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Research Article

Cost-benefit evaluation of repairable retrial systems with warm standby components and setup times

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ABSTRACT

Power supply system models are commonly used in industrial and manufacturing sectors to assess performance metrics. This paper conducts a cost-benefit analysis of four different unreliable retrial systems featuring warm standby components and setup times. The systems differ in the quantity of primary (operative) and warm standby (backup) components they contain. The failure and repair times for both primary and standby components are assumed to follow an exponential distribution. For each system, we present the state-transition diagram along with the corresponding differential-difference equations. Employing the Laplace stieltjes transform and the matrix-analytical method, we derive explicit formulas for key performance metrics, including the mean time to failure (MTTF) and steady-state availability, which serve as vital indicators of system reliability. Numerical examples are provided to examine the influence of system parameters on steady state availability, mean time to failure, and the cost-benefit ratio. The results are presented in both tabular and graphical formats, accompanied by a detailed cost-benefit comparative analysis. The findings provide decision-makers with valuable insights for ensuring system stability and optimizing cost efficiency.

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INTRODUCTION

In many industries, such as manufacturing, computer networks, communication systems, and production systems, reliability and availability of the power supply system with favorable cost-effectiveness are crucial. Standby power source units emerge as critical components of the overall energy strategies for cement plants. Standby power source units are crucial in safeguarding operations against unforeseen outages and mitigating risks associated with power fluctuations. Several studies address the overall power consumption within cement plants. Cement plants can have their own waste heat recovery to produce electric power by using hybrid renewable energy systems investigated by [1]. Demand response in energy-intensive industry analysed by [2] cement production is a complicated, energy-intensive process that relies massively on a stable supply of electric power to run the equipment which ranges from crushers, conveyors that handle raw materials to kilns, mills that convert materials into cement. Cement plants rely largely on

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utility companies and own electric power sources such as diesel, solar or natural gas powered standby generator to stabilise electric power source and have a power monitoring system to switch over the source. This paper to study reliability and availability factors relating to the suitable repairable retrial system of electric power sources within a medium scale cement producing plant.

Multiple articles with retrial queueing systems in which the servers are subjected to unreliable repair stations and setup times are considered. Ram *et a.l* [3] analysed the use of Markov process for evaluating reliability metrics, cost and sensitiveness of performance measures to system parameters while Aghsami and Jolai [4] studied equilibrium balking strategies in a single-server Markovian queue including studies on intermittent breakdowns and flexible shut down policy while setup time and maintenance procedures were observed by Zhang and Xu [5] in context of customer behavior under queuing system. Interested researchers may read on Sun *et al.* [6] and Wang *et al.* [7] for more comprehensive concepts on the queue systems with setup times.

Related Work

Literature concerning reliability and availability evaluation in retrial systems is extensive and comprehensive. Yen et al. [8] proposed retrial machine repair model considering the aspect of intermittent breakdowns and the probabilistic F-policy is implemented. Reliability evaluation considering equipment maintenance featuring a single repairman and associated recovery policy was explored by [9]. Wang et al. [10] studied the unreliable equipment maintenance system with warm standby failures. Complex systems with two connected systems have been analyzed by Mailhulla et al. [11]. Yang and Tsao [12] carried out research on reliability and availability evaluation of the maintainable system consisted of standby units and component failure in different modes considering retrials. Redundant series configuration under single-repair technician where each working unit has a backup unit which failures can also occur has been studied by Gao et al. [13]. The computer network system containing two clients, load balancers, two fog nodes and two cloud servers connected in series parallel configuration has been presented by Wu and Isa [14], Kumar and Jain [15] have discussed on reliability study under queue with an unstable server considering scheduled breaks with working failure. Shekhar et al. [16] discussed multi-component systems with backup considering breakdowns, delays and random flaws. Iggi and Yusuf [17] and Isa et al. [18] proposed an evaluation to determine performance of the system. Bai et al. [19] studied performance optimization model for periodic switching method. Kuo and Ke [20] researched into availability of two distinct configurations of a series system in an imperfect coverage environment where the system is serviced by an unstable server. Juyibari *et al.* [21] studied repairable system using a mixed strategy that focuses on vacation interruption method and working vacation system. Adan and Filik [22] had assessed renewable energy models to identify most economical and ecological designs. [23-25] studied systems with several types of standby and maintenance facility where these units undergo periodic maintenance or may have active failure. Yen and Wang [26] dealt with reliability and availability for four retrial systems. Wu et al. [27] dealt with reliability and availability for four retrial systems of power supply where imperfect coverage, warm standby and general repair-times could be taken care of. Kang et al. [28] present a model of warm standby retrial system with non-identical units undergoing component breakdown with a warm spare, where some of the operations were assumed to follow a priority. Yen et al. [29] analyzed three different availability models in which warm stand-by units along with delays in the fault detection and different repair times are incorporated. Gao [30] examined maintainable retrial system with warm standby units and a two-stage repair process.

This article presents the performance evaluation of the cement producing plant. Previous studies have independently investigated power plant systems considering unreliable retrial models that adopt various applications. However, to the best of the authors' knowledge, there has been no research specifically addressing unreliable retrial systems incorporating warm standby components, and setup times, in which maintenance and repair are effective. Hence, to address this gap, the present study investigates four repairable retrial systems, where the repair facility requires setup time and failed components request repair services at a later time if the facility is busy upon their arrival.

The summarised key features, approaches and comparative analytical goals of important investigations considered by various scholars are shown in Table 1 below. Therefore, the important objectives of this research study were: To investigate the transient behaviour of repairable cement manufacturing plant power system and accounting setup times, to evaluate its performance with increase of time. To forecast the operational efficiency of the retrial system by evaluating steady-state availability, MTTF. To choose optimum cost-effectiveness ratio and to decide for the most favorable decision factors. Studying cement power plant reliability metrics and carrying comparative analysis for these reliability metrics of the cement power system can effectively support all decision making by the management for replacements, equipment upgrades and for capacity requirements. The decision makers-plant managers and government policymakers can choose performance criteria, along with the suitable cost and risks involved in a more balanced ratio for their options along about organisational strategy.

The subsequent sections of the paper are organized as follows: Section 2 outlines the potential application and description of the model. In Section 3, a systematic approach is presented for deriving detailed expressions for the MTTF and steady-state availability for a given retrial system. Section 4 provides detailed numerical findings

Table 1. Summary of similar literature studies

Reference	Warm standby	Setup times	Unreliable retrial system	Matrix analytical approach	Cost-benefit ratio
Rehman and Farzaneh [1]	✘	✘	✘	✘	✓
Zhang and Xu [5]	✘	✓	✓	✘	✘
Wang et al. [10]	✓	✘	✓	✓	✓
Wu and Isa [14]	✘	✘	✘	✓	✓
Wang et al. [25]	✓	✘	✓	✓	✓
Yen and Wang [26]	✓	✘	✘	✓	✓
Proposed model	✓	✓	✓	✓	✓

from sensitivity analysis and comparisons to establish the optimal retrial system. Finally, the summary and conclusion are detailed in Section 5.

POTENTIAL APPLICATION AND MODEL DESCRIPTION

Potential Application

This section presents a possible application of a reliable power supply system in a cement manufacturing plant where the proposed model can be utilized. With the increased demand for modern construction in civil works, cement manufacturing plants are of great need to ensure an adequate supply of cement. In their quest for reliable electrical systems to ensure sustainable production operations in modern cement plants Shirumalla and Chatterjee [31] explored on the reliability of the former. Ali et al. [32] explored options for energy generation systems tailored to the cement industry. In a medium-sized cement manufacturing industry that requires 48MW of electric power and the production capacity of sources is available in units of 48MW, 24MW, and 16MW at different costs. To provide sufficient electric power, repairable service stations, and standby components are available to guarantee system uptime. All components (operating and standby) are unreliable and may fail at any time. A fault detective device is installed to monitor the electric power system. When an operating component fails and is detected, a standby component is switched into operation (if available). At the same time, the failed component is repaired. However, the service station may break down during the busy period. Through the application of queueing theory, we can assess system availability and evaluate the cost-effectiveness of various configurations, forming our issue as an unreliable repair problem that includes spare parts.

Model Description

We take a consideration of a medium cement manufacturing industry that requires 48MW of electrical power generation capacity. To ensure consistent and dependable system availability, there are primary and standby generators. Standby generators are assumed to be susceptible to failure while inactive, before being fully operational. Both primary and standby generators can fail and are subsequently repaired. The interval times between failures of the primary and warm standby generators are exponentially distributed with failure rates λ and α $(0 < \alpha < \lambda)$ respectively. There is a repair facility with a single operator who handles repairs of malfunctioning components and there is no waiting area in front of it and the repairman can attend to only one failed component at a time. As the operating components fail, they are replaced by a standby component (if available). If the service station is open and idle, the component is repaired immediately upon arrival, otherwise, it enters the orbit of retrials and awaits a random duration that is exponentially distributed with a rate γ. The repair times are assumed to be exponentially distributed with a rate μ. If there are no pending repair tasks in the system after a service has been completed, the service station will be turned off at once to save energy. A newly arrived failed component can reactivate the off-server. Before servicing a newly arrived failed component, the service station requires a setup time that follows an exponential distribution with a rate η. The failed component that triggers the server will promptly enter the orbit and request service. The server is not completely reliable and may experience random breakdowns while repairing the failed component, it can be repaired perfectly according to an exponential distribution with rates β and δ respectively. Once a component has been repaired, it is as good as new. Lastly, the retrial processes, the repair service, and component failures are all independent. We consider four retrial systems as follows: The first retrial system comprises one 48MW primary component and one 48MW warm standby component. The second retrial system includes two 24MW primary components and one 24MW warm standby component. The third retrial system contains two 24MW primary components and two 24MW warm standby components. The fourth retrial system is composed of three 16MW primary components and two 16MW warm standby components

Table 2. The acquisition cost for the primary and warm standby components

Component	Cost (in $)
Primary 48MW	48×10^6
Primary 24MW	24×10^6
Primary 16MW	16×10^6
Warm standby 48MW	24×10^6
Warm standby 24MW	12×10^6
Warm standby 16MW	8×10^6

Table 3. The cost for each retrial system

Retrial system	Primary	Warm standby	Cost (in $)
1	1 × 48	1 × 48	72×10^6
2	2 × 24	1 × 24	60×10^6
3	2 × 24	2 × 24	72×10^6
4	3 × 16	2 × 16	64×10^6

Cost-Benefit Factor

We assume that the costs, depending on the size, for both primary and warm standby components are specified in Table 2. Using this information, we compute the costs related to each retrial system as presented in Table 3. Let C_i represent the cost and BF_i be the benefit of the retrial system $i(i = 1,2,3,4)$, where BF_i represents either MTTF or steady-state availability.

