



Research Article

Entropy generation analysis of hydromagnetic flow on nonlinearly radiating Walters' B nanofluid with heat generation and viscous dissipation

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ABSTRACT

The paper analyzed the relevance of irreversibility analysis amidst the nonlinear radiative impact on a Walters' B nano-liquid model. The mechanism of the model, which acts as a catalyst in understanding the behaviours of the associated thermophysical properties in science and engineering fields, is examined through a horizontal stretching surface, taken into account the volumetric heat source/sink terms and viscous dissipation effects. The geometric flow model expressed in partial differential equations are reconstructed into ordinary differential equations by relevant conversion quantities and computed through the Galerkin Weighted Residual Method. The work is validated by the Spectral Collocation Method and the Wolfram language function NDSolve through the average deviation. The significance of the indicated parameters are interpreted through graphs and tables. The novel findings substantiate that the magnitude of Brinkman number invoke higher viscous heating through molecular conduction around the system consequently improves entropy production. This is vital in polymer processing, alongside the interactive application of the Weissenberg number. Also, variations in the heat production and convective heat parameters contribute significantly as heat distributing functions across the system which made way for the temperature distribution effect to the quiescent fluid. These parameters have a useful contribution, leading to the desiccation of the components in the engineering sectors.

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INTRODUCTION

An investigation into thermodynamic fluid model in science and engineering spheres have gained remarkable attention in the literature due to their undeniable usefulness in the fields. Little among these applications are: glass fiber, polymer extrusion, crystal growth and cooling of nuclear reactors, among others. Counting on its wide application in various disciplines, it has been broadly pronounced within the research circles. Jonnadula et al. [1] examined the mechanics of radiative heat emission on hydromagnetic fluid alongside destructive chemical reactions whose impact decays the concentration field. This similar report is buttressed by Javed et al. [2] while investigating with melting heat, the activation energy influence on chemically reacting magnetohydrodynamics flow. Chatterjee and Gupta [3] reported suppressive influence over the magnetic intensity with the contributing strength to the thermal field. Accounting for viscous dissipation, Swain et al. [4] explored Joule's heating influence in an electrically conducting fluid. The finding emphasized the aftermath impact of a rising thermal diffusivity over momentum diffusivity as strengthening factor to the temperature field. Akinbo and Olajuwon [5-7] investigated both the Newtonian model and non-Newtonian model on hydromagnetic flow, which has a useful application in power engineering. Good agreement is observed between the Variational Iteration Method and Akbari-Ganji's method on the non-Newtonian model, involving Micropolar fluid with a bounded domain by Gholinia et al [8] who also asserted that Microrotation is a decreasing function of coupling parameter. Mohyud-Din et al. [9] reported that surface heat and mass transport can be regulated with the passive control model while studying the unsteady case model of heat and mass transport with nanoparticles. Özen and Derun [10] reported inhibited hydration with the mixture of nanoparticles and borogypsum while invoking various nanoparticles on borogypsum's properties involving mortar. Nanoparticles have gained increasing attention within technical and scientific sectors, due to their higher thermal conductivity. This property offers a significant advantage in thermal performance compared to conventional fluids. Consequently, their potential has been explored in existing literature by Gupta [11], Gupta and Saxena [12], Gupta and Sharma [13], Gupta and Dixit [14], and Gupta et al. [15] among other researchers in the field.

In fluid dynamics, heat transfer is generally inevitable with its multiples usages in science and engineering fields. On this note, analyzing irreversibilities plays an indispensable role in analyzing the amount of entropy generated during the heat transfer. Thus, the ideas of entropy generation help to deduce irreversibility identical to friction and heat dissipation among others. It is a useful tool for assessing the effectiveness of different thermal systems, notably engines and heat exchangers. Based on the forward-going importances in the related fields of study, entropy

generation has been investigated in the literature, among the notable contributions is the examination of irreversibility analysis of thermophysical quantities on Newtonian model involving hydromagnetic flow as reported by many researchers in the literature [16-21], where the frictional heating irreversibility due to the magnetic effect strengthens the entropy production. In addition, Non-Newtonian models exhibit viscosities that change in response to variations in shear rate.

leading to more complex entropy generation influenced by fluid rheology. Entropy generation in non-Newtonian fluid aids better representation of real-world applications like blood flow, polymer formation, improving system efficiency and design through more accurate thermodynamic predictions. Investigations of entropy generation on non-Newtonian include but not limited to the studies of micropolar fluid, power law fluids, Eyring-Powell fluid, Walters' B fluid, second grade fluid, as reported by [22-26].

Admitting the useful contributions of other researchers in the literature and industrial applications of entropy generation in the field, the knowledge of irreversibility analysis on the model of non-linear viscous fluid is still very shallow. Keeping in mind the existing work in the literature, entropy generation analysis is yet to be studied with the collective contributions of viscous heating, internal heat production/sink and nonlinear radiation on hydromagnetic Walters' B nanofluid under the benchmark of passive nanoparticles alongside convective heat domain. The objective here is to examine the significance of the embedded parameters associated with the model for industrial applications. Thus, this research is particularly valuable for polymer applications and helps in optimizing the cooling systems, enhancing energy efficiency and controlling thermal management in various industrial applications.

MATHEMATICAL FORMULATION

Here, we look into irreversibility analysis on a steady model, involving an incompressible Walters' B viscoelastic nanofluid through a two-dimensional boundary layer on an expanding sheet. The dynamics of the fluid are examined with magnetic strength B_0 . In addition, the induced magnetic field is considered unimportant due to the diminutive magnetic Reynolds [27]. The non-Newtonian liquid radiates nonlinearly amidst the heat source/sink effects and viscous energy dissipation under the influence of convective heat domain. The fluid with the nanoparticles is considered in the region $y \geq 0$ while the x-axis corresponds to the parallel direction of the stretching sheet with velocity $u_w(x) = ax$, $a > 0$ as indicated in Figure 1. The convective fluid temperature is denoted by T_f , while concentration at the wall is represented by C_w with their respective corresponding far fields, which are specified as T_∞ and C_∞ .

Subject to the mathematical formulation stated above, the resulting equations of the model are presented in

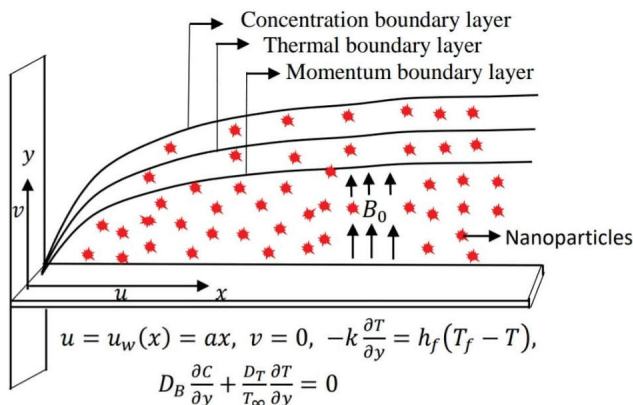


Figure 1. Model geometry.

accordance with Qayyum et al. [28], Saini et al. [29] and Hayat et al. [30] as follows

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_f \frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho} \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{(\rho C)_f} \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{(\rho C)_f} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{(\rho C)_f} \frac{\partial q_r}{\partial y} - \frac{k_0}{(\rho C)_f} \left[\frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial y} \right)^2 + u \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial v}{\partial y} \right] + \frac{Q_0}{(\rho C)_f} (T - T_\infty) \quad (3)$$

Concentration Equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

and the corresponding boundary conditions are

$$u = u_w(x) = ax, \quad v = 0, \quad -k_f \frac{\partial T}{\partial y} = h_f(T_f - T), \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \quad (5)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{at } y \rightarrow \infty \quad (6)$$

The velocities (u, v) are utilized in the directions of (x, y) while other notations are contained at the nomenclatures.

In agreement with Roseland's approximation, heat transfer via thermal radiation is designated as $q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$. Here, the symbol σ^* reflects the Stefan-Boltzmann constant, while k^* corresponds to the absorption coefficient.

