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Research Article

An innovative group decision-making approach utilizing picture fuzzy hypersoft expert set

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ABSTRACT

The goal of this study is to propose a novel hybrid model called picture fuzzy hypersoft expert sets. This hybrid model is a combination of picture fuzzy sets with hypersoft expert sets for dealing with uncertainties in various real-world group decision-making problems. Picture fuzzy sets are an extension of intuitionistic fuzzy sets, and their theory takes into account the degree of refusal during a decision-making process in addition to the degrees of acceptance and rejection. A more comprehensive version of intuitionistic fuzzy hypersoft expert sets is the hybrid model that has been suggested. Along with matching examples, certain novel desirable features of the proposed model namely, subset, equality, complement, union, and intersection are examined. For the constructed model, two renowned operations AND and OR are also examined. Additionally, a decision-making process under the suggested strategy is described, supported by an algorithmic framework. Additionally, an example application is offered for a more thorough explanation; however, it is contingent upon the choice of an appropriate virtual reality device provider. Lastly, the contrast between the started approach and a few current models such as intuitionistic fuzzy hypersoft expert sets is investigated.

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INTRODUCTION

An effective process that may produce rankings for a finite group of items according to several criteria related to these items is multi-attribute group decision making, or MAGDM. How to more precisely and effectively assign a numerical value to a given choice is a major issue in real-world decision-making procedures. It was impossible to characterise items with precise values since fuzziness existed in many difficult practical decision-making situations. Zadeh [1] was the first to introduce the idea of a fuzzy set (FS). This set is a superset of the classical set, as a solution to this problem. The idea of partial truth between ”absolute true” and ”absolute false” is what FS genuinely addresses. It is crucial that the membership function provide objects from closed unit intervals with their membership values. In certain real-world scenarios, the notion of FSs is inapplicable. If a specialist renders a decision regarding the non-membership degree of an item, for example.

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Then the non-membership degree is derived by taking into account the standard negation of the membership degree. As a generalisation of fuzzy sets (FSs), Atanassov [2] introduced the theory of intuitionistic fuzzy sets (IFSs) to address these kinds of issues. The description of FSs was altered by the addition of a new element (which determines the non-membership grade). As a result, an IFS includes two ambiguous components: membership grade and non-membership grade. This is a crucial point to demonstrate the IFS's expanding range of practical uses. There is a restriction on membership and non-membership grades in an IFS setting; that is, the total of the two grades is constrained by one. Many academics have focused on the aforementioned ideas during the past 20 years. Their efforts have produced a lot of successful outcomes in a variety of fields, such as taking decisions, health care diagnosis, and analysis of clustering [3–6]. While the aforementioned theories have proven useful in various real-world contexts, there are certain circumstances that IFSs are unable to fully explain. For instance, in a voting scenario when there are human viewpoints with more than two possible responses yes, no, abstain, or refusal. In these situations, IFSs are unable to appropriately represent the circumstance. Furthermore, an expert can provide their assessment of an object belonging to a universal set concerning a specific attribute. For example, an object's membership value with regard to a given property could be 0.5, its non-membership value could be 0.2, and its neutral behaviour could be 0.3, which indicates that the expert is unsure about the value. Since the current FSs and IFSs are unable to address this issue, Cuong [7] introduced the concept of picture fuzzy sets (PFSs). This structure is a direct generalisation of FS and IFS that incorporates the notion of positive, negative, and neutral membership degrees of an alternative. In contrast to FSs and IFSs, a PFS includes membership, neutral, and non-membership values. Due to these things, it is much better suited to illustrate ambiguous information. Numerous research have been proposed to date to address various everyday issues within the helpful framework of PFSs. Zhu et al. [8] introduced new distance measure for PFSs with applications. Singh [9], for instance, developed a correlation coefficient method for the PFSs. Son [10] developed a generalised distance measure for PFSs and used it to apply the solution to the clustering issue. Garg [11] constructed a few picture fuzzy aggregation operations employing t-norm and t-conorm. Then he applied them to multi-criteria decision making (MCDM). Following that, Wei [12] presented the TODIM approach for MCDM in a PFS setting. Some MAGDM techniques were developed by Ashraf et al. [13] and their applications to group decision-making were investigated. From an alternative angle, Sahu et al. [14] employed a hybridised distance measure for students' job choosing that was based on the theories of PFSs and rough sets. For other key terms pertaining to PFSs, readers should consult [15–24]. Following the development of fuzzy sets, Pawlak [25] developed the rough set model in 1982. This model proved very helpful tool for handling imprecise information. Due to their inability to accept parameterized values, the fuzzy and the rough sets theories are both ineffective when evaluating opinions based on disparate parameters. Verma, and Rohtagi [26] developed the similarity measures by using picture fuzzy sets for the different applications. Khan et al. [27] gave the idea of generalised picture fuzzy set with applications in different areas. Molodtsov [28] in troduced the idea of soft sets (SSs) to close this gap. The SS model functions as a powerful parameterization tool when handling uncertainties. This model is composed of the parameterized families of a universal set. Ali et al. [29] expanded on the work by introducing a few new SS concepts and features. An application of SSs for decision-making was presented by Maji et al. [30]. In order to integrate the advantages and capacities of current models, more effective models are required as uncertain circumstances get more complicated. Hybrid designs, which are merely mixtures of current models, effectively accomplish the goal. Many hybrid models have been developed in the past to address MAGDM scenarios that are unclear [31,32]. For example, Yang et al. [33] introduced picture fuzzy SSs (PFSSs) and their decision-making applications by combining SSs with PFSs. Asgher et al. [34] used the structure of complex picture fuzzy soft set in MADM methods. Asgher et al. [35] again used this structure to give example of MADM. Mutlu et al. [36] developed MADM methods for health systems. Yolcu [37] gave MCDM methods by using intuitionistic fuzzy soft sets. In relation to hesitating, Akram et al. [38] examined N-soft sets and their uses in MAGDM. Additionally, Alcantud and Giarlotta [39] provided possible fuzzy sets that are both necessary and feasible for group decision making. Alkhazalah and Salleh [40] developed the idea of soft expert sets (SESs), which can handle several expert opinions on a single platform, to get over this restriction. Subsequently, fuzzy SESs were introduced by Alkhazalah and Salleh [41] by extending the notions of SESs by their combination with fuzzy theory. Numerous scholars have used the powerful notion of SESs to solve a variety of group making choices difficulties, as seen in [42-45]. For example, intuitionistic fuzzy SESs (IFSESs) were introduced and their applications were examined by Broumi and Smarandache [46]. The bipolar fuzzy SES model was presented by Qudah and Hassan [47] along with certain applications. To describe the certain important characteristics of convexity, Ihsan et al. [48,49] made use of SES and FSES in the convexity. Ali et al. [50] have recently introduced a new hybrid model known as fuzzy bipolar SESs and examined its use in different fields. Soft set like models have been introduced only for the one expert opinion and have no idea about the partitioning of attributes. To solve the problem of partitioning of parameters, Smarandache [51] made extension in the soft set to established the hypersoft set (HS). He also made the combination of this model with other fuzzy set like structures to form fuzzy, intuitionistic fuzzy hypersoft sets.

