



Research Article

Thermal analysis of hybrid Casson nanofluid ($\text{AlN}+\text{Al}_2\text{O}_3/\text{C}_6\text{H}_9\text{NaO}_7$) with Lorentz force effect over a linearly stretching sheet

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ABSTRACT

The current numerical study analyses the flow of 2D, steady-state, MHD Hybrid Casson nanofluid ($\text{AlN}+\text{Al}_2\text{O}_3/\text{C}_6\text{H}_9\text{NaO}_7$) over a linearly stretching sheet by incorporating the Lorentz force, heat generation and radiation effects. The study not only explores to achieve superior heat transmission compared to that of traditional nanofluids, it also highlights the improvement of the fuel efficacy and engine process in automotive engineering, also productivity for the cooling arrangements used in food preservation and processing. This investigation offers vision for the progress of effective energy storage systems, heat exchangers, also cooling systems of different manufacturing applications. The nonlinear coupled partial differential equations are explained with Runge-Kutta system using shooting method. Higher values of radiation and heat generation parameters, strengthens the thermal process and weakens the concentration diffusion. Sherwood number had risen from 0.876 to 1.423 for nanofluids and 1.023 to 1.652 for hybrid nanofluids. Novelty of the research article is to streamline the earlier works by incorporating radiation, Brownian motion, heat source, thermophoresis, and magnetic effects in the equivalent governing equations leading to an achievement of effective hybrid nanofluid flow.

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INTRODUCTION

Nanofluid consists of particles that are nano-meter sized identified as nanoparticle. These nano-meter sized solid nanoparticles are dispersed in base fluids like propylene glycol, synthetic oils, water, etc. They are made up of

carbides (SiC), nitrides (AlN), oxides (Al_2O_3 , SiO_2 , TiO_2), metals (Cu , Al), etc. and they have exclusive physical and chemical characteristic properties. The smoothness of nanofluid flow without any clogging is due to their small sized nanoparticles. Nanofluids are extensively used for the

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improved heat transfer development materials and hence, it captivated many scientists and researchers to discover the heat transfer features. The base fluids possess low heat transfer properties. One among the latest technologies, heat transfer across a sheet stretching or otherwise, demanded an understanding of the flow dynamics, especially near the boundary of these sheets as the optimum changes occur in these layers. This approach gave a deep vision into the features of fluid/heat transfer which hugely supported the engineering processes. The flow over the boundary layer which arises because of a stretching sheet which moves linearly with the changing velocity, was carried out in literature of [1]. As there was advancement in nanotechnology, nanofluid originated which researchers in Fluid Dynamics incorporated into their works.

The name “Nanofluid” has been initiated by [2]. He projected a new classification of fluids which has been observed by suspension of small nano-sized units in the heat transfer of fluids. Nanoparticles are naturally made of carbon nanotubes, metals, carbides, or oxides. Nanofluids possesses very useful properties in several applications of heat transfer in microelectronics, artificial fibers, extrusion of polymers, copper wire drawing, processes in treatments, and coolant for engines.

[3] analyzed a model of Casson fluid considering the heat transmission with injection/suction. The viscosity of the flow across a horizontal stretching surface was studied. [4] concluded that Nusselt number is becoming a property that is decreasing for each and every dimensionless number. Also, it was found that Sherwood number is increasing with a high value of Prandtl number and is a function that is decreasing with the lower value of Prandtl for individual values of Le and Nb . [5] studied the impact of boundary conditions with a partial slip of nanofluid on a stretching surface keeping the wall temperature as constant. The result showed that the shear stress, flow velocity are highly affected by the slip parameters. In the literature of [6], the 3D MHD Casson Carreau nanofluid flow with thermal radiation was studied. It was observed that with the rise in Prandtl number along with buoyancy ratio, the boundary layer thickness had raised. The researchers in [7] probed the effects of Ohmic heating and transverse current on an exponential stretching sheet with embedded porous media.

In literature of [8], the heat absorption effect of Casson nanofluid on a cylinder with the effects of the chemical reactions, was studied. The mathematical model was numerically solved in COMSOL. [9] studied a heat sink/source in MHD Casson fluid and was found that the velocity profiles were higher and the fields of concentration and temperature had an opposite phenomenon. [10] considered the thermal radiation of Casson nanofluid. By keeping only, the Casson fluid parameter constant, it was concluded that an upsurge in Brownian flow motion reduces the temperature. [11] found the point of stagnation of a Casson fluid with variable thickness and thermal radiation. Various parameters like wall thickness, velocity ratio and velocity power index were

studied. [12] analyzed the transmission of heat which flows across a vertical cylinder and having exponential stretching. Influences of parameters like the Lewis number, Prandtl number, and Reynolds number were studied.

Hybrid nanofluids are advanced heat transmission fluids formed by dissolving two different kinds of nanoparticles in the base fluid. They offer improved thermal conductivity, viscosity control, and steadiness related to conventional nanofluids containing only a single type of nanoparticle. Experimentally it is proven that the non-metallic nanoparticles are more stable than metallic nanoparticles. Casson hybrid nanofluids is a non-Newtonian extension of hybrid nanofluids where the base fluid obeys the Casson fluid model. This model accounts for yield stress behavior observed in materials like chocolate, blood, and toothpaste. When nanoparticles are suspended in such fluids, the resulting combination is a Casson hybrid nanofluid. These fluids are useful for simulating real-life complex fluids in biomedical, manufacturing and industrial applications. The authors in [13-15] studied the MHD effects of hybrid Casson nanofluid along by taking into consideration porosity, viscous dissipation, Joule heating and heat sink/source

