Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2026.2174

Research Article

A mathematical analysis of the suitability of using fractal and non-fractal patterns based materials for manufacturing musical instruments to produce natural/green music signals

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ARTICLE INFO

Article history
Received: 18 May 2025
Revised: 11 July 2025
Accepted: 13 April 2026

Keywords:
Ergonomics of Music Signals; Fractal Geometry; Fractal Pattern-Based Materials, Green Computing; Musical Instruments; Natural/Green Music Signals; Non-Fractal Pattern-Based Materials

ABSTRACT

The design and manufacture of musical instruments are fundamentally rooted in materials science, where the properties of the materials used directly influence the instrument's acoustical characteristics and performance. The motivation behind this study was the need to analyse the different materials used for manufacturing musical instruments. This need arose as a result of a partial hearing impairment experienced by one of the authors of the paper at a young age, while listening to high volume music using earphones. The research investigates the suitability of non-fractal and fractal pattern-based materials for manufacturing musical instruments, guided by a mathematical rationale. The significance of this work is that the fractality of the material of which a musical instrument is made can determine the ergonomics of the music signal of the instrument. The approach taken was to perform a mathematical analysis and comparison of the suitability of a material for manufacturing a musical instrument to explore how these structural forms can become the basis for the selection of materials for producing natural/green music signals. Natural/green music signals are those which are based on a fractal pattern, and which are calming to the listener, allowing him to relax mentally by listening to music signals with echoes and timbre. This analysis proved theoretically that a fractal pattern will continue to form at smaller and smaller scales upto an infinitesimal scale without end. The conclusion was that a fractal-pattern based material, being in coherence with the fractal patterns of the brain of the listener, will produce a relaxing experience when he listens to music produced by it. As a consequence of this research work it can be inferred that the choice of material for the manufacturing of musical instruments is simply dependent on the fact that the material should have fractal patterns in its internal structure. The work is novel in terms of the fact that the link between the material structure, the music signal produced and the ergonomic or green nature of the impact has never been explored in the context of fractal geometry, and this link is the focus of this research work. The work is novel in that till date there have been no linkages identified between the ergonomics of a musical instrument's material and the signals produced by playing it, bridged by fractal geometry.

Cite this article as: Batra N, Lakra S. A mathematical analysis of the suitability of using fractal and non-fractal patterns based materials for manufacturing musical instruments to produce natural/green music signals. Sigma J Eng Nat Sci 2026;44(5):3527–3542.

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This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

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INTRODUCTION

The construction of musical instruments is an art that intertwines with the science of materials. The choice of material significantly influences not only the aesthetic appeal but also the acoustic properties of an instrument. This paper aims to explore the diverse palette of materials that have been historically and contemporarily used in creating musical instruments, examining their acoustic characteristics, and cultural significance.

As a child the second author of this paper, experienced a hearing impairment while listening to loud music. The author used to listen to music from an audio cassette player using earphones. While listening to loud music one day the author plugged an earphone in his right ear but not in the left ear and increased the volume to experience better enjoyment. However, the result was that the right ear suddenly experienced a strange sound and was no longer able to hear low volume sounds leaving the author's right ear damaged. This somewhat damaging event came to be the motivation behind this research work.

Over the years, musical instruments were made out of materials which were available and which met the norms of the musicians' culture. Some of the common materials used included bone, wood, animal skin, etc., since they were easy to find [1]. Certain characteristics of wood such as good resonance, tone and echo helped to make it the most common material used for different types of instruments such as for those using strings [2]. A recent study [3], found that instruments made of wood calm and relax listeners.

On the other hand, stretched and dried animal skin became useful for drum-beat based instruments. The hollow structure of certain bones and reeds made them suitable for wind instruments. Due to industrialization and the increased use of metals and alloys, brass and steel came to be used for musical instruments which included pianos and trumpets [4].

Recently, green computing is an area which has begun to be linked to the field of musical instruments manufacturing. This field is focussed on understanding and reducing the environmental impact of using electronic and computing devices. Similarly, ergonomics is another area which helps in understanding the impact of device usage by human beings on themselves. This paper links these two fields to understand how listening to music played by musical instruments impacts the human brain/mind. Natural/green music signals are those produced by a musical instrument made of natural materials such as wood. These materials have fractal patterns in their internal structure and these signals also are made of fractal patterns. The authors initially hypothesized that since these fractal patterns match with the fractal patterns in the human brain they are relaxing to listen to. In turn, this indicates that these musical instruments and signals are green and may be considered ergonomic also.