Rah- man et al. [52] made use of HS with the concept of mappings in supply chain management. Saeed et al. [53] made use of the fuzzy hypersoft set for renewable energy resources with the help of an example. After this Saeed et al. [54] introduced the fundamentals of picture fuzzy hypersoft sets. To handle the situation of multi-opinions of multi-experts having in a single, Ihsan et al. [55] gave the structure of hypersoft expert set(HSES). Ihsan et al. [56] also made combination of this model with fuzzy environment to form fuzzy [57] and intuitionistic fuzzy hypersoft expert sets (IFHSES) [58]. According to the aforementioned research, a number of models, such as PFSs or IFHSESs, have been put forth to efficiently synthesise picture fuzzy information; yet, there hasn't been any attention paid to the development of an effective hybrid model that combines PFSs and HESs [59-61].

Research Gap, Motivation and Novelty

The following is a summary of the primary motivations and novelty for this construction:

(1) In reality, the combination of models, known as IFSESs [46], deals with two dimensional data. Such kind of a data has been assessed by several specialists in relation to various parameters. This approach falls short in addressing the crucial concept of neutrality degree. This degree is evident in a number of real-world scenarios. In such scenarios, we are presented with expert judgements in the form of yes, no, abstain, and refusal. For example, the neutrality degree can be taken into account while diagnosing medical conditions. Certain illnesses (such as chest or heart problems) may not present with symptoms like fever and headache. Similarly, the symptoms of stomach and chest pain are inert against a variety of illnesses, such as viral fever, typhoid, and malaria.

(2) Yang et al. [33] integrated the ideas of PFS and SES to develop an innovative hybrid approach called PFSESs. Yet this model is unable to appropriately handle the partitioning of parameters into sub-parameters with non-overlapping parametric set values. To effectively handle many experts, we combine PFSESs with HSs to develop a novel hybrid model we call picture fuzzy PFHSESs.

The developed picture fuzzy HSES (PFHSES) model has the following main contributions:

1. Motivated by PFSs' ability to handle ambiguous and uncertain data in practical situations, this work aims to introduce a novel hybrid model PFHSESs that combines PFSEs with HSs.
2. Corresponding examples are used to examine some of its desired qualities, including subset complement, union, intersection, OR operation, and AND operation.
3. Based on PFHSESs, a decision-making algorithm is developed.
4. To better illustrate the suggested methodology, an illustrative application is included.
5. In addition, the advantages and a comparison of the suggested hybrid model with a few other modelsincluding intuitionistic fuzzy SESs are examined in order to demonstrate the effectiveness and dependability of the model.

This paper's remaining sections are organised as follows: A thorough explanation of several basic concepts, such as SES, PFS, scoring function, and accuracy function for PFSs and PFSSs, is given in Section 2. An effective expansion of PFSSs or IFSESs, the new hybrid model known as PFHSESs is presented in Section 2. Through illustrative numerical examples, various fundamental features and operations, including subset, complement, union, intersection,

OR operation, and AND operation, are also examined for PFHSESs. For a more thorough explanation of the suggested method, Section 4 offers an application in real life. In order to demonstrate the effectiveness and dependability of our developed model, Section 5 examines the advantages and contrasts the model with others already in use, including IFSESs. The paper's conclusion and recommendations for future research are provided in Section 6.

PRELIMINARY KNOWLEDGE

This part goes over a few fundamental ideas, such as PFSES, and their basic operations, this will come in handy for the paper's remaining sections.

Definition 2.1. [62] Suppose there are sets Υ, Δ, Ω and Γ which are named as a universe of discourse, collection of parameters, experts and opinions respectively. Take $R = \Delta \times \Omega \times \Gamma$, then picture fuzzy soft expert can be considered as a pair (ζ, R) such that: $\zeta : R \to P^{F(\Upsilon)}$.

In set form it can be shown as $(\zeta, R) = \{\langle b, \zeta(b)\rangle : b \in R\}$, where $\zeta(b) = \{y, \langle \eta_R(b), \beta_R(b), \theta_R(b)\rangle : y \in \Upsilon\}$ fulfilling the condition that their sum lies between 0 and 1. While $\eta_R(b)$, $\beta_R(b)$ and $\theta_R(b)$ are used here for truth, neutrality and falsity degrees respectively and each of which lies between 0 and 1.

Definition 2.2 [62] The subset relation between two PFSESs (ζ, R) and (α, P) can be seen as: (1) $R \subseteq P$, (2) $\zeta(b) \subseteq \alpha(b)$, whenever $\eta R(b) \leq \eta P(b)$, $\beta R(b) \leq \beta P(b)$ and $\theta R(b) \geq \theta P(b)$.

Definition 2.3 [62] The equality of two PFSESs (ζ, R) and (α, P) i. e., $(\zeta, R) = (\alpha, P)$ can be seen iff $(\zeta, R) \subseteq (\zeta, P)$ and $(\zeta, P) \subseteq (\zeta, R)$.

Definition 2.4 [62] The union operation between two PFSESs (ζ, R) and (α, P) defines an- other PFSESs $(\beta, Q) = (\zeta, R) \cup (\alpha, P)$ with $Q = R \cup P$ and for all $b \in Q$,

$$\beta(b) = \begin{cases} \zeta(b) & ; \ b \in R - P \\ \alpha(b) & ; \ b \in P - R \\ \cup(\zeta(b), \alpha(b)) & ; \ b \in R \cap P, \end{cases}$$

where $\cup(\zeta(b), \alpha(b)) = \{< u, \max\{\mu 1(b), \mu 2(b)\}, \min\{\nu 1(b), \nu 2(b)\}, \min\{\omega 1(b), \omega 2(b)\} >: u \in \Upsilon\}$.

