



## Research Article

# Solution to a make-or-buy decision problem based on intuitionistic fuzzy sets and grey relational analysis

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## ABSTRACT

This research introduces an integrated decision-support framework for make-or-buy decisions by combining intuitionistic fuzzy sets with grey relational analysis. Since make-or-buy selection involves multiple evaluation criteria and substantial uncertainty in expert judgments, a single decision technique may not adequately capture all aspects of the problem. Therefore, intuitionistic fuzzy sets are employed to represent uncertainty in the evaluation process, whereas grey relational analysis is utilized to rank the available alternatives. To further demonstrate the robustness of the proposed framework, its performance is compared with the results obtained from well-established MCDM techniques. The proposed model was further validated by comparing its outcomes with those generated using the TOPSIS and WASPAS methods, both of which are extensively employed in MCDM studies.

Make-or-buy decisions are strategically important in manufacturing industries due to their direct impact on cost control, resource allocation, and competitiveness. The aim of this study is to add a new one to the multi-criteria decision-making methods by using more than one type of uncertainty and to apply this method in an area that has never been used before in the literature. This method is applied to a real-world make-or-buy decision problem in the metal manufacturing sector, where selecting the optimal sourcing strategy has a direct impact on operational efficiency and cost control. In this study, an extension of the technique developed for ordering preferences by similarity to the ideal solution.

The results show that this integrated approach is effective in handling uncertainty and can be a valuable decision-support tool for practitioners dealing with complex procurement or sourcing decisions in real-world industrial settings. The proposed method ranked “full outsourcing” as the most suitable option, with a grey relational grade of 0.798, indicating strong alignment with the selected criteria under uncertainty. Among the three evaluated alternatives, full outsourcing achieved the highest grey relational grade (0.798), while hybrid and in-house production scored 0.691 and 0.598, respectively. These findings suggest that the proposed integrated approach can be used effectively for complex sourcing decisions in uncertain environments across different industries.

The novelty of this study is that instead of dealing with the whole decision-making process with the same uncertainty theory, this work employs the most appropriate uncertainty methods at different decision stages. In addition, this study is the first of its kind in which the grey theory and fuzzy set theory are used together in make-or-buy decision as far as our literature review demonstrates. So, the contributions of the study to the literature is using the different uncertainty kinds in multicriteria decision making problems.

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## INTRODUCTION

To meet increasing customer demands and expectations, businesses are compelled to adopt scientific decision-making methods. Firms may choose to outsource a product or service instead of producing it themselves for such reasons as taking advantage of economies of scale and focusing on core competencies. These are referred to as make-or-buy decisions in supply chain management. Such decisions are one of the strategic decisions that affect the long-term competitiveness and profitability of a company. It has become increasingly evident that resource allocation decisions significantly influence a business's future [1]. If used correctly, outsourcing has the potential to create a competitive advantage for companies. It is especially emphasized that outsourcing will provide significant benefits to businesses in all jobs that are not considered core competence for the organization.

Decision-makers tend to prefer the lowest cost alternative among the available production and purchasing alternatives. In this comparison, if the cost of the purchasing alternative is lower than that of the production alternative, then the purchasing alternative will be chosen; otherwise, the production alternative will be preferred. If purchasing is the choice in a make-or-buy decision, the enterprise often focuses on purchasing large quantities at the best price possible while ignoring other factors, such as quality and delivery times. However, businesses should also consider such factors as delivery reliability, technical capacity, cost capacity and financial conditions in the decision of purchase.

Make-or-buy decisions are frequently made under conditions where decision makers have only limited or incomplete information. In such situations, grey system theory provides an appropriate analytical framework because it is specifically designed for problems characterized by information deficiency. Introduced by Julong Deng, the theory classifies systems according to the availability of information [2]. A system with complete information is referred to as a *white system*, whereas a *black system* represents a situation in which no information is available. Most real-world decision problems, however, fall between these two extremes and are therefore considered *grey systems*, where only part of the available information is known. Grey system theory is one of the methods developed for studying uncertainties with the superiority in the mathematical analysis of systems with uncertain information. It has been shown [3, 4] to be an effective method that can help solve uncertainty problems with discrete data and incomplete information.

In this study, we use the make-or-buy decision approach to analyze the components needed by an enterprise operating in the metal industry. Due to the uncertainty in decision-makers' evaluations and the inherent subjectivity of the criteria, the intuitionistic fuzzy set (IFS) theory and methods of grey relational analysis (GRA) were used together. Intuitionistic fuzzy set is used because it can reflect the

linguistic preferences of the decision makers more easily by showing the membership, non-membership functions and hesitation index. Grey relational analysis is subsequently used to evaluate the similarity between alternatives based on their performance across multiple criteria. By measuring the degree of closeness between the reference sequence and each alternative, GRA enables the ranking of candidate components even when the available information is limited or incomplete. The complementary strengths of IFS and GRA make the proposed framework suitable for solving complex multi-criteria decision problems characterized by uncertainty and interdependent evaluation factors.

The results show that IFS theory and GRA can be used successfully in solving problems involving different kinds of uncertainties. To enhance the reliability of the obtained results, it is a common practice to validate proposed MCDM approaches using more than one decision-making method. In this study, the technique for ordering preferences by similarity to ideal solution (TOPSIS) and WASPAS, which are most preferred methods to solve MCDM problems, were used to test the accuracy of the results.

This study goes beyond the traditional approaches that typically rely on a single uncertainty theory by proposing an integrated method that applies the most suitable type of uncertainty at each stage of the decision-making process. The combined use of Grey Relational Analysis (GRA) and Intuitionistic Fuzzy Sets (IFS) enables effective modeling of both data incompleteness and subjective evaluations. By overcoming the limitations of classical models, this approach offers a more flexible and realistic solution for complex real-life decision problems. Notably, the application of GRA to a make-or-buy decision problem for the first time also marks a novel contribution to the literature. The proposed model not only delivers theoretical innovation but also demonstrates practical effectiveness through its implementation in a real industrial setting. Therefore, an integrated approach combining Intuitionistic Fuzzy Sets (IFS) and Grey Relational Analysis (GRA) is proposed to effectively address the uncertainties present in real-world make-or-buy decision problems.

The rest of the paper is organized into the following sections. Section 2 reviews the principles of IFS and GRA; Section 3 outlines the application to a real-life case; and Section 4 discusses the results and conclusions.

## Literature Review

With increasing competition, businesses need to use more scientific methods in their decision-making processes. The decision whether the production of goods or services required by businesses will be carried out in their organization or in another supplier company is called "a make-or-buy decision problem" in the literature. The conceptual basis of a make-or-buy decision is based on Williamson's theory of job-cost analysis [5]. The job-cost analysis combines economic theory and production theory that a business should use in developing market share. It deals with

establishing a purchasing discipline that limits the internal and external constraints and analyzes other factors.

### Multi-Criteria Decision Analysis

A review of the literature reveals that Multi-Criteria Decision-Making (MCDM) approaches are frequently applied to solve make-or-buy decision problems. In this context, Padillo and Diaby [6] developed a multi-criteria decision analysis methodology for evaluating make-or-buy strategies. Their study evaluated make-or-buy alternatives using a dynamic framework that simultaneously considered multiple features and objectives. They identified four goals: maximizing strategic competitive performance; maximizing managerial performance; minimizing resource risks; and maximizing financial performance. The model applies an analytic hierarchy process to create measures of importance for criteria. These goals are traded-off based on composite programming, resulting in a ranking of make-or-buy alternatives.