[16] reviewed the transmission of heat features of a 2D Casson nano fluid flow over the wedge. Taking the nanoparticle's volume fraction flux as zero, the impact of flow parameters like volume fraction and temperature of the nanoparticle were studied. [17] comprehensively analysed the Maxwell convective hybrid nanofluid flow on a stretching sheet. [18] analyzed the Casson fluid model on a symmetrical wedge and found that with an increase in the Falkner-Skan exponent, there was an augmentation in velocity and decay in temperature. With an upsurge in Casson parameter, velocity got amplified and a decline in temperature. [19] conducted investigation of a 2D flow of unsteady Casson fluid. It was reported that fluid velocity declines with amplification of unsteadiness parameter. An upsurge in the Casson parameter caused the velocity to get suppressed and there was an enhancement in temperature. [20-22] analyzed the inertia point flow features of hybrid Williamson Casson nanofluid flow with viscous dissipation and injection effects. [23] studied the features of an isotropic incompressible Casson nanofluid with thermal and solute boundary conditions over an oscillatory plate. It was found that the concentration of species and fluid velocity increased whenever there was an increment in the coefficient of mass transfer. [24, 25] combined the Yamada Ota model into Casson Maxwell quadra-hybrid nanofluid framework over an exponential stretching sheet which explores the endothermic and exothermic reactions.

Sodium alginate is a salt which is a natural polymer present in the alginic acid, and is been detached from the brown algae. This is utilized in textile production, industrial, medications and cosmetics. The utilities of sodium alginate include radioembolization [26, 27], medication delivery system [28-31] encapsulation of cells [32-35] cellular therapeutics Barnett [36]. The authors in the literature

[37] carried out a computational study to comprehend the Darcy-Forchheimer EMHD Williamson flow which is sodium alginate-based nanofluid across a vertically stretching cylinder.

On the basis of above-mentioned literature, the originality and novelty of the current work simplify the previous work by including Lorentz effect, Brownian motion, thermophoresis, heat source and radiation effects over a linearly stretching sheet of a Casson hybrid nanofluid flow. Existing work displays, the magnetic parameter has a noteworthy part in the momentum and heat transmission of hybrid nanofluids. The present outcomes have been compared with previous outcomes to show correctness of achieved similar outcomes and were in upright agreement.

PROPERTIES OF BASE FLUID AND HYBRID NANOFLUIDS

The base fluid is sodium alginate ($C_6H_9NaO_7$) and nanoparticles Alumina (Al_2O_3) and Aluminum Nitride (AlN) mixture are considered as nanoparticles. Elementary equations [38] for Casson fluid model:

$$\tau_{ij} = \begin{cases} \left(\mu_c + \frac{P_s}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi > \pi_0 \\ \left(\mu_c + \frac{P_s}{\sqrt{2\pi_0}} \right) 2e_{ij}, & \pi < \pi_0 \end{cases} \quad (1)$$

Fluid yield stress $P_s = e_{ij} e_{ij}$;

π - Rate of deformation that is multiplied to itself.

π_0 - Critical value.

From (1), (for $\pi < \pi_0$) we have

$$\tau_{ij} = \mu_c \left(1 + \frac{1}{\beta} \right) 2e_{ij} \quad (2)$$

where $\beta = \mu_c \sqrt{2\pi_0} / P_s$ is Casson parameter.

The Thermal characteristics viz. Thermal conductivity, density, and specific heat are [39]:

$$\rho_{hnf} = \left(\frac{\phi}{100} \right) \rho_p + \left(1 + \frac{\phi}{100} \right) \rho_f, \quad (3)$$

$$\left. \begin{aligned} \frac{k_{hnf}}{k_f} &= 1.2035 \left(1 + \frac{\phi}{100} \right)^{0.0098} \left(0.01 + \frac{T_{hnf}}{100} \right)^{0.1331} \\ &\quad \left(0.001 + \frac{dp}{100} \right)^{-0.0001} \left(0.01 + \frac{\alpha_p}{\alpha_f} \right)^{0.0153} \end{aligned} \right\} \quad (4)$$

$$(\rho C_p)_{hnf} = \left(\frac{\phi}{100} \right) (\rho C_p)_p + \left(1 - \frac{\phi}{100} \right) (\rho C_p)_f, \quad (5)$$

$$\frac{\mu_{hnf}}{\mu_f} = 0.3659 * C1 * \exp \left\{ \left(1 + \frac{\phi}{100} \right)^{10.83} \left(\frac{T_{hnf}}{90} \right)^{-0.0238} \left(1 + \frac{dp}{170} \right)^{-0.1609} \right\}, \quad (6)$$

$$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}, \quad (7)$$

$$\vartheta_f = \frac{\mu_f}{\rho_f} \quad (8)$$

Problem Formulation

Consider an incompressible, 2D, laminar, viscous, steady, boundary layer flow over a horizontal stretching sheet. The following assumptions were made:

- Two kinds of nanoparticles viz. Al_2O_3 and AlN were dispersed in sodium alginate in order to produce a new composition $Al_2O_3 + AlN / C_6H_9NaO_7$ as depicted in Figure 1.
- We assumed thermal equilibrium between base fluid and nanoparticles.
- x -axis is been aligned with edge of the sheet and y -axis is perpendicular to it and the representation diagram is shown in Figure 2.
- The Brownian flow motion, magnetic effect, Thermophoretic effect, heat generation and radiation are incorporated in the flow system.
- Fluid flow is confined to a section $y > 0$.

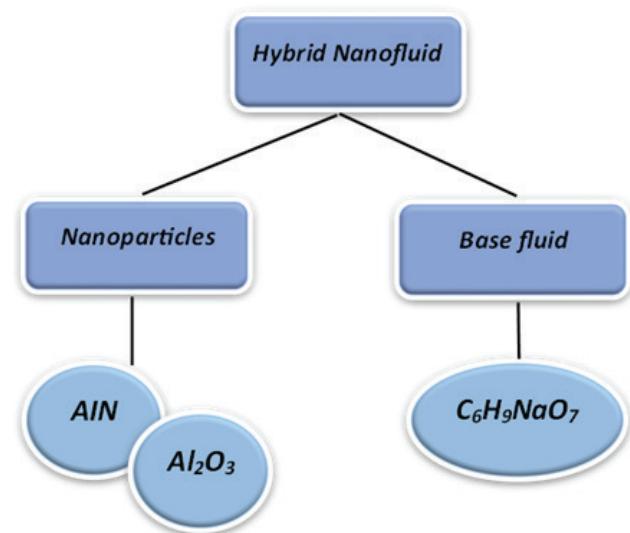


Figure 1. Outline of hybrid nanoparticles.

