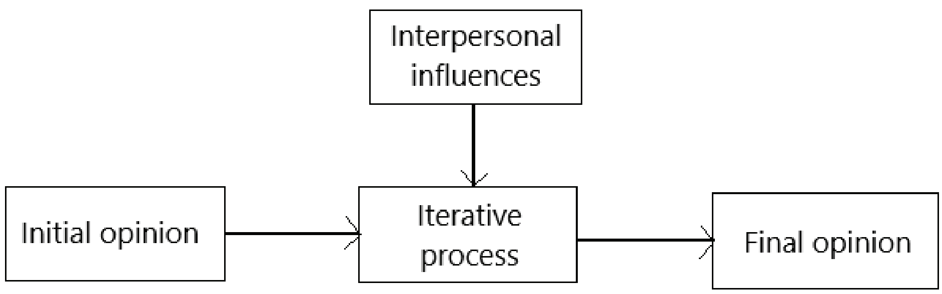

Figure 1. Consensus reaching process.

obtained when initial opinions and IIs of experts are given. Stepwise procedure is as follows:

Step 1:

A fuzzy matrix

$$\tilde{W} = \begin{bmatrix} \tilde{w}_{11} & \tilde{w}_{12} & \cdots & \tilde{w}_{1n} \\ \tilde{w}_{21} & \tilde{w}_{22} & \cdots & \tilde{w}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{n1} & \tilde{w}_{n2} & \cdots & \tilde{w}_{nn} \end{bmatrix}$$

is taken as a matrix of interpersonal influences, where $\tilde{w}_{ij}$ are assumed to be triangular fuzzy numbers and

$\tilde{w}_{i1} \oplus \tilde{w}_{i2} \oplus \cdots \oplus \tilde{w}_{in} = \tilde{1}$ for $i = 1,2,\cdots,n$. And

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & 0 & \cdots & 0 \\ 0 & \tilde{a}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{a}_{nn} \end{bmatrix}$$

is a matrix of sensitivities of experts to the interpersonal influences, $\tilde{a}_{ii} = \tilde{1} - \tilde{w}_{ii}$.

Step 2:

A fuzzy column matrix

$$\tilde{y}^{(1)} = \begin{pmatrix} \tilde{y}_1^{(1)} \\ \tilde{y}_2^{(1)} \\ \vdots \\ \tilde{y}_n^{(1)} \end{pmatrix}$$

is taken where its entries are the initial opinions of n experts.

Step 3:

Compute $\tilde{A}\tilde{W}$

$$\tilde{A}\tilde{W} = \begin{bmatrix} \tilde{w}_{11} \otimes \tilde{a}_{11} & \tilde{w}_{12} \otimes \tilde{a}_{22} & \cdots & \tilde{w}_{1n} \otimes \tilde{a}_{nn} \\ \tilde{w}_{21} \otimes \tilde{a}_{11} & \tilde{w}_{22} \otimes \tilde{a}_{22} & \cdots & \tilde{w}_{2n} \otimes \tilde{a}_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{n1} \otimes \tilde{a}_{11} & \tilde{w}_{n2} \otimes \tilde{a}_{22} & \cdots & \tilde{w}_{nn} \otimes \tilde{a}_{nn} \end{bmatrix},$$

then $\tilde{I} - \tilde{A}\tilde{W}$ and $\tilde{I} - \tilde{A}$ by using $\tilde{I}$ as defined in Definition 4.

Step 4:

Find $(\tilde{I} - \tilde{A}\tilde{W})^{-1}$ by the procedure described in section 3.1 and then final opinions y^{∞} by using the following equation:

$$y^{(\infty)} = \left(\tilde{I} - \tilde{A}\tilde{W}\right)^{-1}\left(\tilde{I} - \tilde{A}\right)y^{(1)}$$

Fuzzy Inverse Matrix

Following is a step-wise procedure to find fuzzy inverse of a fuzzy matrix with the entries represented as triangular fuzzy numbers:

Step 1:

Let $\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} \end{bmatrix}$

be a matrix with entries of triangular fuzzy numbers $\tilde{x}_{ij} = \left(x_{ij} - u_{ij}^L, x_{ij}, x_{ij} + u_{ij}^R\right)$, $i, j = 1,2,3$. u_{ij}^L and u_{ij}^R represent left and right spreads of $\tilde{x}_{ij}$ respectively. Let

$$\tilde{Z} = \begin{bmatrix} \tilde{z}_{11} & \tilde{z}_{12} & \tilde{z}_{13} \\ \tilde{z}_{21} & z_{22} & \tilde{z}_{23} \\ \tilde{z}_{31} & \tilde{z}_{32} & \tilde{z}_{33} \end{bmatrix}$$

be the inverse of $\tilde{X}$ where $z_{ij} = \left(z_{ij} - v_{ij}^L, z_{ij}, z_{ij} + v_{ij}^R\right)$, $i, j = 1,2,3$. v_{ij}^L and v_{ij}^R represent left and right spreads of $\tilde{z}_{ij}$ respectively. Then $\tilde{X} \otimes \tilde{Z} = \tilde{I}$.