Platts et al. [7] developed a structured framework to support make-or-buy decisions by identifying the key factors influencing this process in industrial organizations. Their approach incorporated a multi-attribute decision-making technique to generate comprehensive recommendations for make-or-buy selection. The proposed framework was demonstrated through an industrial case study, highlighting both its practical applicability and its contributions to the literature and industrial decision-making practices.

Hwang [8] developed an integrated decision analysis model for the make-or-buy decision problem for school foodservice. He created a decision support system with three stages: brainstorming to identify decision alternatives, fuzzy analytic hierarchy process (AHP) application as a multi-criteria decision-making method, and application of an integrated priority method.

Golec et al. [9] used the Choquet integral in the make-or-buy decision process. They determined the degree of importance by using the interaction index value and then brought it together with the Choquet integral. They compared this method with other similar methods. The criteria used in this study consist of design, engineering, materials, manufacturing, and price or profit. Four alternatives were evaluated by using AHP, Choquet integral and weighted arithmetic mean method, and then the results were compared.

Water and Peet [10] developed a multi-criteria decision-making method based on the analytical hierarchy process to take on the make-or-buy decision that is encountered in the manufacturing sector. This study addressed two aspects often overlooked in previous research: the content of the decision and the transactional details involved.

Hwang [11] proposed a two-stage method for resolving the make-or-buy decision problem. In the first stage, a multi-criteria analysis method was developed by using the AHP method; and in the second stage, problems involving

multi-purposes, multi-criteria and multi-featured topics were integrated by utilizing the fuzzy set ranking method.

Kahraman et al. [12] focused on outsourcing information systems by using group decision making approach. The work aimed to selecting one of the eight information system providers under seven criteria. Sensitivity analysis was conducted to assess the impact of parameter variations on the final decision.

Cheshmberah et al. [13] investigated the make-or-buy-decision problem in the context of production activities. The study consists of two stages. The first stage investigated whether the activity handled is among the core competencies of the company. In The second stage developed a decision-making mechanism based on the fuzzy TOPSIS approach for activities that are not considered as core competencies.

Minh [14] proposed a model for the make-or-buy decision in an experimental point of view. In the study, the results of some surveys carried out by Japanese automobile manufacturers were used in the calculation of the weights of the criteria and sub-criteria used in the analytical hierarchy process.

Cheshmberah et al. [15] examined outsourcing decisions in manufacturing by applying the PROMETHEE method to an aviation industry case. Their framework incorporated fuzzy numbers to estimate the relative importance of the evaluation criteria, allowing uncertainty in expert judgments to be considered during the decision-making process.

Seyedhosseini et al. [16] studied the make-or-buy decision in order to find which strategy should have been chosen to minimize the total costs of supply chain. For this purpose, two discrete models were created for each strategy and then solved sequentially with many numerical examples. They claimed that computational experiments showed the efficiency of the proposed models.

Sillanpää [17] analyzed factors related to make-or-buy decisions. This research focused on decision making model in the manufacturing industry supply chain. Within the scope of the research, a tool was created for make-or-buy decisions which could be used as a help to evaluate outsourcing analytically.

Park [18] examined the risks associated with make-or-buy decisions under conditions of radical technological change. The study focused on identifying strategies that enable firms to mitigate the trade-off and governance-related risks encountered when selecting an appropriate make-or-buy strategy in rapidly evolving technological environments.

Serrano et al. [19] developed a theoretical framework that can help managers evaluate their sourcing decisions. As a result of a comprehensive literature search, these scholars tried to determine which factors and theories were needed or appropriate for make-or-buy decisions. They suggested that practitioners should combine theories of the

resource-based view, strategic management, and transaction cost economics to evaluate plant location decisions.

Serrano et al. [20] proposed a decision-support framework for make-or-buy analysis to assist managers in evaluating sourcing alternatives. The development of the make-or-buy framework is explained based on the results of qualitative interviews with practitioners and an internal case study. Their results suggest that practitioners should combine the proposed framework with multi-criteria decision analysis based on the pairwise comparison matrix and TOPSIS methodology.

Omerali and Kaya [21] purposed a Type-2 Fuzzy COPRAS methodology to support firms' buy-or-implement decisions in the Internet of Things (IoT) domain. The researchers stated that deciding on the digitalization strategy is not different from the current purchasing decision that companies have faced in the past, but this time they are faced with increasing complexity.

More recently, Klein and Mazzoni [22] revisited the make-or-buy literature from a theoretical standpoint, offering updated insights on vertical integration strategies and outsourcing decisions, and emphasizing future research directions. Another recent study [23] explored the impact of learning-by-doing on make-or-buy decisions, especially in high-tech sectors like semiconductors, introducing a dynamic modeling perspective. Additionally, Brem and Elsner [24] analyzed make-or-buy decisions in products characterized by high technological complexity, underlining the strategic implications of sourcing choices in the consumer goods industry.

While previous studies primarily focus on a single uncertainty theory or isolated decision models, our study aligns different uncertainty methods across multiple decision stages, providing a more holistic and flexible framework.

### Grey System Theory

Grey system theory provides a technique that can be used successfully in studying processes that involve partial uncertainties. However, it is interesting to note that the grey theory has not been used to solve the make-or-buy problems until today. There are some review papers on grey theory in the literature. For example, Yin [25] covered fifteen years of gray system research and produced a historical review and bibliometric analysis.

Li and Lin [26] submitted a review article on grey system theory with the aim of informing the readers about the development history, main content and main difficulties of the grey system theory. They stated that their main motivation was to draw the attention of the international academic community to this theory.

A literature review study [27] is investigated on the use of grey system theory usage of grey systems theory in the supply chain management, credit risk, financial performance evaluation, decision-making process, investment efficiency, energy consumption, etc by Delcea. Unlike other

review articles, this paper focuses solely on economic applications where grey theory has been used successfully, while providing the reader with an overview of the field, as well as new research perspectives.

Another study [28] on grey system theory is reviewed by Pan et al. They have investigated 4859 publications published on Web of Science Core Collection between 1991 and 2018. Bibliometric analysis and visualization on grey system theory is conducted. They provided quantitative insights into the theory's outlook over the past 27 years, while pointing to develop a significant overview for scientists interested in studying and/or employing grey systems theory.

In another review, Jayant et al. [29] conducted a state-of-the-art literature survey to taxonomy the research on the applications and methodologies of grey relational analysis (GRA) in reasonable business environments. They concluded their study with recommendations for future research directions in GRA decision making that is both forward-looking and practically oriented.

Javanmardi et al. [30] provided an updated review of the literature on the roles and impacts of grey system theory based methods in sustainability studies. Although these review papers covered a extensive range of studies of grey system theory, none of them touched on who grey theory could be used to solve make-or-buy problems.

Make-or-buy decisions stand for one of the challenging and very complex tasks facing businesses. Detailed analyses are required to learn all alternatives and to understand short- and long-term company needs. Besides, as the company's needs and market conditions change, a decision that is appropriate for the past can have a completely different meaning for the future. Since the future is unpredictable in all its aspects, the process of make-or-buy decision includes many different types of uncertainties. When the existing literature is examined, it is seen that the uncertainties are mostly ignored, or the process is tried to be handled with a single uncertainty theory regardless of whether it is appropriate. Unlike the literature, in this study, each stage of the make-or-buy decision is handled with the most appropriate uncertainty type. The novelty of this study is that instead of dealing with the whole process of a make-or-buy decision with a chosen uncertainty theory, this paper uses appropriate uncertainty methods in different steps. Another very important role of the study to the literature is that the grey system theory is used for the first time in solving the make-or-buy decision problem. It is expected that these related aspects of the study will help open new territories in decision making in general and make-or-buy decisions in particular.