(σ^*, k^*) signifies Stefan-Boltzmann constant and absorption coefficient respectively. Obviously, Eq. (1) becomes zero by the application of the stream function which simplifies fluid flow analysis by automatically satisfying the continuity equation and reducing the governing equations into dimensionless and solvable form [28, 31]. Thus, applying $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ via the introduction of conversion variables listed below

$$\psi = x(av_f)^{1/2} f(\xi), \quad \xi = y \left(\frac{a}{\nu_f} \right)^{1/2}, \quad \theta(\xi) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\xi) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

into Eqs. (2-6), for ν_f being the kinematics viscosity, produce the dimensionless equations of the form

$$\frac{d^3 f(\xi)}{d\xi^3} + f(\xi) \frac{d^2 f(\xi)}{d\xi^2} - \left(\frac{df(\xi)}{d\xi} \right)^2 + We \left[\left(\frac{d^2 f(\xi)}{d\xi^2} \right)^2 - 2 \frac{df(\xi)}{d\xi} \frac{d^3 f(\xi)}{d\xi^3} + f(\xi) \frac{d^4 f(\xi)}{d\xi^4} \right] - Mn \frac{df(\xi)}{d\xi} = 0 \quad (8)$$

$$\frac{1}{Pr} \left(1 + Rd(1 + (\theta_w - 1)\theta(\xi))^3 \frac{d\theta(\xi)}{d\xi} \right)' - WeEc \left[\frac{df(\xi)}{d\xi} \left(\frac{d^2 f(\xi)}{d\xi^2} \right)^2 - f(\xi) \frac{d^2 f(\xi)}{d\xi^2} \frac{d^3 f(\xi)}{d\xi^3} \right] + Ec \left(\frac{d^2 f(\xi)}{d\xi^2} \right)^2 + f(\xi) \frac{d\theta(\xi)}{d\xi} + Nb \frac{d\theta(\xi)}{d\xi} \frac{d\phi(\xi)}{d\xi} + Nt \left(\frac{d\theta(\xi)}{d\xi} \right)^2 + Q\theta(\xi) = 0 \quad (9)$$

$$\frac{1}{Sc} \left(\frac{d^2 \phi(\xi)}{d\xi^2} + \frac{Nt}{Nb} \frac{d^2 \theta(\xi)}{d\xi^2} \right) + f(\xi) \frac{d\phi(\xi)}{d\xi} = 0 \quad (10)$$

under the following domains

$$f(\xi) = 0, \quad \frac{df(\xi)}{d\xi} = 1, \quad \frac{d\theta(\xi)}{d\xi} = -Bi(1 - \theta(\xi)), \quad Nb \frac{d\phi(\xi)}{d\xi} + Nt \frac{d\theta(\xi)}{d\xi} = 0; \quad \text{at } \xi = 0 \quad (11)$$

$$\frac{df(\xi)}{d\xi} \rightarrow 0, \quad \theta(\xi) \rightarrow 0, \quad \phi(\xi) \rightarrow 0; \quad \text{at } \xi \rightarrow \infty \quad (12)$$

$$\text{Here, } Mn = \frac{\sigma B_0^2}{\rho a}, \quad We = \frac{k_0 a}{\mu}, \quad Rd = \frac{4\sigma^* T_\infty^3}{k_f k^*},$$

$$Pr = \frac{\mu C_p}{k_f}, \quad Nt = \frac{(\rho C)_p D_T (T_f - T_\infty)}{T_\infty \nu_f (\rho C)_f},$$

$$Ec = \frac{U_w^2}{C_f (T_f - T_\infty)}, \quad Nb = \frac{(\rho C)_p D_B (C_w - C_\infty)}{\nu_f (\rho C)_f},$$

$$Q = \frac{Q_0}{a(\rho C)_f}, \quad Bi = \frac{h_f}{k_f} \sqrt{\frac{\nu_f}{a}}, \quad Sc = \frac{\nu_f}{D_B}, \quad \theta_w = \frac{T_f}{T_\infty}$$

The notations Mn connotes magnetic parameter, We portrays Weissenberg number, Rd depicts radiation parameter, Pr reflects Prandtl number, Nt stands for thermophoresis parameter, Ec signifies Eckert number, Nb denotes Brownian motion, Q represents heat generation/absorption, Bi illustrates convective heat parameter, Sc stands for Schmidt number, θ_w denotes temperature ratio parameter. The engineering applications of the study triggers the consideration of

$$Cf = \frac{\tau_w}{\frac{1}{2} \rho u_w^2}, \quad Nu = \frac{x q_w}{k_f (T_f - T_\infty)}, \quad Sh = \frac{x q_m}{D_B (C_w - C_\infty)} \quad (13)$$

where (τ_w, q_w, q_m) correspond to the skin friction/shear stress, surface thermal flux, and surface mass flux, respectively

$$\begin{aligned} \tau_w &= \left[\mu \frac{\partial u}{\partial y} - k_0 \left(u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} + 2 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right) \right]_{y=0}, \\ q_w &= \left[-k_f \frac{\partial T}{\partial y} \right]_{y=0} = \left[-\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \right]_{y=0}, \quad q_m = \left[-D_B \frac{\partial C}{\partial y} \right]_{y=0} \end{aligned} \quad (14)$$

Substitution of Eq. (14) in Eq. (13) with the application of Eq. (7) result in

$$\begin{aligned} Cf Re_x^{\frac{1}{2}} &= (1 - 3We) f''(\xi), \\ Nu Re_x^{-\frac{1}{2}} &= -((1 + Rd(1 + (\theta_w - 1)\theta)^3 \theta'(\xi))), \\ Sh Re_x^{-\frac{1}{2}} &= \phi'(\xi) \quad \text{at } \xi = 0 \end{aligned} \quad (15)$$

for the Skin-friction, Nusselt and Sherwood numbers while $Re_x = ax^2/\nu$ stands local Reynolds number

Entropy Generation

This concern with the irreversibility that occurs due to friction and heat transport, which plays a substantial role in analyzing the performances of thermal systems in engines and heat exchangers, among others. On this note, investigating its rate in a system is of great importance to Science and Engineering related disciplines. In accordance with Eqs. (2-4), the entropy generation suitable for the foregoing model is formulated as [26, 28]

$$\begin{aligned} E_G &= \frac{k_f}{T^2} \left[1 + \frac{16\sigma^* T_\infty^3}{3k_f k^*} \right] \left(\frac{\partial T}{\partial y} \right)^2 + \frac{k_0}{T} \left[\frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial y} \right)^2 + u \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial v}{\partial y} \right] \\ &\quad + \frac{RD_B}{C_\infty} \left(\frac{\partial C}{\partial y} \right)^2 + \frac{RD_B}{T_\infty} \left(\frac{\partial T}{\partial y} \right) \left(\frac{\partial C}{\partial y} \right) + \frac{\sigma B_0^2}{\rho} u^2 \end{aligned} \quad (16)$$

Heat transfer irreversibility Viscous dissipation irreversibility Diffusive irreversibility Joule heating irreversibility