SYSTEMATIC METHODOLOGY

The service station has four different statuses: idle (i = 0), busy (i = 1), set-up process (i = 2), and breakdown (i = 3). At time t, the variable $(i,n{:}t)$ precisely describes the system states where i represents the state of the service station and n denotes the number of failed components in the retrial orbit. Using these notations as a foundation, we can thoroughly examine the four systems and derive their respective performance metrics.

Calculations for Retrial System 1

In the context of reliability, Figure 1 illustrates the state-transition diagram for retrial system 1, electricity drops below 48MW. States (1,1) and (3,1) are therefore regarded as absorbing states in the event that the system cannot be restored. In this case, the system's MTTF is analyzed without taking into account transitions from state (1,1) to (0,1) and (3,1) or from state (3,1) to (1,1).

Let $\Pi_{i,n}(t)$ = the probability at time t that there are n failed components in the retrial orbit when the service station is in state i. Let $\Pi_{i,}(t)$ represent the probability column vector at time t,

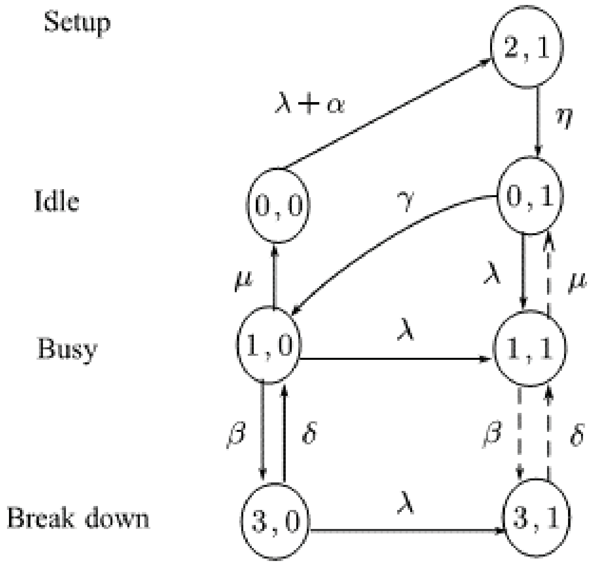

Figure 1. State-transition diagram of retrial system 1.

$$\Pi(t) = [\Pi_{0,0}(t), \Pi_{1,0}(t), \Pi_{3,0}(t), \Pi_{0,1}(t), \Pi_{1,1}(t), \Pi_{2,1}(t), \Pi_{3,1}(t)]^T$$

The initial probability for this system is given by

$$\Pi(0) = [\Pi_{0,0}(0), \Pi_{1,0}(0), \Pi_{3,0}(0), \Pi_{0,1}(0), \Pi_{1,1}(0), \Pi_{2,1}(0), \Pi_{3,1}(0)]^T = [1,0,0,0,0,0,0]^T,$$

where T indicates transpose. For retrial system 1, we obtain the following set of differential-difference equations:

$$\frac{d\Pi_{0,0}(t)}{dt} = -(\lambda+\alpha)\Pi_{0,0}(t) + \mu\Pi_{1,0}(t), \quad (1)$$

$$\frac{d\Pi_{1,0}(t)}{dt} = -(\lambda+\beta+\mu)\Pi_{1,0}(t) + \delta\Pi_{3,0}(t) + \gamma\Pi_{0,1}(t), \quad (2)$$

$$\frac{d\Pi_{3,0}(t)}{dt} = \beta\Pi_{1,0}(t) - (\lambda+\delta)\Pi_{3,0}(t), \quad (3)$$

$$\frac{d\Pi_{0,1}(t)}{dt} = -(\lambda+\gamma)\Pi_{0,1}(t) + \eta\Pi_{2,1}(t), \quad (4)$$

$$\frac{d\Pi_{1,1}(t)}{dt} = \lambda\Pi_{1,0}(t) + \lambda\Pi_{0,1}(t), \quad (5)$$

$$\frac{d\Pi_{2,1}(t)}{dt} = (\lambda+\alpha)\Pi_{0,0}(t) - \eta\Pi_{2,1}(t), \quad (6)$$

$$\frac{d\Pi_{3,1}(t)}{dt} = \lambda\Pi_{3,0}(t). \quad (7)$$

To solve Eq. (1) through (7), we define the Laplace-stieltjes transform (LST) as:

$\Pi^*_{i,n}(s) = \int_0^\infty e^{-st}\,\Pi_{i,n}(t)\,dt, i = 0,1,2;\ n = 0,1$. Applying the LST on both sides of Eq. (1) to Eq. (7) results in

$$(\lambda+\alpha+s)\Pi^*_{0,0}(s) - \mu\Pi^*_{1,0}(s) = \Pi_{0,0}(0),$$

$$(\lambda+\beta+\mu+s)\Pi^*_{1,0}(s) - \delta\Pi^*_{3,0}(s) - \gamma\Pi^*_{0,1}(s) = \Pi_{1,0}(0),$$

$$(\lambda+\delta+s)\Pi^*_{3,0}(s) - \beta\Pi^*_{1,0}(s) = \Pi_{3,0}(0),$$

$$(\lambda+\gamma+s)\Pi^*_{0,1}(s) - \eta\Pi^*_{2,1}(s) = \Pi_{0,1}(0),$$

$$s\Pi^*_{1,1}(s) - \lambda\Pi^*_{1,0}(s) - \lambda\Pi^*_{0,1}(s) = \Pi_{1,1}(0),$$

$$(\eta+s)\Pi^*_{2,1}(s) - (\lambda+\alpha)\Pi^*_{0,0}(s) = \Pi_{2,1}(0),$$

$$s\Pi^*_{3,1}(s) - \lambda\Pi^*_{3,0}(s) = \Pi_{3,1}(0).$$

In matrix form

$$Q_1\Pi^*(s) = \Pi(0), \tag{8}$$

where

$$Q_1 = \begin{pmatrix} \lambda+\alpha+s & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda+\beta+\mu+s & -\delta & -\gamma & 0 & 0 & 0 \\ 0 & -\beta & \lambda+\delta+s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda+\gamma+s & 0 & -\eta & 0 \\ 0 & -\lambda & 0 & -\lambda & s & 0 & 0 \\ -(\lambda+\alpha) & 0 & 0 & 0 & 0 & \eta+s & 0 \\ 0 & 0 & -\lambda & 0 & 0 & 0 & s \end{pmatrix},$$

and

$$\Pi^*(s) = [\Pi^*_{0,0}(s), \Pi^*_{1,0}(s), \Pi^*_{3,0}(s), \Pi^*_{0,1}(s), \Pi^*_{1,1}(s), \Pi^*_{2,1}(s), \Pi^*_{3,1}(s)]^T.$$

Solving Eq. (8) yields $\Pi^*_{0,0}(s)$, $\Pi^*_{1,0}(s)$, $\Pi^*_{3,0}(s)$, $\Pi^*_{0,1}(s)$, $\Pi^*_{1,1}(s)$, $\Pi^*_{2,1}(s)$, $\Pi^*_{3,1}(s)$. Let X be the time to failure of the system, thus, the probabilities $\Pi_{1,1}(t)$ and $\Pi_{3,1}(t)$ suggest that the system cannot meet the minimum requirements before time t, therefore, the system reliability function is derived as follows:

$$R_X(t) = 1 - \Pi_{1,1}(t) - \Pi_{3,1}(t) = \Pi_{0,0}(t) + \Pi_{1,0}(t) + \Pi_{3,0}(t) + \Pi_{0,1}(t) + \Pi_{2,1}(t).$$

$MTTF_1$ of retrial syatem 1

The MTTF of retrial system 1 is obtained by

$$MTTF_1 = \int_0^\infty R_X(t)\,dt = \lim_{s\to 0} R^*_X(s),$$

$$= \lim_{s\to 0}\int_0^\infty e^{-st}\left[\Pi_{0,0}(t)+\Pi_{1,0}(t)+\Pi_{3,0}(t)+\Pi_{0,1}(t)+\Pi_{2,1}(t)\right]dt,$$

$$= \lim_{s\to 0}\left[\Pi^*_{0,0}(s)+\Pi^*_{1,0}(s)+\Pi^*_{3,0}(s)+\Pi^*_{0,1}(s)+\Pi^*_{2,1}(s)\right].$$

The explicit expression of $MTTF_1$ for the retrial system 1 is too extensive to present here. However, a numerical example is provided in Section 4 to evaluate it using appropriate computational software.

