



Research Article

Study of heat flux of tri-hybrid ($\text{TiO}_2+\text{SiO}_2+\text{Al}_2\text{O}_3/\text{C}_6\text{H}_9\text{NaO}_7$) nanofluids over a linearly stretching sheet

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ABSTRACT

The current numerical study investigates the impacts of heat source and radiation effects on thermo-fluidic behavior of trihybrid Casson ($\text{TiO}_2+\text{SiO}_2+\text{Al}_2\text{O}_3/\text{C}_6\text{H}_9\text{NaO}_7$) nanofluids over a linearly stretching sheet. The study not only explores a method to achieve superior heat conduction compared to hybrid nanofluids but also highlights the environmental purification potential, decomposition of destructive materials, and effective cooling capabilities offered by the considered nanoparticle combination. The governing equations were converted into ordinary differential equations and are then explained using Runge-Kutta scheme with a Shooting algorithm. The temperature profile shifts upwards and concentration gradient reduces for higher values of heat source and radiation. Nusselt number decreases by 23.52% when radiation doubles from 1 to 2. The uniqueness of the work lies in its generalization of previous studies by considering heat source, radiation, thermophoresis and Brownian motion effects over a linear stretching sheet leading to the achievement of an effective tri-hybrid nanofluid flow.

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INTRODUCTION

One among the latest technologies, heat transfer across a sheet stretching or otherwise, demanded an understanding of the flow dynamics, especially near the boundary of these sheets as the optimum changes occur in these layers. This approach gave a deep vision into the features of fluid/heat transfer which hugely supported the engineering

processes. In the literature of Crane [1], the boundary layer flow which arises because of a stretched sheet that moves linearly with the changing velocity, was carried out. As there was advancement in nanotechnology, nanofluid originated which researchers in Fluid Dynamics incorporated into their works.

The industrial development increases pollution in the air caused due to dust particles, emissions of oxides of

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sulphur, fumes from the factory, etc. Consequently, issues like global warming are emerging in full scale. Other forms of pollution are caused by power generation, vehicle movements and industrial boilers. These sources form nitrogen oxides which are harmful air pollutants. Lately, the Nitrogen oxide emission had reached distressing level in big cities because of the excessive use of motor driven vehicles. This led to chemical smog, respiratory problems and acid rain. In order to reduce Nitrogen oxide, there are several ways. Some of which are detoxification and purification through photocatalytic reactions. The photocatalyst possess various properties such as high absorbance of visible light, higher optical activity chemical inertness, optical stability, biological inertness, low cost, etc. All the above-mentioned properties are available in Titanium dioxide (TiO_2) and hence are used in various applications.

The name “Nanofluid” has been initiated by [1] who projected a new classification of fluids which has been observed by suspension of nanoparticles in the heat transfer of fluids. These nanoparticles are been made of carbon nanotubes, carbides, metals, or oxides. And these nanofluids have useful properties in several applications of heat transfer in microelectronics, artificial fibers, extrusion of polymers, copper wire drawing, processes in treatments, and coolant for engines. The study of effects of heat transmission over a stretched sheet has been analyzed by various researchers. [2-4] analyzed a nanofluid flow fluid with heat generation/absorption effects. The boundary layer flow across a vertically stretching external surface was understood and concluded that the Natural convection dominated the regime. [5] explored the influence of nanoparticle accumulation in a parabolic solar trough collector for enhancing the current transfer. [6] studied the Taguchi strategy optimization and neural networks of flush fraction impacts over heat degeneracy in Bodewadt movement.

Nanofluid composites are been synthesize in order to augment the rheological properties and thermal conductivity. This has been achieved by making a flawless combination of varying nanoparticles. A nanofluid which is formed by the addition of nanoparticle may not possess rheological properties, hence addition of nanoparticles with different rheological properties would be a better choice to attain overall ability and effectiveness which is termed as Hybrid nanofluid. Hybrid nanofluids improvise the chemical, physical properties of materials, the heat transfer rate because they provide more thermal conductance. For instance, Al_2O_3 has low thermal conductivity despite having effective stability and chemical inertness. Whereas, aluminum, copper, silver, etc possess good thermal conductivity but are chemically reactive and unstable. Scientists have immense interest in monitoring the thermal characteristics and stability of the system by mixing nano particles in the base fluid. [7-9] Therefore, by mixing these nanoparticles with varying chemical and physical bonds give raise to nanofluids known as hybrid nanofluids. Scientists [7-10] have analyzed the thermal and concentration features of hybrid

nanofluids along with different MHD effects. They are been utilized in various applications like pharmaceutical industry, cooling of the electronic heaters, nuclear safety, etc. Various researchers [11] studied MHD effects of parameters of a ternary nanofluid flow on a convective stretched sheet.

The pioneering solution recorded by [12] was found to be worth as it augmented the heat transmission rate by the process of hybridization. [13] presented the betterment of thermal and mechanical features of base fluid by the adding two nanoparticles. [14] performed computational analyses of shape of nanoparticle, mass, and thermal transport on radiative nanofluid flow across a stretching Riga plate. The researchers [15-18] studied the convective fluid flow with magnetic field and heat source. Convective steadiness in porous spongy frameworks for energy and biosensor applications using viscous driven control flow was analyzed by [19, 20]

The importance of Titanium dioxide is seen in various photosensitive applications [21-25]. Higher photo catalysis can be obtained with fabrication of TiO_2 materials by considering higher specific surface areas. Precisely, the nano-structured Titanium dioxide materials which are made by template synthesis method was presented by [26, 27], which caught researchers' attention in the recent years. These materials have greater surface areas and are been used in numerous applications. Metallic nanoparticles like Ag, Au, Cu, CuO etc. possess improved thermal conductance in comparison to fluids. The researchers were then motivated and identified the necessity for the suspension of metallic nano-sized particles in nanofluids.

