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Research Article

An abstract approach to sequences and series using soft complex numbers

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ABSTRACT

The idea of a soft set offers a consistent structure for combining multiple data types and supporting a range of sources and representations. Because of its adaptability, it can be used in a variety of industries where precise and ambiguous information supervision is crucial for reliable and productive evaluation. This article delves into the interplay between sequences and series within the domain of soft complex numbers. It establishes a thorough understanding of soft complex boundedness, offering precise definitions for soft complex convergent sequences, soft complex limits, and soft complex series. Our exploration lays the groundwork with a broad perspective, paving the way for a detailed examination of specific properties embedded in the Bolzano-Weierstrass theorems. Additionally, we unravel the intricacies of soft complex Cauchy sequences and scrutinize the soft complex limits associated with both convergent sequences and series. This comprehensive discussion sheds light on the nuanced aspects of these mathematical concepts, providing a deeper insight into their implications.

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INTRODUCTION

An adaptable scheme for handling ambiguity and uncertainty in a variety of everyday circumstances is provided by fuzzy set [1]. One benefit is that it may depict the differentiation between membership and non-membership, enabling more complex conceptual depictions. Because of its versatility, complex systems with ill-defined constraints can be better modeled. This theory has successfully been applied in various fields of study like differential equations [2–5], and decision-making [6,7]. To equip fuzzy set with parameterization mode, Molodtsov [8] introduced a groundbreaking mathematical approach known as soft set theory (SST), designed to address uncertainties. It has multiple applications in the visualization of knowledge, data mining, and governance. Its capacity to manage the confusion and unpredictability present in practical problem data is a significant benefit. A more adaptable and truthful depiction of intricate systems is made possible by soft sets (SSETs), which permit the inclusion of elements with incomplete memberships. In decision support systems,

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where precise data may be omitted or it may be challenging to collect, adaptability is especially essential. Subsequent research by Maji et al. [9,10] expanded the scope of SST, presenting various operations applied to decision-making problems. Rahman et al. [11] employed parameterized approach to decision making using extension of fuzzy SSETs. Vimala et al. [12] investigated some algebraic structures and their relating results using multi aspects of fuzzy SSETs. Further contributions by Shabir and Naz [13] introduced soft topological spaces, while Majumdar and Samanta [14] explored mappings between SSETs. The study by Feng et al. [15] investigated the intersection of SSETs with fuzzy and rough sets. Researchers in [16,17] introduced soft real sets (SRSs), soft real numbers (SRNs), soft complex sets (SCSs), and soft complex numbers (SCNs), demonstrating their properties and practical applications. Saeed et al. [18] introduced the notions of soft elements and members. Later on, they [19] introduced the notions of soft algebraic structures using soft elements and members. A subsequent study in 2016, focused on fundamental operations in a soft context, discussing soft functions and including results like the Bolzano theorem [20]. Irfan [21] studied various properties of convexity and its inverse in soft and fuzzy SSETs environments. Salih and Sabir [22] explored specific characteristics related to convexity and concavity within the context of SSETs. The researchers [23–27] extended the previously defined knowledge of convexity to some of its other variants in soft and fuzzy SSET environments. Further study related to ambiguous focus on fuzzy complex numbers [28] that extend traditional complex numbers to accommodate uncertainty and imprecision, enabling degrees of membership and uncertainty in both real and imaginary components. Demir [29] discussed the idea of soft complex valued metric spaces and looked at a few topological facets of them. Next, for a variety of soft mappings on soft complex valued metric spaces, certain fixed soft element theorems were developed. Selvachandran et al. [30,31] studied the relations and distance measures between complex vague SSETs. Sabir [32] studied the extension of complex fuzzy set. This extension enhances the representation of fuzzy information within mathematical contexts, particularly in systems where imprecision and ambiguity play a significant role. Wardowski [33] demonstrated the natural relationship between soft operations and soft objects in soft topological spaces and proposed a novel concept of soft element of a SSET. Öztürk [34] examined the connection between bipolar SSETs and bipolar soft points and originated the idea of bipolar soft points. A novel interpretation of soft point was introduced by Senel [35], which allows one to construct all soft points that vary with any parameter that occurs in a SSET. Taşköprü and Altntaş [36] provided a topology whose members are sets of soft elements and explained how this topology relates to basic soft topology. Allam et al. [37] first proposed the idea of a soft element and then modified the idea of a soft point to eliminate any connected issues with the soft point. Polat et al. [38] introduced a number of properties on soft topological spaces, based on the notion of soft elements, which provide us with an alternative viewpoint for the advancement of soft set theory. These qualities include the neighbourhood structure of a soft element, soft interior, and among others. Hameed and Khalil [39] proved some new theorems on the equality of infimum soft sequences. Asma et al. [40] and Kirişçi [41] discussed decision making applications based on uncertain environments.

Novelty and Motivation

The scientific literature reviewed above makes it evident that while certain scholars have studied soft elements, soft members, and soft complex numbers, there needs to be moreresearch on the description of soft complex sequences and series in the literature. The SCNs provide an effective tool for handling equivocal and imperfect data in an organized way. Their capacity to express indeterminate values withreal and imaginary aspects is one of its advantagessince it enables a more thorough explanation of intricate situations. Because of its adaptability, it may be utilized for analyzing systems with multifaceted unpredictability, which is prevalent in a wide range of practical uses. Furthermore, while maintaining uncertain knowledge, SCNs offer an adaptable structure for carrying out computations. The SCNs and soft complex sequence (SCSQs) are useful in many domains where handling ambiguous or imprecise data is crucial. They can be employed to simulate systems with ambiguous or fuzzy parameters in engineering, enabling more reliable and flexible control schemes. The SCNs, which take into account erratic characteristics and unclear market data, can help in risk evaluation and investment optimization in the financial sector. The SCSQs can also be used in medical diagnosis to examine patient information as well as medical records, taking into account the inherent uncertainty in diagnosis and treatment results. Additionally, SCNs can also improve the processing of ambiguous or partial data in artificial intelligence and machine learning, resulting in more trustworthy and precise decisions. All things considered, SCNs have implications in a variety of domains, such as financial services, engineering, and artificial intelligence, where managing uncertainty is essential to solving problems successfully. Thus, keeping in mind above mentioned advantages and adaptability of SCNs, this study is aimed to introduce the novel concept of a SCSQ, examining its convergent components using SCNs and soft real sequences (SRSQ). Additionally, it establishes a convergent series on SCNs, exploring their relationship.

Section 2 reviews some basic concepts related to soft sets, soft elements, soft members, soft complex numbers, etc., and this is how the rest of the article is structured. In Section 3, the soft complex limit and associated theorems have been characterized and described using SCNs and SCSQs. Section 4 has a discussion of the boundedness of SCSQs, soft complex Cauchy sequences using SCNs, and their corresponding results. In Section 5, the convergence of these SCNs soft complex series and the corresponding outcomes have been examined. Lastly, Section 6 presents the overview, future scope, and implications of the anticipated effort

Preliminary Knowledge

The following section is meant to review fundamental definitions and operations for SSETs, SRSs, and SCSs, as outlined in [13,16,17,42].

Definition 1 Let Ü be an introduction to the universe of discussion and $\widetilde{\wp}$ be a set of parameters. Let $2^{(\ddot{U})}$ shows the set of power of Ü and $\widetilde{\Theta}$ be a subset that is not empty of $\widetilde{\wp}$. A pair $(\tilde{\zeta}, \widetilde{\Theta})$ is called a SSET over Ü, where $\tilde{\zeta}$ is a mapping given by $\tilde{\zeta}: \widetilde{\Theta} \to 2^{(\ddot{U})}$.

Put otherwise, a set above Ü is a family of parameterized discourse universe subsets Ü. For $\ddot{q} \in \widetilde{\Theta}, \tilde{\zeta}(\ddot{q})$ might be seen as a set of $\ddot{q}$ approximation SSETs elements $(\tilde{\zeta}, \widetilde{\Theta})$ and if $\ddot{q} \notin \widetilde{\Theta}$ then $\tilde{\zeta}(\ddot{q}) = \emptyset$, that is $\tilde{\zeta}_{\widetilde{\Theta}} = \tilde{\zeta}(\ddot{q}): \ddot{q} \in \widetilde{\Theta} \,\tilde{\subseteq}\, \widetilde{\wp}$, $\tilde{\zeta}: \widetilde{\Theta} \to 2^{(\ddot{U})}$. The family of all these SSETs over Ü denoted by $\widetilde{\Omega}_{\widetilde{\Theta}}(\ddot{U})$.