## MATERIALS AND METHODS

### Methods

Decision-making problems often involve uncertainty due to incomplete, imprecise, or subjective information. In such contexts, different modeling techniques are required

depending on the type of uncertainty encountered. Grey System Theory is used to analyze problems under incomplete and uncertain information proposed by Deng [31]. It classifies systems into three categories: white (fully known), black (completely unknown), and grey (partially known). This theory is especially effective in dealing with systems that involve incomplete data [32]. Grey Relational Analysis (GRA), one of the principal techniques derived from Grey System Theory, is a widely used tool for evaluating alternatives by measuring the similarity between data sequences. It is particularly effective in situations with limited quantitative information. In parallel with grey systems, fuzzy set theory has been used to model uncertainty, particularly when dealing with linguistic or subjective assessments. Unlike conventional fuzzy sets, Intuitionistic Fuzzy Sets (IFS) explicitly account for membership, non-membership, and hesitation degrees, allowing uncertainty associated with human evaluations to be represented more effectively.

In our proposed model, IFS is applied to model the subjective linguistic evaluations provided by decision-makers, while GRA is employed to rank the alternatives based on limited numerical data. This hybrid structure ensures that each type of uncertainty is addressed using the most appropriate method. Fuzzy set theory and grey system theory were developed to analyze systems characterized by uncertainty. Although these theories address different types of uncertainty, both are capable of handling situations involving incomplete or imprecise information. From a technical standpoint, fuzzy set theory addresses conceptual uncertainty using membership functions, often informed by expert experience. For example, the term “young person” is considered fuzzy — while generally understood, its exact age range is difficult to define precisely. Grey system theory was developed as a means to quantify and manage uncertainty. The less information is available, the more effectively grey system theory demonstrates its strength. Grey system characteristics cannot be fully explained through fuzzy set theory, probability, or traditional statistical methods. Grey system theory has found successful application in various fields, including industry, social and ecological systems, economics, transportation, management, education, environmental studies, and defense. Grey Relational Analysis (GRA), an extension of grey system theory, serves as a highly effective tool for analysis in data-limited environments. It measures the degree of similarity between sequences to evaluate relationships under uncertainty.

These concepts and their integration into the proposed decision-making model are explained in the following sections in detail.

### Intuitionistic Fuzzy Set (IFS) Theory

Proposed by Atanassov [33], Intuitionistic Fuzzy Set (IFS) theory extends the fuzzy set framework introduced by Zadeh [34]. By explicitly representing membership, non-membership, and hesitation degrees, IFS offers an effective means of handling uncertainty in decision-making

problems. The basic concepts of IFS relevant to this study are briefly presented below [35].

**Definition 1.** Let  $X$  denote a non-empty universe of discourse and let  $I = [0, 1]$  represent the unit interval. An intuitionistic fuzzy set  $S$  defined on  $X$  is expressed as:

$$S = \{(x, \mu_s(x), \vartheta_s(x)) : x \in X\} \quad (1)$$

where

$\mu_s(x) : X \rightarrow I$ ; specifies the membership degree of element  $x$  in the set  $S$ ;

$\vartheta_s(x) : X \rightarrow I$ ; denotes the non-membership degree associated with the same element.

To ensure a valid intuitionistic fuzzy representation, the membership and non-membership degrees must satisfy the following condition:

$$0 \leq \mu_s(x) + \vartheta_s(x) \leq 1 \quad (2)$$

**Definition 2.** Let  $X$  be a nonempty set and let  $S$  be an IFS defined on  $X$ . The hesitation degree associated with an element is defined as

$$\pi_s(x) = 1 - (\mu_s(x) + \vartheta_s(x)) \quad (3)$$

The index, indicating the hesitation degree about whether an element  $x$  belongs to the set  $S$ , is specified as the index of hesitation. According to the value of  $\pi_s(x)$ , comment can be made about the information of element  $x$ . If the value of  $\pi_s(x)$  is large, information about the element is more certain; if the value of  $\pi_s(x)$  is small, information about the element is more precise. If the value of  $\pi_s(x)$  equals to zero, information about the element is more precise and, in this case, the intuitionistic fuzzy set turns into fuzzy set.

**Definition 3.**  $\alpha = (\mu_s(x), \vartheta_s(x), \pi_s(x))$  is defined as an intuitionistic fuzzy number (IFN), where

$$\mu_s(x) \in [0, 1], \vartheta_s(x) \in [0, 1], 0 \leq \mu_s(x) + \vartheta_s(x) \leq 1 \\ \text{and } \pi_s(x) = 1 - (\mu_s(x) + \vartheta_s(x))$$

**Definition 4.** Consider two intuitionistic fuzzy numbers (IFNs),  $\alpha$  and  $\beta$ , defined on the universe  $X$ , where  $\gamma \in [0, 1]$ . The basic arithmetic operations associated with IFNs are given as follows:

$$\alpha \otimes \beta = (\mu_s(x) * \mu_t(x), \vartheta_s(x) + \vartheta_t(x) - \vartheta_s(x) * \vartheta_t(x)) \quad (4)$$

$$\gamma \alpha = (1 - (1 - (\mu_s(x))^\gamma), ((\vartheta_s(x))^\gamma)) \quad (5)$$

$$\alpha^\gamma = ((\mu_s(x))^\gamma, 1 - (1 - (\vartheta_s(x))^\gamma)) \quad (6)$$

**Definition 5.** Let  $\alpha_i = (\mu_{s_i}, \vartheta_{s_i}, \pi_{s_i})$  be intuitionistic fuzzy numbers, for  $i = 1, 2, \dots, n$ , and  $w = (w_1, w_2, \dots, w_n)$  be weighted vectors of fuzzy numbers. Then, the arithmetic mean is calculated as:

$$AO(\alpha_1, \alpha_2, \dots, \alpha_n) = (1 - \prod_{i=1}^n (1 - \mu_i)^{w_i}, \prod_{i=1}^n \vartheta_i^{w_i}) \quad (7)$$

### GREY RELATIONAL ANALYSIS (GRA)

Grey System Theory was introduced to address decision problems characterized by incomplete and uncertain information. A system with complete information is referred to as a *white system*, whereas a *black system* represents a situation in which no information is available. Grey systems refer to systems in which only part of the available information is known [3,4]. Accordingly, if all the information about a system is known, then the system is seen as a white system; if no information is known; then the system is known as a black system.

The main feature of grey system theory is its ability to make the best use of the limited number of information available. The core feature that distinguishes grey system theory from traditional methods, such as statistics, is that it needs only a very limited amount of data to predict the behavior of an uncertain system. The less information is available, the more effectively grey system theory demonstrates its strength. The amount of information that can be obtained is the most important criterion used to distinguish between grey and non-grey scenarios.

Grey system theory was developed to find solutions to problems involving small sample and weak information. Grey system characteristics cannot be fully explained through fuzzy set theory, probability, or traditional statistical methods. From a technical standpoint, fuzzy set theory addresses conceptual uncertainty using membership functions, often informed by expert experience. Probability and statistics, on the other hand, require special distributions and a reasonably sized sample to make realistic inferences [3].

Grey system theory has found successful applications in various fields, including industry, social and ecological systems, economics, transportation, management, education, environmental studies, and defense [36]. Grey production; grey modeling; grey decision making; grey control; grey

prediction and grey relational analysis are the main fields of grey system theory [37].