Definition. 2.5. [62] The operation intersection between two PFSESs (ζ, R) and (α, P) defines another PFSESs (β, Q) = (ζ, R) ∩ (α, P) with Q = R ∩ P and for all b ∈ Q,

$$\beta(b) = \begin{cases} \zeta(b) & ; \; b \in R - P \\ \alpha(b) & ; \; b \in P - R \\ \cap(\zeta(b), \alpha(b)) & ; \; b \in R \cap P, \end{cases}$$

where ∩(ζ(b), α(b)) = {< u, min {μ1(b), μ2(b)}, min {ν1(b), ν2(b)}, max {ω1(b), ω2(b)} >: u ∈ ϒ}.

PICTURE FUZZY HYPERSOFT EXPERT SETS

This part presents the central idea of the study, which is PFHSESs, along with a few basic model characteristics that are clarified through examples of illustration.

Definition.3.1. Let Ŵ = {$\bar{S}_1$, $\bar{S}_2$, $\bar{S}_3$…$\bar{S}_n$} be the set of different sub-classes corresponds to distinct attributes $\bar{s}_1$, $\bar{s}_2$, $\bar{s}_3$… $\bar{s}_n$, and $\hat{B}$ = Ŵ × Ω × Γ, then picture fuzzy hypersoft expert set can be defined by a mapping $\chi : \hat{B} \to P^{F(\Upsilon)}$. In set form it can be written as: $(\chi, \hat{B}) = \{\langle b, \chi(b)\rangle : b \in \hat{B}\}$ where $\chi(b) = \{\langle g, \eta\hat{B}(g), \beta\hat{B}(g), \theta\hat{B}(g)\rangle : g \in \Upsilon\}$ having the condition $0 \le \eta\hat{B}(g) + \beta\hat{B}(g) + \theta\hat{B}(g) \le 1$. Some hesitancy degree can also be calculated as $1 - (\eta\hat{B}(g) + \beta\hat{B}(g) + \theta\hat{B}(g))$. While $\eta\hat{B}(g)$, $\beta\hat{B}(g)$, and $\theta\hat{B}(g) \in [0, 1]$ represent the satisfying, neutral, and non-satisfying degrees respectively.

Example.3.2. Considering the three alternatives ϒ = {KIA = δ_1, City Honda = δ_2, PASSO Japan = δ_3} available to an organization looking to purchase electric automobiles, the global market for commercial electric vehicles is growing at a rapid pace. The company must consider the advice of two experts Ω = {$\hat{p}_1$, $\hat{p}_2$} in the field to determine which electric vehicle is ideal. Let Δ = {s_1=milage capacity, s_2 = price } be a helpful set of guidelines supplied by the business to assist professionals in the assessment process so they can appropriately meet their needs. Let S_{11} = 15 km, S_{12} = 20 km and S_{21} = 50, 000 Dollar, S_{22} = 60, 000 Dollar, be the different sub-classes corresponding to the attributes {s_1= milage capacity, s_2= price. $\hat{W} = s_1 \times s_2 = \{\partial_1, \partial_2, \partial_3, \partial_4\}$. we get 4 pairs and now $\hat{W} \times \Omega \times \Gamma = \{\xi_1, \xi_2, \xi_3, ..., \xi_{16}\} = \{\xi_i = (\partial_i, \hat{p}_i, 1 \text{ or } 0)\}$. We get 16 pairs and each pair is a 3-triplet. The PFHSES can be established on the basis of. the experts judgements which is given in Table 1.

Table 1. Tabular form of PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩
$(\partial_2, \hat{p}_1, 1)$	⟨0.15, 0.37, 0.24⟩	⟨0.15, 0.31, 0.25⟩	⟨0.23, 0.33, 0.25⟩
$(\partial_2, \hat{p}_2, 1)$	⟨0.42, 0.15, 0.35⟩	⟨0.43, 0.17, 0.32⟩	⟨0.42, 0.11, 0.32⟩
$(\partial_3, \hat{p}_1, 1)$	⟨0.25, 0.32, 0.32⟩	⟨0.21, 0.35, 0.30⟩	⟨0.32, 0.32, 0.21⟩
$(\partial_3, \hat{p}_2, 1)$	⟨0.25, 0.15, 0.15⟩	⟨0.23, 0.13, 0.13⟩	⟨0.30, 0.12, 0.21⟩
$(\partial_4, \hat{p}_1, 1)$	⟨0.27, 0.17, 0.37⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.03⟩
$(\partial_4, \hat{p}_2, 1)$	⟨0.32, 0.04, 0.35⟩	⟨0.42, 0.02, 0.42⟩	⟨0.32, 0.21, 0.42⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩
$(\partial_1, \hat{p}_2, 0)$	⟨0.45, 0.25, 0.25⟩	⟨0.42, 0.43, 0.13⟩	⟨0.40, 0.32, 0.11⟩
$(\partial_2, \hat{p}_1, 0)$	⟨0.27, 0.17, 0.27⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.43⟩
$(\partial_2, \hat{p}_2, 0)$	⟨0.43, 0.16, 0.31⟩	⟨0.42, 0.32, 0.12⟩	⟨0.42, 0.21, 0.13⟩
$(\partial_3, \hat{p}_1, 0)$	⟨0.24, 0.36, 0.38⟩	⟨0.22, 0.34, 0.31⟩	⟨0.23, 0.33, 0.30⟩
$(\partial_3, \hat{p}_2, 0)$	⟨0.36, 0.23, 0.17⟩	⟨0.33, 0.26, 0.12⟩	⟨0.31, 0.23, 0.11⟩
$(\partial_4, \hat{p}_1, 0)$	⟨0.13, 0.37, 0.29⟩	⟨0.16, 0.38, 0.21⟩	⟨0.13, 0.36, 0.22⟩
$(\partial_4, \hat{p}_2, 0)$	⟨0.43, 0.15, 0.35⟩	⟨0.44, 0.18, 0.32⟩	⟨0.43, 0.13, 0.32⟩

In this Table 1, ⟨0.33, 0.23, 0.32⟩ the values 0.33, 0.23, 0.32 represent the satisfaction, neutral and non-satisfaction degrees respectively.

Definition 3.3. Consider two PFHSESs (χ_1, $\hat{B}_1$) and (χ_2, $\hat{B}_2$). The subset relation between two these sets can be seen as if

1. $\hat{B}1 \subseteq \hat{B}_2$,
2. $\chi_1(b) \subseteq \chi_1(b)$ for all b ∈ $\hat{B}_1$ $\Longleftrightarrow \eta\hat{B}_1 \le \eta\hat{B}_2, \beta\hat{B}_1 \le \beta\hat{B}_2$ and $\theta\hat{B}_1 \ge \theta\hat{B}_2$.

The representation for the subset relation is (χ_1, $\hat{B}_1$) ⊆ (χ_2, $\hat{B}_2$).