Definition 2 Let $\tilde{\zeta}_{\widetilde{\Theta}}, \tilde{\Gamma}_{\widetilde{\Sigma}} \in \widetilde{\Omega}_{\widetilde{\wp}}(\ddot{U})$ then $\tilde{\zeta}_{\widetilde{\Theta}}$ is a soft subset of $\tilde{\Gamma}_{\widetilde{\Sigma}}$, symbolized by $\tilde{\zeta}_{\widetilde{\Theta}} \subseteq \tilde{\Gamma}_{\widetilde{\Sigma}}$, if $\widetilde{\Theta} \subseteq \widetilde{\Sigma}$ and $\tilde{\zeta}_{\widetilde{\Theta}} \subseteq \tilde{\Gamma}_{\widetilde{\Sigma}} \forall\, \ddot{q} \in \widetilde{\Theta}$. In this case $\tilde{\zeta}_{\widetilde{\Theta}}$ is described as a soft subset of $\tilde{\Gamma}_{\widetilde{\Sigma}}$, and $\tilde{\Gamma}_{\widetilde{\Sigma}}$ is described as a soft super set, $\tilde{\zeta}_{\widetilde{\Theta}}, \tilde{\Gamma}_{\widetilde{\Sigma}} \supseteq \tilde{\zeta}_{\widetilde{\Theta}}$.

Definition 3 A pair of soft subsets $\tilde{\zeta}_{\widetilde{\Theta}}$ and $\tilde{\Gamma}_{\widetilde{\Sigma}}$ over Ü are considered equal if $\tilde{\zeta}_{\widetilde{\Theta}}$ is a soft subset of $\tilde{\Gamma}_{\widetilde{\Sigma}}$ and $\tilde{\Gamma}_{\widetilde{\Sigma}}$ is a soft subset of $\tilde{\zeta}_{\widetilde{\Theta}}$.

Definition 4 The soft subsets complement $(\tilde{\zeta}, \widetilde{\Theta})$ denoted by $(\tilde{\zeta}, \widetilde{\Theta})^c$ is determined by $(\tilde{\zeta}, \widetilde{\Theta})^c = (\tilde{\zeta}, \widetilde{\Theta})$: $\tilde{\zeta}: \widetilde{\Theta} \to 2^{(\ddot{U})}$ is a mapping provided by $\tilde{\zeta}^c(\ddot{q}) = \ddot{U} - \tilde{\zeta}(\ddot{q}), \forall\, \ddot{q} \in \widetilde{\Theta}$ and $\tilde{\zeta}^c$ and $\tilde{\zeta}$ is said the soft complement function is equivalent to $\tilde{\zeta}$. Clearly $(\tilde{\zeta})$ is similar to $\tilde{\zeta}$ and $((\tilde{\zeta}, \widetilde{\Theta})^c)^c = (\tilde{\zeta}, \widetilde{\Theta})$.

Definition 5 The difference between SSETs $(\tilde{\zeta}, \widetilde{\wp})$ and $(\tilde{\Gamma}, \widetilde{\wp})$ over Ü displayed by $(\tilde{\zeta}, \widetilde{\wp})$ - $(\tilde{\Gamma}, \widetilde{\wp})$ is the SSET $(D, \widetilde{\wp})$ such that $D(\ddot{q}) = \tilde{\zeta}(\ddot{q}) \backslash \tilde{\Gamma}(\ddot{q}), \forall\, \ddot{q} \in \widetilde{\wp}$.

Definition 6 The union of SSETs and $(\tilde{\Gamma}, \widetilde{\Sigma})$ Over Ü is the SSET $(D, \tilde{C})$, where, $\tilde{C} = \widetilde{\Theta} \cup \widetilde{\Sigma}$ for all $\ddot{q} \in \tilde{C}$,

$$D(\ddot{q}) = \begin{cases} \tilde{\zeta}(\ddot{q}) & if \quad \ddot{q} \in \alpha \\ \tilde{\Gamma}(\ddot{q}) & if \quad q \,\ddot{\in}\, \beta \\ \tilde{\zeta}(\ddot{q}) & if \quad \ddot{q} \in \gamma \end{cases} \tag{1}$$

where, $\alpha = \widetilde{\Theta} - \widetilde{\Sigma}$, $\beta = \widetilde{\Sigma} - \widetilde{\Theta}$ and $\gamma = \widetilde{\Theta} \,\tilde{\cap}\, \widetilde{\Sigma}$ and written as $(\tilde{\zeta}, \widetilde{\Theta}) \cup (\tilde{\Gamma}, \widetilde{\Sigma}) = (D, \tilde{C})$.

Definition 7 The intersection $(D, \tilde{C})$ between two SSETs $(\tilde{\zeta}, \widetilde{\Theta})$ and $(\tilde{\Gamma}, \widetilde{\Sigma})$ over Ü, denoted $(\tilde{\zeta}, \widetilde{\Theta}) \,\tilde{\cap}\, (\tilde{\Gamma}, \widetilde{\Sigma})$, $(\tilde{\zeta}, \widetilde{\Theta})$ and $(\tilde{\Gamma}, \widetilde{\Sigma})$ can be defined as $\tilde{C} = \widetilde{\Theta} \,\tilde{\cap}\, \widetilde{\Sigma}$ and $D(\ddot{q}) = \tilde{\zeta}(\ddot{q}) \,\tilde{\cap}\, \tilde{\Gamma}(\ddot{q}), \forall\, \ddot{q} \in \tilde{C}$.

Definition 8 Given a non-empty parameter set $\widetilde{\Theta}$ and a non-empty set $\ddot{V}$. Then a function $\tau: \widetilde{\Theta} \to \ddot{U}$ is defined as a soft element in Ü. A soft element τ of Ü belongs to a SSET $(\tilde{\zeta}, \widetilde{\Theta})$ of Ü which is symbolized by $\tau \in (\tilde{\zeta}, \widetilde{\Theta})$ if $\tau(\ddot{q}) \in \tilde{\zeta}(\ddot{q}), \forall\, \ddot{q} \in \widetilde{\Theta}$. Thus for a SSET $(\tilde{\zeta}, \widetilde{\Theta})$ of Ü concerning the index set $\widetilde{\Theta}$, it gives $\tilde{\zeta}(\ddot{q}) = \{\tau(\ddot{q}): \tau \in (\tilde{\zeta}, \widetilde{\Theta}), \forall\, \ddot{q} \in \widetilde{\Theta}\}$. In that case, τ is said to be a soft element of the SSET $(\tilde{\zeta}, \widetilde{\Theta})$.

For SSETs $(\tilde{\zeta}, \widetilde{\Theta})$ and $(\tilde{\Gamma}, \widetilde{\Theta})$. $(\tilde{\Gamma}, \widetilde{\Theta})$ within $\widetilde{\Omega}_{\widetilde{\Theta}}(\ddot{U})$ the relation $(\tilde{\zeta}, \widetilde{\Theta}) \,\tilde{\subseteq}\, (\tilde{\Gamma}, \widetilde{\Theta})$. holds iff each soft element belonging to $(\tilde{\zeta}, \widetilde{\Theta})$ is also a soft element in $(\tilde{\Gamma}, \widetilde{\Theta})$.

Definition 9 Consider R as the collection of real numbers, let $2^{(R)}$ be the set of all bounded, non-empty subsets of R, and $\widetilde{\Theta}$ be a set of parameters. In this context, a mapping $\tilde{\zeta}: \widetilde{\Theta} \to 2^{(P)}$ is termed a SRS, denoted as $(\tilde{\zeta}, \widetilde{\Theta})$.

In particular if $(\tilde{\zeta}, \widetilde{\Theta})$ is a singleton, then recognizing $(\tilde{\zeta}, \widetilde{\Theta})$ together with the equivalent soft element, it will be defined as a SRN.

Definition 10 The sequence $\{\tilde{\mu}_n\}$ it is claimed that a set of SRNs is convergent to the soft real limit $\tilde{\mu}$ in $R(\widetilde{\Theta})$, when it has the soft real limit $\tilde{\mu}$, otherwise it is claimed as divergent.

Lemma 1 [42] There is a soft convergent sub sequence for every SRSQ.

This outcome confirms clearly that there is a convergent sub sequence for any bounded SRSQ.