Step 2:

We will solve the following system of equations:

$$(\tilde{x}_{11} \otimes \tilde{z}_{11}) \oplus (\tilde{x}_{12} \otimes \tilde{z}_{21}) \oplus (\tilde{x}_{13} \otimes \tilde{z}_{31}) = \tilde{1},$$
$$(\tilde{x}_{11} \otimes \tilde{z}_{12}) \oplus (\tilde{x}_{12} \otimes \tilde{z}_{22}) \oplus (\tilde{x}_{13} \otimes \tilde{z}_{32}) = \tilde{0},$$
$$(\tilde{x}_{11} \otimes \tilde{z}_{13}) \oplus (\tilde{x}_{12} \otimes \tilde{z}_{23}) \oplus (\tilde{x}_{13} \otimes \tilde{z}_{33}) = \tilde{0},$$

$$(\tilde{x}_{21} \otimes \tilde{z}_{11}) \oplus (\tilde{x}_{22} \otimes \tilde{z}_{21}) \oplus (\tilde{x}_{23} \otimes \tilde{z}_{31}) = \tilde{0},$$
$$(\tilde{x}_{21} \otimes \tilde{z}_{12}) \oplus (\tilde{x}_{22} \otimes \tilde{z}_{22}) \oplus (\tilde{x}_{23} \otimes \tilde{z}_{32}) = \tilde{1},$$
$$(\tilde{x}_{21} \otimes \tilde{z}_{13}) \oplus (\tilde{x}_{22} \otimes \tilde{z}_{23}) \oplus (\tilde{x}_{23} \otimes \tilde{z}_{33}) = \tilde{0},$$

$$(\tilde{x}_{31} \otimes \tilde{z}_{11}) \oplus (\tilde{x}_{32} \otimes \tilde{z}_{21}) \oplus (\tilde{x}_{33} \otimes \tilde{z}_{31}) = \tilde{0},$$
$$(\tilde{x}_{31} \otimes \tilde{z}_{12}) \oplus (\tilde{x}_{32} \otimes \tilde{z}_{22}) \oplus (\tilde{x}_{33} \otimes \tilde{z}_{32}) = \tilde{0},$$
$$(\tilde{x}_{31} \otimes \tilde{z}_{13}) \oplus (\tilde{x}_{32} \otimes \tilde{z}_{23}) \oplus (\tilde{x}_{33} \otimes \tilde{z}_{33}) = \tilde{1}.$$

Step 3:

Following system is solved for center part z_{ij} of the inverse matrix:

$$x_{11}z_{11} + x_{12}z_{21} + x_{13}z_{31} = 1,$$
$$x_{11}z_{12} + x_{12}z_{22} + x_{13}z_{32} = 0,$$
$$x_{11}z_{13} + x_{12}z_{23} + x_{13}z_{33} = 0,$$

$$x_{21}z_{11} + x_{22}z_{21} + x_{23}z_{31} = 0,$$
$$x_{21}z_{12} + x_{22}z_{22} + x_{23}z_{32} = 1,$$
$$x_{21}z_{13} + x_{22}z_{23} + x_{23}z_{33} = 0,$$

$$x_{31}z_{11} + x_{32}z_{21} + x_{33}z_{31} = 0,$$
$$x_{31}z_{12} + x_{32}z_{22} + x_{33}z_{32} = 0,$$
$$x_{31}z_{13} + x_{32}z_{23} + x_{33}z_{33} = 1.$$