Here is a mathematical illustration of an picture fuzzy soft expert subset relation to help understand it:

Example 3.4. Let (χ_1, $\hat{B}_1$) and (χ_2, $\hat{B}_2$) be two PFHSESs over ϒ. These two sets have been shown in Tables 2, and 3, respectively such that χ_1 and χ_2 have been below: $\chi_1 = \{(\partial_1, \hat{p}_1, 1)(\partial_1, \hat{p}_1, 0)\}$ and $\chi_2 = \{(\partial_1, \hat{p}_1, 1)(\partial_1, \hat{p}_1, 0)(\partial_1, \hat{p}_2, 1)\}$.

Table 2. Tabular form of PFHSES (χ_1, $\hat{B}1$)

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩

Table 3. Tabular form of PFHSES (χ_2, $\hat{B}_2$)

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.21, 0.30, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩

From above two tables, it is clear that (χ_1, $\hat{B}_1$)⊆ (χ_2, $\hat{B}_2$).

Definition 3.5. Two PFHSESS (χ_1, $\hat{B}_1$) and (χ_2, $\hat{B}_2$) are said to be equal iff (χ_1, $\hat{B}_1$) ⊆ (χ_2, $\hat{B}_2$) and (χ_2, $\hat{B}_2$) ⊆ (χ_1, $\hat{B}_1$).

Definition 3.6. Another subset relation called agree-PFHSES $(\chi, \hat{B})_1$ which can be stated as: $(\chi, \hat{B})_1 = \{\chi(b) : b \in \hat{W} \times \Omega \times \{1\}\}$.

Retaking the PFHSES (χ, $\hat{B}$) in Example 3.2. Then, we can find the agree-PFHSES as shown in Table 4.

Table 4. Tabular form of agree-PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩
$(\partial_2, \hat{p}_1, 1)$	⟨0.15, 0.37, 0.24⟩	⟨0.15, 0.31, 0.25⟩	⟨0.23, 0.33, 0.25⟩
$(\partial_2, \hat{p}_2, 1)$	⟨0.42, 0.15, 0.35⟩	⟨0.43, 0.17, 0.32⟩	⟨0.42, 0.11, 0.32⟩
$(\partial_3, \hat{p}_1, 1)$	⟨0.25, 0.32, 0.32⟩	⟨0.21, 0.35, 0.30⟩	⟨0.32, 0.32, 0.21⟩
$(\partial_3, \hat{p}_2, 1)$	⟨0.25, 0.15, 0.15⟩	⟨0.23, 0.13, 0.13⟩	⟨0.30, 0.12, 0.21⟩
$(\partial_4, \hat{p}_1, 1)$	⟨0.27, 0.17, 0.37⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.03⟩
$(\partial_4, \hat{p}_2, 1)$	⟨0.32, 0.04, 0.35⟩	⟨0.53, 0.02, 0.42⟩	⟨0.32, 0.21, 0.42⟩

Definition 3.7. Another subset relation called disagree-PFHSES $(\chi, \hat{B})_0$ which can be stated as: $(\chi, \hat{B})_0 = \{\chi(b) : b \in \hat{W} \times \Omega \times \{0\}\}$.

Retaking the PFHSES $(\chi, \hat{B})$ in Example 3.2. Then, we can find the disagree-PFHSES as shown in Table 5.

Table 5. Tabular form of disagree-PFHSES

$(\partial_i, \hat{p}_i, 0)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩
$(\partial_1, \hat{p}_2, 0)$	⟨0.45, 0.25, 0.25⟩	⟨0.42, 0.43, 0.13⟩	⟨0.40, 0.32, 0.11⟩
$(\partial_2, \hat{p}_1, 0)$	⟨0.27, 0.17, 0.27⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.43⟩
$(\partial_2, \hat{p}_2, 0)$	⟨0.43, 0.16, 0.31⟩	⟨0.42, 0.32, 0.12⟩	⟨0.42, 0.21, 0.13⟩
$(\partial_3, \hat{p}_1, 0)$	⟨0.24, 0.36, 0.38⟩	⟨0.22, 0.34, 0.31⟩	⟨0.23, 0.33, 0.30⟩
$(\partial_3, \hat{p}_2, 0)$	⟨0.36, 0.23, 0.17⟩	⟨0.33, 0.26, 0.12⟩	⟨0.31, 0.23, 0.11⟩
$(\partial_4, \hat{p}_1, 0)$	⟨0.13, 0.37, 0.29⟩	⟨0.16, 0.38, 0.21⟩	⟨0.13, 0.36, 0.22⟩
$(\partial_4, \hat{p}_2, 0)$	⟨0.43, 0.15, 0.35⟩	⟨0.44, 0.18, 0.32⟩	⟨0.43, 0.13, 0.32⟩

Definition 3.8. Here we define the union operation between two PFHSESs $(\chi_1, \hat{B}_1)$ and $(\chi_2, \hat{B}_2)$, which is again PFHSES $(\chi_1, \hat{B}_1) \cup (\chi_2, \hat{B}_2) = (\chi_3, \hat{B}_3)$, such that $\hat{B}_3 = \hat{B}_1 \cap \hat{B}_2$

$$\hat{B}_3(b) = \begin{cases} \hat{B}_1(b) & ; \ b \in \chi_1 - \chi_2 \\ \hat{B}_2(b) & ; \ b \in \chi_2 - \chi_1 \\ \cup(\hat{B}_1(b), \hat{B}_2(b)) & ; \ b \in \chi_1 \cap \chi_2, \end{cases}$$

Where $\cup(\hat{B}_1(b), \hat{B}_2(b)) = \{< u, max\{\mu_1(b), \mu_2(b)\}, min\{\nu_1(b), \nu_2(b)\}, min\{\omega_1(b), \omega_2(b)\} >: u \in \Upsilon\}$.

Example 3.9. Consider two two PFHSESs $(\chi_1, \hat{B}_1)$ and $(\chi_2, \hat{B}2)$ in example 3.2 such that $\hat{B}_1 = (\partial_1, \hat{p}_1, 1), (\partial_1, \hat{p}_1, 0), (\partial_1, \hat{p}_2, 1)$ and $\hat{B}_2 = (\partial_1, \hat{p}_1, 1), (\partial_1, \hat{p}_1, 0)$.

Now we see the tabular form of these two sets given by as:

Table 6. Tabular form of PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩

Table 7. Tabular form of PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩

Now tabular form of their union is given by in a table 8.