System availability ($Av_{T_1}(\infty)$)

In the availability scenario, we analyze the state-transition diagram of retrial system 1 depicted in Figure 1, focusing transition from state (1,1) to (0,1), from (1,1) to state (3,1), and from state (3,1) to (1,1). Then, the governing differential-difference equations can be expressed in matrix form as:

$$\begin{pmatrix} \frac{d\Pi_{0,0}(t)}{dt} \\ \frac{d\Pi_{1,0}(t)}{dt} \\ \frac{d\Pi_{3,0}(t)}{dt} \\ \frac{d\Pi_{0,1}(t)}{dt} \\ \frac{d\Pi_{1,1}(t)}{dt} \\ \frac{d\Pi_{2,1}(t)}{dt} \\ \frac{d\Pi_{3,1}(t)}{dt} \end{pmatrix} = \begin{pmatrix} -(\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\lambda+\gamma) & \mu & \eta & 0 \\ 0 & \lambda & 0 & \lambda & -(\beta+\mu) & 0 & \delta \\ \lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & \lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix}.$$

Let T_1 be the time to failure of the system 1. To investigate the availability aspects of this system, we will consider the following procedure. The derivatives of the state probability become zero (0) in a steady state, that is $\frac{d\Pi(t)}{dt} = 0$. Hence, $Av_{T_1}(\infty)$ can be obtained as

$Av_{T_1}(\infty) = 1 - \Pi_{1,1}(\infty) - \Pi_{3,1}(\infty)$, and $Q_2\Pi(\infty) = 0$. The matrix form is given by;

$$\begin{pmatrix} -\ \lambda+\alpha & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\lambda+\gamma) & \mu & \eta & 0 \\ 0 & \lambda & 0 & \lambda & -(\beta+\mu) & 0 & \delta \\ \lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & \lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{9}$$

The normalizing condition is

$$\Pi_{0,0}(\infty)+\Pi_{1,0}(\infty)+\Pi_{3,0}(\infty)+\Pi_{0,1}(\infty)+\Pi_{1,1}(\infty)+\Pi_{2,1}(\infty)+\Pi_{3,1}(\infty) = 1. \tag{10}$$

We replace one of the redundant rows in Eq. (9) with Eq. (10) to obtain:

$$\begin{pmatrix} -(\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\lambda+\gamma) & \mu & \eta & 0 \\ 0 & \lambda & 0 & \lambda & -(\beta+\mu) & 0 & \delta \\ \lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{11}$$

The solution to Eq. (11) gives the availability scenario's steady-state probabilities. For retrial system 1, the detailed expression for $Av_{T_1}(\infty)$ is too lengthy to be displayed here. In Section 4, we use specific numerical values to solve an example and compare this retrial system with others.

Calculations for Retrial System 2

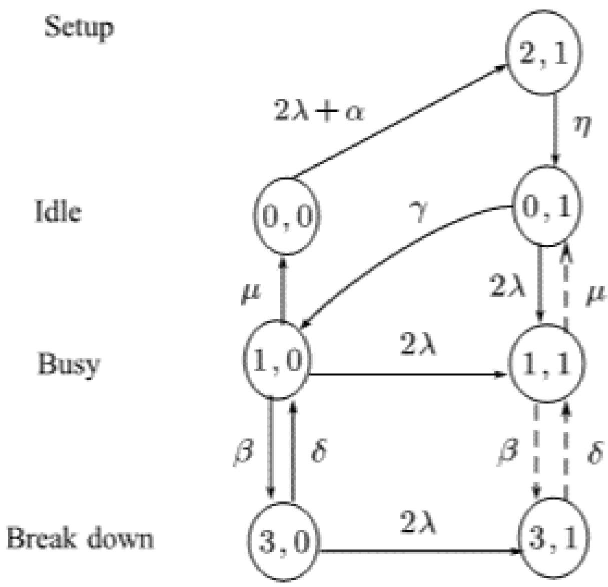

Figure 2. State-transition diagram of retrial system 2.

MTTF$_2$ of retrial syatem 1

Similarly, in the evaluation of system reliability, the system does not transition from state (1,1) to states (0,1) and (3,1) nor from state (3,1) to state (1,1). The corresponding probability vector $\Pi(t)$ for retrial system 2 is given by:

$$\Pi(t) = [\Pi_{0,0}(t), \Pi_{1,0}(t), \Pi_{3,0}(t), \Pi_{0,1}(t), \Pi_{2,1}(t)]^T.$$

The differential-difference equations can be expressed in matrix form as:

$$\frac{d\,\Pi(t)}{dt} = Q_3\,\Pi(t).$$

where

$$Q_3 = \begin{pmatrix} -(2\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(2\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(2\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(2\lambda+\gamma) & 0 & -\eta & 0 \\ 0 & 2\lambda & 0 & 2\lambda & 0 & 0 & 0 \\ 2\lambda+\alpha & 0 & 0 & 0 & 0 & \eta & 0 \\ 0 & 0 & 2\lambda & 0 & 0 & 0 & 0 \end{pmatrix},$$

Obtaining explicit transient solutions poses a challenge. We present a systematic approach for computing MTTF for a retrial system 2. Initially, we neglect the absorbing state's fifth and last rows and their respective columns. Subsequently, we transpose the resulting matrix to obtain matrix A_2, as follows:

$$A_2 = \begin{pmatrix} -(2\lambda+\alpha) & 0 & 0 & 0 & \lambda+\alpha \\ \mu & -(2\lambda+\beta+\mu) & \beta & 0 & 0 \\ 0 & \delta & -(2\lambda+\delta) & 0 & 0 \\ 0 & \gamma & 0 & -(2\lambda+\gamma) & 0 \\ 0 & 0 & 0 & \eta & -\eta \end{pmatrix}.$$

Then, the expected time until reaching a terminal state is derived from

$$E\left[T_{\Pi(0)\to\Pi(absorbing)}\right] = \Pi(0)^T\left(-A_2^{-1}\right)[1,1,1,1,1]^T,$$

where the initial conditions, $\Pi(0)$ for retrial system 2 are given by:

$$\Pi(0) = [\Pi_{0,0}(0), \Pi_{1,0}(0), \Pi_{3,0}(0), \Pi_{0,1}(0), \Pi_{2,1}(0)]^T = [1,0,0,0,0]^T,$$

As mentioned before, the explicit expression for $MTTF_2$ is very complex to be displayed here. A specific numerical example is given in Section 4 and a comparison of the four retrial systems is performed.

System availability ($Av_{T_2}(\infty)$)

For the availability scenario, the state-transition diagram of retrial system 2 is illustrated in Figure 2 (with the transition from state (2,1) to (1,1) and (3,1), and from state (3,1) to (1,1). The differential-difference equations can be expressed in matrix form:

$$\begin{pmatrix} \frac{d\,\Pi_{0,0}(t)}{dt} \\ \frac{d\,\Pi_{1,0}(t)}{dt} \\ \frac{d\,\Pi_{3,0}(t)}{dt} \\ \frac{d\,\Pi_{0,1}(t)}{dt} \\ \frac{d\,\Pi_{1,1}(t)}{dt} \\ \frac{d\,\Pi_{2,1}(t)}{dt} \\ \frac{d\,\Pi_{3,1}(t)}{dt} \end{pmatrix} = \begin{pmatrix} -(2\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(2\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(2\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(2\lambda+\gamma) & \mu & \eta & 0 \\ 0 & 2\lambda & 0 & 2\lambda & -(\beta+\mu) & 0 & \delta \\ 2\lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 2\lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix}.$$

Let T_2 represent the time to failure of the system for retrial system 2. In steady-state, the derivatives of the state probabilities become zero. This allows us to calculate the steady-state availability as $Av_{T_2}(\infty) = 1 - \Pi_{1,1}(\infty) - \Pi_{3,1}(\infty)$, and $Q_4\Pi(\infty) = 0$, in matrix form:

$$\begin{pmatrix} -(2\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(2\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(2\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(2\lambda+\gamma) & \mu & \eta & 0 \\ 0 & 2\lambda & 0 & 2\lambda & -(\beta+\mu) & 0 & \delta \\ 2\lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 2\lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (12)$$

The normalizing condition is given by

$$\Pi_{0,0}(\infty)+\Pi_{1,0}(\infty)+\Pi_{3,0}(\infty)+\Pi_{0,1}(\infty)+\Pi_{1,1}(\infty)+\Pi_{2,1}(\infty)+\Pi_{3,1}(\infty)=1. \quad (13)$$

We substitute Eq. (13) into one of the duplicated rows in Eq. (12) to obtain

$$\begin{pmatrix} -(2\lambda+\alpha) & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -(2\lambda+\beta+\mu) & \delta & \gamma & 0 & 0 & 0 \\ 0 & \beta & -(2\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(2\lambda+\gamma) & \mu & \eta & 0 \\ 0 & 2\lambda & 0 & 2\lambda & -(\beta+\mu) & 0 & \delta \\ 2\lambda+\alpha & 0 & 0 & 0 & 0 & -\eta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(t) \\ \Pi_{1,0}(t) \\ \Pi_{3,0}(t) \\ \Pi_{0,1}(t) \\ \Pi_{1,1}(t) \\ \Pi_{2,1}(t) \\ \Pi_{3,1}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (14)$$

The solution derived from Eq. (14) yields the steady-state probabilities in the availability scenarios. For retrial system 2 the explicit expression of $Av_{T2}(\infty)$ is very complicated to be displayed here. However, a specific numerical example is provided in the next section.

Calculations for Retrial System 3

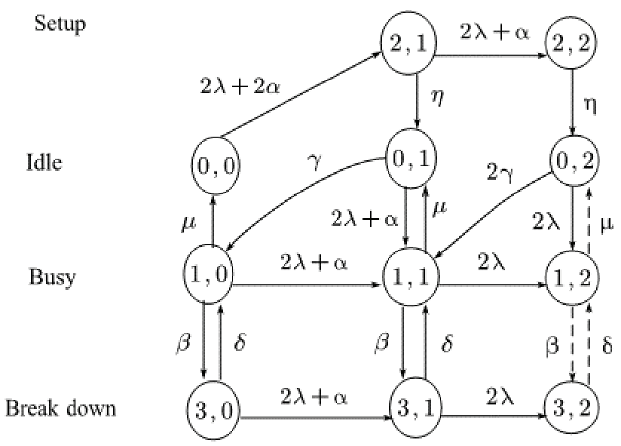

Figure 3. State-transition diagram of retrial system 3.