By the introduction of transformation techniques, Eq. (16) becomes

$$\begin{aligned} E_G &= \frac{E_G^*}{E_0} = Re(1 + Rd(1 + (\theta_w - 1)\theta(\xi)))^3 (\theta'(\xi))^2 \alpha_1 \\ &\quad + \frac{Re \lambda \beta \phi'(\xi)}{(\theta_w - 1)} \left(\theta'(\xi) + \frac{\lambda \phi'(\xi)}{(\theta_w - 1)} \right) \\ &\quad + \frac{Re Br}{(\theta_w - 1)^2} ((f''(\xi))^2 + Mn(f'(\xi))^2) \\ &\quad + \frac{Re Br We}{(\theta_w - 1)^2} (f(\xi) f''(\xi) f'''(\xi) - f'(\xi) (f''(\xi))^2) \end{aligned} \quad (17)$$

while the associated Bejan Number, which defines the reversibility ratio of thermal to the combined irreversibility in Eq. (17) is defined as

$$\begin{aligned} Be &= \frac{(1 + Rd(1 + (\theta_w - 1)\theta(\xi)))^3 (\theta'(\xi))^2 \alpha_1}{(1 + Rd(1 + (\theta_w - 1)\theta(\xi)))^3 (\theta'(\xi))^2 \alpha_1 + \frac{\beta \lambda \phi'(\xi)}{(\theta_w - 1)} \left(\theta'(\xi) + \frac{\lambda \phi'(\xi)}{(\theta_w - 1)} \right)} \\ &\quad + \frac{Re Br}{(\theta_w - 1)^2} ((f''(\xi))^2 + Mn(f'(\xi))^2) + \frac{Re Br We}{(\theta_w - 1)^2} f(\xi) f''(\xi) f'''(\xi) \\ &\quad - \frac{Re Br We}{(\theta_w - 1)^2} f'(\xi) (f''(\xi))^2 \end{aligned} \quad (18)$$

$$\text{where } E_0 = \frac{k_f (\Delta T)^2}{x^2 T_\infty^2}, \quad Br = \frac{\mu u_w^2(x)}{k_f T_\infty}, \quad \alpha_1 = \frac{\Delta T}{T_\infty},$$

$$\beta = \frac{C_\infty RD_B}{k_f}, \quad \lambda = \frac{\Delta C}{C_\infty}$$

stand for the characteristic rate of entropy generation, Brinkman number, non-dimensional temperature difference, and a constant parameter respectively.

Solution of the Problem

The transformed Eqs. (8-10) together with the boundary condition in Eqs. (11-12) are executed, using the Galerkin Weighted Residual Method (GWRM). The method offers spectral accuracy and very efficient in providing solutions to the model of nonlinear differential equations involving unbounded domain. Given reference to the method of solution with the Eqs. (11-12) the trial functions are defined as [32]

$$f(\xi) = \sum_{i=0}^{12} p_i e^{-\frac{i\xi}{4}}, \quad \theta(\xi) = \sum_{i=1}^{13} q_i e^{-\frac{i\xi}{4}}, \quad \phi(\xi) = \sum_{i=1}^{13} r_i e^{-\frac{i\xi}{4}}. \quad (19)$$

Invoking Eq. (19) in the domain (11) at $\xi = 0$ produce

$$\begin{aligned}
 f(\xi=0) &= 0 \Rightarrow \sum_{i=0}^{12} p_i = 0, \quad f'(\xi=0) = 1 \Rightarrow \sum_{i=0}^{12} -\frac{ip_i}{4} = 1 \\
 \theta'(\xi=0) &= -Bi(1 - \theta(\xi=0)) \Rightarrow \sum_{i=1}^{13} -\frac{iq_i}{4} - Bi\left(-1 + \sum_{i=1}^{13} q_i\right) \quad (20) \\
 Nb\theta'(\xi=0) + Nt\phi'(\xi=0) &\Rightarrow Nt \sum_{i=1}^{13} -\frac{iq_i}{4} + Nb \sum_{i=1}^{13} -\frac{ir_i}{4} = 0
 \end{aligned}$$

The far field conditions in Eq. (12) is executed at three hundred thousand of maxiterations via 13.2 Wolfram Mathematica whose residual functions R_f , R_θ , and R_ϕ are taken in multiplication with the $e^{-\frac{i\xi}{4}}$, $i \in \mathbb{Z}$ and integrated under the associated conditions via 13.2 Wolfram Mathematica.

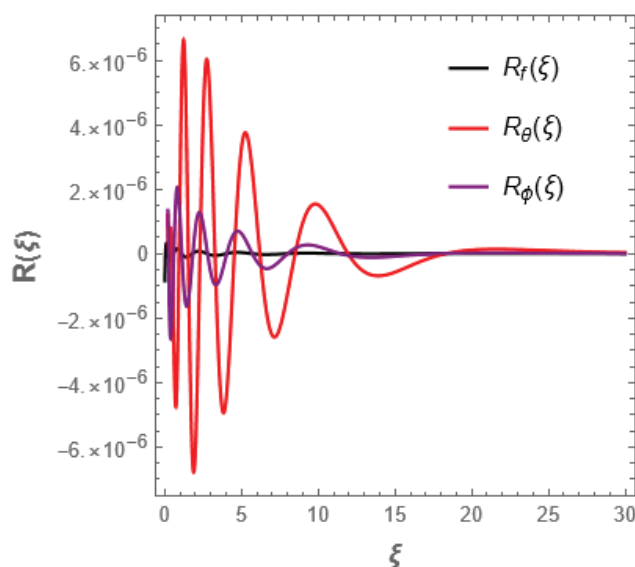


Figure 2. Residual error $R(\xi)$.

In order to ascertain the successful implementation of the computation of this model, the expressions R_f , R_θ , and R_ϕ for residual terms are plotted in Figure 2. The error/residual functions, which are minimized for all positive values, displayed a very small (negligible) residual errors, indicating that the GWRM is very efficiency in finding solutions to nonlinear differential equations within the designated domain.

RESULTS AND DISCUSSION

The model Eqs. (8-10) under the domain of Eqs. (11-12) produce the behaviours of each varying parameter for velocity, temperature and concentration profiles on Walters' B fluid by holding the following parameters $Rd = 0.5$, $Pr = 1.2$, $\theta_w = 0.5$, $Nt = 0.1$, $Nb = 2$, $Q = 0.1$, $Sc = 0.62$, $Bi = 2$, $Ec = 0.1$ constant upon varying each parameter.

Table 1-2 demonstrated that the model perfectly agreed with the existing literature via the conditions imposed by Khan and Pop [33] and Makinde and Aziz [34]. The results are also validated, using numerical integration Mathematica's NDSolve function, computed based on the outputs of the GWRM and considered the average deviation between the functions over a representative interval, in the case, $L = 10$. The average deviations for the velocity distribution, thermal and concentration fields are defined as follows:

$$\Delta f'(\xi) = \frac{1}{L} \int_0^L |(f'(\xi))_1 - (f'(\xi))_2| d\xi \quad (21)$$

$$\Delta \theta(\xi) = -\frac{1}{L} \int_0^L |(\theta(\xi))_1 - (\theta(\xi))_2| d\xi \quad (22)$$

Table 1. Assessing the validity of the Nusselt and Sherwood numbers at $Pr = 10$, $Sc = 10$, $Rd = 0$, $We = \theta_w = Q = Ec = Mn = 0$

Khan and Pop [33]						Present Result					
(a)											
$Nb = 0.1$		$Nb = 0.2$		$Nb = 0.3$		$Nb = 0.1$		$Nb = 0.2$		$Nb = 0.3$	
Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$
0.1	0.9524	0.1	0.5056	0.1	0.2522	0.1	0.95242	0.1	0.50565	0.1	0.25224
0.2	0.6932	0.2	0.3654	0.2	0.1816	0.2	0.69320	0.2	0.36542	0.2	0.18161
0.3	0.5201	0.3	0.2731	0.3	0.1355	0.3	0.52013	0.3	0.27314	0.3	0.13554
(b)											
$Nb = 0.1$		$Nb = 0.2$		$Nb = 0.3$		$Nb = 0.1$		$Nb = 0.2$		$Nb = 0.3$	
Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$	Nt	$-\theta'(0)$
0.1	2.1294	0.1	2.3819	0.1	2.4100	0.1	2.12941	0.1	2.38192	0.1	2.41001
0.2	2.2740	0.2	2.5152	0.2	2.5150	0.2	2.27404	0.2	2.51520	0.2	2.51503
0.3	2.5286	0.3	2.6555	0.3	2.6088	0.3	2.52861	0.3	2.65551	0.3	2.60881

Table 2. Assessing the validity of the Nusselt and Sherwood numbers at $Nt = Nb = 0.5, Rd = We = \theta_w = Q = Ec = Mn = 0$