Definition 11 Let C denote the collection of complex numbers, Λ (C) be the collection of all bounded non-empty subsets of the complex number set and $\widetilde{\Theta}$ be a set of parameters. Then a mapping $\tilde{\zeta}: \widetilde{\Theta} \to 2^{(X)}$ is said to be a SCS. It's shown by $(\tilde{\zeta}, \widetilde{\Theta})$.

Definition 12 In particular, if $(\tilde{\zeta}, \widetilde{\Theta})$ a SSET with a singleton, then identifying $(\tilde{\zeta}, \widetilde{\Theta})$it will be defined as a SCN together with the corresponding soft element. We used the symbols $\tilde{\varpi}_1$, $\tilde{\varpi}_2$ and $\tilde{\varpi}_3$, to denote SCNs.

Example 1 Let C be the set of complex numbers. The $\widetilde{\Theta}$ be a set of parameters given by $\widetilde{\Theta} = \{\ddot{q}_1, \ddot{q}_2, \ddot{q}_3, \ddot{q}_4, \ddot{q}_5\}$. Then $(\tilde{\zeta}, \widetilde{\Theta})$ can be considered as a SCN where

$$(\tilde{\zeta}, \widetilde{\Theta}) = \{(\ddot{q}_1, \{2+i\}), (\ddot{q}_2, \{2i\}), (\ddot{q}_3, \{2-i\}), (\ddot{q}_4, \{4i\}), (\ddot{q}_5, \{5-6i\})\}.$$

Definition 13 A SCS $(\tilde{\zeta}, \widetilde{\Theta})$ is claimed as soft pure imaginary set if $\tilde{\zeta}(\lambda)$ is a subset of the set of pure imaginary numbers for each $\lambda \in \widetilde{\Theta}$.

Let us denote the set of all SCSs by $X_{(\widetilde{\Theta})}$ and the set of all SCNs by $X^{(\widetilde{\Theta})}$.

Definition 14 Assume that $\tilde{\zeta}$ is a SCS or number. Subsequently, $Re\,\tilde{\zeta}$ is a real part and $lm\,\tilde{\zeta}$ is an imaginary part of $\tilde{\varpi}$ and are stated as follows:

$$Re\,\tilde{\zeta}(\ddot{q}) = \{Re(\tilde{\varpi}): \tilde{\varpi} \in \tilde{\zeta}(\ddot{q})\}, Re\,\tilde{\zeta}(\ddot{q}) = Re(\tilde{\zeta}(\ddot{q})) \tag{2}$$

and

$$Im\,\tilde{\zeta}(\ddot{q}) = \{Im(\tilde{\varpi}): \tilde{\varpi} \in \tilde{\zeta}(\ddot{q})\}, Im\,\tilde{\zeta}(\ddot{q}) = Im(\tilde{\zeta}(\ddot{q})) \tag{3}$$

For every $\ddot{q} \in \widetilde{\wp}$. It is shown that $Re\,\tilde{\zeta}$ and $lm\,\tilde{\zeta}$ are SRSs or numbers.

Definition 15 Let $(\tilde{\zeta}, \widetilde{\Theta})$ be a collection of SCS. Then the modulus of $(\tilde{\zeta}, \widetilde{\Theta})$ is symbolized by $(\tilde{\zeta}, \widetilde{\Theta})$ and is defined by $|\tilde{\zeta}|(\lambda) = \{|\tilde{\varpi}|: \tilde{\varpi} \in \tilde{\zeta}(\lambda); \lambda \in \widetilde{\Theta}\}$.

Limits of a Sequence in SCNs

Definition 16 If $\tilde{\mu}_n$ and $\tilde{\nu}_n$ be any two SRSQs then $\tilde{\varpi}_n = \tilde{\mu}_n + i\tilde{\nu}_n$ is a SCSQ denoted by

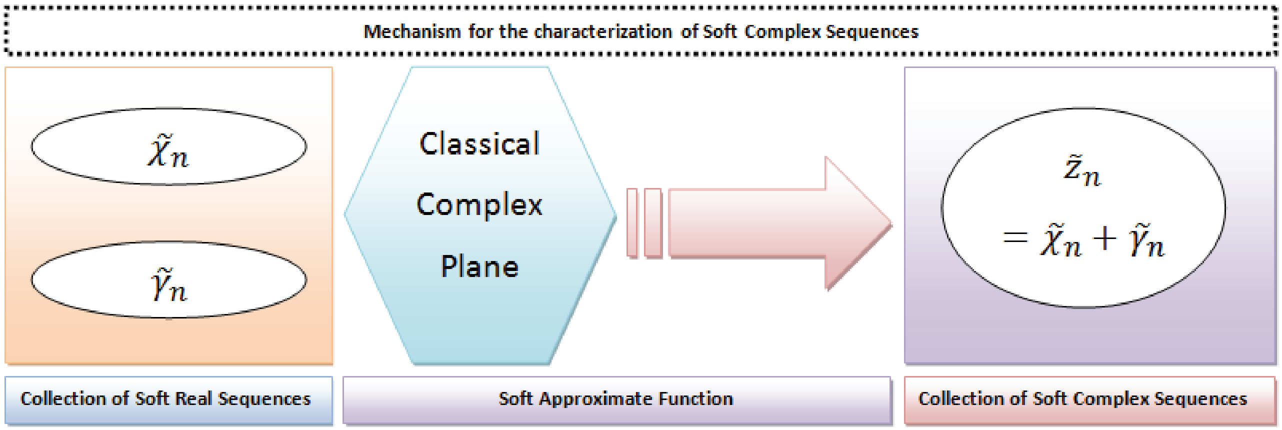

Figure 1. Mechanism for the characterization of SCSQs.

$\{\tilde{\varpi}_n\}_{n=1}^{\infty} = \{\tilde{\varpi}_1, \tilde{\varpi}_2, \tilde{\varpi}_3, \dots, \tilde{\varpi}_n, \dots\}, \tilde{\varpi}_n \in X^{(\tilde{\Theta})}$ for all $n = \{1,2,3, \dots\}$.

Mechanism for the characterization of SCSQs can be viewed from Figure 1.

Definition 17 The sequence $\{\tilde{\varpi}_n\}$ of SCNs is called convergent to soft complex limit $\ddot{\varrho} \in X^{(\tilde{\Theta})}$, when it has the soft complex limit $\ddot{\varrho}$, otherwise it is claimed as divergent.

Definition 18 If $\{\tilde{\varpi}_n\}$ represents a sequence of SCNs. We conclude that $\tilde{\varpi}_n$ approaches the soft complex limit $\ddot{\varrho}$ (as $n \to \infty$) whenever for every SRN $\ddot{\eta} > \bar{0} \ \exists \ \ddot{\hbar}(\ddot{\eta})$, a soft positive integer (SPI) depending on $\ddot{\eta}$ such that

$$|\tilde{\varpi}_n - \ddot{\varrho}| < \ddot{\eta}, \text{ for all } n \geq \ddot{\hbar}(\ddot{\eta}) \tag{4}$$

If $\tilde{\varpi}_n$ approaching the limit of the soft complex $\ddot{\varrho}$, we write $\lim_{n\to\infty} \tilde{\varpi}_n = \ddot{\varrho}$ or $\tilde{\varpi}_n \to \ddot{\varrho}$ as $n \to \infty$.

Example 2 Consider $\tilde{\varpi}_n = \tilde{\mu}_n + i\tilde{\nu}_n \in X^{(\tilde{\Theta})}$ with $\tilde{\Theta}$ is a finite set of parameters where $\tilde{\mu}_n = \bar{0}$ and $\tilde{\nu}_n = \overline{\left(\frac{1}{2n}\right)}, n \in \mathbb{N}$.

Then, we claim that $\lim_{n\to\infty} \tilde{\varpi}_n = \tilde{0} = \bar{0} + i\bar{0}$ is the number of the soft complex, where $\tilde{0}(\ddot{q}) = \bar{0}(\ddot{q}) + i\bar{0}(\ddot{q}) = \bar{0} + i\bar{0}$. In fact, given $\ddot{\eta} > \bar{0}$ it gives that $k = min\{\ddot{\eta}(\ddot{q}) : \ddot{q} \in \tilde{\Theta}\} > 0$. Since $\tilde{\Theta}$ is a finite set of parameters. Because of the Archimedean characteristic of real numbers, $n_0 \in \mathbb{N}$, such that $\frac{1}{n_0} < \ddot{\hbar}$. Given the fact that

$$\left|\tilde{\varpi}_n - \tilde{0}\right|(\ddot{q}) = \left|\overline{\left(\frac{1}{2n}\right)} - \bar{0}\right|(\ddot{q}) = \left|\overline{\left(\frac{1}{2n}\right)}(\ddot{q}) - \bar{0}(\ddot{q})\right|$$
$$= \left|\frac{1}{2n} - 0\right| = \frac{1}{2n} \leq \frac{1}{n_0} < k \leq \ddot{\eta}(\ddot{q}),$$

for every $\ddot{q} \in \tilde{\Theta}$ and $n \geq n_0$. It means that $|\tilde{\varpi}_n - \tilde{0}| < \ddot{\eta}$.