Table 8. Tabular form of union of two PFHSES sets

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩

Definition S.10. Here we define the intersection operation between two PFHSESs $(\chi_1, \hat{B}_1)$

and $(\chi_2, \hat{B}_2)$, which is again PFHSES $(\chi_1, \hat{B}_1) \cap (\chi_2, \hat{B}_2) = (\chi_3, \hat{B}_3)$, such that $\hat{B}_3 = \hat{B}_1 \cap \hat{B}_2$ with b $\in \chi_3$,

$$\hat{B}_3(b) = \begin{cases} \hat{B}_1(b) & ; \ b \in \chi_1 - \chi_2 \\ \hat{B}_2(b) & ; \ b \in \chi_2 - \chi_1 \\ \bigcap(\hat{B}_1(b), \hat{B}_2(b)) & ; \ b \in \chi_1 \bigcap \chi_2, \end{cases}$$

Where $\bigcap(\hat{B}_1(b), \hat{B}_2(b)) = \{< u, max\{\mu_1(b), \mu_2(b)\}, min\{\nu_1(b), \nu_2(b)\}, min\{\omega_1(b), \omega_2(b)\} >: u \in \Upsilon\}$.

Example 3.11. Consider two PFHSESs $(\chi_1, \hat{B}_1)$ and $(\chi_2, \hat{B}_2)$ in example 3.9, then the tabular form of their intersection is given by in a table 9.

Table 9. Tabular form of intersection of two PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.23, 0.32⟩	⟨0.22, 0.31, 0.30⟩	⟨0.32, 0.31, 0.31⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩

Definition 3.12. Here we define the AND operation between two PFHSESs $(\chi_1, \hat{B}_1)$ and $(\chi_2, \hat{B}_2)$, which is shown

as $(\chi_1, \hat{B}_1) \wedge (\chi_2, \hat{B}_2) = (\chi_3, \hat{B}_3)$, such that $\hat{B}_3 = \hat{B}_1 \infty \hat{B}_2$ with $\chi_3 = \chi_1(b_1) \cap \chi_2(b_2)$ and, $\chi_3(b_1, b_2)(u) = \{< u, min\{\mu_1(u), \mu_2(u)\}, min\{\nu_1(u), \nu_2(u)\}, max\{\omega_1(u), \omega_2(u)\} >: u \in \Upsilon\}, \forall(b_1, b_2) \in \hat{B}_1 \infty \hat{B}_2$.

Definition 3.13. Here we define the OR operation between two PFHSESs $(\chi_1, \hat{B}1)$ and $(\chi_2, \hat{B}_2)$, which is shown as $(\chi_1, \hat{B}_1) \vee (\chi_2, \hat{B}_2) = (\chi_3, \hat{B}_3)$, such that $\hat{B}_3 = \hat{B}_1 \infty \hat{B}_2$ with $\chi_3 = \chi_1(b_1) \cup \chi_2(b_2)$ and, $\chi_3(b_1, b_2)(u) = \{< u, max\{\mu_1(u), \mu_2(u)\}, min\{\nu_1(u), \nu_2(u)\}, min\{\omega_1(u), \omega_2(u)\} >: u \in \Upsilon\}, \forall(b_1, b_2) \in \hat{B}_1 \infty \hat{B}_2$.

DECISION SUPPORT SYSTEM

Algorithm: Different parts of the algorithm have been shown here.

1. **First part: Input**

1.1 Group of renowned business VR devices as a set of alternatives.
1.2 Group of parameters used for the evaluation of devices.
1.3 Sub-classes having different parametric valued sets.
1.4 Group of experts for the evaluation of parameters and group of opinions {0, 1} of experts.

2. **Second part: Construction and computations**

2.1 Construction of FPHSESs by the use of input data.
2.2 Separation of agree and disagree-FPHSESs.
2.3 Find the score values for the both agree and disagree-FPHSESs by formula $(\eta - \beta - \theta)$.
2.4 Calculate the difference of scores values of agree and disagree-FPHSESs.
2.5 Calculation of maximum score value.

3. **Third part: Output**

3.1 Selection of best alternative based on the maximum value.

The pictorial version of the algorithm have been shown in the Figure 1.

Validation of Algorithm

The steps of the algorithm that is suggested here are detailed in the example that follows.

Example 3.14. Morton Heilig invented virtual reality (VR) in 1957. His VR system, the Sen- sorama, is considered to be among the best entertainment devices. In actuality, analyst Jaron Lanier coined the word ”virtual reality” (VR) in 1987. His research and analysis helped to ad- vance a number of VR-related sectors. Virtual reality technology mostly relies on computer engineering to develop a virtual environment. Instead of having a screen in front of them, users are able to interact and become fully engaged in a three-dimensional (3D) world. The computer is transformed into a doorkeeper to this strange world by mimicking as many per- ceptions as possible, including listening, feel good, gaze, and even taste. The most recognis- able component of virtual reality is the head-mounted display. Virtual reality (VR) is making significant contributions to several major scientific sectors, including medical care, schooling, and the armed forces. Virtual reality is used in several health-care domains. With virtual reality (VR), any kind of medical situation may be replicated, allowing students to handle it just like in real life. VR can be used to enhance students' capacity for learning. Virtual reality education has the potential to significantly alter the way that educational content is delivered; it works by developing a virtual reality that users may engage with in addition

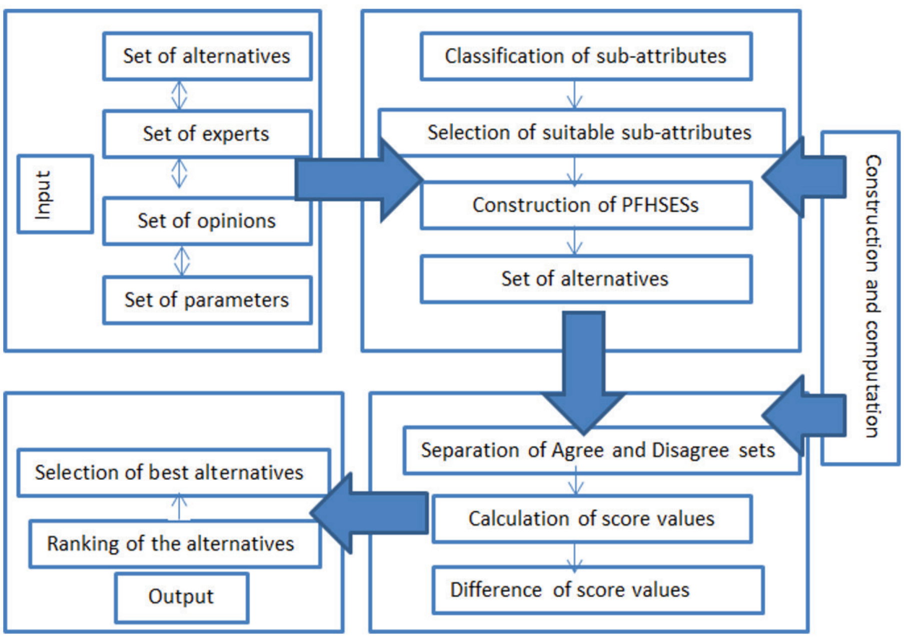

Figure 1. Flow chart of the suggested algorithm.