To find the left spread part v_{ij}^L of the matrix, system is written as follows:

$$x_{11}v_{11}^L + u_{11}^L z_{11} + x_{12}v_{21}^L + u_{12}^L z_{21} + x_{13}v_{31}^L + u_{13}^L z_{31} = \delta,$$
$$x_{11}v_{12}^L + u_{11}^L z_{12} + x_{12}v_{22}^L + u_{12}^L z_{22} + x_{13}v_{32}^L + u_{13}^L z_{32} = \alpha,$$
$$x_{11}v_{13}^L + u_{11}^L z_{13} + x_{12}v_{23}^L + u_{12}^L z_{23} + x_{13}v_{33}^L + u_{13}^L z_{33} = \alpha,$$

$$x_{21}v_{11}^L + u_{21}^L z_{11} + x_{22}v_{21}^L + u_{22}^L z_{21} + x_{23}v_{31}^L + u_{23}^L z_{31} = \alpha,$$
$$x_{21}v_{12}^L + u_{21}^L z_{12} + x_{22}v_{22}^L + u_{22}^L z_{22} + x_{23}v_{32}^L + u_{23}^L z_{32} = \delta,$$
$$x_{21}v_{13}^L + u_{21}^L z_{13} + x_{22}v_{23}^L + u_{22}^L z_{23} + x_{23}v_{33}^L + u_{23}^L z_{33} = \alpha,$$

$$x_{31}v_{11}^L + u_{31}^L z_{11} + x_{32}v_{21}^L + u_{32}^L z_{21} + x_{33}v_{31}^L + u_{33}^L z_{31} = \alpha,$$
$$x_{31}v_{12}^L + u_{31}^L z_{12} + x_{32}v_{22}^L + u_{32}^L z_{22} + x_{33}v_{32}^L + u_{33}^L z_{32} = \alpha,$$
$$x_{31}v_{13}^L + u_{31}^L z_{13} + x_{32}v_{23}^L + u_{32}^L z_{23} + x_{33}v_{33}^L + u_{33}^L z_{33} = \delta.$$

Similarly, right spread part v_{ij}^R of the matrix, system is written as follows:

$$\begin{aligned}
x_{11}v_{11}^R + u_{11}^R z_{11} + x_{12}v_{21}^R + u_{12}^R z_{21} + x_{13}v_{31}^R + u_{13}^R z_{31} &= \delta,\\
x_{11}v_{12}^R + u_{11}^R z_{12} + x_{12}v_{22}^R + u_{12}^R z_{22} + x_{13}v_{32}^R + u_{13}^R z_{32} &= \alpha,\\
x_{11}v_{13}^R + u_{11}^R z_{13} + x_{12}v_{23}^R + u_{12}^R z_{23} + x_{13}v_{33}^R + u_{13}^R z_{33} &= \alpha,
\end{aligned}$$

$$\begin{aligned}
x_{21}v_{11}^R + u_{21}^R z_{11} + x_{22}v_{21}^R + u_{22}^R z_{21} + x_{23}v_{31}^R + u_{23}^R z_{31} &= \alpha,\\
x_{21}v_{12}^R + u_{21}^R z_{12} + x_{22}v_{22}^R + u_{22}^R z_{22} + x_{23}v_{32}^R + u_{23}^R z_{32} &= \delta,\\
x_{21}v_{13}^R + u_{21}^R z_{13} + x_{22}v_{23}^R + u_{22}^R z_{23} + x_{23}v_{33}^R + u_{23}^R z_{33} &= \alpha,
\end{aligned}$$

$$\begin{aligned}
x_{31}v_{11}^R + u_{31}^R z_{11} + x_{32}v_{21}^R + u_{32}^R z_{21} + x_{33}v_{31}^R + u_{33}^R z_{31} &= \alpha,\\
x_{31}v_{12}^R + u_{31}^R z_{12} + x_{32}v_{22}^R + u_{32}^R z_{22} + x_{33}v_{32}^R + u_{33}^R z_{32} &= \alpha,\\
x_{31}v_{13}^R + u_{31}^R z_{13} + x_{32}v_{23}^R + u_{32}^R z_{23} + x_{33}v_{33}^R + u_{33}^R z_{33} &= \delta.
\end{aligned}$$

Step 4:

Form the matrix $\tilde{Z}$ by substituting the values of z_{ij}, v_{ij}^L and v_{ij}^R.

Remark 1:

If only center parts

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \text{ and } Z = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix}$$

of $\tilde{X}$ and $\tilde{Z}$ respectively are considered, then $XZ = I$ or $ZX = I$ i.e. Z is inverse of X.

Remark 2:

The final opinion obtained in this way is not unique since the uniqueness of fuzzy inverse matrix is not guarenteed.

Example

Consider the initial opinion of 3 experts:

$$\tilde{y}^{(1)} = \begin{pmatrix} (0.5,1,2) \\ (1,3,4) \\ (2,3,5) \end{pmatrix}$$

about some alternative, and

$$\widetilde{W} = \begin{pmatrix} (1,3,5) & (2,4,5) & (0,2,3) \\ (0,1,2) & (2,4,6) & (3,4,7) \\ (1,2,5) & (3,4,6) & (0,2,5) \end{pmatrix}$$

be the matrix of interpersonal influences of these experts. Computation of final opinions is required.