<i>Pr</i>	<i>Bi</i>	<i>Sc</i>	Makinde and Aziz [34]		Present findings	
			$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0.1	5	0.0789	1.5477	0.0789	1.5477
2			0.0806	1.5554	0.0806	1.5554
5			0.0735	1.5983	0.0735	1.5983
	1.0		0.1476	1.6914	0.1476	1.6914
	10.0		0.1550	1.7122	0.1550	1.7122
	100		0.1557	1.7144	0.1557	1.7144
		10	0.0647	2.3920	0.0647	2.3920
		15	0.0600	2.9899	0.0600	2.9899
		20	0.0570	3.4881	0.0570	3.4881

$$\Delta\phi(\xi) = -\frac{1}{L} \int_0^L |(\phi(\xi))_1 - (\phi(\xi))_2| d\xi \quad (23)$$

In Table 3, drag intensity is pronounced on the fluid by the plate, owing to the negative values that impede the flow. The Prandtl (*Pr*) effect strengthens the Nusselt number as indicated in Table 4, thereby energies the rate of heat transfer for engineering applications. Contrastingly, the Sherwood number slightly decreases (Table 5) in the presence of passive control nanoparticles which slowly declines

the surface mass transport in application. In Table 3-5, the average deviation for velocity, temperature and concentration fields as mentioned earlier in accordance with the Eqs. (21-23) portrayed excellent agreement between GWRM and NDSolve, as the highest difference is minimal with 10^{-5} . In addition, the validation also justified excellent agreement with a very minimal error with Spectral Collocation Method (SCM) as indicated in Table 3-5. Thus, this study is unique as the highest errors between the methods are very small.

Table 3. Assessment of Skin Friction and validation of the obtained results

GWRM Results			SCM Results	Average Deviation between GWRM and NDSolver at L = 10	GWRM - SCM
<i>Mn</i>	<i>We</i>	$Cf Re_x^{\frac{1}{2}}$	$Cf Re_x^{\frac{1}{2}}$	$\Delta f''(\xi)$	
0.1	0.1	-0.7738791	-0.7738791	4.334×10^{-5}	-
0.3		-0.8412952	-0.8412952	9.447×10^{-5}	-
0.5		-0.9036961	-0.9036961	2.112×10^{-5}	-
	0.2	-0.4690415	-0.4690415	7.627×10^{-5}	-
	0.3	-0.1253566	-0.1253566	1.612×10^{-5}	-

Table 4. Assessment of Nusselt number and validation of the obtained results

GWRM Results		SCM Results	Average Deviation between GWRM and NDSolver at L = 10	GWRM - SCM
<i>Pr</i>	$Nu Re_x^{-\frac{1}{2}}$	$Nu Re_x^{-\frac{1}{2}}$	$\Delta\theta(\xi)$	
1.2	0.370856	0.370870	1.344×10^{-5}	-1.4×10^{-5}
1.5	0.435392	0.435404	1.18×10^{-5}	-1.2×10^{-5}
2.0	0.518989	0.519012	1.019×10^{-5}	-2.3×10^{-5}
3.0	0.638235	0.638233	9.529×10^{-6}	2×10^{-6}

Table 5. Assessment of Sherwood number and validation of the obtained results

	GWRM Results	SCM Results	Average Deviation between GWRM and NDSolver at L=10	GWRM - SCM
Sc	$Sh Re_x^{\frac{1}{2}}$	$Sh Re_x^{\frac{1}{2}}$	$\Delta\phi(\xi)$	
0.24	0.0168880	0.0169624	9.845×10^{-6}	-7.44×10^{-5}
0.62	0.0168764	0.0168616	4.657×10^{-5}	-1.58×10^{-5}
0.78	0.0168294	0.0168334	3.478×10^{-5}	-4×10^{-6}
2.62	0.0167308	0.0166815	2.345×10^{-5}	4.93×10^{-5}

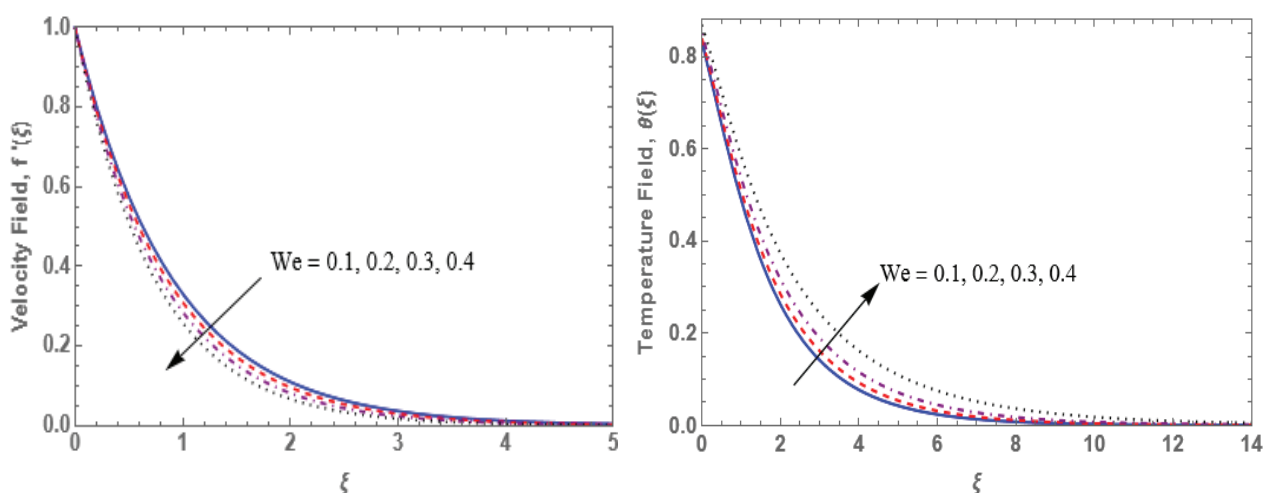
Figure 3 produces the physical interpretation of the Weissenberg number (We) on $f'(\xi)$ and $\theta(\xi)$. Various values of $We \in [0.1, 0.2, 0.3, 0.4]$ invoke the dominance of the viscoelasticity by means of stresses intensities which reduces the movement of the fluid and weakens the momentum layer thickness. In contrast, the tensile stress serves as a heat booster in the thermal field due to the heat generated internally across the boundary layer, thereby solidifying the thermal layer thickness [28]. Thus, We is used to analyze non-Newtonian fluid flow behaviour, crucial in polymer processing and lubrication in the engineering field.

Figure 4 presents the Magnetic effect (Mn) on $f'(\xi)$ and $\theta(\xi)$ where the Lorentz force is more pronounced as Mn varied. The force acts against the motion of the Walters' B liquid and decays the thickness of the corresponding momentum layer. In addition, the presence of (Mn) boosts the dominance of heat distribution by means of frictional heating whose aftermath strengthens the temperature field $\theta(\xi)$ and the thickness of the corresponding layer. This outcome agreed with Shit et al. [19] and is crucial for energy efficiency in conductive fluids, which are essential for electromagnetic applications. At various values of $Rd \in [0.1, 0.4, 0.8, 1.2]$, the temperature field $\theta(\xi)$ is discussed in Figure

5. The interactive contribution of (Rd) energies radiative heat flux on conduction heat transport, which adds thermal energy to the polymeric liquid that consequently augments the temperature field and the associated thermal layer thickness. On this note, Rd contributes to heat transfer efficiency, influencing thermal management in engineering systems. On a similar note, various values of temperature ratio magnified the temperature field $\theta(\xi)$ (Fig. 5). This outcome agreed with the expectation since $\theta_w > 1$. Thus, the thermal boundary layer thickness is boosted. This result agreed with Saranya et al. [35].

Figure 6. is plotted to analyse the Prandtl number (Pr) on $\theta(\xi)$. Taking into consideration the non-Newtonian liquid, with the low thermal diffusivity, strengthening values of (Pr) declines the thermal conductivity which minimizes the temperature gradient, make low the heat field and its layer depth [36]. In applications, non-Newtonian liquid diffuses more rapidly on the surface with smaller values of (Pr).