to viewing. The Link Company's flight instructors were the first to use simulators in a military setting in the late 1920s and early 1930s. The army, navy, and air force are among the armed forces branches that have utilised virtual reality (VR) for instructional reasons. Moreover, one of the applications for VR that is most passionate about VR is the tourism sector. Beginning in 2017, the British Museum unveiled a virtual reality (VR) experience that allowed visitors to fully in teract with a VR headset and have an unmatched digital experience on a computer or mobile device. The discussion above highlights a crucial point: because VR systems have a variety of features, or parameters, choosing the optimal manufacturer is an unpredictable process. Selecting the optimal solution is therefore a crucial task for the consumers. The assessment of various experts about VR systems in accordance with the advantageous characteristics of buyers (wholesale dealers) can assist in the selection of a suitable company producing VR sys- tems. Assume that the group of renowned businesses producing VR equipment is denoted by $\Upsilon = \{\delta_1 = Unity, \delta_2 = Meta, \delta_3 = Apple, \delta_4 = Talespin\}$. To find the top firm that produces VR systems, Think about $\Delta = \{s_1 = PriceValue, s_2 = Warrantytime, s_3 = Re f reshrate, s_4 = Headsetweight, s_5 = Fieldo f viewrange\}$ is the collection of criteria that a dealer uses to deter- mine which manufacturer is most suited to produce VR equipment. The sub-classes having different values sets are $s_1 = \{s_{11} = \$400, s_{12} = \$500\}$, $s_2 = \{s_{21} = 2years, s_{22} = 3years\}$, $s_3 = \{s_{31} = 90Hz, s_{32} = 120Hz\}$, $s_4=\{s_{41}=500\ gram, s\ s_{42}=600grmas\}$, $s_5=\{s_{51}=-90degree\ and s_{52}=120degree\}$.

pairs and each pair is a 5-tuple element. Again $\Psi_{CP} \times \Omega \times \Gamma$ = We will get 40 pairs and each one is a triplet. On preferential basis, we select any 16 pairs. The PFHSES can be established on the basis of the experts judgements which is given in table 10 by as:

Table 10. Tabular form of PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3	δ_4
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.21, 0.12⟩	⟨0.22, 0.31, 0.10⟩	⟨0.32, 0.31, 0.31⟩	⟨0.12, 0.31, 0.21⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩	⟨0.22, 0.31, 0.11⟩
$(\partial_2, \hat{p}_1, 1)$	⟨0.15, 0.37, 0.24⟩	⟨0.15, 0.31, 0.25⟩	⟨0.23, 0.33, 0.25⟩	⟨0.12, 0.31, 0.01⟩
$(\partial_2, \hat{p}_2, 1)$	⟨0.42, 0.15, 0.35⟩	⟨0.43, 0.17, 0.32⟩	⟨0.42, 0.11, 0.32⟩	⟨0.02, 0.31, 0.11⟩
$(\partial_3, \hat{p}_1, 1)$	⟨0.25, 0.32, 0.32⟩	⟨0.21, 0.35, 0.30⟩	⟨0.32, 0.32, 0.21⟩	⟨0.32, 0.01, 0.31⟩
$(\partial_3, \hat{p}_2, 1)$	⟨0.25, 0.10, 0.05⟩	⟨0.23, 0.13, 0.13⟩	⟨0.30, 0.12, 0.21⟩	⟨0.02, 0.31, 0.01⟩
$(\partial_4, \hat{p}_1, 1)$	⟨0.27, 0.17, 0.37⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.03⟩	⟨0.30, 0.01, 0.31⟩
$(\partial_4, \hat{p}_2, 1)$	⟨0.32, 0.04, 0.35⟩	⟨0.53, 0.02, 0.42⟩	⟨0.32, 0.21, 0.42⟩	⟨0.32, 0.11, 0.31⟩
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩	⟨0.32, 0.21, 0.31⟩
$(\partial_1, \hat{p}_2, 0)$	⟨0.45, 0.25, 0.25⟩	⟨0.53, 0.43, 0.13⟩	⟨0.40, 0.32, 0.11⟩	⟨0.02, 0.31, 0.31⟩
$(\partial_2, \hat{p}_1, 0)$	⟨0.27, 0.17, 0.27⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.43⟩	⟨0.32, 0.31, 0.11⟩
$(\partial_2, \hat{p}_2, 0)$	⟨0.43, 0.16, 0.31⟩	⟨0.42, 0.32, 0.12⟩	⟨0.42, 0.21, 0.13⟩	⟨0.21, 0.11, 0.31⟩
$(\partial_3, \hat{p}_1, 0)$	⟨0.24, 0.36, 0.38⟩	⟨0.22, 0.34, 0.31⟩	⟨0.23, 0.33, 0.30⟩	⟨0.32, 0.21, 0.31⟩
$(\partial_3, \hat{p}_2, 0)$	⟨0.36, 0.23, 0.17⟩	⟨0.33, 0.26, 0.12⟩	⟨0.31, 0.23, 0.11⟩	⟨0.32, 0.11, 0.52⟩
$(\partial_4, \hat{p}_1, 0)$	⟨0.13, 0.37, 0.29⟩	⟨0.16, 0.38, 0.21⟩	⟨0.13, 0.36, 0.22⟩	⟨0.02, 0.11, 0.71⟩
$(\partial_4, \hat{p}_2, 0)$	⟨0.43, 0.15, 0.35⟩	⟨0.44, 0.18, 0.32⟩	⟨0.43, 0.13, 0.32⟩	⟨0.02, 0.91, 0.01⟩