Solution

Step 1: Normalized matrix of interpersonal influences is obtained by using operations defined in section 2.1.1 (see appendix):

$$\widetilde{W}^{(N)} = \begin{pmatrix} \left(\frac{1}{13},\frac{1}{3},\frac{5}{3}\right) & \left(\frac{2}{13},\frac{4}{9},\frac{5}{3}\right) & \left(0,\frac{2}{9},1\right) \\ \left(0,\frac{1}{9},\frac{2}{5}\right) & \left(\frac{2}{15},\frac{4}{9},\frac{6}{5}\right) & \left(\frac{1}{5},\frac{4}{9},\frac{7}{5}\right) \\ \left(\frac{1}{16},\frac{1}{4},\frac{5}{4}\right) & \left(\frac{3}{16},\frac{1}{2},\frac{3}{2}\right) & \left(0,\frac{1}{4},\frac{5}{4}\right) \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} \left(\frac{2}{13},\frac{2}{3},\frac{8}{3}\right) & (0,0,0) & (0,0,0) \\ (0,0,0) & \left(\frac{1}{5},\frac{5}{9},\frac{9}{5}\right) & (0,0,0) \\ (0,0,0) & (0,0,0) & \left(\frac{1}{4},\frac{3}{4},\frac{11}{4}\right) \end{pmatrix}$$

In this step identity matrix is considered while in the next step, fuzzy identity matrix is considered.

Step 2:

$$\tilde{A}\widetilde{W} = \begin{pmatrix} \left(\frac{2}{169},\frac{2}{9},\frac{40}{9}\right) & \left(\frac{4}{169},\frac{8}{27},\frac{40}{9}\right) & \left(0,\frac{4}{27},\frac{8}{3}\right) \\ \left(0,\frac{5}{81},\frac{18}{25}\right) & \left(\frac{2}{75},\frac{20}{81},\frac{63}{25}\right) & \left(\frac{1}{25},\frac{20}{81},\frac{63}{25}\right) \\ \left(\frac{1}{64},\frac{3}{16},\frac{55}{16}\right) & \left(\frac{3}{64},\frac{3}{8},\frac{33}{8}\right) & \left(0,\frac{3}{16},\frac{55}{16}\right) \end{pmatrix},$$

$$\left(\tilde{I} - \tilde{A}\right) = \begin{pmatrix} (1-\delta,1,1+\delta) & (-\alpha,0,\alpha) & (-\alpha,0,\alpha) \\ (-\alpha,0,\alpha) & (1-\delta,1,1+\delta) & (-\alpha,0,\alpha) \\ (-\alpha,0,\alpha) & (-\alpha,0,\alpha) & (1-\delta,1,1+\delta) \end{pmatrix}$$

$$- \begin{pmatrix} \left(\frac{2}{13},\frac{2}{3},\frac{8}{3}\right) & (0,0,0) & (0,0,0) \\ (0,0,0) & \left(\frac{1}{5},\frac{5}{9},\frac{9}{5}\right) & (0,0,0) \\ (0,0,0) & (0,0,0) & \left(\frac{1}{4},\frac{3}{4},\frac{11}{4}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{11}{13}-\delta,\frac{1}{3},\delta-\frac{5}{3}\right) & (-\alpha,0,\alpha) & (-\alpha,0,\alpha) \\ (-\alpha,0,\alpha) & \left(\frac{4}{5}-\delta,\frac{4}{9},\delta-\frac{4}{5}\right) & (-\alpha,0,\alpha) \\ (-\alpha,0,\alpha) & (-\alpha,0,\alpha) & \left(\frac{3}{4}-\delta,\frac{1}{4},\delta-\frac{7}{4}\right) \end{pmatrix},$$

$$\tilde{I} - \tilde{A}\widetilde{W} = \begin{pmatrix} \left(\frac{167}{169}-\delta,\frac{7}{9},\delta-\frac{31}{9}\right) & \left(-\alpha-\frac{4}{169},\frac{-8}{27},\alpha-\frac{40}{9}\right) & \left(-\alpha,\frac{-4}{27},\alpha-\frac{8}{3}\right) \\ \left(-\alpha,\frac{-5}{81},\alpha-\frac{18}{25}\right) & \left(\frac{73}{75}-\delta,\frac{61}{81},\delta-\frac{38}{25}\right) & \left(-\alpha-\frac{1}{25},\frac{-20}{81},\alpha-\frac{63}{25}\right) \\ \left(-\alpha-\frac{1}{64},\frac{-3}{16},\alpha-\frac{55}{16}\right) & \left(-\alpha-\frac{3}{64},\frac{-3}{8},\alpha-\frac{33}{8}\right) & \left(1-\delta,\frac{13}{16},\delta-\frac{39}{16}\right) \end{pmatrix}.$$