Figure 7 reports the outcome of the Eckert number (Ec) on Temperature field $\theta(\xi)$. The presence of (Ec) at $0.1 \leq Ec \leq 1.2$ energies irreversible process of the dissipation of kinetic energy to internal energy. This process infuses

**Figure 3.** Reaction of We on velocity field $f'(\xi)$ and temperature field $\theta(\xi)$.

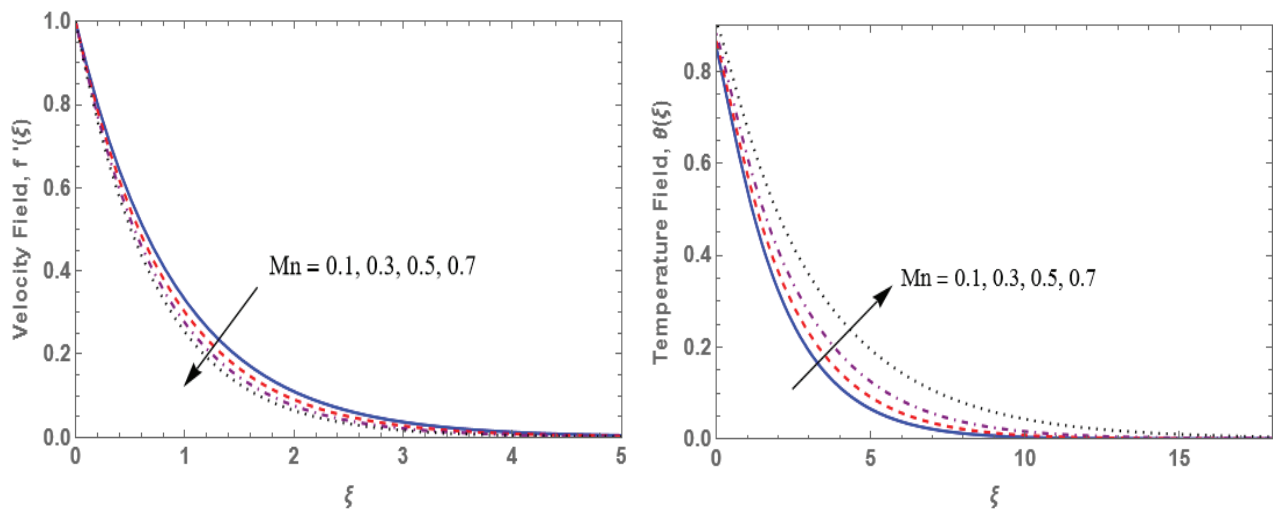


Figure 4. Reaction of Mn on velocity and temperature fields.

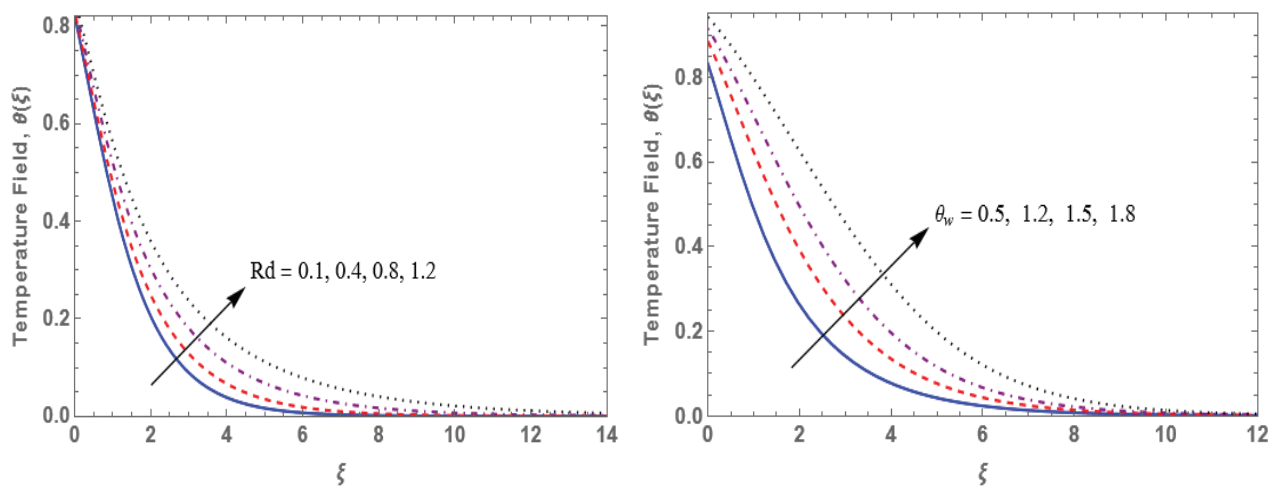


Figure 5. Reaction of Rd and θ_w on temperature field.

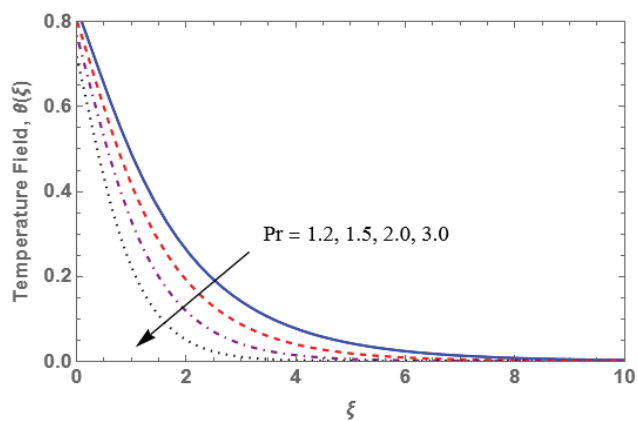


Figure 6. Reaction of Pr on temperature field $\theta(\xi)$.

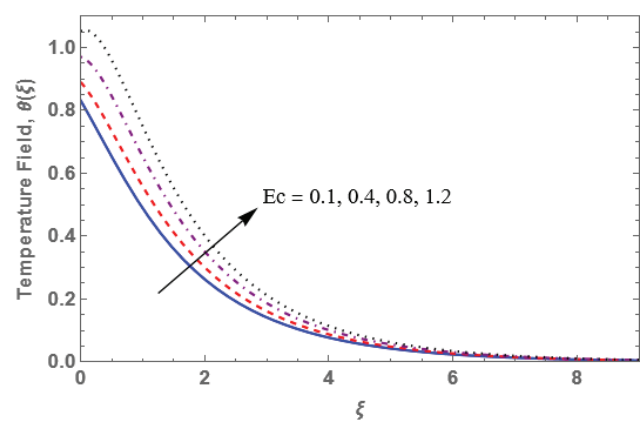


Figure 7. Reaction of Ec on temperature field $\theta(\xi)$.

more heat into the thermal gradient and strengthens the associated thermal boundary layer thickness. Thus, this phenomenon agreed with the findings of Megahed and Reddy [37]. It is interesting to note that Ec quantifies viscous dissipation in high-speed flows, crucial for heat transmission in aerospace and combustion. The significance of heat generation/absorption (Q) is portrayed in Figure 8 for Temperature field $\theta(\xi)$ at $Q \in [0.06, 0.10, 0.13, 0.16]$. The generation/absorption, in the presence of non-linear radiative heat transfer, energies spontaneous movement of particles within the fluid, which strengthens the temperature distribution and consequently solidifies the thermal boundary thickness.

Figure 9 reflects the behaviours of convective heat parameter (Bi) on $\theta(\xi)$. As Bi varied at $0.1 \leq Bi \leq 0.4$, temperature distributing impact is magnified across the boundary layer due to the strong convective heating by virtue of $Bi > 0$. This process improves the temperature boundary layer depth and solidifies heat-transfer effects in the free stream.