Table 11. Tabular form of agree-PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3	δ_4
$(\partial_1, \hat{p}_1, 1)$	⟨0.33, 0.21, 0.12⟩	⟨0.22, 0.31, 0.10⟩	⟨0.32, 0.31, 0.31⟩	⟨0.12, 0.31, 0.21⟩
$(\partial_1, \hat{p}_2, 1)$	⟨0.35, 0.21, 0.15⟩	⟨0.33, 0.22, 0.13⟩	⟨0.30, 0.21, 0.11⟩	⟨0.22, 0.31, 0.11⟩
$(\partial_2, \hat{p}_1, 1)$	⟨0.15, 0.37, 0.24⟩	⟨0.15, 0.31, 0.25⟩	⟨0.23, 0.33, 0.25⟩	⟨0.12, 0.31, 0.01⟩
$(\partial_2, \hat{p}_2, 1)$	⟨0.42, 0.15, 0.35⟩	⟨0.43, 0.17, 0.32⟩	⟨0.42, 0.11, 0.32⟩	⟨0.02, 0.31, 0.11⟩
$(\partial_3, \hat{p}_1, 1)$	⟨0.25, 0.32, 0.32⟩	⟨0.21, 0.35, 0.30⟩	⟨0.32, 0.32, 0.21⟩	⟨0.32, 0.01, 0.31⟩
$(\partial_3, \hat{p}_2, 1)$	⟨0.25, 0.10, 0.05⟩	⟨0.23, 0.13, 0.13⟩	⟨0.30, 0.12, 0.21⟩	⟨0.02, 0.31, 0.01⟩
$(\partial_4, \hat{p}_1, 1)$	⟨0.27, 0.17, 0.37⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.03⟩	⟨0.30, 0.01, 0.31⟩
$(\partial_4, \hat{p}_2, 1)$	⟨0.32, 0.04, 0.35⟩	⟨0.53, 0.02, 0.42⟩	⟨0.32, 0.21, 0.42⟩	⟨0.32, 0.11, 0.31⟩

Table 12. Tabular form of disagree-PFHSES

$(\partial_i, \hat{p}_i, 1)$	δ_1	δ_2	δ_3	δ_4
$(\partial_1, \hat{p}_1, 0)$	⟨0.23, 0.43, 0.32⟩	⟨0.11, 0.31, 0.40⟩	⟨0.32, 0.32, 0.41⟩	⟨0.32, 0.21, 0.31⟩
$(\partial_1, \hat{p}_2, 0)$	⟨0.45, 0.25, 0.25⟩	⟨0.53, 0.43, 0.13⟩	⟨0.40, 0.32, 0.11⟩	⟨0.02, 0.31, 0.31⟩
$(\partial_2, \hat{p}_1, 0)$	⟨0.27, 0.17, 0.27⟩	⟨0.25, 0.35, 0.15⟩	⟨0.13, 0.23, 0.43⟩	⟨0.32, 0.31, 0.11⟩
$(\partial_2, \hat{p}_2, 0)$	⟨0.43, 0.16, 0.31⟩	⟨0.42, 0.32, 0.12⟩	⟨0.42, 0.21, 0.13⟩	⟨0.21, 0.11, 0.31⟩
$(\partial_3, \hat{p}_1, 0)$	⟨0.24, 0.36, 0.38⟩	⟨0.22, 0.34, 0.31⟩	⟨0.23, 0.33, 0.30⟩	⟨0.32, 0.21, 0.31⟩
$(\partial_3, \hat{p}_2, 0)$	⟨0.36, 0.23, 0.17⟩	⟨0.33, 0.26, 0.12⟩	⟨0.31, 0.23, 0.11⟩	⟨0.32, 0.11, 0.52⟩
$(\partial_4, \hat{p}_1, 0)$	⟨0.13, 0.37, 0.29⟩	⟨0.16, 0.38, 0.21⟩	⟨0.13, 0.36, 0.22⟩	⟨0.02, 0.11, 0.71⟩
$(\partial_4, \hat{p}_2, 0)$	⟨0.43, 0.15, 0.35⟩	⟨0.44, 0.18, 0.32⟩	⟨0.43, 0.13, 0.32⟩	⟨0.02, 0.91, 0.01⟩

Table 13. Score values of agree-PFHSES

Pairs	δ_1	δ_2	δ_3	δ_4
$(\partial_1, \hat{p}_1, 1)$	0.0	-0.19	-0.30	-0.30
$(\partial_1, \hat{p}_2, 1)$	-0.01	-0.02	-0.02	-0.20
$(\partial_2, \hat{p}_1, 1)$	-0.46	-0.41	-0.35	-0.20
$(\partial_2, \hat{p}_2, 1)$	-0.08	-0.06	-0.01	-0.40
$(\partial_3, \hat{p}_1, 1)$	-0.39	-0.34	-0.21	0.00
$(\partial_3, \hat{p}_2, 1)$	-0.10	-0.03	-0.03	-0.30
$(\partial_4, \hat{p}_1, 1)$	-0.27	-0.25	-0.13	-0.02
$(\partial_4, \hat{p}_2, 1)$	-0.07	0.09	-0.31	-0.1
$t_{ij} = \Sigma\,\hat{p}_{ij}$	$t_1 = -1.38$	$t_2 = -0.98$	$t_3 = -1.36$	$t_4 = -1.52$

Table 14. Score values of disagree-PFHSES

Pairs	δ_1	δ_2	δ_3	δ_4
$(\partial_1, \hat{p}_1, 0)$	-0.32	-0.60	-0.41	-0.20
$(\partial_1, \hat{p}_2, 0)$	-0.25	-0.03	-0.19	-0.60
$(\partial_2, \hat{p}_1, 0)$	-0.17	-0.25	-0.53	-0.11
$(\partial_2, \hat{p}_2, 0)$	-0.04	0.02	0.08	-0.21
$(\partial_3, \hat{p}_1, 0)$	-0.50	-0.43	-0.40	-0.42
$(\partial_3, \hat{p}_2, 0)$	-0.04	-0.05	-0.03	-0.31
$(\partial_4, \hat{p}_1, 0)$	-0.53	-0.43	-0.45	-0.08
$(\partial_4, \hat{p}_2, 0)$	-0.07	-0.06	-0.02	-0.9
$y_{ij} = \Sigma\,\hat{p}_{ij}$	$y_1 = -1.92$	$y_2 = -1.83$	$y_3 = -1.31$	$y_4 = -2.83$

Table 15. Final score values

$t_{ij} = \Sigma\,\hat{p}_{ij}$	$y_{ij} = \Sigma\,\hat{p}_{ij}$	$\delta_{ij} = \lvert t_{ij} - y_{ij} \rvert$
$t_1 = -1.38$	$y_1 = -1.92$	3.3
$t_2 = -0.98$	$y_2 = -1.83$	2.81
$t_3 = -1.36$	$y_3 = -1.31$	2.67
$t_4 = -1.52$	$y_4 = -2.83$	4.35

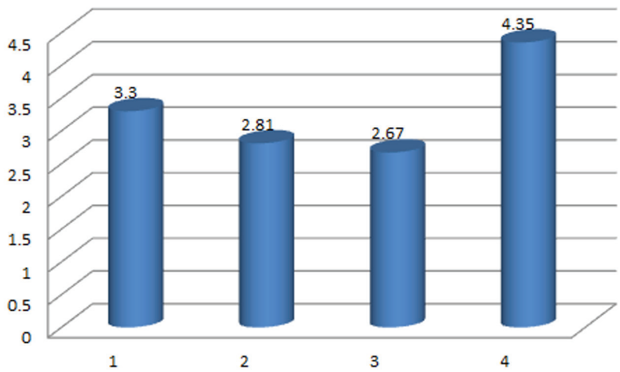

Figure 2. Representation of alternatives's rankings.