In the industries, the requirement of a coolant with better capability of heat transfer has motivated the scientists to design a new concept for nanofluid for enhancement of heat transfer. Hence the concept of hybrid nanofluids was discovered which enhances the heat transfer characteristics. With this inspiration, researchers have conducted experimental studies and this led to the discovery of tri-hybrid nanofluids. Here, three kinds of nanoparticles having varying chemical and physical bonds are suspended. For example, TiO_2 possesses higher acid strength while treated with sulphuric acid, and then forms TiSO_4 . In the same manner, Al_2O_3 reacts with sulphuric acid to form $\text{Al}_2(\text{SO}_4)_3$. Weaker acid strengths are produced due to the reaction between sulphuric acid and alumina. They further increase the stability and chemical inertness of composition of nanoparticle. Whereas SiO_2 cannot form sulfatase, due to its high electron negativity than Ti and Al. This study led to the development of a coolant which cools appliances using TiO_2 . Therefore, to develop coolant using TiO_2 , the supportive agents were SiO_2 and Al_2O_3 . [28] reviewed tertiary nanofluids in between rotating parallel plates with the effects of varying Lorentz effects and electric fields. [29] discussed the flow of (Al_2O_3 -water) time-dependent mixed convection of nanofluids with a stagnation-point for a spinning sphere with motile microorganisms. [30] comprehensively

reviewed the mass and heat transfer through a stagnation point double-diffusion Cattaneo-Christov flow by means of non-Newtonian Sutterby nanofluids. [31-34] studied approximation properties of Bernstein polynomials and modified Jain Gamma operators.

Sodium alginate is a salt which is a natural polymer present in the alginic acid, and is been detached from the brown algae. This is utilized in textile production, industrial, medications and cosmetics. The functions of sodium alginate involve embolotherapy [30, 35], medication delivery system [36-39] encapsulation of cells [40-43].

On the basis of above-mentioned literature, uniqueness and originality for the existing work is to clarify the earlier studies by including the Brownian motion, thermophoresis, heat source and radiation parameters along with appropriate boundary conditions over a linearly stretching sheet producing more refined outcomes of tri-hybrid ($\text{TiO}_2 + \text{SiO}_2 + \text{Al}_2\text{O}_3 / \text{C}_6\text{H}_9\text{NaO}_7$) nanofluid flow. The current work indicates that heat flux play a noteworthy part in the species concentration and thermal transmission on trihybrid nanofluids. A comparative analysis with the previously published results to measure the assurance and accuracy of obtained similar outcomes and were in good agreement.

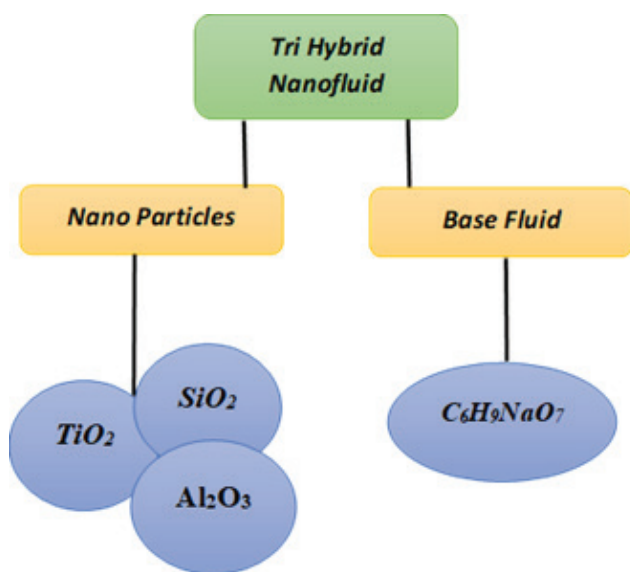


Figure 1. Preparation scheme of tri-hybrid nanoparticles.

PROBLEM FORMULATION

Consider two-dimensional, incompressible, steady-state, laminar, viscous ternary nanofluid flow past linearly stretching sheet. Following assumptions were considered in the fluid flow problem:

- Three kinds of nanoparticles viz. TiO_2 , SiO_2 and Al_2O_3 are been dispersed in water (base fluid), to produce a new configuration $\text{TiO}_2 + \text{SiO}_2 + \text{Al}_2\text{O}_3 / \text{C}_6\text{H}_9\text{NaO}_7$ as portrayed in Figure 1.
- The rectangular co-ordinate system has been preferred such that x -axis is considered lengthways of the sheet, y -axis is perpendicular to it, and the Schematic representation is portrayed in Figure 2.
- Brownian motion, thermophoresis, and radiation parameter are been incorporated in the flow regime.
- Flow occurs in a section $y > 0$.
- The sheet has a growth that is exponential with a velocity given as $u_w(x) = ax$. Here 'a' is taken as constant, and V_w is considered to be the mass transfer at the wall.
- Temperature and Concentration near the stretching surface is $T_w = T_\infty + b(x/l)^2$,
- $C_w = C_\infty + c(x/l)^2$ and far away are given is C_∞ and T_∞ .
- Table 1 shows the experimental values of $\text{C}_6\text{H}_9\text{NaO}_7$, TiO_2 , SiO_2 and Al_2O_3 [44-46], and Table 2 shows the Thermodynamic characteristics of Tri-Hybrid Nanofluids [47, 48].
- Base fluid and nanoparticles were in thermal equilibrium.