Theorem 1 Let $\{\tilde{\varpi}_n\}$ be a sequence in the SCNs. Write $\tilde{\varpi}_n = \tilde{\mu}_n + i\tilde{\nu}_n$ where $\tilde{\mu}_n$ and $\tilde{\nu}_n$ are sequences of SRNs. Then the given sequence convergent in $X^{(\tilde{\Theta})}$ iff $\{\tilde{\mu}_n\}$ and $\{\tilde{\nu}_n\}$ converge in R $(\tilde{\Theta})$.

Proof. Suppose $\tilde{\mu}_n \to \tilde{\mu}$ and $\tilde{\nu}_n \to \tilde{\nu}$. We now prove that $\tilde{\varpi}_n = \tilde{\mu} + i\tilde{\nu}$. Indeed, fix $\ddot{\eta} > \bar{0}$ and find, by the stated convergence, there exists $\ddot{\hbar}$ such that $n > \ddot{\hbar}$. It given that $|\tilde{\mu}_n - \tilde{\mu}| < \ddot{\eta}/2$ and $|\tilde{\nu}_n - \tilde{\nu}| < \ddot{\eta}/2$. Then, for $n > \ddot{\hbar}$, it gives

$$\left|\tilde{\varpi}_n - (\tilde{\mu} + i\tilde{\nu})\right| = \left|(\tilde{\mu}_n - \tilde{\mu}) + i(\tilde{\nu}_n - \tilde{\nu})\right| \leq \left|\tilde{\mu}_n - \tilde{\mu}\right|$$
$$+ \left|\tilde{\nu}_n - \tilde{\nu}\right| < \ddot{\eta}/2 + \ddot{\eta}/2 = \ddot{\eta}.$$

Conversely, suppose that $\tilde{\varpi}_n = \tilde{\mu} + i\tilde{\nu}$. Then, given $\ddot{\eta} > \bar{0}$, we find SPI $\ddot{\hbar}$ such that for $n > \ddot{\hbar}$, it holds that $|\tilde{\varpi}_n - (\tilde{\mu} + i\tilde{\nu})| < \ddot{\eta}$. So clearly

$$\left|\tilde{\mu}_n - \tilde{\mu}\right| \leq \left|\tilde{x}_n - \tilde{\mu}\right| + \left|\tilde{\nu}_n - \tilde{\nu}\right| = \left|\tilde{\varpi}_n - (\tilde{\mu}_n + i\tilde{\nu}_n)\right| < \ddot{\eta}, \tag{5}$$

and

$$\left|\tilde{\nu}_n - \tilde{\nu}\right| \leq \left|\tilde{\nu}_n - \tilde{\nu}\right| + \left|\tilde{\mu}_n - \tilde{\mu}\right| = \left|\tilde{\varpi}_n - (\tilde{\mu}_n + i\tilde{\nu}_n)\right| < \ddot{\eta}. \tag{6}$$

Proposition 1 Let $\{\tilde{\varpi}_{1n}\}$ and $\{\tilde{\varpi}_{2n}\}$ be convergent infinite sequences of SCNs. Then the sequences $\{\tilde{\varpi}_{1n} + \tilde{\varpi}_{2n}\}$, $\{\tilde{\varpi}_{1n} - \tilde{\varpi}_{2n}\}$, and $\{\tilde{\varpi}_{1n} \,.\, \tilde{\varpi}_{2n}\}$ are convergent, and

$$\lim_{n\to\infty}(\tilde{\varpi}_{1n} + \tilde{\varpi}_{2n}) = \lim_{n\to\infty}\tilde{\varpi}_{1n} + \lim_{n\to\infty}\tilde{\varpi}_{2n},$$
$$\lim_{n\to\infty}(\tilde{\varpi}_{1n} - \tilde{\varpi}_{2n}) = \lim_{n\to\infty}\tilde{\varpi}_{1n} - \lim_{n\to\infty}\tilde{\varpi}_{2n},$$
$$\lim_{n\to\infty}(\tilde{\varpi}_{1n} \cdot \tilde{\varpi}_{2n}) = \left(\lim_{n\to\infty}\tilde{\varpi}_{1n}\right)\left(\lim_{n\to\infty}\tilde{\varpi}_{2n}\right).$$

In addition, if $\tilde{\varpi}_{2n} = 0$, for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \tilde{\varpi}_{1n} \neq \bar{0}$, then the sequence $\{\tilde{\varpi}_{1n} / \tilde{\varpi}_{2n}\}$ of SCNs is convergent, and

$$\lim_{n\to\infty}\left(\frac{\tilde{\varpi}_{1n}}{\tilde{\varpi}_{2n}}\right) = \frac{\lim_{n\to\infty}\tilde{\varpi}_{1n}}{\lim_{n\to\infty}\tilde{\varpi}_{2n}}.$$

Proof. Let $\lim_{n\to\infty} \tilde{\varpi}_{1n} = \ddot{\varrho}_1$ and $\lim_{n\to\infty} \tilde{\varpi}_{2n} = \ddot{\varrho}_2$. Suppose some SRN $\ddot{\eta} > \bar{0}$ be given. It follows from the idea of limits sequence that there are SPI $\ddot{\hbar}_1(\ddot{\eta})$ and $\ddot{\hbar}_2(\ddot{\eta})$ depending on $\ddot{\eta}$ such that $|\widetilde{\varpi}_{1n} - \ddot{\varrho}_1| < \ddot{\eta}/2$ whenever $n \geq \ddot{\hbar}_1$ and $|\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \ddot{\eta}/2$ whenever $n \geq \ddot{\hbar}_2$. If $n > \ddot{\hbar}$, where $\ddot{\hbar}$ is the maximum of $\ddot{\hbar}_1$ and $\ddot{\hbar}_2$, then

$$|\widetilde{\varpi}_{1n} + \widetilde{\varpi}_{2n} - (\ddot{\varrho}_1 - \ddot{\varrho}_2)| \leq |\widetilde{\varpi}_{1n} - \ddot{\varrho}_1| + |\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \ddot{\eta}/2 + \ddot{\eta}/2 = \ddot{\eta}.$$

Thus, $\lim_{n\to\infty}(\widetilde{\varpi}_{1n} + \widetilde{\varpi}_{2n}) = \lim_{n\to\infty}\widetilde{\varpi}_{1n} + \lim_{n\to\infty}\widetilde{\varpi}_{2n}$.

Let $\ddot{\vartheta}$ be some complex number. We prove that $\lim_{n\to\infty}\ddot{\vartheta}\,\widetilde{\varpi}_{2n} = \ddot{\vartheta}\ddot{\varrho}_2$. Now, given any SPI $\ddot{\hbar}$, we can choose a soft positive $\ddot{\delta}$ small enough to ensure that $|\ddot{\varrho}|\ddot{\delta} < \ddot{\eta}$. Now $\lim_{n\to\infty}\widetilde{\varpi}_{2n} = \ddot{\varrho}_2$, and therefore, there exists some SPI $\ddot{\hbar}$ such that $|\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \ddot{\delta}$ whenever $n \geq \ddot{\hbar}$. But then $\left|\ddot{\vartheta}\widetilde{\varpi}_{2n} - \ddot{\vartheta}\ddot{\varrho}_2\right| = |\ddot{\vartheta}||\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \ddot{\delta}\left|\ddot{\vartheta}\right|$. This gives the fact that $\lim_{n\to\infty}\ddot{\vartheta}\,\widetilde{\varpi}_{2n} = \ddot{\vartheta}\ddot{\varrho}_2$.