$MTTF_3$ of retrial system 3

The state-transition diagram of retrial system 3 is given in Figure 3. For system reliability case, the system does not transit from state (1,2) to (3,2) and (0,2) and from state (3,2) to (1,2). The corresponding probability vector $\Pi(t)$ for retrial system 3 is presented as:

$$\Pi(t) = [\Pi_{0,0}(t), \Pi_{1,0}(t), \Pi_{3,0}(t), \Pi_{0,1}(t), \Pi_{1,1}(t), \Pi_{2,1}(t), \Pi_{3,1}(t), \Pi_{0,2}(t), \Pi_{1,2}(t), \Pi_{2,2}(t), \Pi_{3,2}(t)]^T.$$

The governing differential-difference equations of retrial system 3 can be expressed in matrix form as:

$$\frac{d\,\Pi(t)}{dt} = Q_5\,\Pi(t),$$

where

$$Q_5 = \begin{pmatrix} -a_{11} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{12} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{14} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{15} & 0 & a_{15} & -a_{16} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{11} & 0 & 0 & 0 & 0 & -a_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{15} & 0 & \beta & 0 & -(2\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda+\gamma) & 0 & \eta & 0 \\ 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 2\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{15} & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 0 & 0 \end{pmatrix},$$

and

$$a_{11} = 2(\lambda + \alpha),\ a_{12} = 2\lambda + \alpha + \beta + \mu,\ a_{13} = 2\lambda + \alpha + \delta,$$
$$a_{14} = 2\lambda + \alpha + \gamma,\ a_{15} = 2\lambda + \alpha,\ a_{16} = 2\lambda + \beta + \mu,\ a_{17} = 2\lambda + \alpha + \eta.$$

To compute $MTTF_3$, we neglect the rows and columns of the matrix Q_5 that pertain to the absorbing states and transpose the resulting matrix to form a new matrix A_3, where

$$A_3 = \begin{pmatrix} -a_{11} & 0 & 0 & 0 & 0 & a_{11} & 0 & 0 & 0 \\ \mu & -a_{12} & \beta & 0 & a_{15} & 0 & 0 & 0 & 0 \\ 0 & \delta & -a_{13} & 0 & 0 & 0 & a_{15} & 0 & 0 \\ 0 & \gamma & 0 & -a_{14} & a_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & -a_{16} & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & \eta & 0 & -a_{17} & 0 & 0 & a_{15} \\ 0 & 0 & 0 & 0 & \delta & 0 & -(2\lambda+\delta) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\gamma & 0 & 0 & -2(\lambda+\gamma) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta & -\eta \end{pmatrix}.$$

Then, $MTTF_3$ is obtained from

$$MTTF_3 = E\left[T_{\Pi(0)\to\Pi(absorbing)}\right] = \Pi(0)^T\left(-A_3^{-1}\right)[1,1,1,1,1,1,1,1,1]^T,$$

where the initial conditions $\Pi(0)$ of retrial system 3 is given by

$$\Pi(0) = [\Pi_{0,0}(0), \Pi_{1,0}(0), \Pi_{3,0}(0), \Pi_{0,1}(0), \Pi_{1,1}(0), \Pi_{2,1}(0), \Pi_{3,1}(0), \Pi_{0,2}(0), \Pi_{2,2}(0)]^T = [1,0,0,0,0,0,0,0,0]^T,$$

The explicit expression for $MTTF_3$ is too extensive to be shown here. In Section 4, a numerical example is provided for this aspect to compare this system with the other systems.

System availability ($Av_{T3}(\infty)$)

For the availability case, the state- transition diagram of retrial system 3 is shown in Figure 3 (with transition from state (1,2) to (3,2) and (0,2) and from state (3,2) to (1,2)). Let T_3 be the time to failure of the system for retrial system 3 in steady-state, we obtainn $Av_{T3}(\infty)$ from $Av_{T3}(\infty)$ = 1 - $\Pi_{1,2}(\infty)$ - $\Pi_{3,2}(\infty)$, and $Q_6\Pi(\infty) = 0$ or expressed in matrix notation:

$$\begin{pmatrix} -a_{11} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{12} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{14} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{15} & 0 & a_{15} & -a_{16} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{11} & 0 & 0 & 0 & 0 & -a_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{15} & 0 & \beta & 0 & -\left(2\lambda+\delta\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\left(\lambda+\gamma\right) & \mu & \eta & 0 \\ 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 2\lambda & -\left(\beta+\mu\right) & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & a_{15} & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(\infty) \\ \Pi_{1,0}(\infty) \\ \Pi_{3,0}(\infty) \\ \Pi_{0,1}(\infty) \\ \Pi_{1,1}(\infty) \\ \Pi_{2,1}(\infty) \\ \Pi_{3,1}(\infty) \\ \Pi_{0,2}(\infty) \\ \Pi_{1,2}(\infty) \\ \Pi_{2,2}(\infty) \\ \Pi_{3,2}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (15)$$

The normalizing condition is denoted by

$$\Pi_{0,0}(\infty)+\Pi_{1,0}(\infty)+\Pi_{3,0}(\infty)+\Pi_{0,1}(\infty)+\Pi_{1,1}(\infty)+\Pi_{2,1}(\infty) \\ +\Pi_{3,1}(\infty)+\Pi_{0,2}(\infty)+\Pi_{1,2}(\infty)+\Pi_{2,2}(\infty)+\Pi_{3,2}(\infty)=1. \quad (16)$$

Substituting Eq. (16) into any one of the duplicated rows of Eq. (15) to yield

$$\begin{pmatrix} -a_{11} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{12} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{14} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{15} & 0 & a_{15} & -a_{16} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{11} & 0 & 0 & 0 & 0 & -a_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{15} & 0 & \beta & 0 & -\left(2\lambda+\delta\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\left(\lambda+\gamma\right) & \mu & \eta & 0 \\ 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 2\lambda & -\left(\beta+\mu\right) & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & a_{15} & 0 & 0 & 0 & -\eta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(\infty) \\ \Pi_{1,0}(\infty) \\ \Pi_{3,0}(\infty) \\ \Pi_{0,1}(\infty) \\ \Pi_{1,1}(\infty) \\ \Pi_{2,1}(\infty) \\ \Pi_{3,1}(\infty) \\ \Pi_{0,2}(\infty) \\ \Pi_{1,2}(\infty) \\ \Pi_{2,2}(\infty) \\ \Pi_{3,2}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (17)$$

Again, the explicit expression for $Av_{T3}(\infty)$ is overly extensive to be displayed here. However, in Section 4 a numerical example with specific values will be solved to compare the systems.

Calculations for Retrial System 4

MTTF$_4$ of retrial syatem 4

For the reliability case, the state-transition diagram of retrial system 4 is shown in Figure 4 (without transition from state (1,2) to (3,2) and (0,2) and from state (3,2) to

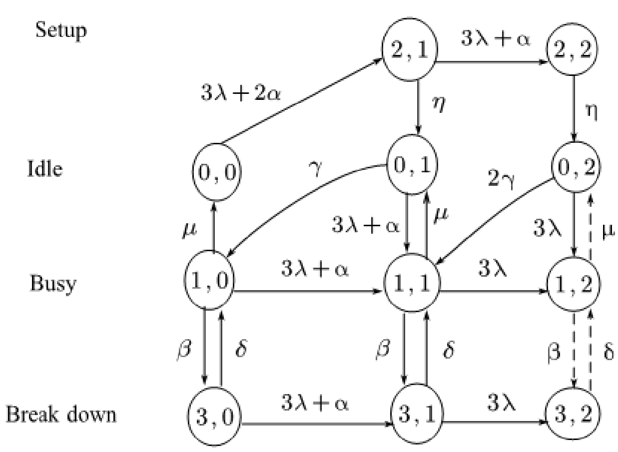

Figure 4. State-transition diagram of retrial system 4.

(1,2)). The corresponding probability vector $\Pi(t)$ for retrial system 4 is presented as:

$$\Pi(t) = [\Pi_{0,0}(t), \Pi_{1,0}(t), \Pi_{3,0}(t), \Pi_{0,1}(t), \Pi_{1,1}(t), \Pi_{2,1}(t), \\ \Pi_{3,1}(t), \Pi_{0,2}(t), \Pi_{1,2}(t), \Pi_{2,2}(t), \Pi_{3,2}(t)]^T.$$

The differential-difference equations can be formulated in matrix notation as

$$\frac{d\,\Pi(t)}{dt} = Q_7\,\Pi(t), \quad (18)$$

with

$$Q_7 = \begin{pmatrix} -a_{21} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{22} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{24} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{25} & 0 & a_{25} & -a_{26} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & -a_{27} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{25} & 0 & \beta & 0 & -\left(3\lambda+\delta\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\left(3\lambda+2\gamma\right) & 0 & \eta & 0 \\ 0 & 0 & 0 & 0 & 3\lambda & 0 & 0 & 3\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{25} & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3\lambda & 0 & 0 & 0 & 0 \end{pmatrix},$$

where

$$a_{21} = (3\lambda + 2\alpha),\ a_{22} = 3\lambda + \alpha + \beta + \mu,\ a_{23} = 3\lambda + \alpha + \delta, \\ a_{24} = 3\lambda + \alpha + \gamma,\ a_{25} = 3\lambda + \alpha,\ a_{26} = 3\lambda + \beta + \mu,\ a_{27} = 3\lambda + \alpha + \eta.$$

To compute $MTTF_4$, we omit the rows and columns of the matrix Q_7 pertaining to the absorbing state(s) and transpose resulted matrix to a new matrix A_4, from which we have

$$A_4 = \begin{pmatrix} -a_{21} & 0 & 0 & 0 & 0 & a_{21} & 0 & 0 & 0 \\ \mu & -a_{22} & \beta & 0 & a_{25} & 0 & 0 & 0 & 0 \\ 0 & \delta & -a_{23} & 0 & 0 & 0 & a_{25} & 0 & 0 \\ 0 & \gamma & 0 & -a_{24} & a_{25} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & -a_{26} & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & \eta & 0 & -a_{27} & 0 & 0 & a_{25} \\ 0 & 0 & 0 & 0 & \delta & 0 & -\left(3\lambda+\delta\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\gamma & 0 & 0 & -\left(3\lambda+2\gamma\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta & -\eta \end{pmatrix}.$$

Then, $MTTF_4$ is derived from

$$MTTF_4 = E\left[T_{\Pi(0)\to\Pi(absorbing)}\right] = \Pi(0)^T\left(-A_4^{-1}\right)\left[1,1,1,1,1,1,1,1,1\right]^T,$$

where, the initial conditions $\Pi(0)$ of retrial system 4 is given by

$$\Pi(0) = [\Pi_{0,0}(0), \Pi_{1,0}(0), \Pi_{3,0}(0), \Pi_{0,1}(0), \Pi_{1,1}(0), \Pi_{2,1}(0), \\ \Pi_{3,1}(0), \Pi_{0,2}(0), \Pi_{2,2}(0)]^T = [1,0,0,0,0,0,0,0,0]^T,$$

The explicit expression for $MTTF_4$ is extensive to be shown here. In Section 4, a particular numerical example is presented for this scenario to compare this system with the other systems.