Sustainable materials are being used more and more for different types of devices. Research in material science has today included the development of composites as a new class of materials which are more stable [5]. These are more sustainable for being used for manufacturing instruments.

The local culture also helps to give direction to the type of material used for an instrument. E.g., using particular types of woods for making certain instruments in Japan is based on the cultural practices of the country [6]. Newer versions of the classical violin use carbon fibres indicating the use of recent technologies in classical music [7].

There is a wide range of materials used in making musical instruments. These vary from wood and skin to modern composites and carbon fibres [8]. This paper will delve into the internal structure of materials, observing that a fractal pattern-based material such as wood is suitable to produce natural/green music signals [9] as compared to a non-fractal pattern-based material such as plastic [10].

A recent study [11] indicated that there is a link between calming effects and listening to repetitive music. Another study [12], concluded that even the selection of the right kind of wood can have an effect on the relaxation which a listener might experience. Another study [13] linked fractal patterns in music to the generation of brain signals, whereas a new study [14] showed that the difference between the morphometry of the brain of musicians and non-musicians is based on the fractal dimensions of neural patterns.

In the open literature, as per an Internet-wide search, there is no single research paper to be found linking together the ergonomics of using a particular material to construct a musical instrument so that the music signals produced by the instrument are relaxing to the listener. There were a few papers to be found relating only any 2 of these 3 areas.

A research paper [15] links together sound quality to the material used to construct a musical instrument. Similarly, other research work links the ergonomics of musical instrument design [16] while discussing human-instrument interaction, and some others present studies on the relaxing impact of music on human listeners on the basis of psychoacoustics [17]. A related study [18] presents an analysis based on fractal theory, of the influence of music on human respiration.

The major gap found in the literature survey was that the impact of the internal structure of the material of which a musical instrument is made, in terms of the ergonomics of the music produced by it on a listener, has not been addressed till date.

To address the research gap, firstly, a mathematical analysis needed to be done to explain the suitability of fractal-pattern based and non-fractal pattern-based materials to manufacture musical instruments with an aim of maintaining an ergonomic or green effect on the listener. With this objective, a comparative study needs to be conducted on the impact of 2 music signals, first, a signal that is produced by a musical instrument made of a natural material having an internal structure based on fractal patterns, and second, a signal produced by a non-fractal pattern based musical instrument. This study needs to be done to bring out the impact on real human listeners of these 2 different types of music signals, to understand which one is ergonomic and which one is not.

The work is novel in the light of the fact that till date there have been no linkages identified between the ergonomics of a musical instrument's material and the signals produced by playing it, specifically in the context of fractal geometry.

The contribution of this paper is that a link was found between the material(s) with which a musical instrument is made and whether the music played by a musical instrument is calming to the human listener or not. Based on the mathematical rationale presented, the link between natural materials with an internal fractal structure and production of calming music, was established.

The paper begins with an introduction which also gives its background. The paper further discusses the materials with which Western musical instruments and Indian musical instruments are made. Then the paper presents the non-fractal based internal structure of plastics. Following this, the paper gives a mathematical rationale behind the reason why fractal-based natural materials such as wood are more suitable for producing music which is calming to the human mind. The paper discusses the fractal internal structure of wood after this and then the paper presents a discussion, followed by conclusions and the future work.

Materials Used for Manufacturing Musical Instruments

As per a study [19] non-electronic musical instruments show chaotic behaviour when they are used to play music. This seems to point in the direction that the physics underlying these natural material based instruments is also chaotic and connects to fractal theory. Another study [20] identified that there are beat-like patterns found in the music of a tabla. A recent research work [21] links the quality of sound of the tabla and the material used to manufacture it.

A wide range of materials are used in making Western musical instruments. In the past wood was the material mostly used to make instruments [22]. However, since wood was not able to last for very long, other materials such as composites were used to make instruments in later years.

Some of the materials used for manufacturing Western musical instruments are discussed briefly below.

Plastic materials such as polybutylene terephthalate (PBT) are being used to make the strings of musical instruments [23].

Traditionally metals such as brass have been used widely in making instruments. Choosing the right metal is important for the quality of music produced by the instrument [15, 24]. For example, iron in the form of steel is used for making strings in pianos and for producing music in musical triangles [25]. Copper and its alloys such as brass and bronze, are used in making trumpets [26].

New developments in material science have led to the use of non-metals such as polymers as materials for making musical instruments.

Some materials used in making Indian musical instruments are discussed briefly next.