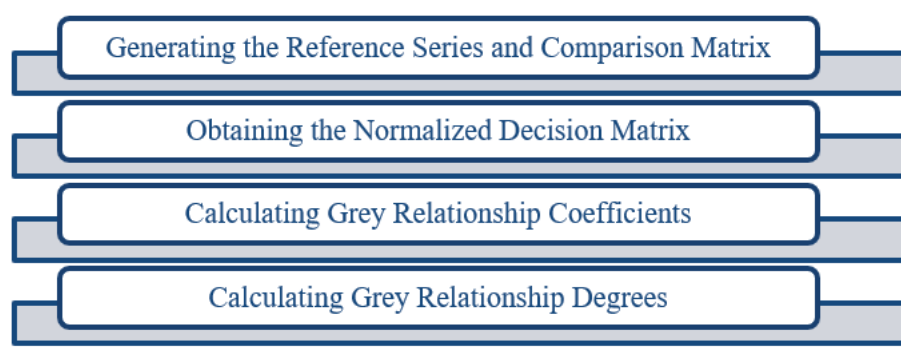
Grey Relational Analysis (GRA) serves as a highly effective tool for analysis in data-limited environments. Based on the ideal series it references, the method of this analysis is developed to reveal the indefinite relationship of this ideal series with other series of concern. The alternative which has a larger grey correlation analysis ratio with the reference series is seen as closer to the ideal [38].

The first step of the grey relational analysis method is to convert the performances of all alternatives into a comparable series. In the second step, the reference (target) series are created by selecting the best values for each criterion. Following the normalization process, the degree of association between the reference sequence and each alternative sequence is evaluated through grey relational coefficients. Lastly, using the resulting coefficients, the degree of grey relation between each comparison series and the reference series is determined. The best option is the alternative with the highest grey relational degree. The procedure of grey relational analysis is shown in Figure 1, where the steps of the grey relational analysis method are explained in detail.

### GENERATING THE REFERENCE SERIES AND COMPARISON MATRIX

Let us consider that there are  $n$  criteria and  $m$  alternatives in a decision problem. And the  $i^{\text{th}}$  alternative is represented as  $Y_i = \{y_{i1}, y_{i2}, \dots, y_{ip}, \dots, y_{in}\}$ , where  $y_{ij}$  denotes the performance value of the  $i^{\text{th}}$  alternative with respect to the  $j^{\text{th}}$  criterion.

Firstly, the creation of the reference series can be obtained by using two methods. In the first method, a reference series can be created with the largest value that a criterion can take. For instance, if the relevant criterion is the level of grades obtained on an exam with a potential total 100 points, then the reference serial value can be taken as 100. The second approach constructs the reference sequence by selecting the optimal value for each evaluation



**Figure 1.** Grey relational analysis procedure.

criterion from the available alternatives. For benefit criteria, the highest observed value is assigned to the reference sequence, whereas for cost criteria, the lowest observed value is selected.

### Obtaining the Normalized Decision Matrix

Normalization is used when the criteria used in the analysis have different units. Normalization is done in two different ways, depending on whether the criteria relate to finding maximum(s) or minimum(s). In the case of maximum, the normalization value is calculated by using Equation (8):

$$x_{ij} = \frac{y_{ij} - \min\{y_{ij}\}}{\max\{y_{ij}\} - \min\{y_{ij}\}}; i = 1, 2, \dots, m \quad (8)$$

and  $j = 1, 2, \dots, n$

In the case of maximum, the normalization value is calculated by using Equation (9):

$$x_{ij} = \frac{\max\{y_{ij}\} - y_{ij}}{\max\{y_{ij}\} - \min\{y_{ij}\}}; i = 1, 2, \dots, m \quad (9)$$

and  $j = 1, 2, \dots, n$

Following the normalization process described in Eq. (8) and/or Eq. (9), all criteria values will be transformed into the interval [0, 1]. The value of  $x_{ij}$  is equal to 1 indicates that alternative  $i$  achieves the best performance for criterion  $j$  [38].

### Calculating Grey Relational Coefficients

After the reference series are defined, to compare the alternatives with each other, grey correlation coefficients are calculated for each alternative.

Let us accept these notations;

$$\Delta_{ij} = |x_{0j} - x_{ij}|,$$

$$\Delta_{\min} = \min_{i,j} \{\Delta_{ij}\},$$

$$\Delta_{\max} = \max_{i,j} \{\Delta_{ij}\},$$

The grey correlation coefficients can be calculated as in Eq. (10).

$$\gamma(x_{0j}, x_{ij}) = \frac{\Delta_{\min} + \varphi \Delta_{\max}}{\Delta_{ij} + \varphi \Delta_{\max}}; i = 1, 2, \dots, m \quad (10)$$

and  $j = 1, 2, \dots, n$

The distinguishing coefficient  $\varphi \in [0, 1]$  in Eq. (10) is used for increasing the range of the grey correlation coefficient. If enough data and experience is not known, this coefficient should be taken 0.5.

### Calculating Grey Relationship Degrees

The grey relation grade is calculated for all alternatives after the grey correlation coefficients are calculated. Taking the average of the obtained grey correlation coefficients is

the common method for obtaining the grey relation grade. Equation (11) is used for this purpose.

$$\delta(X_0, X_i) = \frac{1}{n} \sum_{j=1}^n \gamma(x_{0j}, x_{ij}) \quad (11)$$

The evaluation criteria are not equally important; therefore, different weights are assigned to reflect their relative significance. To account for differences in criterion importance, normalized weights are integrated into the calculation according to Eq. (12). Here,  $w_j$  represents the normalized weight of the  $j^{\text{th}}$  criterion, and  $\sum_{j=1}^n w_j = 1$ .

$$\delta(X_0, X_i) = \frac{1}{n} \sum_{j=1}^n w_j \gamma(x_{0j}, x_{ij}) \quad (12)$$

The alternative with the highest grey relational grade will be the alternative serving the target the most. IFS is employed for modeling subjective linguistic evaluations, while GRA is used to process the limited numerical data available — ensuring alignment between the nature of uncertainty and the method applied.

### Application

This section presents a step-by-step explanation of our proposed integrated MCDM framework for the make-or-buy decision, which combines intuitionistic fuzzy logic and grey relational analysis. And then we will look at its application to a chosen sample problem. In the study, the make-or-buy decision-making process is examined by using real-life data for the components needed by an enterprise operating in the metal sector. The product selected for make-or-buy analysis is the component with the highest usage frequency in the company's operations.

Assume that there are  $l$  decision makers ( $d_1, d_2, \dots, d_l$ ),  $m$  alternatives ( $x_1, x_2, \dots, x_m$ ) and  $n$  criteria ( $c_1, c_2, \dots, c_n$ ) in the decision problem of our concern. The proposed approach includes 8 steps shown in Figure 2.

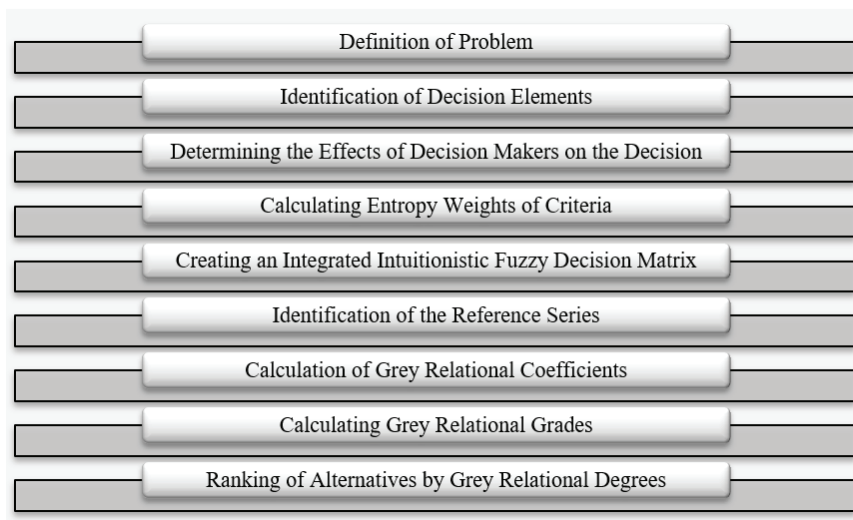
### Stage 1: Definition of Problem

The most critical step in scientific research is the correct formulation of the problem. Accurate solutions cannot be expected from problems that are poorly formulated. For this reason, the first step of any decision problem is the correct formulation of the problem. In this study, it aims to solve the make-or-buy problem for the parts with the highest usage rate in a business operating in the metal sector, based on scientific methods.