System availability ($Av_{T_4}(\infty)$)

For the availability scenario, the state-transition diagram of retrial system 4 is shown in Figure 4 (with transition from state (1,2) to (3,2) and (0,2) and from state (3,2) to (1,2)). Let T_4 be the time to failure of the system for retrial system 4. In steady-state, we obtain the $Av_{T_4}(\infty)$ from:

$Av_{T_3}(\infty) = 1 - \Pi_{1,2}(\infty) - \Pi_{3,2}(\infty)$, and $Q_6\Pi(\infty) = 0$, the matrix form is given by;

$$\begin{pmatrix} -a_{21} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{22} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{24} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{25} & 0 & a_{25} & -a_{26} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & -a_{27} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{25} & 0 & \beta & 0 & -(3\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(3\lambda+2\gamma) & \mu & \eta & 0 \\ 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 3\lambda & -(\beta+\mu) & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & a_{25} & 0 & 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3\lambda & 0 & \beta & 0 & -\delta \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(\infty) \\ \Pi_{1,0}(\infty) \\ \Pi_{3,0}(\infty) \\ \Pi_{0,1}(\infty) \\ \Pi_{1,1}(\infty) \\ \Pi_{2,1}(\infty) \\ \Pi_{3,1}(\infty) \\ \Pi_{0,2}(\infty) \\ \Pi_{1,2}(\infty) \\ \Pi_{2,2}(\infty) \\ \Pi_{3,2}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (19)$$

The normalizing condition is denoted by

$$\Pi_{0,0}(\infty) + \Pi_{1,0}(\infty) + \Pi_{3,0}(\infty) + \Pi_{0,1}(\infty) + \Pi_{1,1}(\infty) + \Pi_{2,1}(\infty) + \Pi_{3,1}(\infty) + \Pi_{0,2}(\infty) + \Pi_{1,2}(\infty) + \Pi_{2,2}(\infty) + \Pi_{3,2}(\infty) = 1. \quad (20)$$

By substituting Eq. (20) into one of the redundant rows of Eq. (19) leads to

$$\begin{pmatrix} -a_{21} & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_{22} & \delta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -a_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{24} & \mu & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{25} & 0 & a_{25} & -a_{26} & 0 & \delta & 2\gamma & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & -a_{27} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{25} & 0 & \beta & 0 & -(3\lambda+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(3\lambda+2\gamma) & \mu & \eta & 0 \\ 0 & 0 & 0 & 0 & 2\lambda & 0 & 0 & 3\lambda & -(\beta+\mu) & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & a_{25} & 0 & 0 & 0 & -\eta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \Pi_{0,0}(\infty) \\ \Pi_{1,0}(\infty) \\ \Pi_{3,0}(\infty) \\ \Pi_{0,1}(\infty) \\ \Pi_{1,1}(\infty) \\ \Pi_{2,1}(\infty) \\ \Pi_{3,1}(\infty) \\ \Pi_{0,2}(\infty) \\ \Pi_{1,2}(\infty) \\ \Pi_{2,2}(\infty) \\ \Pi_{3,2}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (21)$$

Once more, the explicit expression for $Av_{T_4}(\infty)$ is too extensive to present here. However, in Section 4 a numerical example with specific values will be included to compare these systems.

RESULTS AND DISCUSSION

A cost-effective power supply system is crucial across various industries, such as manufacturing, computer networks, communication systems, and production environments. To meet organizational needs requires evaluating and projecting operational efficiency. Our theoretical and numerical findings give managers and policymakers useful information that helps them maintain high levels of quality control, optimize management processes, and anticipate performance with accuracy. Numerical solutions are presented in this section using particular cases with assigned values. We calculate the steady-state availability, and mean-time-to-failure (MTTF), and compare the cost-benefit ratios across all retrial systems to identify the best-performing option. To investigate how various configuration parameters affect these performance metrics, more numerical experiments are carried out.

Comparison of MTTF and Availability

The effects of various system parameters on MTTF and steady-state availability are investigated through numerical analysis of the cases listed below.

Case 1: Fix $\mu = 2$, $\alpha = 2\lambda$, $\eta = 0.3$, $\delta = 0.5$, $\beta = 0.05$, $\gamma = 3$ and alter the values of λ.

Case 2: Fix $\lambda = 0.3$, $\alpha = 2\lambda$, $\eta = 0.3$, $\delta = 0.5$, $\beta = 0.05$, $\gamma = 3$ and alter the values of μ.

Case 3: Fix $\lambda = 0.3$, $\alpha = 2\lambda$, $\mu = 2$, $\delta = 0.5$, $\beta = 0.05$, $\gamma = 3$ and alter the values of η.

Case 4: Fix $\lambda = 0.3$, $\mu = 2$, $\eta = 0.3$, $\delta = 0.5$, $\beta = 0.05$, $\gamma = 3$ and alter the values of α.

Case 5: Fix $\lambda = 0.3$, $\alpha = 2\lambda$, $\eta = 0.3$, $\mu = 2$, $\delta = 0.5$, $\gamma = 3$ and alter the values of β.

Case 6: Fix $\lambda = 0.3$, $\alpha = 2\lambda$, $\eta = 0.3$, $\mu = 2$, $\beta = 0.05$, $\gamma = 3$ and alter the values of δ.

Case 7: Fix $\lambda = 0.3$, $\alpha = 2\lambda$, $\eta = 0.3$, $\mu = 2$, $\delta = 0.5$, $\beta = 0.05$ and alter the values of γ.

The effects of system parameters are examined $MTTF_i$ in Table 4-Table 10 and Av_i, (i = 1,2,3,4) in Table 11 and Figure 5-Figure *7* respectively. Based on the results presented in these Tables and Figures, it is evident that The MTTF and steady-state availability are heavily influenced by the failure rate λ of operating components and the efficiency of repairs, while the retrial rate γ of failed components has a moderate impact. Consequently, to enhance availability and reduce MTTF, maintenance efforts should focus on extending the lifespan of active components and improving repair efficiency. This approach ensures the system maintains an adequate supply of spare parts.

MTTF and AV are both decreasing functions of λ and α, this occurs because larger values of λ or α, result in a faster reduction in the number of operative components, causing

Table 4. Comparison of retrial systems i = 1,2,3,4 for MTTF

Results	
Range of λ	
$0.01 < \lambda < 0.680555$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$
$0.680555 < \lambda < 1.0$	$MTTF_4 < MTTF_3 < MTTF_2 < MTTF_1$
Range of μ	
$0.01 < \mu < 0.161857$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$
$0.161857 < \mu < 0.541816$	$MTTF_4 < MTTF_3 < MTTF_2 < MTTF_1$
$0.541816 < \mu < 3.0$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$
Range of η	
$0.01 < \eta < 0.126025$	$MTTF_4 < MTTF_3 < MTTF_2 < MTTF_1$
$0.126025 < \eta < 1.0$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$
Range of β	
$0.001 < \beta < 0.30448$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$
$0.30448 < \beta < 1.0$	$MTTF_4 < MTTF_3 < MTTF_2 < MTTF_1$
Range of δ	
$0.001 < \delta < 1.0$	$MTTF_4 < MTTF_2 < MTTF_3 < MTTF_1$

Table 5. Values of MTTF for various λ and μ

λ	$MTTF_1$				$MTTF_2$			
	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$	$\mu = 3$	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$	$\mu = 3$
0.1	108.348	126.8930	142.272	155.232	44.9554	51.8556	57.5919	62.4359
0.2	44.9554	51.8556	57.5919	62.4359	20.1821	22.7881	24.9661	26.8136
0.3	27.7549	31.6414	34.8809	37.6225	13.4236	14.9273	16.1910	17.2678
0.4	20.1821	22.7881	24.9661	26.8136	10.3899	11.4172	12.2851	13.0281
0.5	16.0217	17.9435	19.5540	20.9232	8.6931	9.4605	10.1122	10.6726
0.6	13.4236	14.9273	16.1910	17.2678	7.6179	8.2235	8.7404	9.1868
0.7	11.6597	12.8847	13.917	14.7986	6.8795	7.3755	7.8008	8.1695
0.8	10.3899	11.4172	12.2851	13.0281	6.3434	6.7605	7.1197	7.4323
0.9	9.4352	10.3156	11.0614	11.7012	5.9376	6.2954	6.6048	6.8752
1.0	8.6931	9.4605	10.1122	10.6726	5.6206	5.9323	6.2030	6.4403