On apply this result with $\ddot{\vartheta} = -1$, we see that $\lim_{n\to\infty} -\widetilde{\varpi}_{2n} = -\ddot{\varrho}_2$. It follows that $\lim_{n\to\infty}\widetilde{\varpi}_{1n} - \widetilde{\varpi}_{2n} = \ddot{\varrho}_1 - \ddot{\varrho}_2$. Next we prove that if $\{\widetilde{\varpi}_{3n}\}$ and $\{\widetilde{\varpi}_{4n}\}$ are infinite sequences of SCNs, and if $\lim_{n\to\infty}\widetilde{\varpi}_{3n} = \bar{0}$ and $\lim_{n\to\infty}\widetilde{\varpi}_{4n} = \bar{0}$, then $\lim_{n\to\infty}(\widetilde{\varpi}_{3n}\widetilde{\varpi}_{4n}) = \bar{0}$. Suppose some SRN $\ddot{\eta} > \bar{0}$ be given, it follows from the definition of limits sequence that there exists SPSIs $\ddot{\hbar}_3$ and $\ddot{\hbar}_4$ such that $|\widetilde{\varpi}_{3n}| < \sqrt{\ddot{\eta}}$ whenever $n \geq \ddot{\hbar}_3$ and $|\widetilde{\varpi}_{4n}| < \sqrt{\ddot{\eta}}$ whenever $n \geq \ddot{\hbar}_{4n}$. Suppose $\ddot{\hbar}$ be the maximum of $\ddot{\hbar}_3$and $\ddot{\hbar}_4$, if $n \geq \ddot{\hbar}$ then that $|\widetilde{\varpi}_{3n}| < \sqrt{\ddot{\eta}}$ and $|\widetilde{\varpi}_{4n}| < \sqrt{\ddot{\eta}}$. Therefore, $|\widetilde{\varpi}_{3n}\widetilde{\varpi}_{4n}| < \sqrt{\ddot{\eta}}$. It shows that $\lim_{n\to\infty}(\widetilde{\varpi}_{3n}\widetilde{\varpi}_{4n}) = \bar{0}$.

We can apply this result with $\widetilde{\varpi}_{3n} = \widetilde{\varpi}_{1n} - \ddot{\varrho}_1$ and $\widetilde{\varpi}_{4n} = \widetilde{\varpi}_{2n} - \ddot{\varrho}_2$ for all positive integer n, where $\ddot{\varrho}_1 = \lim_{n\to\infty}\widetilde{\varpi}_{1n}$ and $\ddot{\varrho}_2 = \lim_{n\to\infty}\widetilde{\varpi}_{2n}$. Now $\lim_{n\to\infty}\widetilde{\varpi}_{3n} = \bar{0}$ and $\lim_{n\to\infty}\widetilde{\varpi}_{4n} = \bar{0}$ follows that

$$\begin{aligned}\bar{0} &= \lim_{n\to\infty}(\widetilde{\varpi}_{3n}\widetilde{\varpi}_{4n}) = \lim_{n\to\infty}((\widetilde{\varpi}_{1n} - \ddot{\varrho}_1)(\widetilde{\varpi}_{2n} - \ddot{\varrho}_2)) \\ &= \lim_{n\to\infty}(\widetilde{\varpi}_{1n}\widetilde{\varpi}_{2n} - \ddot{\varrho}_2\widetilde{\varpi}_{2n} - \ddot{\varrho}_1\widetilde{\varpi}_{1n} + \ddot{\varrho}_1\ddot{\varrho}_2) \\ &= \lim_{n\to\infty}(\widetilde{\varpi}_{1n}\widetilde{\varpi}_{2n}) - \ddot{\varrho}_2\lim_{n\to\infty}(\widetilde{\varpi}_{2n}) - \ddot{\varrho}_1\lim_{n\to\infty}(\widetilde{\varpi}_{1n}) + \lim_{n\to\infty}(\ddot{\varrho}_1\ddot{\varrho}_2) \\ &= \lim_{n\to\infty}\widetilde{\varpi}_{1n}\widetilde{\varpi}_{2n} - \ddot{\varrho}_1\ddot{\varrho}_2\end{aligned}$$

Therefore $\lim_{n\to\infty}\widetilde{\varpi}_{1n}\widetilde{\varpi}_{2n} = \ddot{\varrho}_1\ddot{\varrho}_2$.

Finally, suppose that $\widetilde{\varpi}_{2n} \neq \bar{0}$ for every positive integers n, and that $\ddot{\varrho}_2 \neq \bar{0}$. Then for any given SRN $\ddot{\eta} > \bar{0}$, there exists some SPIs $\ddot{\hbar}_5$ such that

$|\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \frac{1}{2}|\ddot{\varrho}_2|^2\ddot{\eta}$ and $|\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \frac{1}{2}|\ddot{\varrho}_2|$,

whenever $n \geq \ddot{\hbar}_5$. For $n \geq \ddot{\hbar}_5$, then $|\widetilde{\varpi}_{2n}| \geq |\ddot{\varrho}_2| - |\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| > \frac{1}{2|\ddot{\varrho}_2|}$, and therefore,

$$\left|\frac{1}{\widetilde{\varpi}_{2n}} - \frac{1}{\ddot{\varrho}_2}\right| = \left|\frac{\ddot{\varrho}_2 - \widetilde{\varpi}_{2n}}{\ddot{\varrho}_2\widetilde{\varpi}_{2n}}\right| \leq \frac{2}{|\ddot{\varrho}_2|^2}|\widetilde{\varpi}_{2n} - \ddot{\varrho}_2| < \ddot{\eta}$$

Thus, $\lim_{n\to\infty}\frac{1}{\widetilde{\varpi}_{2n}} = \frac{1}{\ddot{\varrho}_2}$. It given that $\lim_{n\to\infty}\frac{\widetilde{\varpi}_{1n}}{\widetilde{\varpi}_{2n}} = \frac{\ddot{\varrho}_1}{\ddot{\varrho}_2}$, as required.

Theorem 2 If $\ddot{\varrho}$ is the convergent soft complex limit for the SCSQ $\widetilde{\varpi}_n$, then it is also the convergent soft complex limit for $\widetilde{\varpi}_{n+s}(s \in \mathbb{N})$.

Proof. Since $\{\widetilde{\varpi}_n\}$ converges to soft complex limit $\ddot{\varrho}$ then for any SRN $\ddot{\eta} > \bar{0}$, there exists SPI $\ddot{\hbar}(\ddot{\eta})$ such that $|\widetilde{\varpi}_n - \ddot{\varrho}| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}(\ddot{\eta})$. If we take $m = n + s$, for all $s \in \mathbb{N}$, then $m > n > \ddot{\hbar}(\ddot{\eta})$ implies $m = n + s$ and $|\widetilde{\varpi}_{n+s} - \ddot{\varrho}| < \ddot{\eta}$. Hence $\ddot{\varrho}$ is the convergent soft complex limit for the SCSQ $\widetilde{\varpi}_{n+s}$.

Theorem 3 A convergent sequence of SCNs has a unique soft complex limit.

Proof. Let $\{\widetilde{\varpi}_n\}$ be a soft complex convergent sequence. Suppose that $\lim_{n\to\infty}\widetilde{\varpi}_n = \ddot{\varrho}_1$ and $\lim_{n\to\infty}\widetilde{\varpi}_n = \ddot{\varrho}_2$. We wish to prove $\ddot{\varrho}_1 = \ddot{\varrho}_2$. For given $\ddot{\eta} > \bar{0}$, by the Definition 18, there exists a soft real positive integer $\ddot{\hbar}_1$ such that $n \geq \ddot{\hbar}_1$ implies $|\widetilde{\varpi}_n - \ddot{\varrho}_1| < \ddot{\eta}/2$. Moreover, there exists a soft real positive integer $\ddot{\hbar}_2$ such that $n > \ddot{\hbar}_2$ implies $|\widetilde{\varpi}_n - \ddot{\varrho}_2| < \ddot{\eta}/2$. For $n > max\left\{\ddot{\hbar}_1, \ddot{\hbar}_2\right\}$, by the triangle inequality it gives

$$|\ddot{\varrho}_1 - \ddot{\varrho}_2| = |\ddot{\varrho}_1 + \widetilde{\varpi}_n - \widetilde{\varpi}_n - \ddot{\varrho}_2| \leq |\widetilde{\varpi}_n - \ddot{\varrho}_1| + |\widetilde{\varpi}_n - \ddot{\varrho}_2| < \frac{\eta}{2} + \frac{\eta}{2} = \ddot{\eta}.$$

This shows that $|\ddot{\varrho}_1 - \ddot{\varrho}_2| < \ddot{\eta}$ for all $\ddot{\eta} > \bar{0}$. It follows that $|\ddot{\varrho}_1 - \ddot{\varrho}_2| = \bar{0}$ and hence, $\ddot{\varrho}_1 = \ddot{\varrho}_2$.