This PFHSESs has been partitioned into two parts, i. e., agree and disagree-PFHSESs. These two sets have been shown in two different Tables 11 and 12 respectively.

Tables 13 and 14 respectively are created to find the score values of agree and disagree- PFHSESs by using the formula $(\eta - \beta - \theta)$.

Table 15 is created to find the final score values of alternatives using values obtained from the above two tables 13 and 14 respectively and the results obtained from the 15 are shown in Figure 2.

RESULTS AND DISCUSSION

When we examine the past few decades, from the development of FS theory to the present, we can see that numerous researchers from practically every field of study have engaged in a race to produce various natural generalizations of FSs, including complex FSs, generalized FSs, IFSs, PFSs, and so on, or structures combining these extensions with other known un- certainty theories, such rough sets, SESs, and soft sets. It is evident that hybrid models of PFS with SES are still unable to manage multi-expert picture fuzzy soft information more effectively.

Table 16. Computational comparison

Samples	$\delta_1, \delta_2, \delta_3, \delta_4$	Ranking	Best option
IFSESs [46]	4.33, 0.19, 3.21, 7.21	$\delta_4 > \delta_1 > \delta_3 > \delta_2$	δ_4
PFSESs [62]	4.85, -0.26, 5.15, 11.0	$\delta_4 > \delta_3 > \delta_1 > \delta_2$	δ_4
Suggested App.	3.3, 2.81, 2.67, 4.35	$\delta_4 > \delta_1 > \delta_2 > \delta_3$	δ_4

This feature served as the impetus for this work, which combined the previously described existing theories to develop a novel hybrid model known as PFHSESs. Several experts' assessments of each alternative in relation to each parameter under consideration can be handled by our established model. The suggested method works really well and is work- able for handling soft expert information that is ambiguous and imprecise. Especially when the issue being considered is founded on soft, fuzzy picture data that has been gathered using the opinions of various experts. As an extension of PFSs, such as PFSSs, several successful outcomes have been generated to address various issues of multiple scientific domains, including artificial intelligence. We developed our suggested model as an effective generalisation of SESs, FSESs, IFSESs, or PFSSs as PFSS is a soft extension of the PFS model but cannot handle the individual evaluation of multiple experts in a group decision-making scenario. By using the suggested method and the current IFSES model to solve the application in Section 4, we were able to evaluate the efficacy of our proposed model. Although it is obvious that we get the best results, the order in which the sub-optimal decision objects are ranked has changed slightly. Table 16 presents the comparative analysis of the established PFHSES model in both qualitative and quantitative modes. We see that, in the picture fuzzy hypersoft expert environment, the initial strategy is more coherent and feasible to handle various real-world problems. Table 16 shows the values of alternatives obtain through the suggested approach are smaller than the values obtained in [46] and [62] respectively.

Limitations of the Study

The use of uncertainty theories to mathematical modeling-based real-world systems has grown significantly over the last 20 years. Experts can utilise a hybrid model, which combines two or more ideas, to help them make decisions. No matter how good a hybrid model is, models will nearly always have limitations. A good hybrid model can solve some of the problems with existing theories and produce more accurate findings than existing ones. The following talks about the initiated approach's shortcomings that we saw when it was being built.

- Since assessments and input data are the primary sources of information for mathematical modelling. In the case of a big data set, the computational speed of the suggested hybrid model may be slow. This shortcoming exists in nearly all models tha are now in use, but it may be fixed with the aid of tools, such as MATLAB, and the proper coding technique.
- When new parameters or alternatives are added to an existing set in a group decision- making situation, our initial model faces additional challenges related to the ranking of alternatives. The autonomous behaviour of parameters and objects is the primary cause of these issues.

Benefits of the Proposed Study

The described study has the following benefits over the existing structures:

- Multiple levels of uncertainty are incorporated into the PFHSE-set framework through image fuzzy sets, which support refusal degrees in addition to positive, neu- tral, and negative membership degrees. In comparison with intuitionistic fuzzy sets or ordinary fuzzy sets, which can only accommodate two or three degrees of membership, respectively, this offers more versatility.
- The method enables the more structured capturing of expert information by including the expert set element into the PFHSE framework. Experts can indicate their levels of confidence or lack thereof (refusal degree), which enhances decision-making models that depend heavily on expert judgment.
- When dealing with multidimensional characteristics, the PFHSE-set may handle dis- junct sets of parameters thanks to the hypersoft framework. This helps with complex, subtle, or connected features in particular. It provides a more in-depth comprehension of real-world situations, including medical diagnosis or industrial operations, which frequently call for this level of specificity.

CONCLUSION

In this study, a novel hybrid model PFHSESs that is an expansion of PFSSs, FSESs, FH- SESs or IFSESs is structured. This new structure has been used more broadly and effectively to characterise its properties. Specifically, a number of fundamental operations are built and examined with corresponding numerical examples. These include subset, equality, union, intersection, complement, OR operation, AND operation, and agree and disagree-PFHSESs. Additionally, the AND and OR operations confirm several of De Morgan's laws for PFHSESs. Additionally, a PFHSES-based approach to solving the

MAGDM difficulties is described. To describe the validity and cogency of the developed hybrid model that is, a choice of the top VR system manufacturing company, we tested our proposed technique on a real-world application. Lastly, a comparison between the technique that has been offered and a few models that already exist, is given. Then its certain limitations have been discussed. In order to address decision-making under uncertainty, this research may go in other directions in the future by implementing the picture fuzzy hypersoft expert model in a variety of industries, including healthcare, finance, and environmental management. Furthermore, this model's practical applicability could be increased by integrating it with other decision-making approaches, building software tools, and optimizing algorithms. Its performance could also be benchmarked through comparative research with other fuzzy and soft set-based models; additionally, adding feedback mechanisms and expanding the model to dynamic contexts could improve its practicality.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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