### Stage 2: Identification of Decision Elements

Accordingly, the interviews with the company officials, the decision alternatives were determined with details shown in Table 1.

The criteria that are thought to have an effect in the make-or-buy decision were determined as a result of literature research and interviews with company officials [39],[40],[41]. The criteria to be used in the study and explanations regarding these criteria are shown in Table 2.



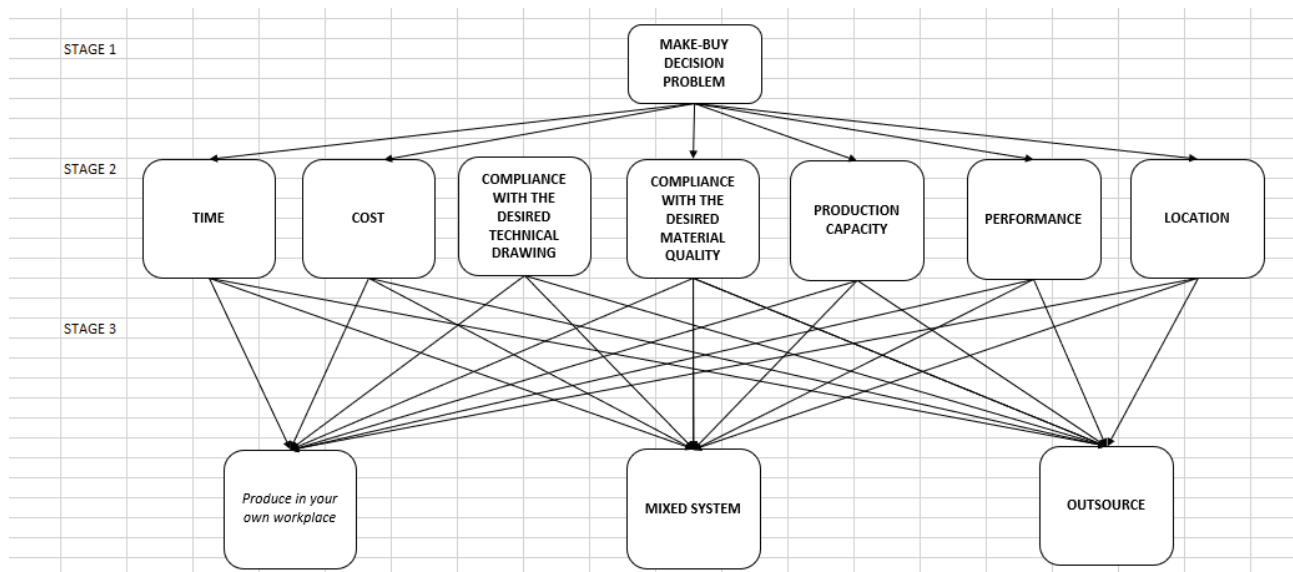
**Figure 2.** The steps of our proposed approach.

**Table 1.** Alternatives considered for the make-or-buy decision

| Code  | Alternatives        | Descriptions                                                                                                                    |
|-------|---------------------|---------------------------------------------------------------------------------------------------------------------------------|
| $x_1$ | In-house production | All operations related to the selected components are performed entirely within the company's own machining department.         |
| $x_2$ | Hybrid system       | 50% of the required quantity is produced internally, while the remaining 50% is outsourced to an external supplier.             |
| $x_3$ | Full Outsourcing    | The entire production process is handled by an external provider, thereby supplementing the company's limited in-house capacity |

**Table 2.** Decision criteria and their descriptions

| Identifier | Criterion                                     | Description                                                                                                                                                                                                                                                               |
|------------|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $c_1$      | Time                                          | Since the production of the final product is affected by every component that makes up the product; the production time of the component is important, and a low amount of time is preferred.                                                                             |
| $c_2$      | Cost                                          | In an increasingly competitive environment, the small amount of cash required to produce a component keeps the company ahead of its rivals. Therefore, the final product cost includes the cost of the component that is decided to be produced in-house or outsourced.   |
| $c_3$      | Compliance with the desired technical drawing | The technical drawing is important for the components produced in the enterprise. Even tolerances are not based on cm; given on an mm basis. For this reason, compliance with the technical drawing is important whether it is produced outside or within the enterprise. |
| $c_4$      | Compliance with the desired material quality  | The company provides raw materials example the products that will be produced by outsiders. In some cases, even if a raw material cannot be sent, material quality is still important.                                                                                    |
| $c_5$      | Production capacity                           | Although there must be a certain production capacity for the in-house production situation within the company, the capacity of the enterprise, which is used as an outside provider, will also enter the total capacity and provide a plus to the production capacity.    |
| $c_6$      | Performance                                   | Preferring a company as an outsourcing means that the work, they have done in the past is suitable for the desired material quality and compliant with the desired technical drawing, time and cost constraints.                                                          |
| $c_7$      | Location                                      | The targeted outsider provider should not be disadvantaged by location.                                                                                                                                                                                                   |



**Figure 3.** The hierarchical structure of the example problem.

Complex decision problems are often represented in a hierarchical structure to facilitate the evaluation process. The hierarchical structure, developed by the decision-makers for solving the most accurate make-or-buy decision problem, is shown in Figure 3.

### Step 3: Determining the Effects of Decision Makers on the Eventual Decision

In most cases, decision makers do not possess the same level of expertise or experience. Several decision makers are in better positions when they are compared to others in terms of experiences, positions, etc. The linguistic expressions, showing the influence levels of the decision makers, and their equivalent intuitionistic fuzzy numbers (IFNs) are shown in Table 3 [42].

Let  $(\mu_d, \vartheta_d, \pi_d)$  be an IFN chosen to present the  $d^{th}$  decision maker's influence level. After that the value  $\delta_d$ , the weight of the  $d^{th}$  decision maker, can be obtained as in Eq. (13) [43, 44]:

$$\delta_d = \frac{\frac{\mu_d}{\mu_d + \vartheta_d}}{\sum_{d=1}^l \frac{\mu_d}{\mu_d + \vartheta_d}} \text{ and } \sum_{d=1}^l \delta_d = 1, \quad (13)$$

$$\delta_d \in [0,1], d = 1,2, \dots, l$$

In the problem of our investigation, 2 decision makers ( $d_1, d_2$ ) who are experts in the make-or-buy decision are assigned by management. The decision maker 1 was defined as *Extremely Important (EI)* due to his past experience, knowledge and position within the company, and the decision maker 2 defined as *Moderately Important (MI)* for the same reasons. The weight scores calculated in accordance with the intuitive fuzzy linguistic variables are shown in Table 4 as in Eq. (13).

The linguistic importance ratings of decision makers and their associated weights are summarized in Table 4.

### Step 4: Calculating Entropy Weights of Criteria

To decide criteria weights, decision makers evaluated the criteria through using the linguistic variables and intuitive fuzzy number equivalents; the outcomes are shown in Table 3. The evaluations made by the decision makers for each criterion are indicated in Table 5.