Step 3:

$$\text{Let } \tilde{Z} = \begin{pmatrix} (z_{11}-v_{11}^L, z_{11}, z_{11}+v_{11}^R) & (z_{12}-v_{12}^L, z_{12}, z_{12}+v_{12}^R) & (z_{13}-v_{13}^L, z_{13}, z_{13}+v_{13}^R) \\ (z_{21}-v_{21}^L, z_{21}, z_{21}+v_{21}^R) & (z_{22}-v_{22}^L, z_{22}, z_{22}+v_{22}^R) & (z_{23}-v_{23}^L, z_{23}, z_{23}+v_{23}^{\square\square}) \\ (z_{31}-v_{31}^L, z_{31}, z_{31}+v_{31}^R) & (z_{32}-v_{32}^L, z_{32}, z_{32}+v_{32}^R) & (z_{33}-v_{33}^L, z_{33}, z_{33}+v_{33}^R) \end{pmatrix},$$

be the inverse of $\tilde{I} - \tilde{A}\widetilde{W}$, described above. Then to find the center part of the matrix, following system of equations is solved:

$$\frac{7}{9}z_{11}-\frac{8}{27}z_{21}-\frac{4}{27}z_{31}=1,$$
$$\frac{5}{81}z_{11}+\frac{61}{81}z_{21}-\frac{20}{81}z_{31}=0,$$
$$\frac{3}{16}z_{11}-\frac{3}{8}z_{21}+\frac{13}{16}z_{31}=0,$$

$$\frac{7}{9}z_{12}-\frac{8}{27}z_{22}-\frac{4}{27}z_{32}=0,$$
$$\frac{5}{81}z_{12}+\frac{61}{81}z_{22}-\frac{20}{81}z_{32}=1,$$
$$\frac{3}{16}z_{12}-\frac{3}{8}z_{22}+\frac{13}{16}z_{32}=0,$$

$$\frac{7}{9}z_{13}-\frac{8}{27}z_{23}-\frac{4}{27}z_{33}=0,$$
$$\frac{5}{81}z_{13}+\frac{61}{81}z_{23}-\frac{20}{81}z_{33}=0,$$
$$\frac{3}{16}z_{13}-\frac{3}{8}z_{23}+\frac{13}{16}z_{33}=1,$$

$$z_{11}=1.4796,\ z_{12}=0.8442,\ z_{13}=0.5263,$$
$$z_{21}=0.2748,\ z_{22}=1.7214,\ z_{23}=0.5732,$$
$$z_{31}=0.4682,\ z_{32}=0.9893,\ z_{33}=1.6168.$$

To find the left spread part, following system of equations is solved:

$$\tfrac{7}{9}v_{11}^L+\left(\tfrac{167}{169}-\delta\right)z_{11}-\tfrac{8}{27}v_{21}^L+\left(-\tfrac{4}{169}-\alpha\right)z_{21}-\tfrac{4}{27}v_{31}^L+(-\alpha)z_{31}=\delta,$$
$$\tfrac{5}{81}v_{11}^L+(-\alpha)z_{11}-\tfrac{61}{81}v_{21}^L+\left(\tfrac{73}{75}-\delta\right)z_{21}-\tfrac{20}{81}v_{31}^L+\left(-\tfrac{1}{25}-\alpha\right)z_{31}=\alpha,$$
$$\tfrac{3}{16}v_{11}^L+\left(-\tfrac{1}{64}-\alpha\right)z_{11}-\tfrac{3}{8}v_{21}^L+\left(-\tfrac{3}{64}-\alpha\right)z_{21}+\tfrac{13}{16}v_{31}^L+(1-\delta)z_{31}=\alpha,$$

$$\tfrac{7}{9}v_{12}^L+\left(\tfrac{167}{169}-\delta\right)z_{12}-\tfrac{8}{27}v_{21}^L+\left(-\tfrac{4}{169}-\alpha\right)z_{21}-\tfrac{4}{27}v_{31}^L+(-\alpha)z_{31}=\alpha,$$
$$\tfrac{5}{81}v_{12}^L+(-\alpha)z_{12}-\tfrac{61}{81}v_{22}^L+\left(\tfrac{73}{75}-\delta\right)z_{22}-\tfrac{20}{81}v_{32}^L+\left(-\tfrac{1}{25}-\alpha\right)z_{32}=\delta,$$
$$\tfrac{3}{16}v_{12}^L+\left(-\tfrac{1}{64}-\alpha\right)z_{12}-\tfrac{3}{8}v_{22}^L+\left(-\tfrac{3}{64}-\alpha\right)z_{22}+\tfrac{13}{16}v_{32}^L+(1-\delta)z_{32}=\alpha,$$