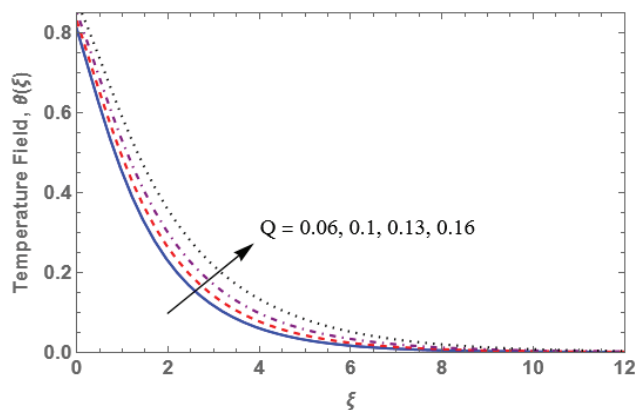


Figure 8. Reaction of Q on temperature field $\theta(\xi)$.

In Figure 10, the enhancing values of Brownian motion (Nb) intensifies diffusion, causing particles to spread more rapidly whose aftermath reduces the concentration field $\phi(\xi)$ and its associated boundary layer thickness. However, the inverse behaviours is observed with the thermophoresis parameter (Nt) on concentration field $\phi(\xi)$ (Fig. 10). The physics behind this behaviour is that thermophoretic force instigates rapid motion away from a stretchable sheet. This rapid motion conveys along with it, a thermophoretic force that consequently improves the associated concentration layer thickness. These outcomes aligned with the findings in Saini et al. [29] and Makinde *et al.* [38]. It should be noted that Nb enhances mixing and diffusion of nanoparticles, while Nt drives particles from hot to cold regions, crucial for controlling concentration in cooling.

The concentration distribution $\phi(\xi)$ is influenced by the Schmidt number (Sc) as presented in Figure 11. Bearing in mind lower molecular diffusivity, improving values of (Sc) weakens the nanomaterial concentration. The aftermath

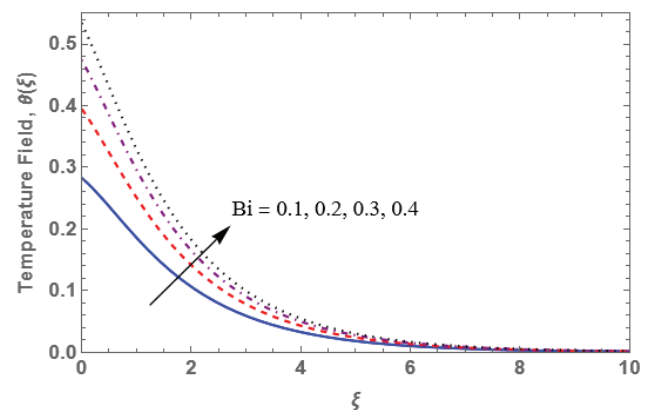


Figure 9. Reaction of Bi on temperature field $\theta(\xi)$.

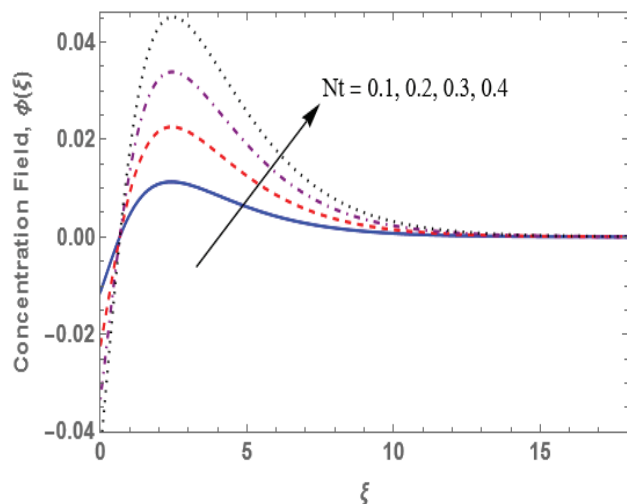
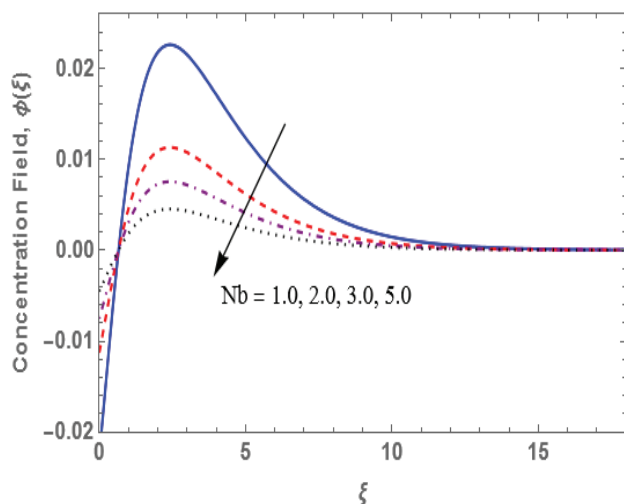


Figure 10. Reaction of Nb and Nt on concentration field $\phi(\xi)$.

reaction of this behaviour makes less the concentration boundary layer thickness. It is worth mentioning that Sc quantifies mass diffusion relative to momentum diffusion, crucial in chemical engineering, fluid mechanics, and heat transfer applications.

Figure 12-14 discusses the skin-friction, Nusselt and Sherwood numbers which are very useful in engineering applications. In Figure 12, both Magnetic (Mn) and Weissenberg (We) decline the skin friction due to the surface drag force applied into the liquid by the sheet. Figure 13 is plotted to report the effect of the Radiation parameter (Rd) against the thermophoresis parameter (Nt) on the Nusselt number. The two parameters exhibit cooling contributions for the Nusselt number, which reduces the rate of heat exchange in practical systems. The Schmidt number (Sc) effects on the Sherwood number is plotted against the thermophoresis parameter (Nt) in Figure 14. As indicated in the figure, variation in Sc reduces the Sherwood number strength whose aftermath made low the rate of mass transfer. The opposite phenomenon is observed with ($Nt > 0$) which consequently improves the rate of mass transfer in application.

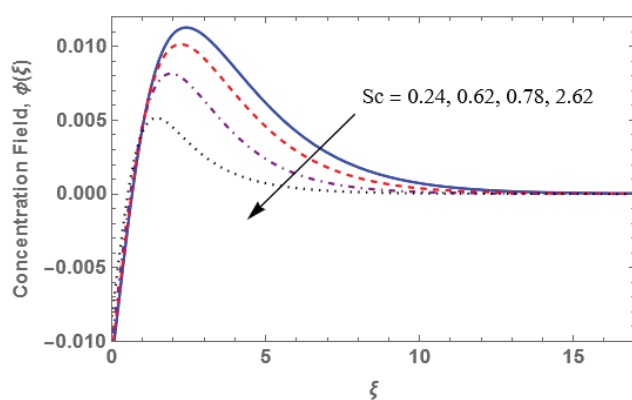


Figure 11. Reaction of Sc on concentration field $\phi(\xi)$.

Figure 15-18 illustrate the effect of various parameters on entropy generation and Bejan number (Be). In Figure 15, the significance of Mn on Entropy Generation and Bejan number is plotted. The interactive impact of the Magnetic parameter (Mn) intensifies the temperature distribution due to the frictional heating irreversibility in the hydromagnetic fluid that consequently boosts the entropy production (N_G). Thus, the Bejan number (Be) declines as viscous effects dominate, thereby reducing the relative contribution of thermal irreversibility. On the other hand, enhancing values of We as reported for Entropy Generation and Bejan number in Figure 16, strengthens the elasticity of the fluid, thereby reducing viscous dissipation whose aftermath lowers the entropy production (N_G). Under the same variation of We (i.e. $0.1 \leq We \leq 0.4$), the Bejan number (Be) rises as shown in Figure 16. This is because thermal irreversibility becomes more significant relative to total entropy generation, indicating a shift toward entropy being influenced primarily by heat transfer mechanisms.