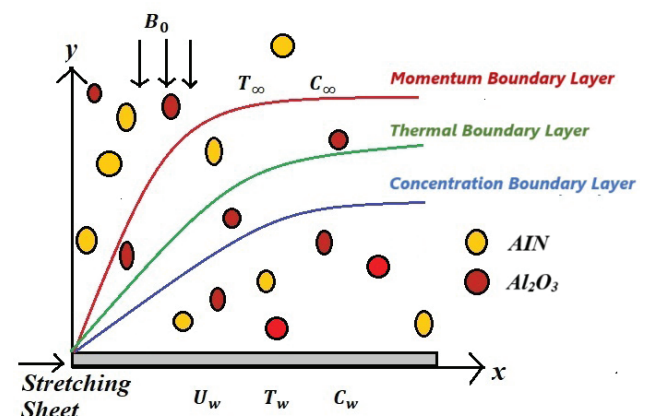


Figure 2. Geometry of the Problem.

Table 1. Thermal characteristic features of base fluid and nanoparticles

Properties	AlN	Al ₂ O ₃	C ₆ H ₉ NaO ₇
Density	3260	3880	989
Specific Heat	735	773	4175
Thermal Conductivity	180	40	0.613
Coefficient of Thermal Expansion	0.00027	0.00028	0.00001

- A uniform transverse magnetic field with strength B_0 is set, and induced magnetic field is ignored owing to the negligible magnetic Reynolds number.
- The stretching velocity of the sheet is $u_w(x) = ax$. a is taken as a constant.
- The temperature and Concentration at the wall and far from the surface are quantified as T_w , C_w , T_∞ , and C_∞ .
- The temperature difference considered in this model is within the range where property variation is relatively small and does not significantly alter the qualitative behaviour of the flow and heat transmission.
- Table 1 summarizes the thermodynamic features of the base fluid and its nanoparticles.

The Casson nano fluid flow gets altered due to the impact of Lorentz effect, radiation and generation of heat effects under the boundary layer approximations. Hence, based on these Prandtl flow assumptions and above-mentioned estimates the demonstrative Navier-Stokes equations of the present study are presented below [2, 40-42]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho_{hnf}} \quad (10)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q_0}{(\rho c p)_{hnf}} (T - T_\infty) - \frac{1}{(\rho c)_{hnf}} \left(\frac{\partial q_r}{\partial y} \right) \quad (11)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (12)$$

The conditions at the boundary: [2, 40-42]

$$u = u_w(x), \quad v = -v_w, \quad T = T_w, \quad \text{and} \quad C = C_w \quad \text{at} \quad y = 0 \quad (13)$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{and} \quad C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty \quad (14)$$

where, u denotes the velocity with x -direction and v represents the velocity along y direction.

$$\text{where } \alpha = \frac{k}{(\rho c p)_{hnf}}, \quad \tau = \frac{(c \rho) p}{(\rho c p)_{hnf}}, \quad \vartheta = \frac{\mu}{\rho_{hnf}},$$

$$u_w = ax, \quad T_w = T_\infty + b \left(\frac{x}{l} \right)^2, \quad C_w = C_\infty + c \left(\frac{x}{l} \right)^2 \quad (15)$$

To simplify the numerical study, the quantities are been presented by means of similarity transforms.

$$\eta = \sqrt{\frac{a}{\vartheta}} y, \quad \Psi = \sqrt{a \vartheta} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (16)$$

$$u = ax f'(\eta), \quad v = -\sqrt{a \vartheta} f(\eta) \quad (17)$$

$\Psi(x, y)$ denotes stream function which is:

$$u = \Psi_y, \quad v = -\Psi_x \quad (18)$$

Radiative flux which is in y direction is dominant and the x direction component can be neglected. Radiative heat flux q_r , with the help of Rosseland approximations is mentioned as,

$$q_r = -\frac{4\sigma^*}{3k^*} T_y^4 \quad (19)$$

where σ^* - Stefan Boltzmann constant. Within the flow, the difference in temperature, is small and hence the term T^4 has been expanded with the aid of Taylor's series.

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (20)$$

It follows on substituting Eq. (20) into Eq. (18),

$$q_r = -\frac{16T_\infty^3 \sigma^*}{3k^*} T_y \quad (21)$$

The governing Eqns. (9) – (12) are altered into ODE by the aid of Similarity transformations. [2, 41-43]

$$\left(1 + \frac{1}{\beta} \right) f''' + A_1 A_2 (f f'' - f'^2 - \frac{M}{A_2} f') = 0 \quad (22)$$

$$\frac{1}{A_4} \left(1 + \frac{4}{3} \frac{R}{A_3} \right) \theta'' + Pr f \theta' + Pr Nb \theta' \phi' + Pr Nt \theta'^2 + \frac{\lambda}{A_3} Pr \theta = 0 \quad (23)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0 \quad (24)$$

Where,

$$\frac{1}{A_1} = 0.3659 * C_1 * \exp \left\{ \left(1 + \frac{\phi}{100} \right)^{10.83} \left(\frac{T_{hnf}}{90} \right)^{-0.06239} \left(1 + \frac{dp}{170} \right)^{-0.1609} \right\},$$

$$A_2 = \left(\frac{\phi}{100} \right) \frac{\rho_p}{\rho_f} + \left(1 + \frac{\phi}{100} \right)$$

$$A_3 = \left(\frac{\phi}{100} \right) \frac{(\rho C_p)_p}{(\rho C_p)_f} + \left(1 - \frac{\phi}{100} \right)$$

$$A_4 = 1.2035 \left\{ \left(0.001 + \frac{\phi}{100} \right)^{0.0098} \left(0.01 + \frac{T_{hnf}}{90} \right)^{0.1331} \left(0.001 + \frac{dp}{170} \right)^{-0.0001} \left(0.01 + \frac{\alpha_p}{\alpha_f} \right)^{0.0153} \right\}$$