Table 6. Values of MTTF for various λ and μ

λ	$MTTF_3$				$MTTF_4$			
	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$	$\mu = 3$	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$	$\mu = 3$
0.1	75.3757	94.4482	111.8790	127.805	40.4587	49.9338	58.6375	66.6191
0.2	26.6649	32.4626	37.8184	42.7503	15.5753	18.5015	21.2380	23.7813
0.3	15.5753	18.5015	21.2380	23.7813	9.8103	11.2876	12.6954	14.0230
0.4	11.1215	12.9253	14.6335	16.2366	7.4676	8.3663	9.2381	10.0721
0.5	8.8271	10.0609	11.2438	12.3647	6.2564	6.8585	7.4522	8.0278
0.6	7.4676	8.3663	9.2381	10.0721	5.5393	5.9673	6.3956	6.8159
0.7	6.5865	7.2691	7.9388	8.5852	5.0759	5.3928	5.7140	6.0328
0.8	5.9785	6.5129	7.0427	7.5584	4.7573	4.9991	5.2469	5.4954
0.9	5.5393	5.9673	6.3956	6.8159	4.5278	4.7167	4.9122	5.1099
1.0	5.2104	5.5594	5.9117	6.2601	4.3564	4.5067	4.6637	4.8237

Table 7. Values of MTTF for various λ and α

λ	$MTTF_1$				$MTTF_2$			
	$\alpha = 0.04$	$\alpha = 0.06$	$\alpha = 0.08$	$\alpha = 0.10$	$\alpha = 0.04$	$\alpha = 0.06$	$\alpha = 0.08$	$\alpha = 0.10$
0.1	137.7835	126.8928	118.4223	111.6459	53.9474	51.8556	50.0627	48.5088
0.2	53.9474	51.8556	50.0627	48.5088	23.1522	22.7881	22.4544	22.1474
0.3	32.3983	31.6414	30.9642	30.3547	15.0568	14.9273	14.8055	14.6906
0.4	23.1521	22.7881	22.4544	22.1474	11.4798	11.4172	11.3574	11.3003
0.5	18.1495	17.9435	17.7517	17.5727	9.4964	9.4605	9.4259	9.3926
0.6	15.0568	14.9273	14.8055	14.6906	8.2464	8.2235	8.2013	8.1798
0.7	12.9724	12.8847	12.8016	12.7226	7.3913	7.3755	7.3602	7.3453
0.8	11.4798	11.4172	11.3574	11.3003	6.7719	6.7605	6.7493	6.7385
0.9	10.3622	10.3156	10.2709	10.2279	6.3040	6.2954	6.2869	6.2786
1.0	9.4964	9.4605	9.4259	9.3926	5.9391	5.9323	5.9257	5.9192

Table 8. Values of MTTF for various λ and α

λ	$MTTF_3$				$MTTF_4$			
	$\alpha = 0.04$	$\alpha = 0.06$	$\alpha = 0.08$	$\alpha = 0.10$	$\alpha = 0.04$	$\alpha = 0.06$	$\alpha = 0.08$	$\alpha = 0.10$
0.1	105.0711	94.4482	86.1901	79.5812	53.4322	49.9338	47.0141	44.5365
0.2	34.0463	32.4626	31.0852	29.8744	19.0272	18.5015	18.0232	17.5855
0.3	19.0272	18.5015	18.0232	17.5855	11.4678	11.2876	11.1183	10.9589
0.4	13.1703	12.9253	12.6970	12.4835	8.4524	8.3663	8.2841	8.2056
0.5	10.1981	10.0609	9.9312	9.8084	6.9074	6.8584	6.8113	6.7658
0.6	8.4524	8.3663	8.2841	8.2056	5.9983	5.9673	5.9373	5.9082
0.7	7.3274	7.2691	7.2132	7.1594	5.4139	5.3928	5.3723	5.3524
0.8	6.5546	6.5130	6.4728	6.4348	5.0142	4.9991	4.9844	4.9700
0.9	5.9983	5.9673	5.9373	5.9082	4.7279	4.7167	4.7057	4.6949
1.0	5.5832	5.5594	5.5363	5.5138	4.5154	4.5067	4.4983	4.4899

Table 9. Values of MTTF for various λ and γ

λ	$MTTF_1$				$MTTF_2$			
	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 4$	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 4$
0.1	119.0643	126.8928	133.2839	138.6002	48.8630	51.8556	54.3106	56.3609
0.2	48.8630	51.8556	54.3106	56.3609	21.6091	22.7881	23.7637	24.5842
0.3	29.9172	31.6414	33.0623	34.2533	14.2245	14.9273	15.5133	16.0093
0.4	21.6091	22.7881	23.7637	24.5842	10.9242	11.4172	11.8310	12.1833
0.5	17.0589	17.9435	18.6782	19.2982	9.0839	9.4605	9.7787	10.0508
0.6	14.2245	14.9273	15.5133	16.0093	7.9206	8.2235	8.4808	8.7021
0.7	12.3042	12.8847	13.3704	13.7828	7.1233	7.3755	7.5909	7.7770
0.8	10.9242	11.4172	11.8310	12.1833	6.5452	6.7605	6.9452	7.1054
0.9	9.8882	10.3156	10.6755	10.9828	6.1083	6.2954	6.4566	6.5971
1.0	9.0839	9.4605	9.7786	10.0508	5.7675	5.9323	6.0751	6.1999

Table 10. Values of MTTF for various λ and γ

λ	$MTTF_3$				$MTTF_4$			
	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 4$	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 4$
0.1	90.7913	94.4482	97.3324	99.6654	48.0586	49.9338	51.4206	52.6283
0.2	31.2754	32.4626	33.4083	34.1796	17.8593	18.5015	19.0178	19.4417
0.3	17.8593	18.5015	19.0178	19.4417	10.9299	11.2876	11.5787	11.8202
0.4	12.5021	12.9253	13.2683	13.5519	8.1290	8.3663	8.5617	8.7253
0.5	9.7529	10.0609	10.3124	10.5218	6.6871	6.8584	7.0012	7.1218
0.6	8.1290	8.3663	8.5616	8.7253	5.8374	5.9673	6.0767	6.1701
0.7	7.0794	7.2691	7.4265	7.5592	5.2911	5.3928	5.4794	5.5539
0.8	6.3574	6.5130	6.6430	6.7533	4.9176	4.9991	5.0691	5.1299
0.9	5.8374	5.9673	6.0767	6.1701	4.6502	4.7167	4.7743	4.8247
1.0	5.4493	5.5594	5.6528	5.7329	4.4518	4.5067	4.5548	4.5971

Table 11. Comparison of retrial systems $i = 1,2,3,4$ for availability

Results	
Range of λ	
$0.01 < \lambda < 0.41850$	$Av_{T_1}(\infty) > Av_{T_3}(\infty) > Av_{T_2}(\infty) > Av_{T_4}(\infty)$
$0.41850 < \lambda < 1.0$	$Av_{T_1}(\infty) > Av_{T_2}(\infty) > Av_{T_3}(\infty) > Av_{T_4}(\infty)$
Range of μ	
$0.01 < \mu < 1.27079$	$Av_{T_1}(\infty) > Av_{T_3}(\infty) > Av_{T_2}(\infty) > Av_{T_4}(\infty)$
$1.27079 < \mu < 3.0$	$Av_{T_1}(\infty) > Av_{T_2}(\infty) > Av_{T_3}(\infty) > Av_{T_4}(\infty)$
Range of η	
$0.01 < \eta < 0.15698$	$Av_{T_1}(\infty) > Av_{T_2}(\infty) > Av_{T_3}(\infty) > Av_{T_4}(\infty)$
$0.15698 < \eta < 1.0$	$Av_{T_1}(\infty) > Av_{T_3}(\infty) > Av_{T_2}(\infty) > Av_{T_4}(\infty)$
Range of β	
$0.001 < \beta < 0.153784$	$Av_{T_1}(\infty) > Av_{T_3}(\infty) > Av_{T_2}(\infty) > Av_{T_4}(\infty)$
$0.153784 < \beta < 1.0$	$Av_{T_1}(\infty) > Av_{T_2}(\infty) > Av_{T_3}(\infty) > Av_{T_4}(\infty)$
Range of δ	
$0.001 < \delta < 0.249205$	$Av_{T_1}(\infty) > Av_{T_2}(\infty) > Av_{T_3}(\infty) > Av_{T_4}(\infty)$
$0.249205 < \delta < 1.0$	$Av_{T_1}(\infty) > Av_{T_3}(\infty) > Av_{T_2}(\infty) > Av_{T_4}(\infty)$

MTTF and steady-state availability to decline as λ or α increase. Furthermore, MTTF and steady-state availability increase as functions of μ and γ. This is because higher values of μ and γ, reduce the time required to restore a failed component to working condition, leading to improved system availability and MTTF.

Comparison Based on Cost-Benefit Ratios

We investigate the impacts of various system parameters on four distinct retrial systems with different costs in Figure 8-Figure 17.

The repair rate of the failed components μ and the recovery rate of the broken service facility δ affect $C_i/MTTF_i$ and C_i/Av_i significantly. However, $C_i/MTTF_i$ and C_i/Av_i are increasing function as functions of the failure rate of operating components λ, setup time rate η, server breakdown rate β, and the retrial rate of a failed component γ. The reason is that larger values of λ, η, β and γ are, the construction cost becomes higher, and then $C_i/MTTF_i$ and C_i/Av_i become bigger. Therefore, according to the analysis, retrial system 2 is recommended as it has the most economical construction cost and an excellent cost-benefit ratio.