In Figure 17, the influence of thermal radiation (Rd) is analyzed on Entropy Generation and Bejan number. Increasing values of (Rd) intensifies the internal energy of the system thereby resulting in a greater thermal

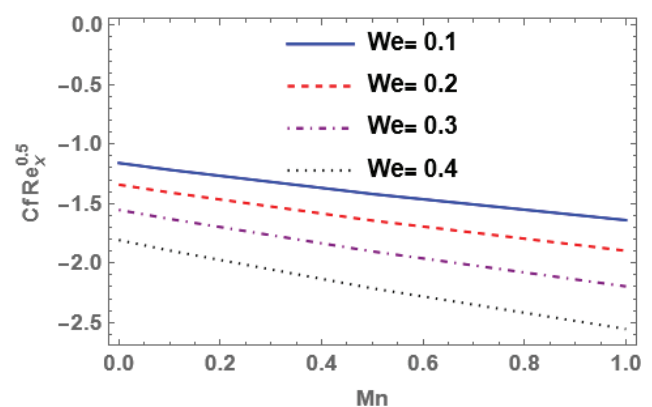


Figure 12. Skin-friction of Mn against We .

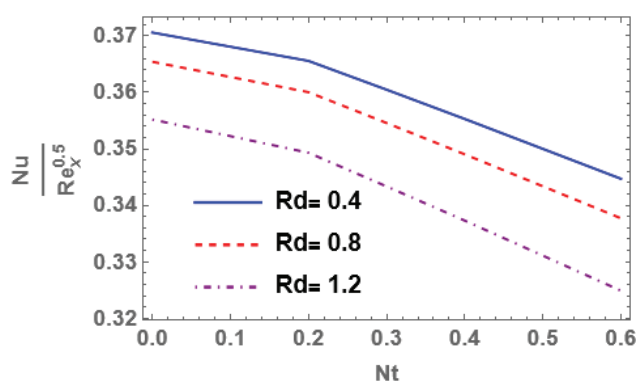


Figure 13. Nusselt number of Nt against Rd .

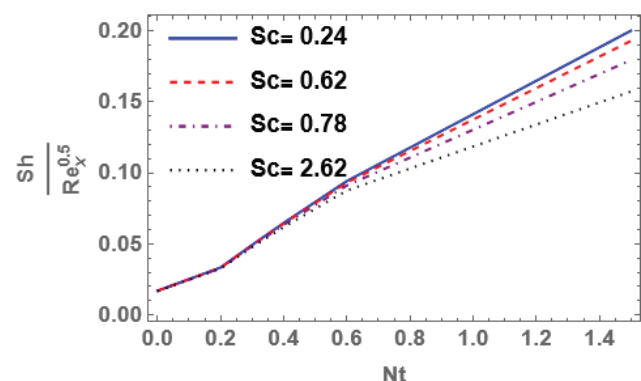


Figure 14. Sherwood number of Nt against Sc .

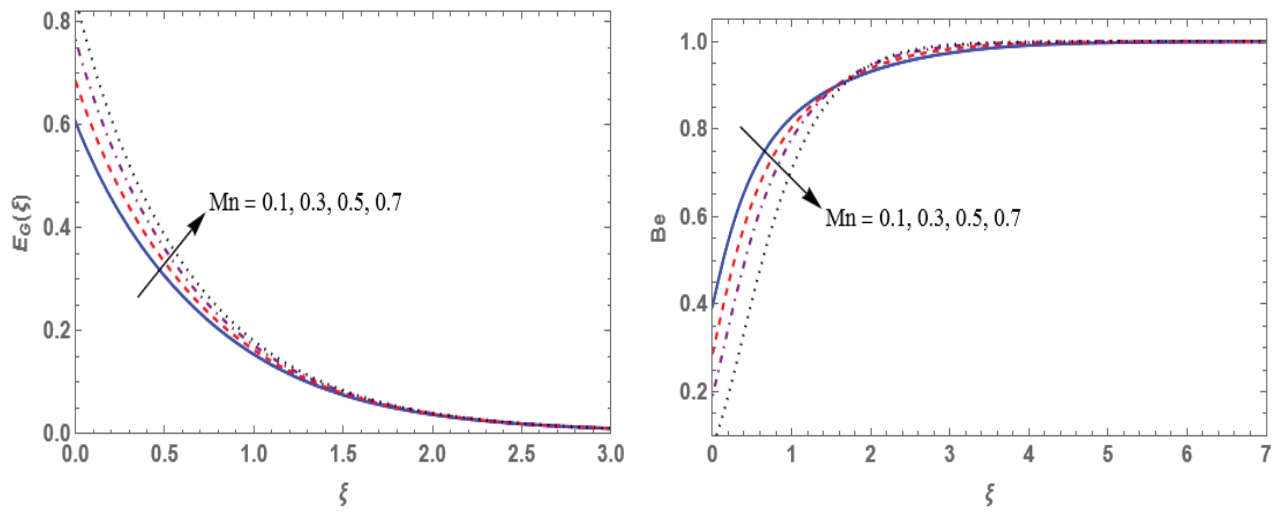


Figure 15. Reaction of Mn on entropy generation and Bejan number.

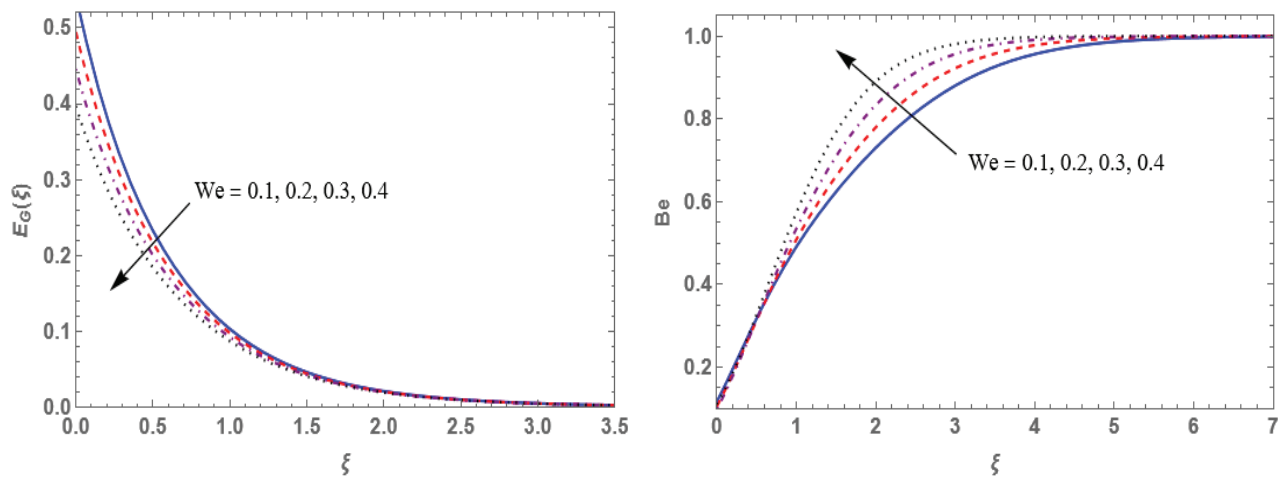


Figure 16. Reaction of We on entropy generation and Bejan number.