**Table 3.** Linguistic assessment scale and their corresponding intuitionistic fuzzy numbers (IFNs)

| Linguistic assessment scale | Intuitionistic fuzzy number $(\mu, \vartheta, \pi)$ |
|-----------------------------|-----------------------------------------------------|
| Extremely Important (EI)    | $(0.90, 0.10, 0.00)$                                |
| Highly Important (HI)       | $(0.75, 0.20, 0.05)$                                |
| Moderately Important (MI)   | $(0.50, 0.40, 0.10)$                                |
| Low Importance (LI)         | $(0.35, 0.60, 0.05)$                                |
| Negligibly Important (NI)   | $(0.10, 0.90, 0.00)$                                |

**Table 4.** Linguistic importance ratings of decision makers and their associated weights

| Decision Makers        | Linguistic Terms | Weights |
|------------------------|------------------|---------|
| Decision Maker 1 $d_1$ | EI               | 0,618   |
| Decision Maker 2 $d_2$ | MI               | 0,382   |

**Table 5.** Decision makers' evaluations of the criteria

| Criteria | Decision maker 1 | Decision maker 2 |
|----------|------------------|------------------|
| $c_1$    | EI               | EI               |
| $c_2$    | MI               | HI               |
| $c_3$    | HI               | HI               |
| $c_4$    | HI               | HI               |
| $c_5$    | LI               | MI               |
| $c_6$    | NI               | LI               |
| $c_7$    | MI               | LI               |

Entropy values and entropy weights are calculated for each criterion by using Eq. (14) and Eq. (15), respectively [45]. The calculated values for both criteria are presented in Table 6.

$$H_j = -\frac{1}{n \ln 2} \sum_{d=1}^l [\mu_{dj} \ln \mu_{dj} + \vartheta_{dj} \ln \vartheta_{dj} - (1 - \pi_{dj}) \ln(1 - \pi_{dj}) - \pi_{dj} \ln 2], \quad (14)$$

$$d = 1, \dots, l; j = 1, \dots, n$$

$$w_j = \frac{1 - H_j}{n - \sum_{j=1}^n H_j}, j = 1, 2, \dots, n, \quad \sum_{j=1}^n w_j = 1 \quad (15)$$

**Table 6.** Entropy values of criteria

| Criteria | Entropy Values |
|----------|----------------|
| $H_1$    | 0.087          |
| $H_2$    | 0.190          |
| $H_3$    | 0.156          |
| $H_4$    | 0.156          |
| $H_5$    | 0.215          |
| $H_6$    | 0.147          |
| $H_7$    | 0.222          |

**Table 7.** Entropy values of criteria

| Criterion | Computed Entropy |
|-----------|------------------|
| $w_1$     | 0.157            |
| $w_2$     | 0.137            |
| $w_3$     | 0.138            |
| $w_4$     | 0.138            |
| $w_5$     | 0.128            |
| $w_6$     | 0.138            |
| $w_7$     | 0.125            |

**Table 8.** Linguistic assessment scale and IFNs for the importance of alternatives

| Linguistic Assessment Scale | IFNs ( $\mu, \vartheta, \pi$ ) |
|-----------------------------|--------------------------------|
| Excellent (E)               | (1.00, 0.00, 0.00)             |
| Outstanding (O)             | (0.95, 0.05, 0.00)             |
| Very high (VH)              | (0.85, 0.10, 0.05)             |
| High (H)                    | (0.70, 0.20, 0.10)             |
| Moderate (M)                | (0.50, 0.35, 0.15)             |
| Low (L)                     | (0.35, 0.55, 0.10)             |
| Very Low (VL)               | (0.25, 0.70, 0.05)             |
| Extremely Low (EL)          | (0.10, 0.90, 0.00)             |

Then entropy weights were calculated for each criterion by using Eq. (15). The values obtained are shown in Table 7.

#### Step 5: Creating an Integrated Intuitionistic Fuzzy Decision Matrix

The alternatives for each criterion are evaluated by decision makers with using the linguistic expressions shown in Table 8. Each decision maker's evaluations are formed as an intuitionistic fuzzy decision matrix ( $R^d$ ). The individual decision matrices provided by all decision makers are subsequently aggregated into a unified intuitionistic fuzzy decision matrix ( $R$ ) using the Induced Generalized-Intuitionistic Fuzzy Ordered Weighted Averaging (IG-IFOWA).

**Definition 6.** The IG-IFOWA operator is defined as follows [42]:

$$\begin{aligned} IG-IFOWA_\delta(\langle u_1, \alpha_1 \rangle, \langle u_2, \alpha_2 \rangle, \dots, \langle u_l, \alpha_l \rangle) = \\ (w_1(\alpha_{\sigma(1)})^\lambda \oplus w_2(\alpha_{\sigma(2)})^\lambda \oplus \dots \oplus w_l(\alpha_{\sigma(l)})^\lambda) \\ = \left( \left( 1 - \prod_{d=1}^l \left( 1 - (\mu_{\alpha_{\sigma(d)}})^\lambda \right)^{\delta_d} \right)^{1/\lambda}, \right. \\ \left. 1 - \left( 1 - \prod_{d=1}^l \left( 1 - (1 - \vartheta_{\alpha_{\sigma(d)}})^\lambda \right)^{\delta_d} \right)^{1/\lambda} \right) \end{aligned} \quad (16)$$

where

$\lambda \in (-\infty, +\infty)$ ;

$\alpha_{\sigma(d)}$ : is the value of the IFOWA pair  $\langle u_d, \alpha_d \rangle$  having the  $d^{th}$  largest  $u_d$

$u_d$ : is referred to as the order inducing variable

$\alpha_d$ : is referred to as the IF argument variable

$\delta = (\delta_1, \delta_2, \dots, \delta_l)^T$ : is an associated weight vector with  $\delta_d \in [0, 1]$ ,  $d = 1, 2, \dots, l$  and  $\sum_{d=1}^l \delta_d = 1$ .

Let  $R^d = [r_{ij}^d]_{m \times n}$  be an IF decision matrix of decision maker  $d$ :

$$R^d = [r_{ij}^d]_{m \times n} = \begin{bmatrix} r_{11}^d & r_{12}^d & r_{13}^d & \cdots & r_{1n}^d \\ r_{21}^d & r_{22}^d & r_{23}^d & \cdots & r_{2n}^d \\ r_{31}^d & r_{32}^d & r_{33}^d & \cdots & r_{3n}^d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1}^d & r_{m2}^d & r_{m3}^d & \cdots & r_{mn}^d \end{bmatrix}$$

Equation (16) is employed to aggregate the decision matrix into an intuitionistic fuzzy decision matrix, denoted by ( $R$ ). The resulting aggregated matrix is expressed as follows:

$$R = [r_{ij}]_{m \times n} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & r_{33} & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & r_{mn} \end{bmatrix}$$

The alternative for each criterion had been evaluated by decision makers using linguistic expressions in Table 8. These evaluations are shown in Tables 9 and 10, respectively.