The boundary conditions [2, 41–43]:

$$\left. \begin{aligned} f(0) = f_w; f'(0) = 1; \theta(0) = 1; \phi(0) = 1; \text{ at } \eta = 0 \\ f'(\infty) = 0; \theta(\infty) = 0; \phi(\infty) = 0; \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (25)$$

Governing parameters are given by:

$$\left. \begin{aligned} f_w = -\frac{v_w}{\sqrt{a\theta}}; Nt = \frac{(\rho c) p D_T (T_w - T_\infty)}{(\rho c)_f \theta T_\infty}; Nb = \frac{(\rho c) p D_B (\phi_w - \phi_\infty)}{(\rho c)_f \theta} \\ Pr = \frac{\theta}{\alpha}; R = \frac{4\sigma^* T_\infty^3}{3k^* k}; Le = \frac{\theta}{D_B}; \lambda = \frac{Q_0}{(\rho c_p)_f \alpha} \end{aligned} \right\} \quad (26)$$

where f represents the velocity (dimensionless), θ represents the temperature, and ϕ represents the nanoparticle concentration.

$$C_f = \frac{T_w}{\rho u_w^2}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x h_m}{D_B(\phi_w - \phi_\infty)} \quad (27)$$

$$q_w = -k(\phi_y)_{y=0}, h_m = -D_B(\phi_y)_{y=0} \quad (28)$$

From the above-mentioned equations, the skin friction coefficient, heat and mass transfer rates in non-dimensional arrangement are:

$$C_f \sqrt{Re_x} = -f''(0), \frac{Nu_x}{\sqrt{Re_x}} = -(1+R)\theta'(0), \frac{Sh_x}{\sqrt{Re_x}} = -h'(0)s \quad (29)$$

Solution Procedure

The numerical outcomes for the ODE's (22) to (24) with the related boundary conditions in Eq. (25) are mathematically resolved using shooting mechanism and Bvp4c solver. To get precise and effective outcomes, choosing proper initial guess values is mandatory. It is crucial to alter this approximation in order to get correctness and assure the convergence in the obtained results. The boundary layer width is selected, as it has an impact on the mathematical outcomes. Criterion for convergence is taken as 10^{-5} to attain a improved degree of accuracy. The functioning process of Bvp4c system is portrayed in Figure 3 [44].

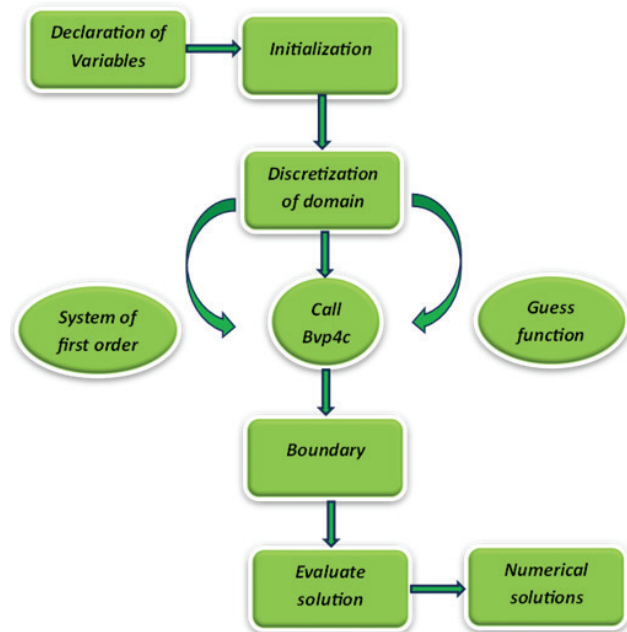


Figure 3. Flow configuration and coordinate system.

The substitutions made were:

$$f_1 = f; f_2 = f'; f_3 = f''; f_4 = \theta; f_5 = \theta'; f_6 = \phi; f_7 = \phi'$$

The reduced equations are:

$$f_3' = -A_1 A_2 \frac{(-f_1 f_3 + f_2^2 - \frac{M}{A_2} f_2)}{(1 + \frac{1}{\beta})}$$

$$f_5' = -A_4 Pr \frac{(-f_1 f_5 - Nb f_5 f_7 - Nt f_5^2 - \frac{1}{A_3} \lambda f_4)}{(1 + \frac{4}{3} \frac{R}{A_3})}$$

$$f_7' = -Le f_1 f_7 + \frac{Nt}{Nb} A_4 Pr \frac{(-f_1 f_5 - Nb f_5 f_7 - Nt f_5^2 - \frac{1}{A_4} \lambda f_3)}{(1 + \frac{4}{3} \frac{R}{A_3})}$$

Conditions at the boundary:

$$f_1(0) = f_w; f_2(0) = 1; f_4(0) = 1; f_6(0) = 1,$$

$$f_1(\infty) = 0; f_2(\infty) = 0; f_6(\infty) = 0$$

The significant feature of the present study is, BVP is altered into an IVP. The solution to the IVP is found by making guess values for the initial values that are missing by using shooting technique. The initial guesses were systematically selected using a continuation approach, wherein solutions at nearby parameter values were used as starting guesses for subsequent computations. The convergence is achieved within a reasonable range of initial estimates, and divergence occurs only when guesses are significantly far from physically admissible solutions. Step size is 0.001 for doing calculations and tolerance for error is taken as 10^{-5} .

RESULTS AND DISCUSSION

A mathematical study was executed to study the thermo-fluidic transport of Casson hybrid nanofluids over a linearly stretching sheet. Runge-Kutta method with shooting process is used to explain the equations governing the transport phenomena. The flow velocity and temperature distributions were determined, followed by evaluating the concentration profile through numerical solutions. The graphical depiction of Hybrid nanofluid motion, thermal

and mass diffusion for numerous parameters are shown below.