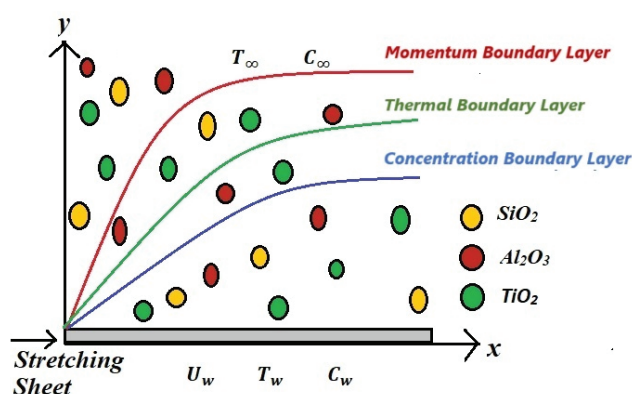


Figure 2. Geometry of the problem.

Table 1. Investigational values of $\text{C}_6\text{H}_9\text{NaO}_7$, TiO_2 , SiO_2 and Al_2O_3

	$\rho(\text{kg/m}^3)$	$\sigma(\text{S/m})$	$K(\text{W/m.K})$	$C_p(\text{J/Kg.K})$
$\text{C}_6\text{H}_9\text{NaO}_7$	989	5.5×10^{-6}	0.6396	4175
TiO_2	4250	2.4×10^6	8.953	683
SiO_2	2270	3.5×10^6	1.4013	680
Al_2O_3	6310	5.96×10^7	32.9	765

Table 2. Thermodynamic properties of tri-hybrid nanofluids

Viscosity	$\frac{\mu_{tnf}}{\mu_f} = \frac{1}{(1-\varphi_{SiO_2})^{2.5}(1-\varphi_{Al_2O_3})^{2.5}(1-\varphi_{TiO_2})^{2.5}}$
Density	$\frac{\rho_{tnf}}{\rho_f} = (1 - \varphi_{Al_2O_3}) \left[(1 - \varphi_{Al_2O_3}) \left\{ (1 - \varphi_{TiO_2}) + \varphi_{TiO_2} \frac{\rho_{TiO_2}}{\rho_f} \right\} + \varphi_{Al_2O_3} \frac{\rho_{Al_2O_3}}{\rho_f} \right] + \varphi_{SiO_2} \frac{\rho_{SiO_2}}{\rho_f}$
Specific Heat	$\frac{(\rho c_p)_{tnf}}{(\rho c_p)_f} = \varphi_{SiO_2} \frac{(\rho c_p)_{SiO_2}}{(\rho c_p)_f} + (1 - \varphi_{SiO_2}) \left[(1 - \varphi_{Al_2O_3}) \left\{ (1 - \varphi_{TiO_2}) + \varphi_{TiO_2} \frac{(\rho c_p)_{TiO_2}}{(\rho c_p)_f} + \varphi_{Al_2O_3} \frac{(\rho c_p)_{Al_2O_3}}{(\rho c_p)_f} \right\} \right]$
Thermal Conductivity	$\frac{k_{tnf}}{k_{hnf}} = \left(\frac{k_{TiO_2} + 2k_{hnf} - 2\varphi_{TiO_2}(k_{hnf} - k_{TiO_2})}{k_{TiO_2} + 2k_{hnf} + \varphi_{TiO_2}(k_{hnf} - k_{TiO_2})} \right), \frac{k_{hnf}}{k_f} = \left(\frac{k_{Al_2O_3} + 2k_{nf} - 2\varphi_{Al_2O_3}(k_{nf} - k_{Al_2O_3})}{k_{Al_2O_3} + 2k_{nf} + \varphi_{Al_2O_3}(k_{nf} - k_{Al_2O_3})} \right),$ $\frac{k_{nf}}{k_f} = \left(\frac{k_{SiO_2} + 2k_f - 2\varphi_{Al_2O_3}(k_f - k_{SiO_2})}{k_{SiO_2} + 2k_f + \varphi_{Al_2O_3}(k_f - k_{SiO_2})} \right)$
Electrical Conductivity	$\frac{\sigma_{tnf}}{\sigma_{hnf}} = \left(1 + \frac{3 \left(\frac{\sigma_{TiO_2}}{\sigma_{hnf}} - 1 \right) \varphi_{TiO_2}}{\left(\frac{\sigma_{TiO_2}}{\sigma_{hnf}} + 2 \right) - \left(\frac{\sigma_{TiO_2}}{\sigma_{hnf}} - 1 \right) \varphi_{TiO_2}} \right), \frac{\sigma_{hnf}}{\sigma_{nf}} = \left(1 + \frac{3 \left(\frac{\sigma_{Al_2O_3}}{\sigma_{nf}} - 1 \right) \varphi_{Al_2O_3}}{\left(\frac{\sigma_{Al_2O_3}}{\sigma_{nf}} + 2 \right) - \left(\frac{\sigma_{Al_2O_3}}{\sigma_{nf}} - 1 \right) \varphi_{Al_2O_3}} \right),$ $\frac{\sigma_{nf}}{\sigma_f} = \left(1 + \frac{3 \left(\frac{\sigma_{SiO_2}}{\sigma_{nf}} - 1 \right) \varphi_{SiO_2}}{\left(\frac{\sigma_{SiO_2}}{\sigma_{nf}} + 2 \right) - \left(\frac{\sigma_{SiO_2}}{\sigma_{nf}} - 1 \right) \varphi_{SiO_2}} \right)$

The Casson trihybrid nano fluid flow gets modified with the influences of radiation and heat source effects under the boundary layer approximations [49].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \vartheta_{tnf} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma_{tnf}}{\rho_{tnf}} B_o^2 u - \vartheta_{tnf} \left(\frac{u}{k_o} \right), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{tnf} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) - \frac{1}{(\rho c_p)_{tnf}} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The conditions at the boundary are

$$v = -v_w, \quad u = u_w(x), \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \quad (5)$$

$$u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \quad (6)$$

$$\text{where } \alpha = \frac{k}{(\rho c_p)_f}, \quad \tau = \frac{(\rho c_p)_p}{(\rho c_p)_f}, \quad \vartheta = \frac{\mu}{\rho_f}, \quad (7)$$

The coupled non-linear PDE is converted to coupled-non-linear ODE with the help of the similarity transformations mentioned as follows:

$$\eta = \sqrt{\frac{a}{\vartheta}} y, \quad \Psi = \sqrt{a \vartheta} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (8)$$

$$u = \alpha x f'(\eta), \quad v = -\sqrt{a \vartheta} f(\eta) \quad (9)$$

In equation (8), $\Psi(x, y)$ denotes the stream function which is given as follows:

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x} \quad (10)$$

Radiative heat flux along y direction is dominant and the x direction component can be neglected. Radiative heat flux q_r , with the help of Rosseland approximations is mentioned as,

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (11)$$

where σ^* - Stefan Boltzmann constant. Within the flow, the difference in temperature, is small and hence the term T^4 has been expanded with the aid of Taylor's series having a T_∞ as the free stream temperature.