Theorem 4 The SCSQ $\{\widetilde{\varpi}_n\}$ is convergent iff the SRSQ $\{|\widetilde{\varpi}_n|\}$ is also convergent.

Proof. Let $\{\widetilde{\varpi}_n\}$ be a soft complex convergent sequence, and let $\lim_{n\to\infty}\widetilde{\varpi}_n = \ddot{\varrho}$. For given $\ddot{\eta} > \bar{0}$, by Definition 18, there exists a soft real positive integer $\ddot{\hbar}$ such that $n > \ddot{\hbar}$ implies $|\widetilde{\varpi}_n - \ddot{\varrho}| < \ddot{\eta}$. So by the triangle inequality we can show that $||\widetilde{\varpi}_n| - |\ddot{\varrho}|| \leq |\widetilde{\varpi}_n - \ddot{\varrho}| < \ddot{\eta}$. Conversely, can be shown similarly.

Remark 1 The SCSQ $\{\widetilde{\varpi}_n\}$ converges to zero iff the SRSQ $\{|\widetilde{\varpi}_n|\}$ converges to zero.

Bounded by the Sequence of SCNs

Definition 19 Soft complex number sequence $\{\widetilde{\varpi}_n\}$ is claimed as bounded if a SRN $\widetilde{M} \in \tilde{P}(\widetilde{M} > 0)$ exists such that $|\widetilde{\varpi}_n| \leq \widetilde{M}\ \forall\, n \in \mathbb{N}$. A sequence is unbounded if it is not bounded.

Example 3 Consider $\widetilde{\varpi}_n = \overline{\iota\left(\frac{1}{n}\right)} \in X^{(\widetilde{\Theta})}(n \in \mathbb{N})$ where $\widetilde{\Theta}$ is a finite set of parameters. Then, it gives that $\widetilde{\varpi}_n$ is bounded since clearly there exist $\tilde{1} \in \tilde{P}$ and $|\widetilde{\varpi}_n|(\ddot{q}) = \left|\overline{\iota\left(\frac{1}{n}\right)}\right|(\ddot{q}) = \left|\overline{\left(\frac{1}{n}\right)}(\ddot{q})\right| = \left|\frac{1}{n}\right| \leq \tilde{1}$ for every $\ddot{q} \in \widetilde{\Theta}$.

Definition 20 Let $\{\widetilde{\varpi}_n\}$ be a SCSQ. We say that $\{\widetilde{\varpi}_n\}$ is a soft complex Cauchy sequence if for all SRNs $\ddot{\eta} > \bar{0}$, there exists a positive soft integer $\ddot{\hbar}$ such that $|\widetilde{\varpi}_n - \widetilde{\varpi}_m| < \ddot{\eta}$, for all $n, m \geq \ddot{\hbar}$.

For $\ddot{\eta}$ to be less than the difference between any two soft elements that occur before and following the term's $\ddot{\hbar}$.

Proposition 2 Let $\{\widetilde{\varpi}_n\}$ be a sequence of SCNs that converges. Then it is bounded.

Proof. Suppose that the SCSQ $\{\widetilde{\varpi}_n\}$ converge to a soft complex limit $\ddot{\varrho} \in X^{(\widetilde{\Theta})}$. Then for $\ddot{\eta} = \tilde{1} > \bar{0}\ \exists\ \ddot{\hbar} \in \mathbb{N}$

such that $|\widetilde{\varpi}_n - \ddot{\varrho}| < \tilde{1}$ for all $n \geq \ddot{\hbar}$. It follows that $|\widetilde{\varpi}_n| < |\ddot{\varrho}| + |\widetilde{\varpi}_n - \ddot{\varrho}| < |\ddot{\varrho}| + \tilde{1}$.

If we set $\widetilde{m} = \max\{|\ddot{\varrho}|, |\widetilde{\varpi}_1|, |\widetilde{\varpi}_2|, |\widetilde{\varpi}_3|, \dots, |\widetilde{\varpi}_{n-1}|\}$ then $|\widetilde{\varpi}_n| < \widetilde{m} + \tilde{1}$ for all $n \in \mathbb{N}$. This implies that the SCSQ $\{\widetilde{\varpi}_n\}$ is bounded.

Theorem 5 Let $\{\widetilde{\varpi}_n\}$ be a bounded sequence of SCNs. Then $\{\widetilde{\varpi}_n\}$ has a convergent subsequence.

Proof. Since $\{\widetilde{\varpi}_n\}$ is bounded sequence of SCNs, then $\{Re\, \widetilde{\varpi}_n\}$ and $\{Im\, \widetilde{\varpi}_n\}$ are bounded sequences of SRNs. By Lemma 1, $\{Re\, \widetilde{\varpi}_n\}$ and $\{Im\, \widetilde{\varpi}_n\}$ has a convergent subsequences $\{Re\, \widetilde{\varpi}_{n_k}\}$ and $\{Im\, \widetilde{\varpi}_{n_k}\}$, respectively. Now $\{Re\, \widetilde{\varpi}_{n_k}\}$ is a subsequence of the convergent sequence $\{Re\, \widetilde{\varpi}_{n_k}\}$, and so converges to the same soft complex limit. Hence, $\{\widetilde{\varpi}_{n_{k_r}}\}$ is a convergent subsequence of $\{\widetilde{\varpi}_n\}$ as its real and imaginary parts are both convergent sequence of SCNs.

Theorem 6 If $\{\widetilde{\varpi}_n\}$ is a SCSQ bounded and converges to soft complex limit $\ddot{\varrho} \in X^{(\widetilde{\Theta})}$. Then ${\widetilde{\varpi}_n}^2 = \ddot{\varrho}^2$.

Proof. Suppose $\{\widetilde{\varpi}_n\}$ be a SCSQ bounded and it is converge to $\ddot{\varrho} \in X^{(\widetilde{\Theta})}$. So, by Definition 18, for a SRN $\ddot{\eta} > \bar{0}$, there is a soft natural integer $\ddot{\hbar}$ in which $|\widetilde{\varpi}_n - \ddot{\varrho}| \leq \frac{\ddot{\eta}}{\widetilde{M} + |\ddot{\varrho}|}$, $n \geq \ddot{\hbar}$, also since $\{\widetilde{\varpi}_n\}$ is bounded, therefore, $|\{\widetilde{\varpi}_n\}| < \widetilde{M} (n \in \mathbb{N})$, this means that

$$|{\widetilde{\varpi}_n}^2 - \ddot{\varrho}^2| = |(\widetilde{\varpi}_n - \ddot{\varrho})(\widetilde{\varpi}_n + \ddot{\varrho})| = |\widetilde{\varpi}_n - \ddot{\varrho}||\widetilde{\varpi}_n + \ddot{\varrho}|.$$

The triangular inequality has given us $|\widetilde{\varpi}_n + \ddot{\varrho}| \leq |\widetilde{\varpi}_n| + |\ddot{\varrho}|$, this implies that $|\widetilde{\varpi}_n + \ddot{\varrho}| \leq \widetilde{M} + |\ddot{\varrho}|, n \in \boldsymbol{N}$. Now it gives

$$|{\widetilde{\varpi}_n}^2 - \ddot{\varrho}^2| \leq |\widetilde{\varpi}_n - \ddot{\varrho}|(\widetilde{M} + |\ddot{\varrho}|), n \geq \ddot{\hbar} \tag{7}$$

implies that

$\left|{\widetilde{\varpi}_n}^2 - \ddot{\varrho}^2\right| \leq \frac{\ddot{\eta}}{\widetilde{M} + |\ddot{\varrho}|} (\widetilde{M} + |\ddot{\varrho}|) = \ddot{\eta}, n \geq \ddot{\hbar}$. This completes the proof.

Theorem 7 If $\{\widetilde{\varpi}_n\} = \{\tilde{\mu} + i\tilde{\nu}\}$ is a sequence of SCNs then $\{\widetilde{\varpi}_n\}$ is a Cauchy sequence iff both sequences of SRNs $\{\tilde{\mu}_n\}$ and $\{\tilde{\nu}_n\}$ are Cauchy sequence of SRNs.