Figure 4 displays that the rise in Casson parameter from 1 to 3.5, leads to decline in motion of the fluid. Rise in β leads to higher yield stress, which then led to a rise in fluid's elasticity which cause the hybrid nanofluid to turn out to be thicker and viscous. The fluid behaves like solid and hence declining the fluid velocity. Similar results regarding velocity profile were obtained by [43]. The suction parameter represents how much fluid is been pulled towards the surface as shown in Figure 5. By increasing suction parameter from 0.5 to 1.5, a part of fluid gets removed from the flow nearby the surface and lessens the kinetic energy, causing the velocity to reduce. In case of hybrid nanofluid (Al_2O_3 and AlN) has an effect on the viscosity and thermal conductivity. Similar outcomes pertaining to velocity profile was found in the literature of [45]. The physical significance of these varying parameters over velocity profile plays a crucial role in various fields. For MHD driven processes porous electrodes, and heat sink with porous structure could affect the fluid velocity management of nanofluids. The velocity variation due to suction and Casson fluid parameters could be harnessed to enhance the heat transfer in various devices like generators, coolant for nuclear reactors, chemical reactors of catalytic processes, etc.

Figure 6 depicts the effect of radiation on thermal profile. Radiation number quantifies the influence of radiative heat transfer. Heat diffuses more easily and temperature remains higher near the surface. Aluminum oxide is chemically stable and has good thermal conductivity. Aluminum Nitride is electrically insulating and has high thermal conductivity. Combining both enhances the thermal conductivity more than using a conventional nanofluid. Comparable trends in temperature profile were

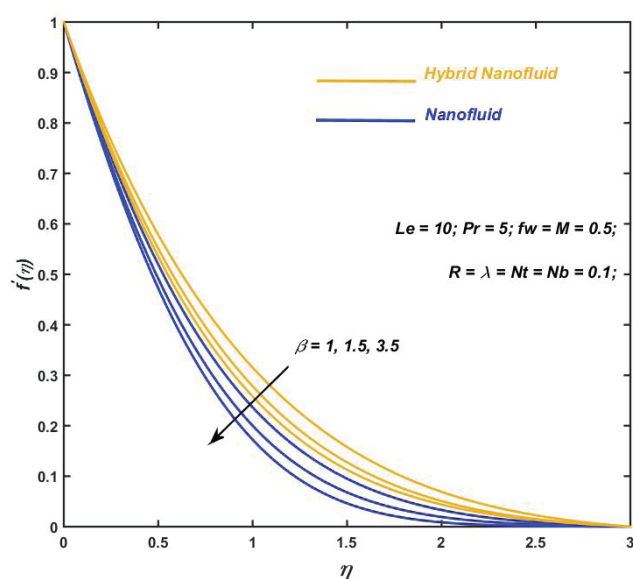


Figure 4. Velocity profile with varying Casson fluid parameter β .

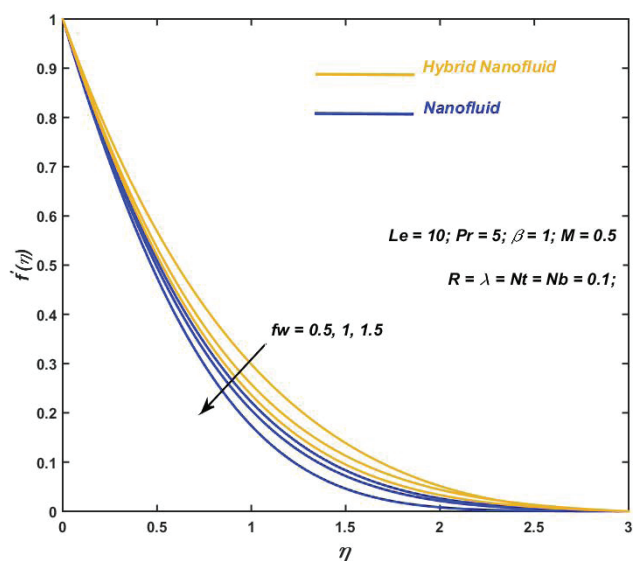


Figure 5. Velocity profile with varying suction parameter.

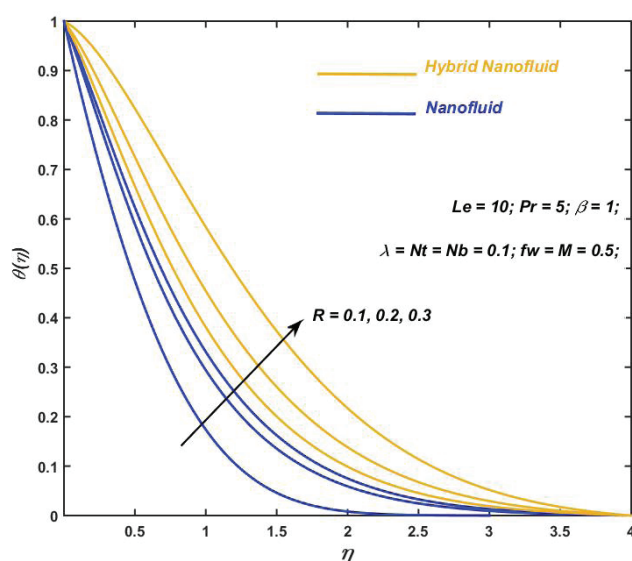


Figure 6. Thermal profile for varying radiation parameter R .

observed in [46]. The influence of heat generation on thermal profile is depicted in Figure 7. For higher values of λ , heat is generated inside the fluid and not just the wall. This directly rises the fluid temperature. Both AlN and Al₂O₃ are high thermal conductivity materials. Thermal conductivity of AlN (~140-180 W/m.K) and Al₂O₃ (~31 W/m.K). Together they distribute generated heat uniformly and maintain high temperature in comparison to that of base fluid and mono nanofluid. The physical significance is of utmost important as it prevents overheating and material degradation. Also reduces energy needed for cooling systems. Proper usage of radiative heat transfer with hybrid