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (12)$$

It follows on substituting Eq. (12) into Eq. (10),

$$q_r = -\frac{16\tau_\infty^3 \sigma^*}{3k^*} \frac{\partial T}{\partial y} \quad (13)$$

The governing Eqns. (2) – (4) are changed into an ordinary differential equation by Similarity transformations. They are as follows:

$$\left(1 + \frac{1}{\beta} \right) f''' - A_1 A_2 (f f'' - 2f'^2) - K f' - A_1 A_2 M f' = 0 \quad (14)$$

$$\frac{1}{A_4} \left(A_3 + \frac{4}{3} R \right) \theta'' + Pr(f\theta' + Nb\theta'\phi' + Nt\theta'^2 + A_4\theta\lambda) = 0 \quad (15)$$

$$\phi'' + Le f\phi' + \frac{Nt}{Nb} \theta'' = 0 \quad (16)$$

The conditions near the boundary:

$$\left. \begin{aligned} f(0) = f_w; f'(0) = 1; \theta(0) = 1; \phi(0) = 1; \text{ at } \eta = 0 \\ f'(\infty) = 0; \theta(\infty) = 0; \phi(\infty) = 0; \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (17)$$

$$\left. \begin{aligned} f_w = -\frac{v_w}{\sqrt{a\theta}}; Nt = \frac{(\rho c) p D_T (T_w - T_\infty)}{(\rho c) f \theta T_\infty}; Nb = \frac{(\rho c) p D_B (\phi_w - \phi_\infty)}{(\rho c) f \theta}; \\ Pr = \frac{\theta}{a}; R = \frac{4\sigma^* T_\infty^3}{3k^* k}; Le = \frac{\theta}{D_B}; \lambda = \frac{Q_0}{(\rho c p)_f a}; \\ A_1 = \frac{\mu_f}{\mu_{tnf}}; A_2 = \frac{\rho_{tnf}}{\rho_f}; A_3 = \frac{(\rho c p)_{tnf}}{(\rho c p)_f}; A_4 = \frac{k_{tnf}}{k_f} \end{aligned} \right\} \quad (18)$$

where f represents the velocity (dimensionless), θ represents the temperature, and ϕ represents the nanoparticle concentration.

$$C_f = \frac{T_w}{\rho u_w^2}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x h_m}{D_B(\phi_w - \phi_\infty)} \quad (19)$$

$$q_w = -k(\phi_y)_{y=0}, h_m = -D_B(\phi_y)_{y=0} \quad (20)$$

By using the above equations, the reduced skin friction coefficient, heat and mass transmission rates in non-dimensional arrangement are:

$$C_f \sqrt{Re_x} = -f''(0), \frac{Nu_x}{\sqrt{Re_x}} = -(1+R)\theta'(0), \frac{Sh_x}{\sqrt{Re_x}} = -h'(0)s \quad (21)$$

where, Re_x , Nu_x , Sh_x and C_f signify the Reynolds number, Nusselt number, Sherwood number and skin friction, respectively.

Solution Procedure

The Runge-Kutta method with 4th order with boundary conditions (17) are used to solve the equations (14), (15) and (16) which are nonlinear coupled in nature. Equations are expressed as follows. Flow configuration is described in Figure 3 [50].

The equations are solved with shooting technique. The shooting technique is utilized to alter the ODE boundary value problem to a matching initial value problem.

New variables are now defined as:

$$f_1 = f; f_2 = f'; f_3 = f''; f_4 = \theta; f_5 = \theta'; f_6 = \phi; f_7 = \phi' \quad (22)$$

The higher ordered coupled non-linear differential equations along with their boundary conditions have been changed into seven differential equations with first order and appropriate boundary conditions, and are given below:

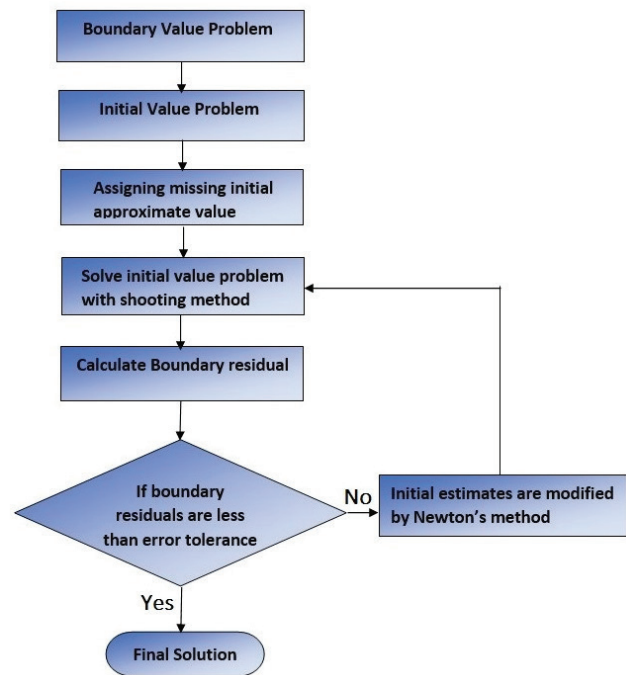


Figure 3. Flow configuration.