Proof. If we suppose $\{\widetilde{\varpi}_n\}$ is a Cauchy sequence, then for every SRN $\ddot{\eta} > \bar{0}$ ∃ a positive soft integer $\ddot{\hbar}$ such that $|\widetilde{\varpi}_n - \widetilde{\varpi}_m| \leq \ddot{\eta}\ (\tilde{n}, m \geq \ddot{\hbar})$. Then for $n \geq \ddot{\hbar}$, it gives that

$$\left|\tilde{\mu}_n - \tilde{\mu}_m\right| = \sqrt{\left(\tilde{\mu}_n - \tilde{\mu}_m\right)^2} \leq \sqrt{\left(\tilde{\mu}_n - \tilde{\mu}_m\right)^2 + \left(\tilde{\nu}_n - \tilde{\nu}_m\right)^2} = \left|\widetilde{\varpi}_{\tilde{n}} - \widetilde{\varpi}_m\right| \leq \ddot{\eta}$$

and

$$\left|\tilde{\nu}_n - \tilde{\nu}_m\right| = \sqrt{\left(\tilde{\nu}_n - \tilde{\nu}_m\right)^2} \leq \sqrt{\left(\tilde{\mu}_n - \tilde{\mu}_m\right)^2 + \left(\tilde{\nu}_n - \tilde{\nu}_m\right)^2} = \left|\widetilde{\varpi}_n - \widetilde{\varpi}_m\right| \leq \ddot{\eta}.$$

So both $\{\tilde{\mu}_n\}$ and $\{\tilde{\nu}_n\}$ are Cauchy sequence of SRNs.

Conversely, suppose that both $\{\tilde{\mu}_n\}$ and $\{\tilde{\nu}_n\}$ are Cauchy sequence of SRNs. Then for every SRN $\ddot{\eta} > \bar{0}$, since $\{\tilde{\mu}_n\}$ is a Cauchy sequence, there exists a positive soft integer $\ddot{\hbar}_1$ such that for all $n, m \geq \ddot{\hbar}_1$, then

$|\tilde{\mu}_n - \tilde{\mu}_m| \leq \ddot{\eta}/2$. Similarly, since $\{\tilde{\nu}_n\}$ is Cauchy then there exists an positive soft integer $\ddot{\hbar}_2$ such that for all $n, m \geq \ddot{\hbar}_2$. Then $|\tilde{\nu}_n - \tilde{\nu}_m| \leq \ddot{\eta}/2$. If $\ddot{\hbar} = max\{\ddot{\hbar}_1, \ddot{\hbar}_2\}$, then for $n, m \geq \ddot{\hbar}$, it gives

$$\left|\widetilde{\varpi}_n - \widetilde{\varpi}_m\right| = \left|\left(\tilde{\mu}_n + i\tilde{\nu}_n\right) - \left(\tilde{\mu}_m + i\tilde{\nu}_m\right)\right| = \left|\left(\tilde{\mu}_n - \tilde{\mu}_m\right) + i\left(\tilde{\nu}_n - \tilde{\nu}_m\right)\right| \leq \left|\tilde{\mu}_n - \tilde{\mu}_m\right| + \left|\tilde{\nu}_n - \tilde{\nu}_m\right| < \ddot{\eta}.$$

So $\{\widetilde{\varpi}_n\}$ is a Cauchy sequence of SCNs.

Theorem 8 Every soft Complex Cauchy sequence has a bound.

Proof. Suppose that $\{\widetilde{\varpi}_n\}$ is a soft Complex Cauchy sequence. Then by Definition 18, for every SRN $\ddot{\eta} = 1 > 0$, there exists a positive integer $\ddot{\hbar}$ such that $|\widetilde{\varpi}_n - \widetilde{\varpi}_m| < 1$, for all $n, m \geq \tilde{k}$. So for $n \geq \ddot{\hbar}$, it gives $|\widetilde{\varpi}_n| \leq 1 + |\widetilde{\varpi}_{\ddot{\hbar}}|$.

Theorem 9 Every convergent SCSQ is a soft complex Cauchy sequence.

Proof. Let $\{\widetilde{\varpi}_n\}$ be a SCSQ and convergent to $\ddot{\varrho} \in X^{(\widetilde{\Theta})}$. For $\ddot{\eta} > \bar{0}$, there exists a positive integer $\ddot{\hbar}$ such that

$$|\widetilde{\varpi}_s - \ddot{\varrho}| \leq \frac{\ddot{\eta}}{2} (s \geq \ddot{\hbar}).$$

This implies that for $n, m \geq \ddot{\hbar}$, it gives

$$|\widetilde{\varpi}_n - \widetilde{\varpi}_m| \leq |\widetilde{\varpi}_n - \ddot{\varrho}| + |\widetilde{\varpi}_m - \ddot{\varrho}| < \ddot{\eta}.$$

Theorem 10 Every soft complex Cauchy sequence is convergent.

Proof. Let $\{\widetilde{\varpi}_n\} = \{\tilde{\mu}_n + i\tilde{\nu}_n\}$ be any soft complex Cauchy sequence, where $\tilde{\mu}_n$ and $\tilde{\nu}_n$ are Cauchy sequences of SRNs. Then $\{\tilde{\mu}_n\}$ and $\{\tilde{\nu}_n\}$ are convergent sequences of SRNs to $\tilde{\mu}_0$ and $\tilde{\nu}_0$, respectively.

Convergence of Series in SCNs

Let $\{\widetilde{\varpi}_n\}$ be a sequence of SCNs, we make a new sequence that is specified by $\ddot{\omega}_1 = \widetilde{\varpi}_1, \ddot{\omega}_2 = \widetilde{\varpi}_1 + \widetilde{\varpi}_2, \ddot{\omega}_3 = \widetilde{\varpi}_1 + \widetilde{\varpi}_2 + \widetilde{\varpi}_3, \dots, \ddot{\omega}_n = \widetilde{\varpi}_1 + \widetilde{\varpi}_2 + \widetilde{\varpi}_3 + \dots + \widetilde{\varpi}_n$, where $\ddot{\omega}_n$ is called the sequence of n^{th} partial sums of sequence of SCNs $\{\widetilde{\varpi}_n\}$. The sequence $\{\ddot{\omega}_n\}$ is symbolized by $\widetilde{\varpi}_1 + \widetilde{\varpi}_2 + \widetilde{\varpi}_3 + \dots = \sum_{n=1}^{\infty} \widetilde{\varpi}_n$ called an infinite series of SCNs. If $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$ exists then the series is claimed as convergent and $\ddot{\omega}$ is its sum, i.e.

$$\sum_{n=1}^{\infty} \widetilde{\varpi}_n = \lim_{n\to\infty} \sum_{k=1}^{n} \widetilde{\varpi}_k = \lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}.$$

If a series of SCNs is not convergent, it is called divergent.

Example 4 The soft complex series $\sum_{n=0}^{\infty} i^n$ does not convergent. Because the SCSQ of its parial sums is periodic and therefore doses not have a limit.

Theorem 11 If the infinite series of SCNs $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ is convergent then $\lim_{n\to\infty} \tilde{\varpi}_n = \bar{0}$.

Proof. Let $\ddot{\omega}_n = \tilde{\varpi}_1 + \tilde{\varpi}_2 + \tilde{\varpi}_3 + \cdots + \tilde{\varpi}_{n-1} + \tilde{\varpi}_n$ be the n^{th} partial sum of the series of SCNs. Suppose that the series is convergent and $\ddot{\omega}$ be the sum $\sum_{n=1}^{\infty} \tilde{\varpi}_n$, this implies that $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$, we get that $\tilde{\varpi}_n = \ddot{\omega}_n - \ddot{\omega}_{n-1}$, by $\tilde{\varpi}_1 + \tilde{\varpi}_2 + \tilde{\varpi}_3 + \cdots + \tilde{\varpi}_{n-1} = \ddot{\omega}_{n-1}$, taking limit on both sides, it gives that

$$\lim_{n\to\infty} \tilde{\varpi}_n = \lim_{n\to\infty} \ddot{\omega}_n - \lim_{n\to\infty} \ddot{\omega}_{n-1} = \ddot{\omega} - \ddot{\omega} = 0.$$

Remark 2 Consider the infinite series of SCNs $\sum_{n=0}^{\infty} \tilde{\varpi}_n = \tilde{\varpi}_0 + \tilde{\varpi}_1 + \tilde{\varpi}_2 + \cdots$ if $\tilde{r}_n = \tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \tilde{\varpi}_{n+2} + \cdots$ then $\tilde{r}_n$ is called remainder of the infinite series of SCNs. If $\ddot{\omega}$ is the sum of infinite series of SCNs then $\ddot{\omega} = \ddot{\omega}_n - \tilde{r}_n$.