$$\tfrac{7}{9}v_{13}^L+\left(\tfrac{167}{169}-\delta\right)z_{13}-\tfrac{8}{27}v_{23}^L+\left(-\tfrac{4}{169}-\alpha\right)z_{23}-\tfrac{4}{27}v_{33}^L+(-\alpha)z_{33}=\alpha,$$
$$\tfrac{5}{81}v_{13}^L+(-\alpha)z_{13}-\tfrac{61}{81}v_{23}^L+\left(\tfrac{73}{75}-\delta\right)z_{23}-\tfrac{20}{81}v_{33}^L+\left(-\tfrac{1}{25}-\alpha\right)z_{33}=\alpha,$$
$$\tfrac{3}{16}v_{13}^L+\left(-\tfrac{1}{64}-\alpha\right)z_{13}-\tfrac{3}{8}v_{23}^L+\left(-\tfrac{3}{64}-\alpha\right)z_{23}+\tfrac{13}{16}v_{33}^L+(1-\delta)z_{33}=\delta.$$

To find the right spread part, following system of equations is solved:

$$\tfrac{7}{9}v_{11}^R+\left(\delta-\tfrac{31}{9}\right)z_{11}-\tfrac{8}{27}v_{21}^R+\left(\alpha-\tfrac{40}{9}\right)z_{21}-\tfrac{4}{27}v_{31}^R+\left(\alpha-\tfrac{8}{3}\right)z_{31}=\delta,$$
$$\tfrac{5}{81}v_{11}^R+\left(\alpha-\tfrac{18}{25}\right)z_{11}-\tfrac{61}{81}v_{21}^R+\left(\delta-\tfrac{38}{25}\right)z_{21}-\tfrac{20}{81}v_{31}^R+\left(\alpha-\tfrac{63}{25}\right)z_{31}=\alpha,$$
$$\tfrac{3}{16}v_{11}^R+\left(\alpha-\tfrac{55}{16}\right)z_{11}-\tfrac{3}{8}v_{21}^R+\left(\alpha-\tfrac{33}{8}\right)z_{21}+\tfrac{13}{16}v_{31}^L+\left(\delta-\tfrac{39}{16}\right)z_{31}=\alpha,$$

$$\tfrac{7}{9}v_{12}^R+\left(\delta-\tfrac{31}{9}\right)z_{12}-\tfrac{8}{27}v_{22}^R+\left(\alpha-\tfrac{40}{9}\right)z_{22}-\tfrac{4}{27}v_{32}^R+\left(\alpha-\tfrac{8}{3}\right)z_{32}=\alpha,$$
$$\tfrac{5}{81}v_{12}^R+\left(\alpha-\tfrac{18}{25}\right)z_{12}-\tfrac{61}{81}v_{22}^R+\left(\delta-\tfrac{38}{25}\right)z_{22}-\tfrac{20}{81}v_{32}^R+\left(\alpha-\tfrac{63}{25}\right)z_{32}=\delta,$$
$$\tfrac{3}{16}v_{12}^R+\left(\alpha-\tfrac{55}{16}\right)z_{12}-\tfrac{3}{8}v_{22}^R+\left(\alpha-\tfrac{33}{8}\right)z_{22}+\tfrac{13}{16}v_{32}^L+\left(\delta-\tfrac{39}{16}\right)z_{32}=\alpha,$$

$$\tfrac{7}{9}v_{13}^R+\left(\delta-\tfrac{31}{9}\right)z_{13}-\tfrac{8}{27}v_{23}^R+\left(\alpha-\tfrac{40}{9}\right)z_{23}-\tfrac{4}{27}v_{33}^R+\left(\alpha-\tfrac{8}{3}\right)z_{33}=\alpha,$$
$$\tfrac{5}{81}v_{13}^R+\left(\alpha-\tfrac{18}{25}\right)z_{13}-\tfrac{61}{81}v_{23}^R+\left(\delta-\tfrac{38}{25}\right)z_{23}-\tfrac{20}{81}v_{33}^R+\left(\alpha-\tfrac{63}{25}\right)z_{33}=\alpha,$$
$$\tfrac{3}{16}v_{13}^R+\left(\alpha-\tfrac{55}{16}\right)z_{13}-\tfrac{3}{8}v_{23}^R+\left(\alpha-\tfrac{33}{8}\right)z_{23}+\tfrac{13}{16}v_{33}^L+\left(\delta-\tfrac{39}{16}\right)z_{33}=\delta.$$