$$f'_1 = f_2$$

$$f'_2 = f_3$$

$$f'_3 = \frac{A_1 A_2 (f_1 f_3 - 2f_2^2) + K f_2 + A_1 A_2 M f_2}{(1 + \frac{1}{\beta})}$$

$$f'_4 = f_5$$

$$f'_5 = -Pr A_4 \frac{(-f_1 f_5 - Nb f_5 f_7 - Nt f_5^2 - \lambda A_4 f_4)}{(A_3 + \frac{4}{3} R)}$$

$$f'_6 = f_7$$

$$f'_7 = -Le f_1 f_7 + \frac{Nt}{Nb} Pr \frac{(-f_1 f_5 - Nb f_5 f_7 - Nt f_5^2 - \lambda A_4 f_4)}{(1 + \frac{4}{3} R)}$$

Boundary conditions are:

$$f_1(0) = f_w; f_2(0) = 1; f_4(0) = 1; f_6(0) = 1,$$

$$f_1(\infty) = 0; f_2(\infty) = 0; f_6(\infty) = 0$$

The significant feature of this study is, BVP is changed into an IVP. Then the result to IVP is found by taking guess values for initial values which are not present by means of shooting technique. The step size taken is 0.01 for doing computations. Tolerance for error of 10^{-3} is used as given in the literature of [51].

RESULTS AND DISCUSSION

A mathematical investigation was made to study the thermo-fluidic transport of tri-hybrid nanofluids on an

exponentially stretching sheet. And the equations governing the transport phenomena are explained using the Runge-Kutta method along with a shooting algorithm. Velocity and thermal distributions were determined, followed by evaluating the concentration profile through numerical solutions for the trihybrid nanofluids. In all the results presented below, reference curves with appropriate parameters are drawn and the obtained results are compared with it.

For the Figures 4 to 6, the standard curve for reference has the following values:

$Le = 10$; $Pr = 5$; $Nt = Nb = 0.1$; $R = \lambda = 0$; $fw = 0.5$; $\beta = 5$; $M = K = 1$

It is found from Figure 4, when suction parameter is reduced from 1.5 to 0.5, suction weakens and hence the boundary layer thickens slightly. This causes the velocity

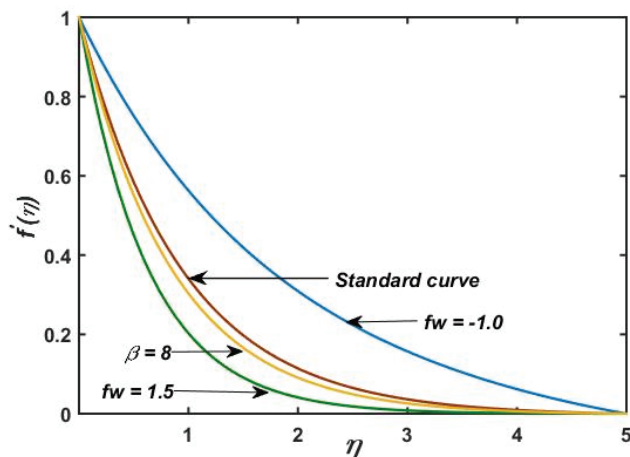


Figure 4. Velocity variation with Suction fw and Casson nanofluid parameter β .

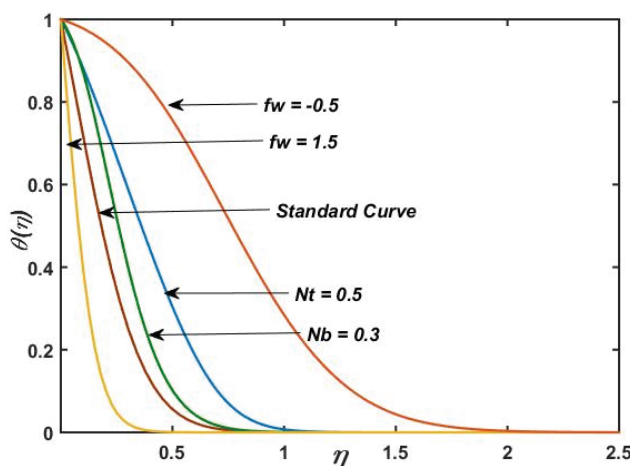


Figure 5. Temperature variation with Thermophoresis Nt , Suction fw , Brownian motion Nb .

to increase and more fluid remains in the boundary layer. When suction is changed from 0.5 to -1, there is a transition from weak suction to injection. At $fw = 0.5$, there is weak suction and at $fw = 0$, there is neither suction nor injection. $fw = -1$ represents injection wherein fluid is pushed into the boundary layer. Therefore, more fluid is added to the flow region which causes the velocity to further rise, especially near the wall. When Casson fluid parameter is raised from 5 to 8, the velocity was found to decrease. Casson fluid parameter β represents non-Newtonian nature of Casson fluid and is inversely related to yield stress. For smaller β ($=5$), the fluid is less viscous and hence moves freely. For larger β ($=8$), the fluid has high resistance to flow and hence the velocity becomes less. Comparable outcomes were found in the literature of [52].

Figure 5 reveals the variations of suction, Brownian flow motion and thermophoresis parameter with temperature profile. When suction parameter fw is 1.5 (strong suction), the boundary layer is thinner as there is less thermal diffusion. When $fw = 0.5$ (weaker suction), there is a moderate boundary layer thinning and hence the temperature is higher. For $fw = -0.5$ (injection), the boundary layer thickens, so there is more thermal diffusion due to injection, meaning higher temperature across the layer. This phenomenon is very useful in applications requiring heat suppression (e.g., electronic cooling) as it maintains thermal stability and enhances surface cooling.