The linguistic evaluation shown in Table 9-10 was transformed into IFNs. Then the IF decision matrices ( $R^d$ ,  $d = 1, 2$ ) of individual decision makers are formed as:

$R^1 =$

$$\begin{matrix} (0.7, 0.2, 0.1) & (0.7, 0.2, 0.1) & (0.35, 0.55, 0.1) & (0.5, 0.35, 0.15) & (0.35, 0.55, 0.1) & (0.5, 0.35, 0.15) & (0.7, 0.2, 0.1) \\ (0.5, 0.35, 0.15) & (0.35, 0.55, 0.1) & (0.35, 0.55, 0.1) & (0.85, 0.1, 0.05) & (0.35, 0.55, 0.1) & (0.85, 0.1, 0.05) & (0.25, 0.7, 0.05) \\ (0.5, 0.35, 0.15) & (0.85, 0.1, 0.05) & (0.85, 0.1, 0.05) & (0.7, 0.2, 0.1) & (0.5, 0.35, 0.15) & (0.85, 0.1, 0.05) & (0.85, 0.1, 0.05) \end{matrix}$$

$R^2 =$

$$\begin{matrix} (0.85, 0.1, 0.05) & (0.85, 0.1, 0.05) & (0.25, 0.7, 0.05) & (0.35, 0.55, 0.1) & (0.25, 0.7, 0.05) & (0.7, 0.2, 0.1) & (0.85, 0.1, 0.05) \\ (0.5, 0.35, 0.15) & (0.7, 0.2, 0.1) & (0.7, 0.2, 0.1) & (0.25, 0.7, 0.05) & (0.5, 0.35, 0.15) & (0.25, 0.7, 0.05) & (0.85, 0.1, 0.05) \\ (0.5, 0.35, 0.15) & (0.7, 0.2, 0.1) & (0.85, 0.1, 0.05) & (0.85, 0.1, 0.05) & (0.7, 0.2, 0.1) & (0.7, 0.2, 0.1) & (0.85, 0.1, 0.05) \end{matrix}$$

Su et al., (2012) suggests using the value  $\lambda = 2$  in Eq. (16). So, we set the parameters  $\lambda = 2$  and  $\delta = (0.618, 0.382)$  by applying the IG-IFOWA operator. Operating the IG-IFOWA operator in Eq. (16), the intuitionistic fuzzy decision matrix is converted to the following combined intuitionistic fuzzy decision matrix.

$R =$

$$\begin{matrix} (0.772, 0.153, 0.075) & (0.596, 0.153, 0.252) & (0.1, 0.598, 0.302) & (0.204, 0.11, 0.385) & (0.1, 0.598, 0.302) & (0.353, 0.28, 0.367) & (0.596, 0.153, 0.252) \\ (0.5, 0.35, 0.15) & (0.287, 0.359, 0.355) & (0.287, 0.359, 0.355) & (0.558, 0.191, 0.251) & (0.174, 0.457, 0.37) & (0.558, 0.191, 0.251) & (0.411, 0.293, 0.296) \\ (0.5, 0.35, 0.15) & (0.55, 0.13, 0.22) & (0.723, 0.1, 0.178) & (0.596, 0.153, 0.252) & (0.353, 0.28, 0.367) & (0.65, 0.13, 0.22) & (0.723, 0.1, 0.178) \end{matrix}$$

**Table 11.** Distance Matrix

|                  | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | min   | max   |
|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\Delta_{01}(j)$ | 1,619 | 1,443 | 1,498 | 1,208 | 1,498 | 0,928 | 0,557 | 0,557 | 1,619 |
| $\Delta_{02}(j)$ | 1,150 | 0,928 | 1,072 | 0,633 | 1,283 | 0,633 | 0,882 | 0,633 | 1,283 |
| $\Delta_{03}(j)$ | 1,150 | 1,520 | 0,378 | 0,557 | 0,928 | 0,480 | 0,378 | 0,378 | 1,520 |

**Table 9.** Evaluation for Alternative of Decision Maker 1

|       | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $x_1$ | H     | H     | L     | M     | L     | M     | H     |
| $x_2$ | M     | L     | L     | VH    | L     | VH    | VL    |
| $x_3$ | M     | VH    | VH    | H     | M     | VH    | VH    |

**Table 10.** Evaluation for alternative of decision maker 2

|       | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $x_1$ | VH    | VH    | VL    | L     | VL    | H     | VH    |
| $x_2$ | M     | H     | H     | VL    | M     | VL    | VH    |
| $x_3$ | M     | H     | VH    | VH    | H     | H     | VH    |

### Step 6: Identification of the Reference Series

The reference series is formed by considering the best values of the criteria. According to the intuitionistic fuzzy decision matrix,  $\alpha^+ = (1, 0, 0)$  can be employed as the reference value for maximum criteria and  $\alpha^- = (0, 1, 0)$  the reference value for minimum criteria. So, the reference series  $x_0$  are created as follows:

$$x_0 = [(0, 1, 0)(0, 1, 0)(1, 0, 0)(1, 0, 0)(1, 0, 0)(1, 0, 0)(1, 0, 0)]_{1 \times 7}$$

### Step 7: Calculation of Grey Relational Coefficients

The grey relational coefficients are calculated by using Eq. (10). While determining the distance matrix between  $x_0$  and  $x_p$ , the hamming distance value is used. Distance matrix is calculated by using the absolute difference between the membership and non-membership degrees is computed

**Table 12.** The grey relational coefficient matrix

|                  | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ |
|------------------|-------|-------|-------|-------|-------|-------|-------|
| $\gamma_{01}(j)$ | 0,489 | 0,527 | 0,514 | 0,589 | 0,514 | 0,683 | 0,869 |
| $\gamma_{02}(j)$ | 0,606 | 0,683 | 0,631 | 0,823 | 0,567 | 0,823 | 0,702 |
| $\gamma_{03}(j)$ | 0,606 | 0,510 | 1,000 | 0,869 | 0,683 | 0,921 | 1,000 |

**Table 13.** Final scores of the alternatives

| Alternatives | Grey relational scores |
|--------------|------------------------|
| $x_1$        | 0,598                  |
| $x_2$        | 0,691                  |
| $x_3$        | 0,798                  |

**Table 14.** Comparison of methods

| Alternatives        | IF-TOPSIS | IF-GRA | IF-WASPAS |
|---------------------|-----------|--------|-----------|
| Full outsourcing    | 1         | 1      | 1         |
| In-house production | 3         | 3      | 3         |
| Hybrid system       | 2         | 2      | 2         |

according to Eq. (17). Accordingly, the distance matrix obtained is shown in Table 11:

$$\Delta_{0i}(j) = |\mu_{ij} - \mu_{0j}| + |\vartheta_{ij} - \vartheta_{0j}|, j=1,2,3,\dots,n \quad (17)$$

The distinguishing coefficient factor ( $\varphi$ ) is set to 0.5 and the grey relational coefficients are computed by applying Eq. (10). The values obtained are shown in Table 12:

#### Step 8: Calculating Grey Relational Grades

The grey relational grades are transformed into the grey relational coefficients by using Eq. (12). Grey relational grades of the individual alternatives are as shown in Table 13.

#### Step 9: Ranking of Alternatives by Grey Relational Degrees

The alternatives are ordered from the highest to the lowest grey relational grade, with the alternative obtaining the highest value being identified as the most suitable option. The three alternatives are ranked as follows: Full outsourcing; Hybrid System; and In-house production ( $\gamma_3 > \gamma_2 > \gamma_1$ ). Thus, the most appropriate alternative is outsourcing.

As discussed previously, employing multiple methods is generally considered beneficial for analyzing multi-criteria decision-making (MCDM) problems. The intuitionistic fuzzy TOPSIS (IF TOPSIS) [46] method and the intuitionistic fuzzy WASPAS [47] are used to test the accuracy of our results. The reason why TOPSIS and WASPAS methods are preferred rather than other MCDM methods is that these methods can be used by using the same data without changing the criterion weights.

Table 14 compares the rankings produced by the IF-TOPSIS and IF-WASPAS methods with those generated by the proposed approach. When the rankings in Table 14 are analyzed, it can be seen that all methods produce same

rankings, suggesting outsourcing as the best option. Based on these results we can suggest that our proposed method can be used safely for the solution of our problem.