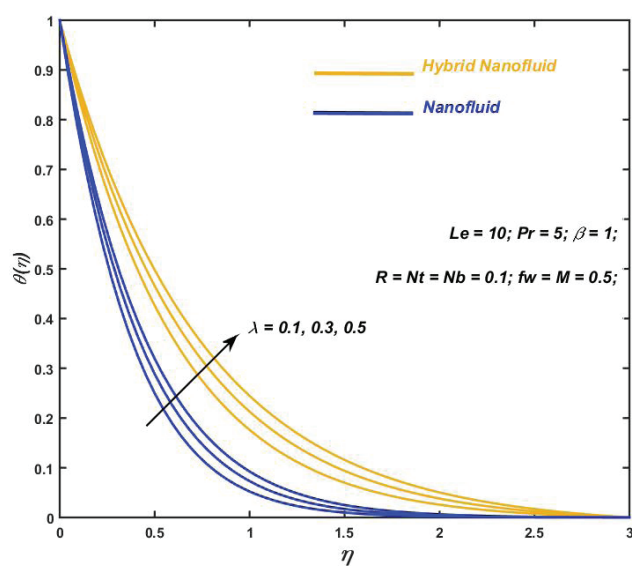


Figure 7. Thermal profile for varying heat generation λ .

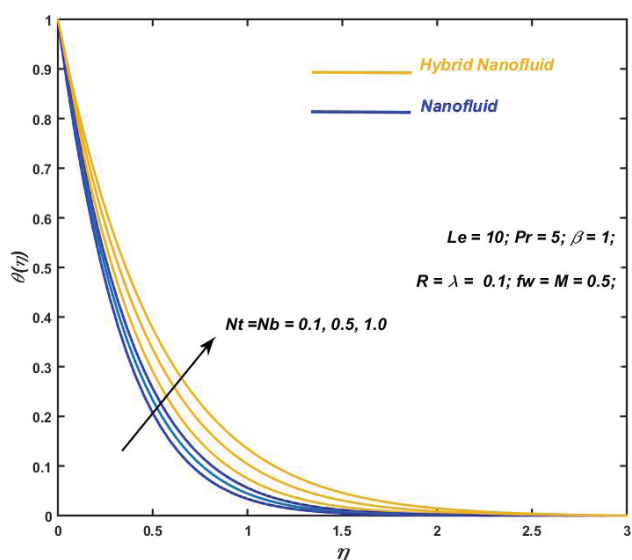


Figure 8. Thermal profile for varying Nt, Nb .

nanofluids can increase energy efficiency in thermal systems. The choice of AlN and Al₂O₃ balances high conductivity with chemical stability which is very ideal for high-temperature or high-radiation environments. Figure 8 displays the impact of thermophoretic and Brownian flow motion on thermal field. Thermophoresis represents the translocation of nanoparticles from hotter to colder region which is driven by temperature gradients. If thermophoretic effect becomes stronger, hybrid nanoparticles (AlN + Al₂O₃) are pushed from hotter to colder regions which led to the decrease in the local concentration near the hot surface. Both the nanoparticles having high thermal conductivity and being stable in water-based fluids, leads to rise in thermal profile. As Brownian motion parameter increases the nanoparticle random motion intensifies as there are more frequent particle-fluid interactions. Change of kinetic to thermal energy increases and hence the fluid temperature increases. Finally, an increase in Nb and Nt enhances nanoparticle-induced energy transport in AlN + Al₂O₃ hybrid nanofluid, resulting in an upsurge in fluid temperature and thickens the boundary layer [47, 48].

Figure 9 illustrates the influence of heat source on species concentration profile. Whenever the heat source strength rises, the internal heat generation raises the thermal energy of the nanofluid which increases the boundary layer thickness. Rising the temperature, Brownian motion of the particles push the nanoparticles far away from the hot surface. So, the particles travel from higher to lower temperature regions. This led to a drop in the concentration gradient at the wall. From Figure 10, when thermal radiation parameter increases, stronger radiative flux adds extra thermal energy to the fluid that thickens the boundary layer. Higher temperature intensifies the

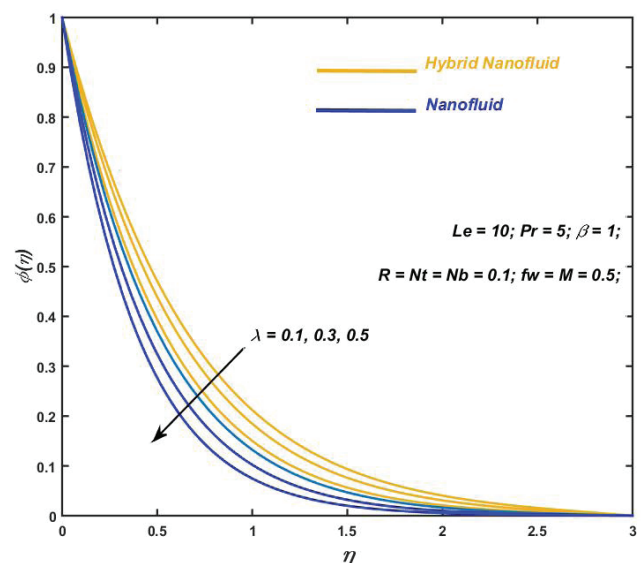


Figure 9. Concentration profile for varying λ .

random motion of nanoparticles pushing them away from the surface. As the nanoparticles travel from hotter to colder regions, the concentration gradient at the surface gets weakened. Thus, for a $\text{AlN} + \text{Al}_2\text{O}_3$ Casson hybrid nanofluid, an increase in radiation parameter generally causes the concentration profile to reduce. Similar results about the concentration profile were seen in the work of [39]. The physical significance of enhanced heat transfer and radiation boosts the thermal diffusion and hence can be used in reactors, cooling systems and solar collectors. Engineers and scientists can use radiation to manipulate particle distribution and optimize performance in thermal management systems.