The Brownian motion Nb parameter measures the effect of randomness of nanoparticle motion on energy transport in trihybrid nanofluids which enhances the energy exchange between base fluid (sodium alginate) and nanoparticles. It increases the thermal energy transport within the fluid, which then leads to an increase in temperature, for advanced values of Nb , especially near the surface. TiO_2 and Al_2O_3 contributes to higher thermal stability, and has good photocatalytic properties, whereas

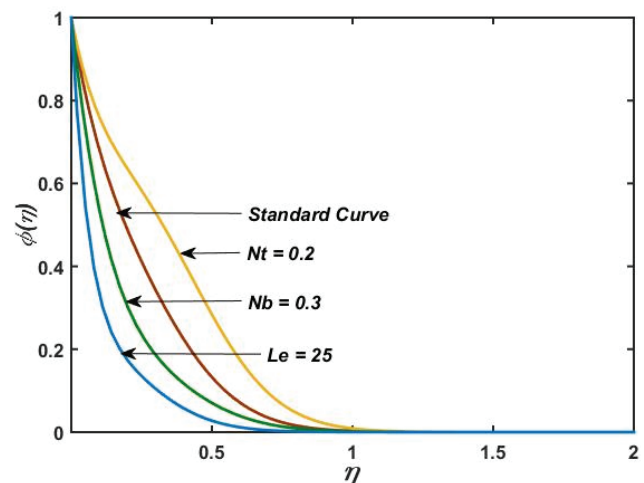


Figure 6. Concentration variation with Thermophoresis Nt , Lewis number Le , and Brownian motion Nb .

SiO_2 enhances thermal suspension ability. When thermophoresis Nt parameter is raised from 0.1 to 0.5, there was a rise in temperature. Thermophoresis refers to motion of particles from a hot to a cold region in a fluid due to the thermal gradient. A high range values of Nt , indicates a strong thermophoretic force, which push a greater number of nanoparticles away from the surface, hence resulting in the rise in temperature within the boundary layer.

Figure 6 depicts the concentration variations of trihybrid nanofluids with various parameters. When Lewis number Le rises, the concentration typically decreases. For a trihybrid nanofluid, higher Lewis number specifies that thermal diffusion is faster than mass diffusion. Therefore, thermal boundary layer develops more rapidly when compared to that of concentration boundary layer. So, the concentration boundary layer had become steep. When thermophoresis Nt , is raised from 0.1 to 0.2, the

concentration profile generally increases. A higher value of Nt indicates a stronger movement of particles away from the hot exterior surface to the bulk fluid. This leads to the accumulation of fluid in the cooler region, which then rises the concentration away from the wall. As a result, overall concentration increases. When Nb , is raised from 0.1 to 0.3, there was a decline in the concentration. Nb signifies the random movement of nanoparticles to thermal agitation. Higher values of Nb enhances the diffusive movement of particles, which spreads the nanoparticle more widely through the fluid domain. Therefore, the concentration gradient becomes more smoother, and overall concentration profile decreases. Alike conclusions were found in the literature of [52].

The physical implications of trihybrid nanofluid flow plays a vital role. In thermal systems, wherein the temperature difference is significant, thermophoresis becomes very important. In $(\text{TiO}_2 + \text{SiO}_2 + \text{Al}_2\text{O}_3 / \text{C}_6\text{H}_9\text{NaO}_7)$ nanofluid, increasing Nb causes greater particle dispersion. Hence it is dominant in micro- and nano-scale flows.

When internal heat generation increases, more thermal energy is added within the fluid which then acts as a source term in the energy equation. TiO_2 and Al_2O_3 possess excellent thermal conductivity and SiO_2 improves thermal stability and controls agglomeration. As a result, more heat accumulates, which then leads to the higher fluid temperature throughout the flow region as depicted in Figure 7. From Figure 8, radiation enhances heat transfer via electromagnetic waves which contributes to additional heat flux. This leads to the enhanced energy transport, effectively raising the temperature of the field. In Figure 9, increased temperature often decreases fluid density and can reduce species diffusion in in trihybrid nanofluid. Solute or nanoparticle concentration tends to decrease with stronger internal heat generation. Hence there is lower concentration near the wall. From Figure 10, higher radiation causes dilution of species concentration, and higher thermal agitation,

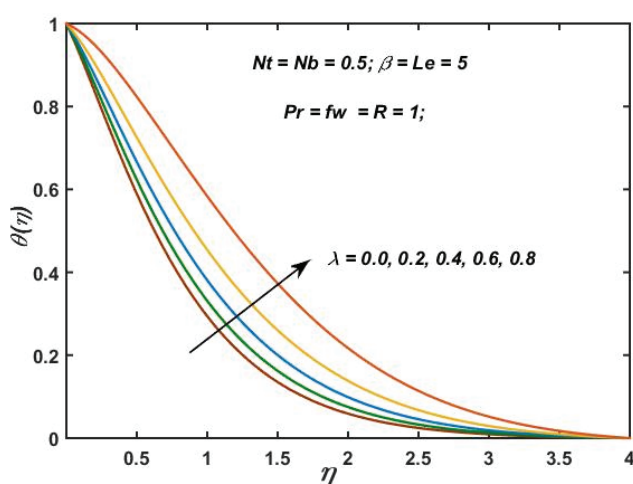


Figure 7. Temperature variation with heat generation λ .