Then, the criterion weights are changed and the alternative to be selected is examined. The rankings obtained according to the changing criteria weights are as seen in Table 15.

It can be seen that in the current situation, full outsourcing alternative has been determined as the best alternative. In case the criteria weights are equal; there is no change in the selection order of the alternatives. If the criteria are the most important criteria one by one, the rankings are as shown in the Table 15.

## RESULTS AND DISCUSSION

In this study, the make-or-buy decision is discussed in a company operating in the metal sector. The literature review shows that different types of uncertainties are often overlooked, and the entire decision-making process is typically addressed using only a single uncertainty theory. However, the levels of influences of individuals on the ultimate decision, criterion weights and evaluations of alternatives on the basis of the chosen criteria generally include different types of uncertainties. Unlike prior studies, this work addresses each stage of the make-or-buy decision using the most appropriate uncertainty model for that stage. As for the adoption of methods, both intuitionistic fuzzy set theory and grey relational analysis were chosen based on the thought that they are most suitable for the structure of the problem. A common approach used by various authors to solve multi-criteria decision problems is to use more than one method jointly. In this direction, we used TOPSIS and WASPAS methods to test the accuracy of our results obtained in this study. As a result, it was seen that all three methods considered, produced the same results. The consistency among the three methods used in this study serves

**Table 15.** The sensitivity analysis of criterion weights

|                                                             | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | First Selection  | Second Selection    | Third Selection     |
|-------------------------------------------------------------|-------|-------|-------|-------|-------|-------|-------|------------------|---------------------|---------------------|
| <b>Current situation</b>                                    | 0.16  | 0.14  | 0.14  | 0.15  | 0.14  | 0.14  | 0.13  | Full outsourcing | Hybrid system       | In-house production |
| <b>Equality of criteria weights</b>                         | 1/7   | 1/7   | 1/7   | 1/7   | 1/7   | 1/7   | 1/7   | Full outsourcing | Hybrid system       | In-house production |
| <b>First criterion being the most important criterion</b>   | 1     | 0     | 0     | 0     | 0     | 0     | 0     | Full outsourcing | In-house production | Hybrid system       |
| <b>Second criterion being the most important criterion</b>  | 0     | 1     | 0     | 0     | 0     | 0     | 0     | Hybrid system    | Full outsourcing    | In-house production |
| <b>Third criterion being the most important criterion</b>   | 0     | 0     | 1     | 0     | 0     | 0     | 0     | Hybrid system    | In-house production | Full outsourcing    |
| <b>Fourth criterion being the most important criterion</b>  | 0     | 0     | 0     | 1     | 0     | 0     | 0     | Hybrid system    | In-house production | Full outsourcing    |
| <b>Fifth criterion being the most important criterion</b>   | 0     | 0     | 0     | 0     | 1     | 0     | 0     | Hybrid system    | In-house production | Full outsourcing    |
| <b>Sixth criterion being the most important criterion</b>   | 0     | 0     | 0     | 0     | 0     | 1     | 0     | Hybrid system    | In-house production | Full outsourcing    |
| <b>Seventh criterion being the most important criterion</b> | 0     | 0     | 0     | 0     | 0     | 0     | 1     | Hybrid system    | Full outsourcing    | In-house production |

as strong evidence that our proposed method can be safely applied in make-or-buy decisions.

The problem handled within the scope of this study represents a real-life problem that is often seen in the operation of a company in the metal sector. Each make-or-buy decision stands for a decision-making process that generally includes different kinds of uncertainties, such as sectorial uncertainties and subjective evaluations of decision makers. This study demonstrates that grey relational analysis and intuitionistic fuzzy set theory can be effectively combined to handle diverse forms of uncertainty. In this context, our proposed method is evaluated to see how it can be used successfully in solving similar problems involving different types of uncertainties.

Our findings are both consistent with and supported by previous studies in literature. For example, Cheshmberah

et al. [15] addressed the make-or-buy decision in the aerospace industry using a fuzzy TOPSIS approach and also concluded that outsourcing was the most effective strategy, particularly under uncertain conditions—similar to our result in the metal manufacturing context. Serrano et al. [30] emphasized the critical role of multi-criteria decision analysis in sourcing strategies, reinforcing the importance of structured evaluation methods when dealing with complex decisions. In terms of methodology, Padillo and Diaby [6] proposed a composite programming-based multi-objective model for make-or-buy decisions but did not account for varying types of uncertainty at different decision stages. Our integrated approach, which employs intuitionistic fuzzy sets (IFS) to handle subjectivity and grey relational analysis (GRA) to deal with incomplete data, goes beyond these earlier models by addressing uncertainty

more flexibly and comprehensively. Thus, our study not only confirms existing conclusions about the advantages of outsourcing in certain industrial contexts but also introduces a novel, hybrid methodology that improves decision reliability under uncertainty.

In addition to the consistent rankings produced across three different multi criteria decision methods (IFS-GRA, IF-TOPSIS, and IF-WASPAS), we observed that removing either intuitionistic fuzzy set theory or grey relational analysis from the process led to more unstable rankings and increased sensitivity in results under changing criterion weights. These findings suggest that the combination of intuitionistic fuzzy numbers and grey relational analysis enhances the robustness and consistency of decision-making under uncertainty. Furthermore, using the appropriate uncertainty method at each step offers a more realistic representation of how information is processed in actual industrial environments.

## CONCLUSION

With increasing levels of competition and accelerating speed of development in all economic sectors, searches for efficient efforts for businesses to meet elevating customer demands and expectations and to produce better customer satisfactions force enterprises to use more effective strategies in their activities. In today's competitive environment, the make-or-buy decision has emerged as a key strategic issue. The make-or-buy decision is a strategic decision that contributes significantly to the profitability of the company. Outsourcing decisions are often made solely based on cost, neglecting other crucial factors such as quality and delivery reliability. However, in addition to costs, many factors such as delivery reliability, technical capacity, and the costs of financial arrangements should be taken into account in the make-or-buy decisions.

In this study, the make-or-buy decision analysis for the components needed by an enterprise operating in the metal industry is examined. Due to the uncertainty naturally existing in decision makers' evaluations regarding the employed criteria and the subjectivity of these criteria, the intuitionistic fuzzy set theory and grey relational analysis methods were used jointly. The findings indicate that the integration of intuitionistic fuzzy set theory and grey relational analysis is effective in managing uncertainty. Applying multiple MCDM techniques is generally considered a good practice for validating decision-making results. To test the accuracy of our results, the intuitionistic fuzzy TOPSIS (IF TOPSIS) and WASPAS methods were used with the same data that was underneath our proposed approach. Analysis of the resulting rankings shows that all three methods produced consistent outcomes. All three methods suggest outsourcing as the best option. These results support the robustness and reliability of our proposed method in solving make-or-buy problems when there are many types of uncertainties involved in the process. In addition, the method's flexible

structure allows it to be adapted to various industries facing similar uncertainty-driven decisions, including automotive, electronics, and logistics sectors. The novelty of this study is to use appropriate methods of uncertainty in different steps of analysis instead of employing the same uncertainty theory throughout the whole selection process. In addition, the use of grey theory for the first time in make-or-buy decisions is considered as an important contribution of the study to literature. Moreover, the robustness of the results under multiple scenarios and the consistent rankings across different multicriteria decision methods confirm the reliability and added value of the proposed integrated approach. The consistency of the results across multiple methods confirms the reliability of our integrated framework and its alignment with the decision-making needs defined at the beginning of the study.

## AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

## DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

## CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## ETHICS

There are no ethical issues with the publication of this manuscript.

## STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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