Finally,

$$\left(\tilde I-\tilde A\widetilde W\right)^{-1}=\begin{pmatrix}
\begin{pmatrix}4.0708-4.1473\delta-5.0379\alpha,\\1.4796,\\1.4813-2.2968\alpha-1.1881\delta\end{pmatrix} & \begin{pmatrix}3.8706-4.0673\delta-8.9151\alpha,\\0.8442,\\31.7316-4.9032\alpha-2.3789\delta\end{pmatrix} & \begin{pmatrix}2.5247-2.6400\delta-7.9523\alpha,\\0.5247,\\22.1241-3.3046\alpha-1.5873\delta\end{pmatrix}\\
\begin{pmatrix}1.3508-1.4229\delta-0.6857\alpha,\\0.2748,\\11.1583-2.2684\alpha-0.8733\delta\end{pmatrix} & \begin{pmatrix}5.2689-5.4839\delta-6.2201\alpha,\\1.7214,\\22.307-4.5240\alpha-2.0410\delta\end{pmatrix} & \begin{pmatrix}2.4682-2.6315\delta-6.9177\alpha,\\0.5732,\\16.7743-2.9252\alpha-1.4850\delta\end{pmatrix}\\
\begin{pmatrix}2.0948-2.1901\delta-7.7177\alpha,\\0.4682,\\18.5478-2.5055\alpha-1.2535\delta\end{pmatrix} & \begin{pmatrix}4.4271-4.6873\delta-9.3167\alpha,\\0.9893,\\32.8976-5.1465\alpha-2.7086\delta\end{pmatrix} & \begin{pmatrix}4.8993-5.0444\delta-6.3812\alpha,\\1.6168,\\24.0659-3.4660\alpha-1.8108\delta\end{pmatrix}
\end{pmatrix},$$

$$\left(\tilde I-\tilde A\widetilde W\right)^{-1}\left(\tilde I-\tilde A\right)=\begin{pmatrix}
\begin{pmatrix}(4.0708-4.1473\delta-5.0379\alpha)\\(0.8461-\delta)-\alpha\\(6.3953-6.7073\delta-16.8674\alpha),\\\frac{1}{3}(1.4796),\\(1.4813-2.2968\alpha-1.1881\delta)\\(\delta-1.6666)+\alpha\\(53.8557-8.2078\alpha-3.9662\delta)\end{pmatrix} & \begin{pmatrix}(3.8706-4.0673\delta-8.9151\alpha)\\(0.8-\delta)-\alpha\\(6.5955-6.7873\delta-12.9902\alpha),\\\frac{4}{9}(0.8442),\\(31.7316-4.9032\alpha-2.3789\delta)\\(\delta-0.8)+\alpha\\(23.6054-5.6014\alpha-2.7754\delta)\end{pmatrix} & \begin{pmatrix}(2.5247-2.64\delta-7.9523\alpha)\\(0.75-\delta)-\alpha\\(7.9414-8.2146\delta-13.953\alpha),\\\frac{1}{4}(0.5263),\\(22.1241-3.3046\alpha-1.5873\delta)\\(\delta-1.75)+\alpha\\(33.2129-7.2\alpha-3.567\delta)\end{pmatrix}\\
\begin{pmatrix}(1.3508-1.4229\delta-0.6857\alpha)\\(0.8461-\delta)-\alpha\\(7.7371-13.1378\alpha-8.1154\delta),\\\frac{1}{3}(0.2748),\\(11.1583-2.2684\alpha-0.8733\delta)\\(\delta-1.6666)+\alpha\\(39.0813-7.4492\alpha-3.526\delta)\end{pmatrix} & \begin{pmatrix}(5.2689-6.2201\alpha-5.4839\delta)\\(0.8-\delta)-\alpha\\(3.819-4.0544\delta-7.6034\alpha),\\\frac{4}{9}(1.7214),\\(22.307-4.5240\alpha-2.0410\delta)\\(\delta-0.8)+\alpha\\(27.9326-5.1936\alpha-2.3583\delta)\end{pmatrix} & \begin{pmatrix}(2.4682-2.6315\delta-6.9177\alpha)\\(0.75-\delta)-\alpha\\(6.6197-6.9068\delta-6.9058\alpha),\\\frac{1}{4}(0.5732),\\(16.7743-2.9252\alpha-1.4850\delta)\\(\delta-1.75)+\alpha\\(23.4653-6.7924\alpha-2.914\delta)\end{pmatrix}\\
\begin{pmatrix}(2.0948-2.1901\delta-7.7177\alpha)\\(0.8461-\delta)-\alpha\\(9.3264-9.7317\delta-15.6979\alpha),\\\frac{1}{3}(0.4682),\\(18.5478-2.5055\alpha-1.2535\delta)\\(\delta-1.6666)+\alpha\\(56.9635-8.6125\alpha-4.5194\delta)\end{pmatrix} & \begin{pmatrix}(4.4271-4.6873\delta-9.3167\alpha)\\(0.8-\delta)-\alpha\\(6.9941-7.2345\delta-14.0989\alpha),\\\frac{4}{9}(0.9893),\\(32.8976-5.1465\alpha-2.7086\delta)\\(\delta-0.8)+\alpha\\(42.6137-5.9715\alpha-3.0643\delta)\end{pmatrix} & \begin{pmatrix}(4.8993-5.0444\delta-6.3812\alpha)\\(0.75-\delta)-\alpha\\(6.5219-6.8774\delta-17.0344\alpha),\\\frac{1}{4}(1.6168),\\(24.0659-3.4660\alpha-1.1881\delta)\\(\delta-1.75)+\alpha\\(51.4454-7.652\alpha-3.9621\delta)\end{pmatrix}
\end{pmatrix},$$