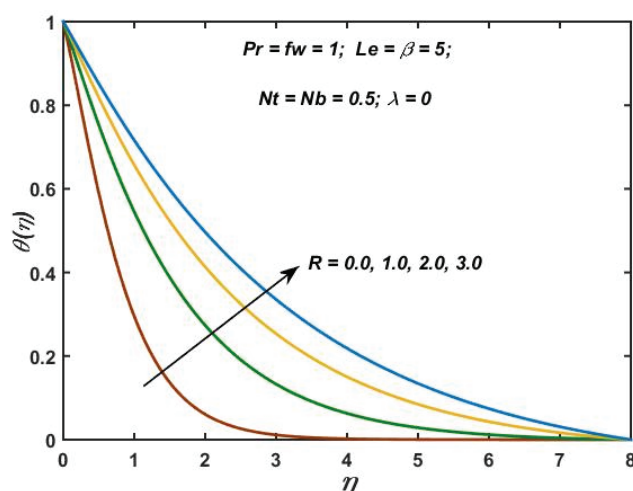


Figure 8. Temperature variation with radiation R .

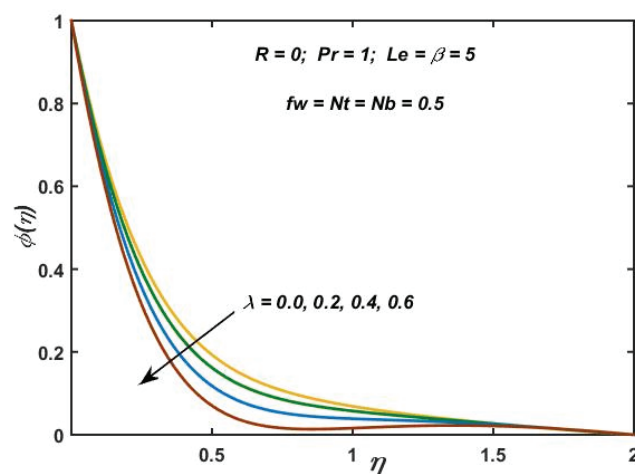


Figure 9. Concentration variation with heat generation λ .

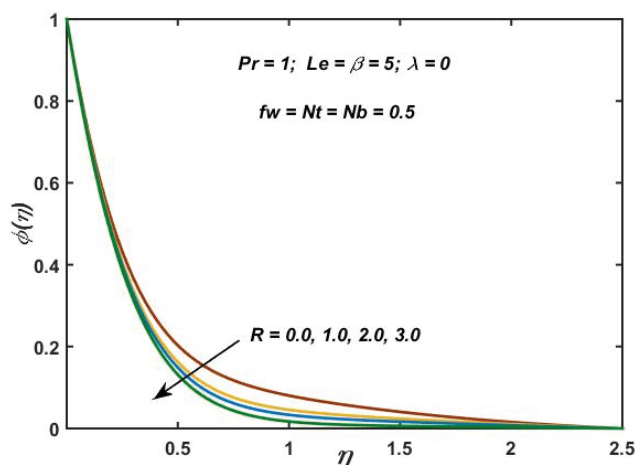


Figure 10. Concentration variation with radiation R .

which reduces the relative particle accumulation, especially in boundary layers. This results in the reduced concentration gradient and overall lower concentration values.

There are various industrial and engineering applications with heat generation and radiation. This can be used in nuclear reactors, power plants because TiO_2 and Al_2O_3 possess excellent thermal conductivity and SiO_2 improves thermal stability. It is also used in solar collectors, solar thermal systems as TiO_2 and SiO_2 are optically active, and are good absorbers of solar radiation. Trihybrid nanofluid can be used as nanofluid-assisted drug delivery as TiO_2 is a Biocompatible oxide used for photothermal effects.

Figures 11 and 12 shows that there is decline for higher value of heat source and radiation parameter. A positive heat source and thermal radiation parameter causes the thermal transport to rise within the fluid, reducing the thermal gradient near the surface. As Nusselt number is

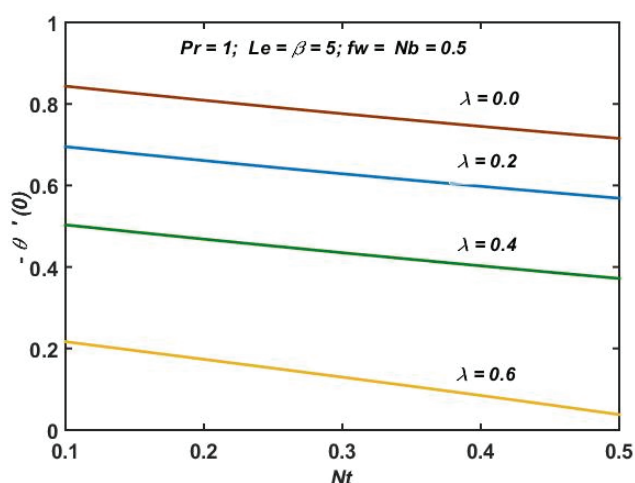


Figure 11. Local Nusselt number on Nt for varying heat generation λ .

directly proportionate to the thermal gradient, it reduces. Figures 13 and 14 depicts the variation of Sherwood number for varying heat source and radiation parameter. The Sherwood number was found to increase and similar results were obtained by [53].

Table 3 presents the present outcomes with the previously published result by taking $Le=10$, $f_w=0$, $\beta \rightarrow \infty$. From Table 4, the Nusselt number typically reduces for advanced values of R and λ for a trihybrid nanofluid flow. Positive heat source and radiation enhances thermal energy to the fluid while increasing the temperature of the fluid near the surface. The thermal gradient at the wall decreases and Nusselt being proportional to the temperature gradient, reduces. Sherwood number was found to decrease for advanced values of heat generation and radiation.

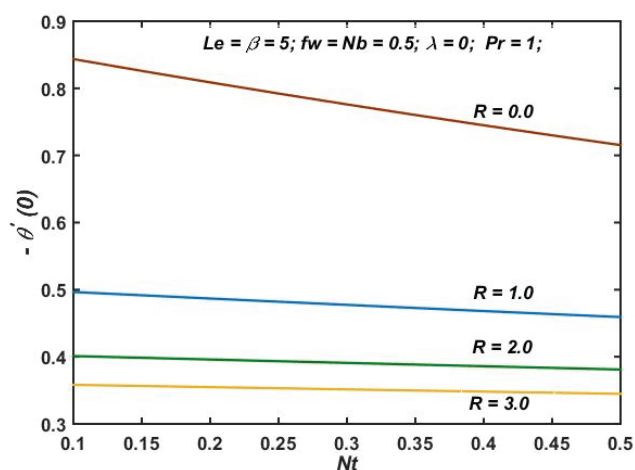


Figure 12. Local Nusselt number on Nt for varying radiation R .