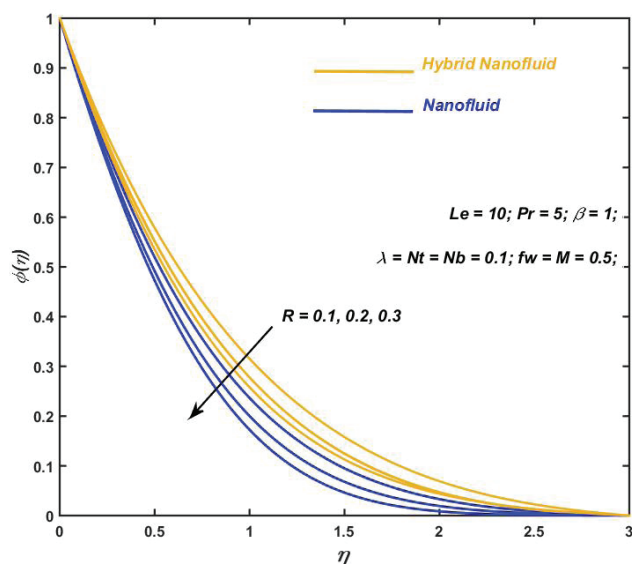


Figure 10. Concentration profile for varying R .

Figure 11 depicts the effects on skin friction coefficient for Casson fluid parameter. Increasing Casson fluid parameter diminishes the yield stress effect. Fluid near the wall deforms more easily. Velocity gradient close to the hot surface and shear stress increases. So, skin friction increases and similar results about skin friction were found in [44]. Figure 12, illustrates the effect of heat generation with Nusselt number. Internal heat generation adds energy to fluid which causes the fluid temperature to rise throughout the fluid. Wall heat flux and temperature gradient close to the wall reduces which led to the Nusselt number to reduce. Figure 13 depicts the impact the radiation parameter has on Nusselt number. Increasing radiation parameter, the radiative heat flux and thermal conductivity becomes more efficient. More heat is distributed away from the wall causing the wall temperature gradient to decline. Consequently, Nusselt number decreases.

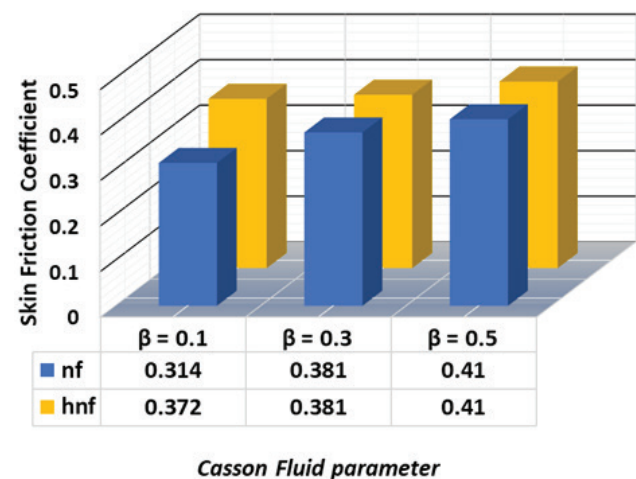


Figure 11. Skin friction variation with Casson fluid parameter.

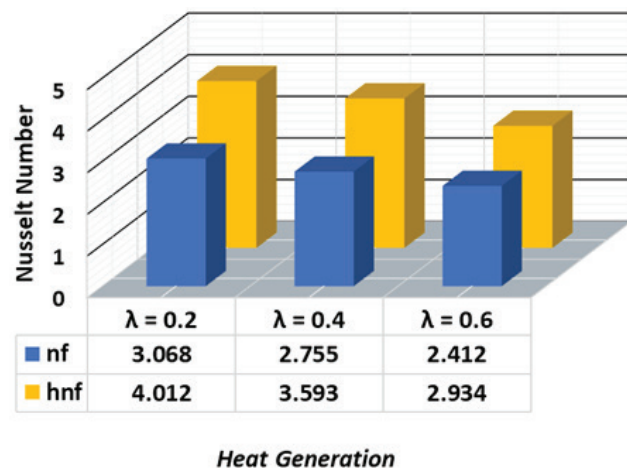


Figure 12. Nusselt number variation with heat generation.

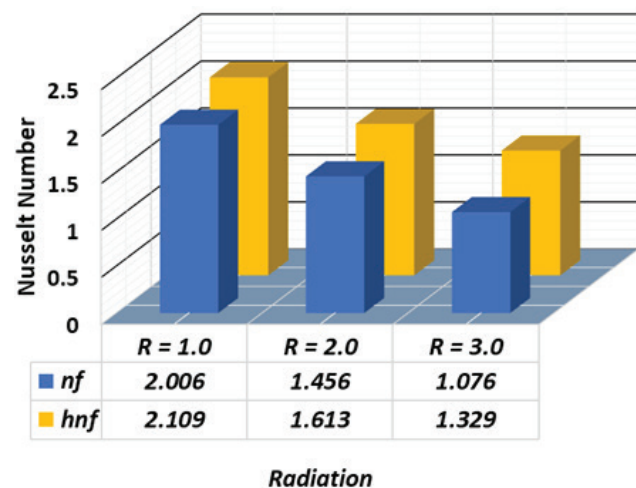


Figure 13. Nusselt number variation with radiation.

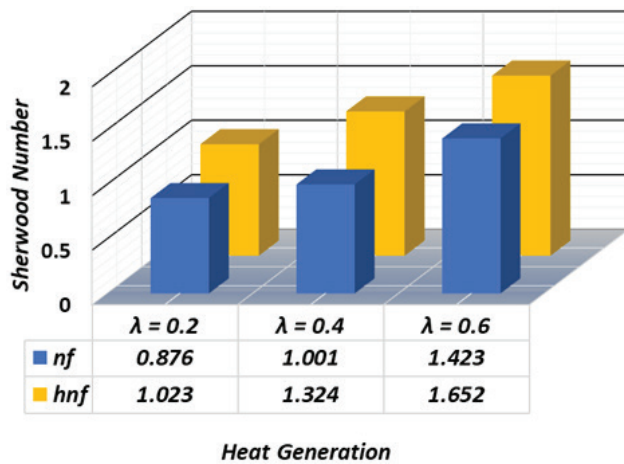


Figure 14. Sherwood variation with heat generation.