Theorem 12 A series $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ of terms with SCNs is convergent iff for every SRN $\ddot{\eta} > \bar{0}$, there exists a positive integer $\ddot{\hbar}$ such that $\left|\tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \cdots + \tilde{\varpi}_{n+p}\right| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$ and $p \geq 0$.

Proof. Suppose $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ is convergent, and let $\tilde{\varpi}_n = \tilde{\varpi}_1 + \tilde{\varpi}_2 + \cdots + \tilde{\varpi}_{n-1}$ be the n^{th} partial sum of the series of SCNs. Suppose that the series is convergent and $\ddot{\omega}$ be a sum of series, it means that $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$, for every SRN $\ddot{\eta} > \bar{0}$ there exists a positive integer $\ddot{\hbar}$ such that $|\ddot{\omega}_n - \ddot{\omega}| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$.

Let $\tilde{r}_n = \tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \tilde{\varpi}_{n+2} + \cdots$ be the remainder of the infinite series of SCNs, we get that $\ddot{\omega} = \ddot{\omega}_n - \tilde{r}_n$, so that $|\ddot{\omega}_n - \ddot{\omega}| = |\ddot{\omega} - \ddot{\omega}_n| = |\tilde{r}_n| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$, or $|\tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \tilde{\varpi}_{n+2} + \cdots| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$. So that, $\left|\tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \tilde{\varpi}_{n+2} + \cdots + \tilde{\varpi}_{n+p}\right| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$ and $p \geq 0$.

Conversely, we know that $\sum_{n=1}^{\infty} \tilde{\varpi}_n = \tilde{\varpi}_1 + \tilde{\varpi}_2 + \cdots + \tilde{\varpi}_n + \cdots$ and if $\ddot{\omega}$ is its partial sum then, $\ddot{\omega} = \ddot{\omega}_n - \tilde{r}_n$ and $|\tilde{r}_n| < |\ddot{\omega}_n - \ddot{\omega}|$. From $|\tilde{r}_n| = \left|\tilde{\varpi}_n + \tilde{\varpi}_{n+1} + \tilde{\varpi}_{n+2} + \cdots + \tilde{\varpi}_{n+p}\right| < \ddot{\eta}$ for all $n \geq \ddot{\hbar}$ and $p \geq 0$ so that $|\ddot{\omega}_n - \ddot{\omega}| < \ddot{\eta}$, it means that $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$.

Definition 21 Let $\tilde{\varpi}_n \in X^{(\tilde{\Theta})}$ $(n \geq \bar{0})$, a series $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ converges to $\tilde{\varpi}_0$, if for every SRN $\ddot{\eta} > \bar{0}$, there exists a positive integer $\ddot{\hbar}$ such that $\left|\sum_{k=1}^{\infty} \tilde{\varpi}_k - \tilde{\varpi}_0\right| < \ddot{\eta}$, for all $n \geq \ddot{\hbar}$.

Definition 22 A SCNs series $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ converges absolutely if $\sum_{n=1}^{\infty}|\tilde{\varpi}_n|$ converges.

Proposition 3 If the series $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ converges absolutely then, $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ converges.

Proof. Let $\ddot{\omega}_n = \tilde{\varpi}_1 + \tilde{\varpi}_2 + \tilde{\varpi}_3 + \cdots + \tilde{\varpi}_{n-1} + \tilde{\varpi}_n$ be the series of partial sums in the context of its convergence. Then for every SRN $\ddot{\eta} > \bar{0}$, there exists a positive integer $\ddot{\hbar}$ such that $\sum_{j=m+1}^{\infty}\left|\tilde{\varpi}_j\right| < \ddot{\eta}, \forall\, n \geq \ddot{\hbar}$. If $n, m \geq \ddot{\hbar}$ then,

$$\left|\ddot{\omega}_n - \ddot{\omega}_m\right| = \left|\tilde{\varpi}_{m+1} + \tilde{\varpi}_{m+2} + \cdots + \tilde{\varpi}_n\right|$$
$$= \left|\sum_{j=m+1}^{\infty} \tilde{\varpi}_j\right| \leq \sum_{j=n+1}^{\infty}\left|\tilde{\varpi}_j\right| \leq \sum_{j=n+1}^{\infty}\left|\tilde{\varpi}_j\right| < \ddot{\eta}.$$

Finally, it gives that $\{\tilde{\varpi}_n\}$ is a Cauchy sequence in SCNs so by Theorem 9 it is a convergent sequence in SCNs and $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ is convergent.

Theorem 13 Let $\tilde{\varpi}_n = \tilde{\mu}_n + i\tilde{\nu}_n$ $(n \in \mathbb{N})$ be a SCSQ and $\ddot{\omega} = \tilde{\mu} + i\tilde{\nu} \in X^{(\tilde{\Theta})}$. Then the soft complex series

$\sum_{n=1}^{\infty} \tilde{\varpi}_n = \ddot{\omega}$ iff the soft real series $\sum_{n=1}^{\infty} \tilde{\mu}_n = \tilde{\mu}$ and $\sum_{n=1}^{\infty} \tilde{\nu}_n = \tilde{\nu}$.

Proof. First we write the sequence of partial sums of the series $\sum_{n=1}^{\infty} \tilde{\varpi}_n$ as $\ddot{\omega}_n = \ddot{\omega}_{1n} + i\ddot{\omega}_{2n}$, where $\ddot{\omega}_{1n} = \sum_{n=1}^{\infty} \tilde{\mu}_n$ and $\ddot{\omega}_{2n} = \sum_{n=1}^{\infty} \tilde{\nu}_n$. This implies that $\sum_{n=1}^{\infty} \tilde{\varpi}_n = \ddot{\omega}$ iff $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$. Now by Theorem 1 on convergent SCSQs we get that $\lim_{n\to\infty} \ddot{\omega}_n = \ddot{\omega}$ iff $\lim_{n\to\infty} \ddot{\omega}_{1n} = \tilde{\mu}$ and $\lim_{n\to\infty} \ddot{\omega}_{2n} = \tilde{\nu}$. Therefore, $\sum_{n=1}^{\infty} \tilde{\varpi}_n = \ddot{\omega}$.

Conversely, since $\ddot{\omega}_{1n} = \tilde{\mu}$ and $\ddot{\omega}_{2n} = \tilde{\nu}$ are partial sums of the real soft series $\sum_{n=1}^{\infty} \tilde{\mu}_n$ and $\sum_{n=1}^{\infty} \tilde{\nu}_n$, respectively, so the result is obtained directly.

CONCLUSION

The SCNs offer a useful tool for organizing and managing ambiguous and incomplete data. One of its benefits is that they may articulate imprecise values with both real and imaginary features, which allows for a more in-depth description of complex circumstances. Due to its flexibility, it can be applied to the analysis of systems with complex randomness, which is frequent in many real-world applications. In light of the aforementioned benefits and flexibility of SCNs, by using SCNs and SRSQ, the novel notion of a SCSQ has been presented, and its link with SCNs has been demonstrated by a convergent series analysis. Further, the soft complex limit of SCSQs has been introduced, which is an initial route for the convergence of SCSQs. Some famous soft convergence theorems are presented. Important theorems such as the Bolzano-Weierstrass theorem have also been explored. Soft complex boundedness of SCSQs is also well described. There are also some related soft complex theorems present. Cauchy Sequences in SCNs have also been reflected, as is their relationship with bounded sequences in SCNs. Furthermore, by using SCSQs with specific characteristics, we introduce the infinite series and its convergent in SCNs. The presented work can be used in order to characterize the various notions of differential and integral calculus in SSET environment. Moreover, the various theorems and ideas in fuzzy measure theory and fuzzy functional analysis can also be modified for SSET environment using the SCNs, and SCSQs. The suggested approach has enormous potential in a number of different fields for its future possibilities. Scholars could further explore ways to improve the suggested approaches to better manage situations in the actual world when specific and vague knowledge coexist. Additionally, there is an exciting opportunity for combining the suggested idea with modern innovations like machine learning and artificial intelligence, where the structure's versatility may help create stronger and more effective algorithms for pattern recognition and data analysis.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

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The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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