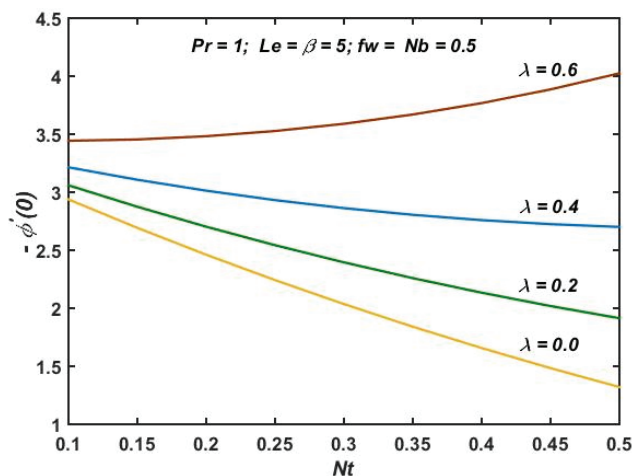


Figure 13. Local Sherwood number on Nt for varying heat generation λ .

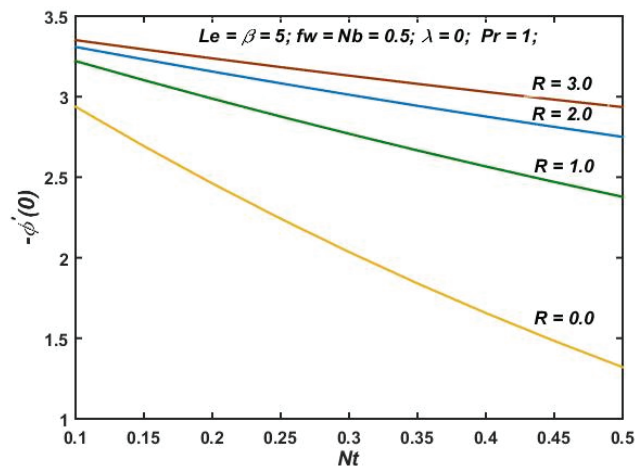


Figure 14. Sherwood number on Nt for varying radiation R .

Table 4. Computed local Nusselt $-\theta'(0)$, local Sherwood $-\phi'(0)$ numbers for changing values of heat generation λ and Radiation R as $Le = 5$, $fw = 0.5$, $\beta = 5$, $Nb = Nt = 0.1$, $Pr = 10$, $M = K = 1$

λ	R	$-\theta'(0)$	$-\phi'(0)$
0.0	0.0	3.35766146	0.74574654
0.2		3.06867167	1.02374100
0.4		2.75576605	1.32413791
0.6		2.41285144	1.65260655
0.8		2.03105529	2.01739063
1.0		1.59669978	2.43120211
0.0	1.0	2.10893932	1.88572612
	2.0	1.61344374	2.31487442
	3.0	1.32981417	2.55088933

Table 3. Comparison table

Pr	Khan and Pop [4]	Gorla and Sidwai [54]	Manjunatha et al. [45]	Current Study
2	0.9113	0.9114	0.9113	0.9114
6.13	1.7597	1.7596	1.7597	1.7596
7	1.8954	1.8954	1.8954	1.8953
20	1.3539	1.3539	1.3539	1.3539

CONCLUSION

The present study investigates two-dimensional, laminar, incompressible, steady, viscous tri-hybrid nanofluid flow on a linearly stretching sheet. Three kinds of nanoparticles called TiO_2 , SiO_2 and Al_2O_3 are been dispersed in (base fluid) sodium alginate to obtain a new configuration $TiO_2+SiO_2+Al_2O_3/C_6H_9NaO_7$. Thermophoretic and Brownian motion parameters are merged in the concentration Navier Stokes equation. The above-mentioned physical illustrations have many usages in the field of industries and manufacturing, particularly in cooling systems. Due to its higher thermal conductivity, lately it has become a very effective method in cooling systems. In the current problem, numerical analysis of tri-hybrid nanoparticles has been conducted to study the pattern of velocity, thermal and concentration profiles. Some of the decisive results of the current study are enumerated below:

1. When the heat generation λ rises, there is an inclination in the temperature.
2. With an inclination in radiation number R , a rise in temperature is found.
3. A decay in the concentration is found, due the increase in heat generation.
4. Concentration has reduced for higher radiation values.
5. A decline in Nusselt number; a rise in Sherwood number were noted with the rise in heat generation.

6. With the rise in radiation, there is a fall in the local Nusselt number, and a rise in the local Sherwood number.

NOMENCLATURE

C	Nanoparticle concentration
C_w	Surface Concentration
C_∞	Concentration near infinity
D_B	Brownian flow motion coefficient
D_T	Thermophoretic motion coefficient
h_f	Coefficient for heat transfer
K	Porosity parameter
K_1	Coefficient of Thermal slip
L	Thermal Slip
M	Magnetic parameter
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Pr	Prandtl number

Greek Symbols

β	Casson fluid parameter
ρ	Fluid density
η	Dimensionless Similarity variable
π	Deformation rate
σ	Electric conductivity

λ Heat source
 θ Non-dimensional temperature

Subscripts

f Nanofluid
 tnf Tri-Hybrid Nanofluid
 ∞ Condition at the free stream
 w Condition at the wall

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The authors declared no conflict of interest.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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