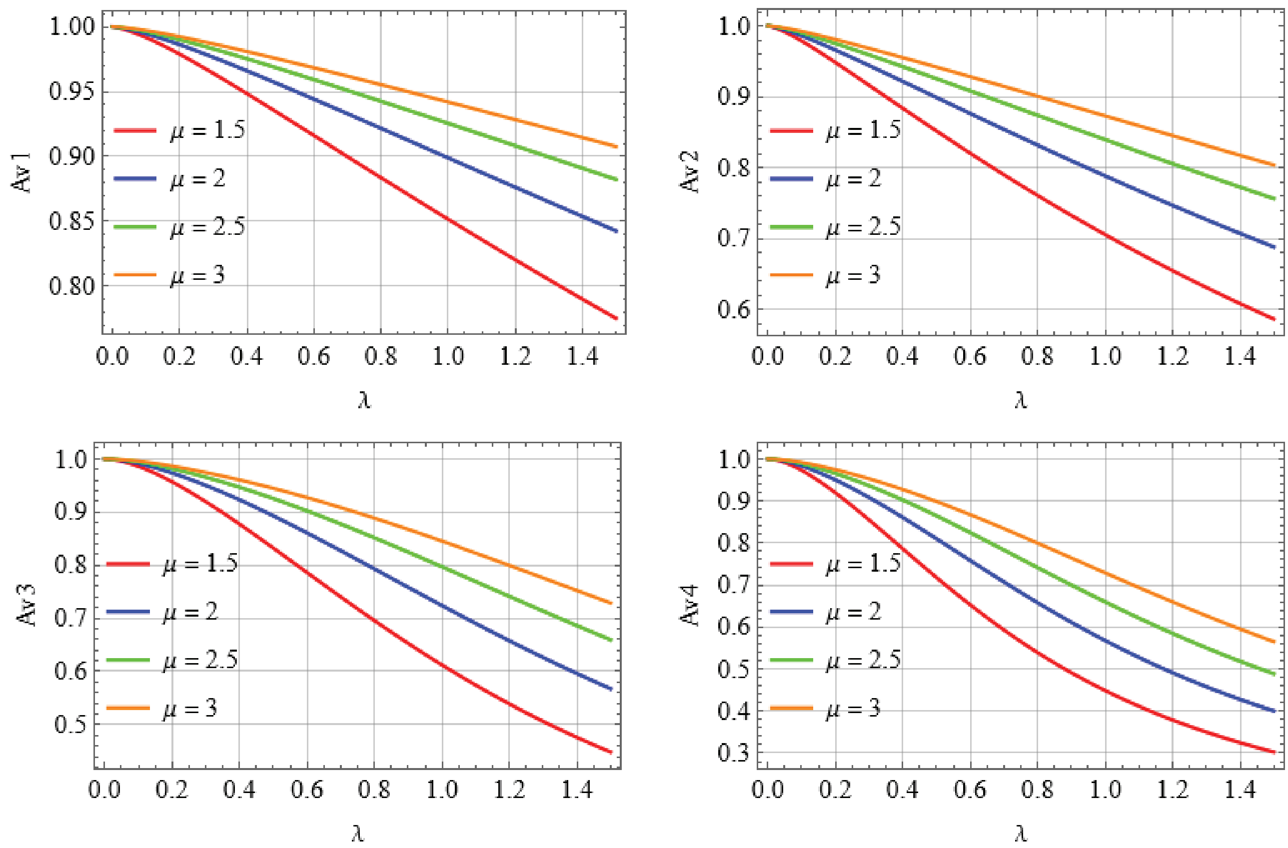

Figure 5. Av_i vs. λ for different values of μ.

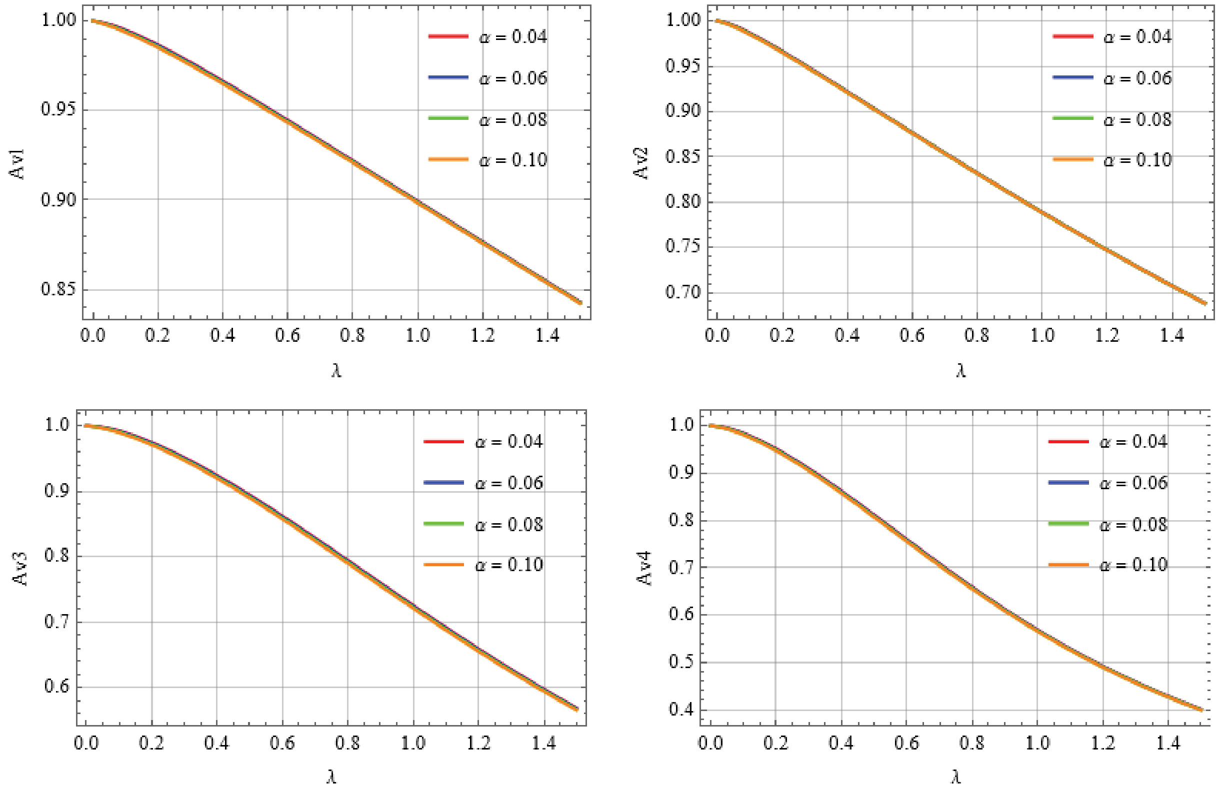

Figure 6. Av_i vs. λ for different values of α.

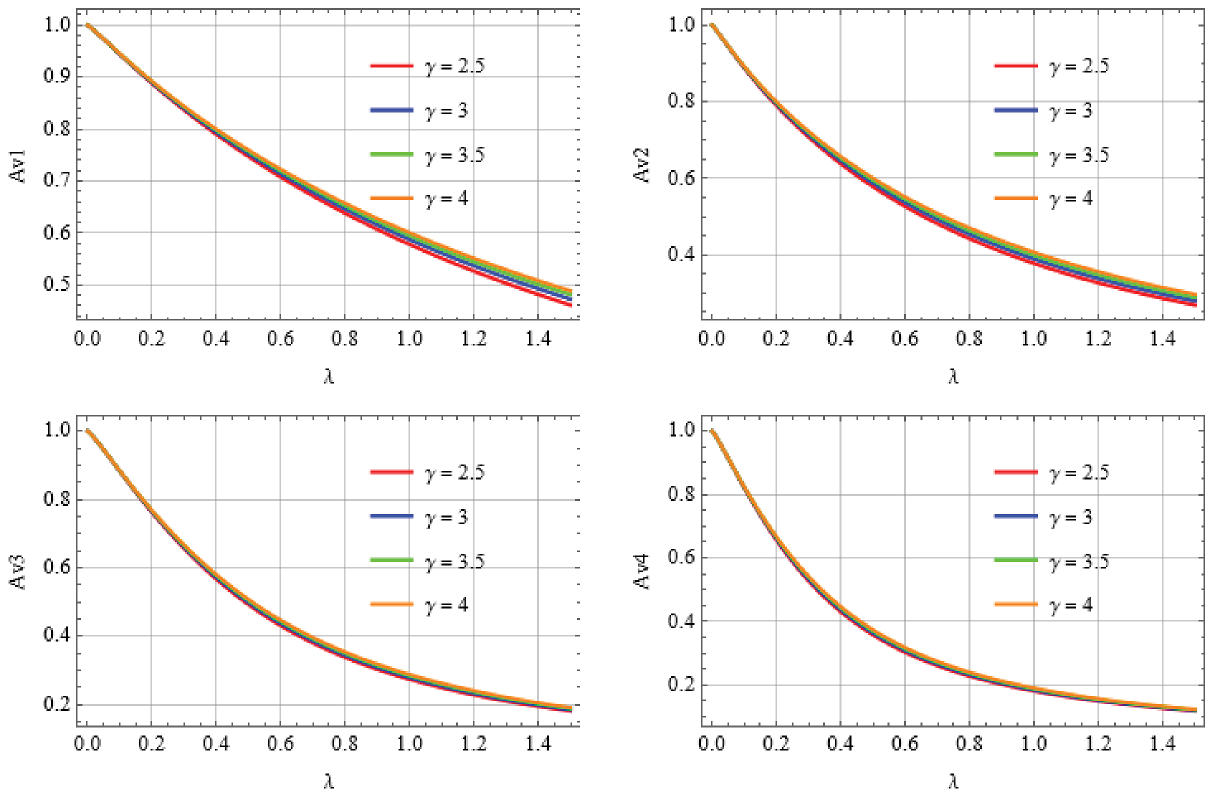

Figure 7. Av_i vs. λ for different values of γ.