$$\left(\tilde I-\tilde A\widetilde W\right)^{-1}\left(\tilde I-\tilde A\right)y^{(1)}=$$
$$\begin{pmatrix}
\begin{pmatrix}8.6056-19.92237\delta-46.8677\alpha+53.9088\delta\alpha+11.4209\delta^2+49.3299\alpha^2\\2.0135\\-300.0644+265.97108\delta+420.4570\alpha-77.5984\delta\alpha-19.8283\delta^2-74.8212\alpha^2\end{pmatrix}\\
\begin{pmatrix}8.4888-19.817\delta-36.5696\alpha+42.3240\delta\alpha+11.4583\delta^2+27.9839\alpha^2\\2.8167\\-255.3503+2178519\delta+354.852\alpha-68.314\delta\alpha-17.3356\delta^2-69.6348\alpha^2\end{pmatrix}\\
\begin{pmatrix}11.7768-27.5159\delta-44.9912\alpha+51.5904\delta\alpha+15.8711\delta^2+56.0166\alpha^2\\2.6877\\-377.6724+317.7056\delta+596.7564\alpha-87.0978\delta\alpha-22.3954\delta^2-79.371\alpha^2\end{pmatrix}
\end{pmatrix},$$

which is the final opinion of experts.

CONCLUSION

In this paper, CRP is remodeled with fuzzy numbers, instead of crisp numbers. There are some limitations in the example given for illustration. In step 1, diagonal entriesentries $\tilde a_{ii}$, $i = 1,2,3$ of the matrix $\tilde A$ are equal to $\tilde 1 - \tilde w_{ii}$. $\tilde 1$ is chosen according to the entries of $\widetilde W^{(N)}$ i.e. $\left(\frac{3}{13},1,\frac{13}{3}\right)$, $\left(\frac{1}{5},1,3\right)$, $\left(\frac{1}{4},1,4\right)$ are the respective fuzzy one numbers for three rows. In step 2, fuzzy identity matrix $\tilde I$ consists of three equal fuzzy one numbers and six equal fuzzy zero numbers. This matrix can also be chosen with different fuzzy one and fuzzy zero numbers (means δ and α are not necessarily same for each entry). As well as decision-making problems are capable to model with fuzzy numbers, interpersonal influences and opinion dynamics have also the potential to process with fuzzy numbers and fuzzy matrices in more general forms. Moreover some other types and generalizations of fuzzy numbers would contribute to the proceedings of influences and opinions in future. This work can facilitate some other consensus-reaching processes with fuzzy numbers and prepare for advancements in fuzzy inverse matrices.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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APPENDIX

Consider the first row of $\tilde{W}$:

$$\begin{aligned}
(1,3,5) \oplus (2,4,5) \oplus (0,2,3) &= (3,9,13) \\
(1,3,5)/(3,9,13) &= (1/13,1/3,5/3) \\
(2,4,5)/(3,9,13) &= (2/13,4/9,5/3) \\
(0,2,3)/(3,9,13) &= (0,2/9,1)
\end{aligned}$$

And $(1/13,1/3,5/3) \oplus (2/13,4/9,5/3) \oplus (0,2/9,1) = (3/13,1,13/3)$, which is a fuzzy one number as defined in Definition 3.