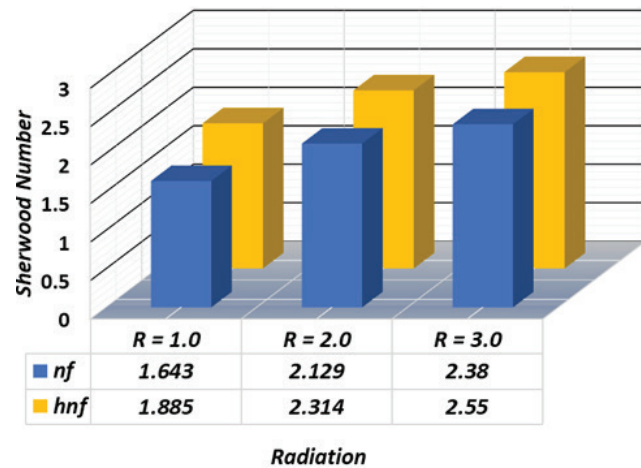


Figure 15. Sherwood variation with radiation.

Figure 14 shows the Sherwood number variation for varying values of heat generation. Greater heat generation leads to enhanced mass transfer and greater diffusion of the solute from solid surface to the fluid leading to an upsurge in Sherwood number. These conclusions have significant implications for optimizing the convective heat transfer which involves hybrid and other intricate fluids where concentration gradients are important. Figure 15 displays the effect of radiation on Sherwood number. Radiation

indirectly enhances fluid motion enough to increase mass transfer. So, Sherwood number rises for advanced values of Radiation parameter.

Table 2 shows that heat generation and radiation rises the fluid temperature near the wall, which reduces the temperature dissimilarity between the wall and the fluid leading to lower surface heat flux. Hence the Nusselt number reduces. In engineering applications, (e.g., nuclear reactors, electronic cooling) higher radiation or internal heating lowers the convective heat removal capacity. So, it is considered in the thermal design. Heat generation and radiation contributes to overall heat transfer and increases temperature which then increases the temperature dependent diffusion coefficient potentially increasing the mass transfer. Hence a rise in Sherwood number is noted. Table 3 compares the present with the previously published outcomes.

Table 4 depicts a Grid Independence Test. A formal mesh (step size) independence test has been incorporated. The numerical solution was computed using progressively refined steps, and the variations in key physical quantities such as Sherwood and Nusselt numbers were monitored. The outcomes demonstrate that beyond a certain value, further grid refinement produces negligible changes (within 10⁻⁵ tolerance), confirming numerical steadiness and independence of the computational grid.

CONCLUSION

Table 2. Computed local Nusselt $-\theta'(0)$, local Sherwood $-\phi'(0)$ numbers for varying values of heat generation λ and radiation R as $Le = 5$, $f_w = 0.5$, $\beta = 5$, $Nb = Nt = 0.1$, $Pr = 10$

λ	R	$-\theta'(0)$	$-\phi'(0)$
0.0	0.0	3.35766146	0.74574654
0.2		3.06867167	1.02374100
0.4		2.75576605	1.32413791
0.6		2.41285144	1.65260655
0.8		2.03105529	2.01739063
1.0		1.59669978	2.43120211
0.0	1.0	2.10893932	1.88572612
	2.0	1.61344374	2.31487442
	3.0	1.32981417	2.55088933

Table 3. The comparative outcomes for $-\theta'(0)$ according to $Le = 5$, $Nb = Nt = 0.1$, $Pr = 2$

F_w	Raju et al. [48]	Shah et al. [49]	Current Outcomes
0.5	1.230791	1.2307914	1.2307916
0	0.764552	0.7643523	0.7643533
-0.5	0.399084	0.3990987	0.3990988

Table 4. Grid independence test

Grid Points	Step Size(h)	Nusselt Number	Sherwood Number
200	0.01	3.35766146	0.74574654
400	0.005	3.35766143	0.74574651
800	0.0025	3.35766136	0.74574645
1200	0.00167	3.35766129	0.74574639

The present numerical study investigates 2D, laminar, incompressible, steady, viscous hybrid nanofluid flow past a linear stretching sheet. Two kinds of nanoparticles AlN and Al_2O_3 were suspended in base fluid (sodium alginate) to obtain a new configuration $AlN+Al_2O_3/C_6H_9NaO_7$. Magnetic field parameter is included in the momentum equation. Heat source and radiation parameters are included in energy equation. due of its higher thermal conductivity, this method is highly effective in cooling systems. In this present problem, mathematical analysis of hybrid nanoparticles has been conducted to study the temperature, velocity, and concentration profiles.

The significant works of the existing study are:

1. Temperature augmented for higher values of heat generation and radiation.
2. A decay in the concentration was found, with an upsurge in heat generation.
3. Concentration boundary layer declined for higher radiation values.
4. A decline in local Nusselt number and a rise in local Sherwood number are observed with the rise in heat source.
5. With the rise in radiation, there is a fall in the local Nusselt number and an upsurge in the local Sherwood number.
6. Hybrid nanofluid has an improved heat transfer rate when compared to that of traditional nanofluid.

Future Work

The current CFD simulation provides the researchers a motivation to the modelling of 2D Casson hybrid nanofluid flow, and heat transfer problems which are pertaining to the fields of Electrical and Machine-driven Engineering. It can also be utilized to examine various aspects related to solar collectors, chemical engineering, thermal industries, medicine, etc. The present problem can be used to study rheological behaviours of viscoelastic or micro hybrid nanofluids, dusty or ternary nanofluids, etc.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw

data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The authors declared no conflict of interest.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

Artificial intelligence was not used in the preparation of the article.

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