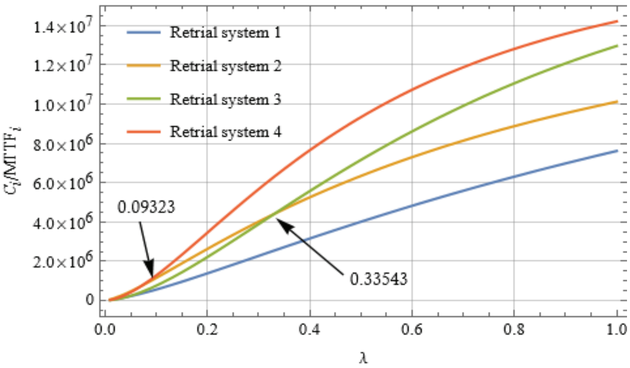

Figure 8. $C_i/MTTF_i$ vs. λ.

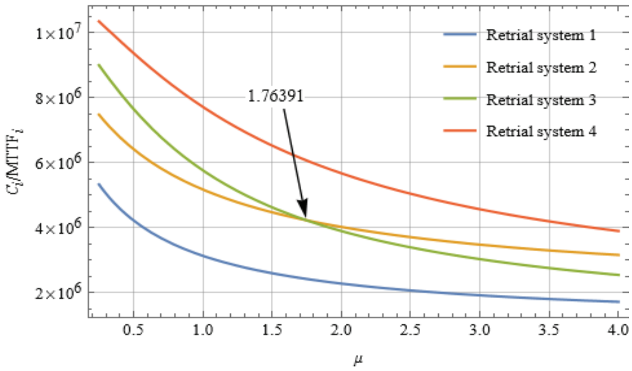

Figure 9. $C_i/MTTF_i$ vs. μ.

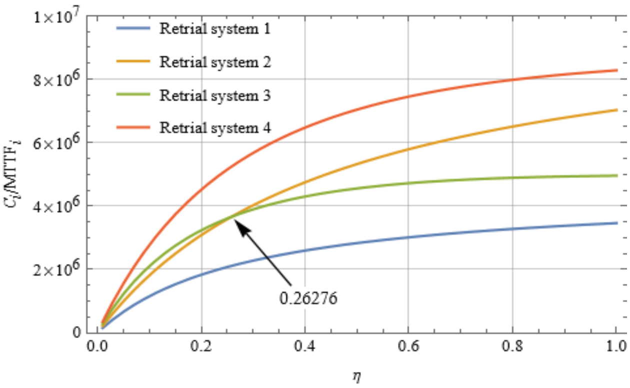

Figure 10. $C_i/MTTF_i$ vs. η.

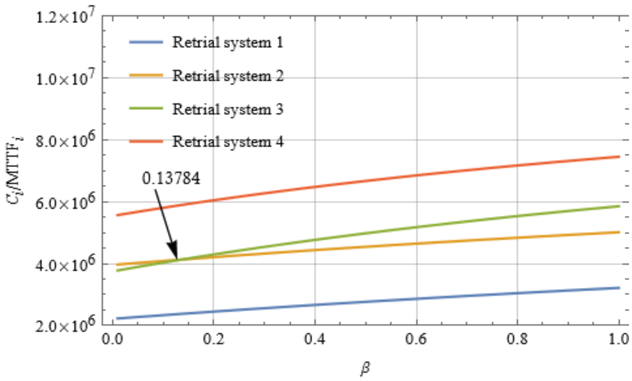

Figure 11. $C_i/MTTF_i$ vs. β.

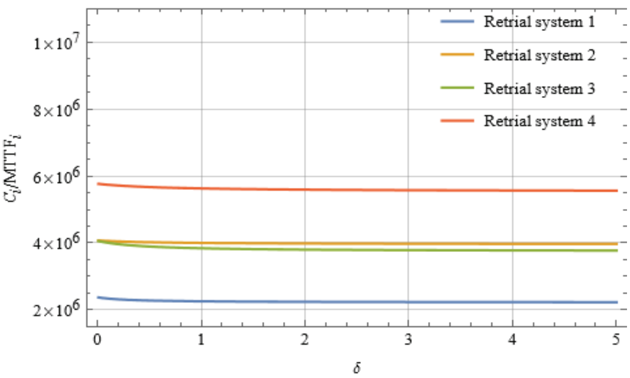

Figure 12. $C_i/MTTF_i$ vs. δ.

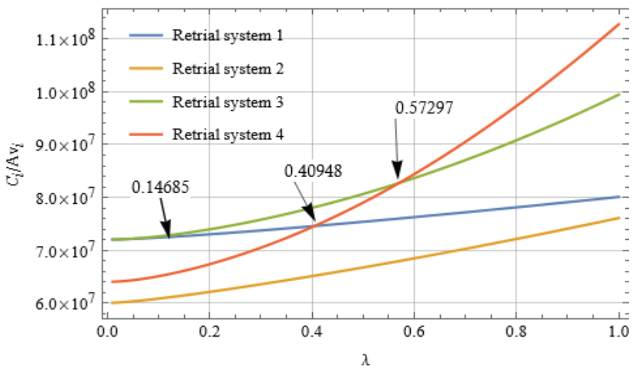

Figure 13. C_i/Av_i vs. λ.

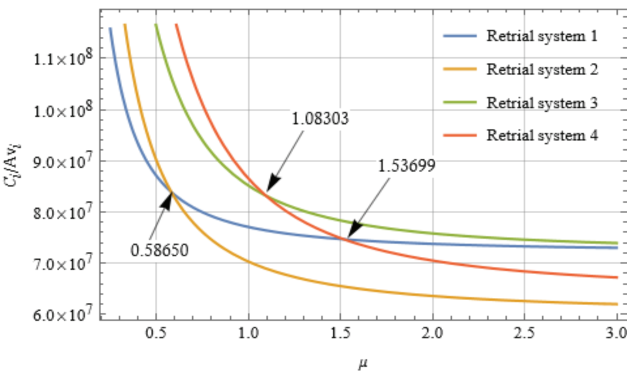

Figure 14. C_i/Av_i vs. μ.

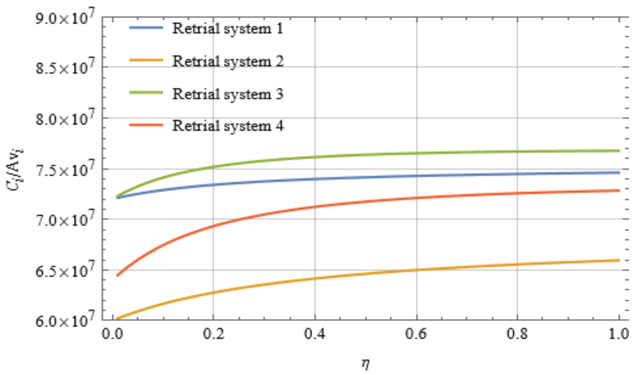

Figure 15. C_i/Av_i vs. η.

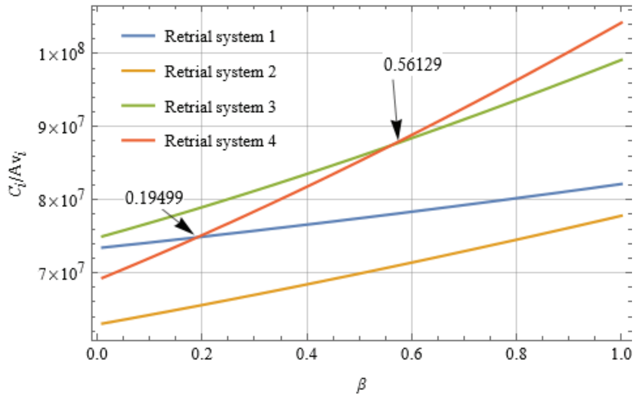

Figure 16. C_i/Av_i vs. β.

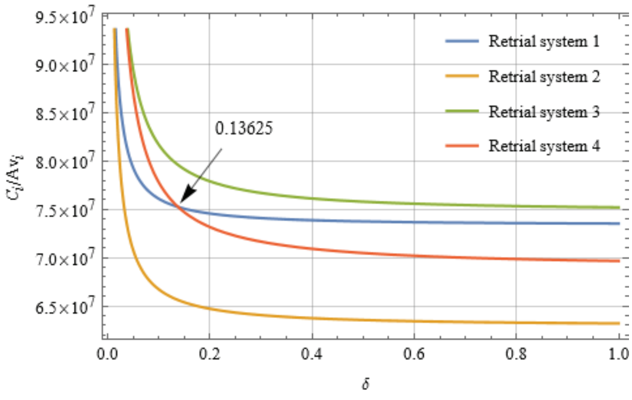

Figure 17. C_i/Av_i against δ.

CONCLUSION

In this paper we studied the cost-benefit evaluation of four unreliable retrial systems exhibiting warm standbys and setup times. Steady-state probability was derived by employing a matrix-analytical technique, and the Laplace transform enabled the derivation of mean time to failure for each retrial system. Furthermore, numerical experiments were performed using Wolfram Mathematica software to analyze numerical results. Four retrial systems were assessed and compared based on their mean time to failure, steady-state availability, and cost-benefit ratio. The key findings show that optimal mean time to failure was noted in retrial system 4 and the optimal steady-state availability was found in retrial system 2. Based on the cost-benefit ratio, retrial system 1 is optimal since it emerges as the best-performing configuration. The proposed method is a useful tool for decision-makers in diverse parameter settings, allowing them to select an appropriate system that maximizes benefits. Numerical analyses have revealed that the system reliability, mean time to failure, and steady-state availability can be improved by increasing the repair service rates, reducing the rate of failures of both active and warm standby components by implementing effective methods, such as routine maintenance service, to prolong the lifespan of the equipment. The current work can be extended in the following ways in the future; (i) considering a situation in which the server takes a vacation when there are no failed components in the system awaiting repair. (ii) investigating cases where the repair times follow general distributions.

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AUTHORSHIP CONTRIBUTIONS

Jones Edward Chiwinga initiate the model and do all the writing and mathematical analysis while Muhammad Salihu Isa and Jinbiao Wu helps in editing and supervision.

CONFLICT OF INTEREST

Authors have declared that there is no conflict of interest with regard to this research.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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