Most traditional Indian musical instruments are made from materials including wood, cotton, bamboo, leather and metals. The body of different instruments are made of wood and cotton ropes have been used to bind parts together [27]. Leather has also been used in certain parts of instruments [28]. Bamboo has been used in giving a glazed look to the surface of an instrument. Overall, the existence of the large number of instruments made of these natural materials in India represents the rich traditions of music in India [25, 29]. Recently, composites such as Carbon Fibre Reinforced Epoxy (CFRE) have also been used to improve the timbre of certain instruments [30, 31].

The material used to make a musical instrument will usually influence the music signals that are created [32]. Ref. [33] describes how the material from which wood is made affect its acoustic properties while also linking it to the internal structure of wood.

Table 1 presents a general overview of the variety of materials used for making musical instruments [5].

Table 1. Materials used to manufacture musical instruments [5]

Material	Properties	Used in Instruments	Natural (Yes/No)	Fractal structure of Material (Yes/No)
Wood	Resonant, durable, aesthetically pleasing	Violins, guitars, percussion	Yes	Yes
Metal	Sonorous, malleable, corrosion-resistant	Trumpets, percussion, piano	Derived from natural ore (can be considered natural)	Yes
Plastic	Lightweight, versatile, cost-effective	Modern flutes, recorders	No	No
Animal Hides	Flexible, natural sound quality	Drumheads, some string instrument parts	Yes	Yes
Bone and Ivory	Dense, hard, historically used	Piano keys (historically), some wind instruments	Yes	Yes
Composites	Customizable properties, innovative designs	Experimental instruments, some high-end models	No	No

Non-Fractal Based Internal Structure of Plastics

Non-fractal internal structure of plastics is widely used in many applications but recently it has been seen that the role of fractal geometries in polymer composites for improving mechanical properties is very useful [34, 35].

Structured materials (SMs) based on fractal curves, known as aperiodic, are also used for the structural construction and have been found to demonstrate promising results for tuning mechanical performance and elasticity of thermoplastic polyurethane composites [36]. Moreover, the morphology of polymer-stabilized liquid crystals (PSLCs) is important in the electro-optic properties of such materials, and the fractal dimension represents the structure of the polymer network and its effect on the properties of the device with different preparation conditions. The study of fractal geometries in polymer processing creates opportunities to enhance the performance and the functionality of polymers in many applications [37].

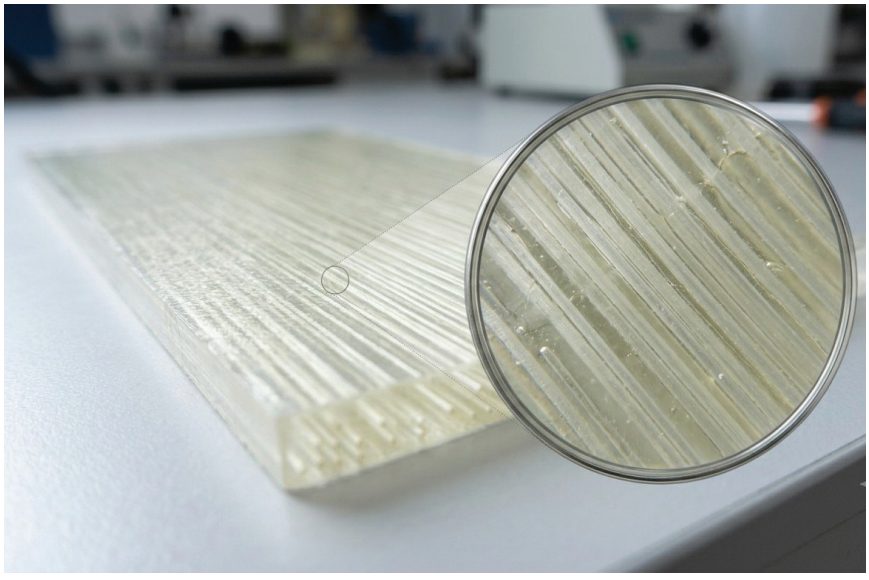
Plastics, traditionally, have been made using straight fibres by aligning them in parallel to prepare planar sheets of plastic material. However, as shown in Figure 1 [38, 39], such a sheet does not have an internal fractal pattern-based structure, which implies that plastic sheets made in this manner cannot be considered as natural material.

Table 2 [40] presents a comparison of wood and plastic on the basis of their acoustic parameters.