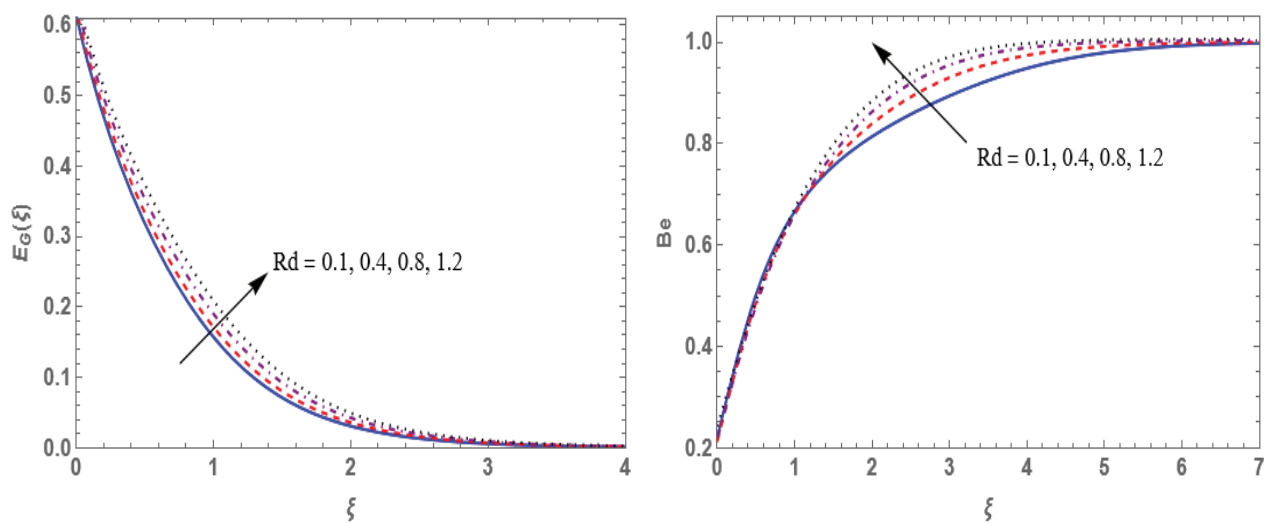


Figure 17. Reaction of Rd on entropy generation and Bejan number.

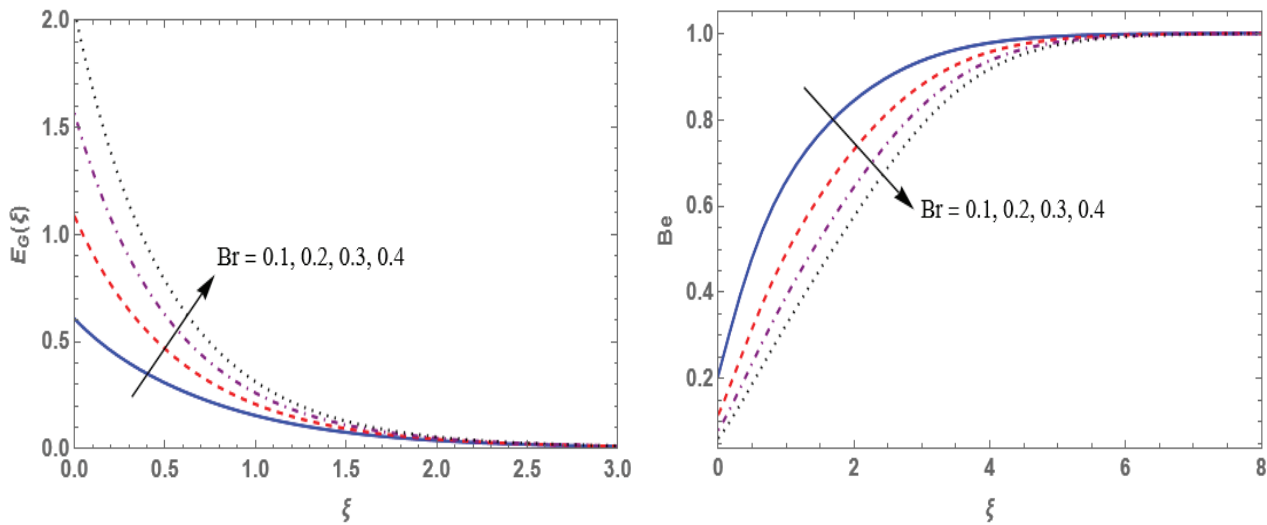


Figure 18. Reaction of Br on entropy generation and Bejan number.

irreversibility whose aftermath improves the entropy production (N_G). In addition, since thermal effects outweigh viscous effects under strong radiation, the Bejan number (Be) also improves, indicating that thermal effects govern the overall irreversibility in radiation-dominated conditions. These outcomes agreed with the findings of Hayat et al. [39] and Alsaadi et al. [40]. The mechanics of Brinkman number (Br) on Entropy Generation and Bejan number is represented in Figure 18. It is noteworthy that Entropy Generation improves over the enhancement in the values of Brinkman number (Br). This outcome indicates that viscous heating dominates by means of heat transmission through the molecular conduction around the vicinity of the sheet since Br quantifies viscous dissipation with respect to fluid friction. Brinkman number is very useful in applications in polymer processing. It helps to determine the significance of viscous heating relative to thermal conduction, influencing the temperature distribution and flow behaviour of the polymer during processing. This phenomenon makes viscous effects the main source of irreversibility, reducing the Bejan number (Be) since it measures the ratio of heat transfer irreversibility to total entropy generation. These outcomes agreed with Qayyum et al. [28] and Jabeen et al. [24].

CONCLUSION

In this study, an entropy-generation investigation is conducted on the influence of a viscous and nonlinearly radiating hydromagnetic Walters' B nanofluid with heat generation/absorption and convective boundary condition. The model's underlying equations that determine the behaviour of the embedded parameter are solved by the Galerkin Weighted Residual Method (GWRM), and the errors in computation are taken into account by validating

the results with two other methods, as indicated in the manuscript. The article presents the following key findings, among others.

1. The findings demonstrate excellent agreement with reference results available in the literature, as well as with the Spectral Collocation Method and NDSolve. The flow of the non-Newtonian liquid is boosted in the presence of strengthening radiative flux. This enhances heat transport by adding thermal energy to support the polymeric flow.
2. In the neighbourhood of the sheet, the viscous heating dominates by means of molecular conduction with the interaction of the Brinkman number. In applications involving polymer, the Brinkman number is quite helpful.
3. Skin friction is a decreasing function of the Magnetic parameter and Weissenberg number influence due to the surface drag force applied into the liquid by the viscous stresses. Friction drag is essential in Aerodynamics.
4. The presence of Eckert number triggers the irreversible conversion of kinetic energy to internal energy, which intensifies the heat distribution across the system. This is crucial in mechanical and chemical engineering fields to analyze viscous heating relative to heat conduction in high viscosity flows, thereby aiding thermal management and reactor design.
5. The interactive impact of heat generation and convective heat parameters causes greater heat across the thermal field, which boots the thermal effect at quiescent fluid and facilitates the drying of the component in the engineering field.

Limitation of the Study/Suggestion for Further Studies

In this study, computational analysis has been examined on two-dimensional boundary layer flow of Walters' B nanofluid with constant thermophysical properties.

Nonetheless, an attempt could be made to look into the unsteady-case model. Also, future investigation may be conducted into the variable thermophysical properties of this model, such as variable dynamic viscosity and variable thermal conductivity.

NOMENCLATURES

u	velocity components along x [$s.m^{-1}$]
v	velocity components along y [$s.m^{-1}$]
T	Temperature of the fluid [K]
ν_f	kinematic viscosity [$m^2.s^{-1}$]
$(\rho C)_f$	Heat capacity of the fluid [$J.kg^{-1}$]
τ	$(\rho C)_p/(\rho C)_f$
ρ	density [kgm^{-3}]
q_r	radiation heat flux [$W.m^{-2}$]
$(\rho C)_p$	Heat capacity of the nanoparticles [$J.kg^{-1}$]
σ	electrical conductivity [$s.m^{-1}$]
D_T	Thermophoretic diffusion coefficient
k^*	mean of the absorption coefficient [m^{-1}]
B_0	magnetic field [Am^{-1}]
k_0	elastic parameter
σ^*	Stefan-Boltzmann constant [$W.m^{-2}.K^{-4}$]
Q_0	Heat generation/absorption coefficient
h_f	heat transfer coefficient [$W.m^{-2}.K$]
k	thermal conductivity [$W.m^{-1}.K$]
D_B	Brownian diffusion coefficient [$m^2.s^{-1}$]
μ	dynamic viscosity [$kgm^{-1}.s^{-1}$]

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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