Similarly, using plastic material based on parallel fibres used as components in a musical instrument will not allow the instrument to play natural fractal-pattern based music, since the material itself does not have an internal structure based on a fractal pattern.

Table 2. A comparison of wood and plastic based on acoustic parameters [40] [Open Access]

Parameter	Wood	Plastic
Isotropy	Anisotropic, where the longitudinal Young's modulus may be 20 times the transverse modulus [40]. Permits a musical instrument made of wood to exhibit resonance in its music signal.	Isotropic, where the Young's modulus is the same in all directions causing the absence of resonance in a musical instrument made of plastic.
Internal Damping	Low, which implies that wood permits a long sustain in music signals produced by a wooden musical instrument since its rate of absorbing vibrations is slow.	High, in which a musical instrument made of plastic absorbed vibrations quickly causing a short sustain.
Timbre	Due to an internal fractal structure, a complex timbre to be formed which lasts over a longer duration	Since the internal structure is uniform, plastic permits only a limited timbre which does not last for long.



(a)



(b)

Figure 1. (a) The non-fractal internal structure of a plastic sheet [38] [AI generated] and (b) A plastic saxophone [39] [Open Access].

MATERIALS AND METHODS

Definition 1. A fractal material is a material which has an internal structure based on a fractal pattern, thus exhibiting self-similarity across scales, e.g, wood.

Definition 2. A non-fractal material is a material whose internal structure is not based on a fractal pattern and self-similarity across scales within it is absent, e.g., a brick made by human beings.

A prominent metric to distinguish a fractal material from a non-fractal material is the fractal dimension, whose value is non-integer in the case of a fractal material and an integer in the case of a non-fractal material. Another metric which can be combined with the fractal dimension is the topological dimension of a material. The fractal dimension of a fractal material is greater than its topological dimension, whereas for a non-fractal material, the fractal dimension is equal to its topological dimension.

Definition 3: A natural signal is a signal which is produced by a human or animal voice or a musical instrument made of a fractal material.

These sources of natural signals usually produce music with the help of vibrations either of strings, a column of air or the entire instrument.

Definition 4: A natural/green music signal is a signal which is produced by a musical instrument made of a natural fractal material, is green for the environment and ergonomic to the listener.

The signal is green since it is natural and will not pollute the environment since it is a signal which is a non-artificial signal. An example of an artificial signal is a synthesized voice signal produced by a mobile phone speaker. The signal is ergonomic in the sense that the signal is safe, respectful of the listener's well-being and does not cause mental fatigue or dislike to the listener - the signal is, instead, soothing and relaxing to the listener. Since such a music signal is produced by a musical instrument made of a fractal material, it naturally has a fractal pattern as part of its signal wave itself. Therefore, these signals may also be termed as fractal music signals.

Definition 5. A non-natural music signal is one which is synthesized by an electronic device using computer algorithms and digital signal processing models and methods.

Most of these models have traditionally not been based on fractal mathematics and therefore the signals synthesized using them are non-fractal pattern-based music signals or simply non-fractal music signals.

Fractal and non-fractal music signals can be distinguished using the following signal metrics:

Fractal dimension: the fractal dimension of a music signal is calculated using the Higuchi method. This method calculates the fractal dimension using the following formula:

$$D_f = \frac{log(L(k))}{log(\frac{1}{k})} \quad (1)$$

where L(k) is the average length of the curve for a given time step k [41].

Hurst Exponent H: is another metric using which a fractal music signal can be distinguished from a non-fractal music signal [42]. This exponent is calculated on the basis of the rescaled range of the fractal music signal using the following formula:

$$R/S = c * t^H \quad (2)$$

where R/S is the rescaled range, t is the time span, and c is a constant. H is calculated from the log-log plot of R/S vs. t, where the slope of the plot is the value of H [43].

Using H, the fractal dimension can also be calculated using the following relation:

$$D_f = 2 - H \quad (3)$$

The value of H determines whether a given music signal is fractal or non-fractal. If H = 0.5, the signal is non-fractal and for 0 < H < 1, where H ≠ 0.5, the signal is fractal [44].

Manufacturing musical instruments using fractal materials is already a practice where the materials used, such as wood, skin, bone, are available in abundance in nature. As such there is no need to create fractal materials artificially since they can be grown in natural environments, where they develop naturally.

To understand the influence of the fractality of music signals on the manufacturability of fractal materials, requires an acoustic response analysis of the frequency response of signals of both the electric guitar, a non-fractal material musical instrument and the *tabla*, a fractal material musical instrument.

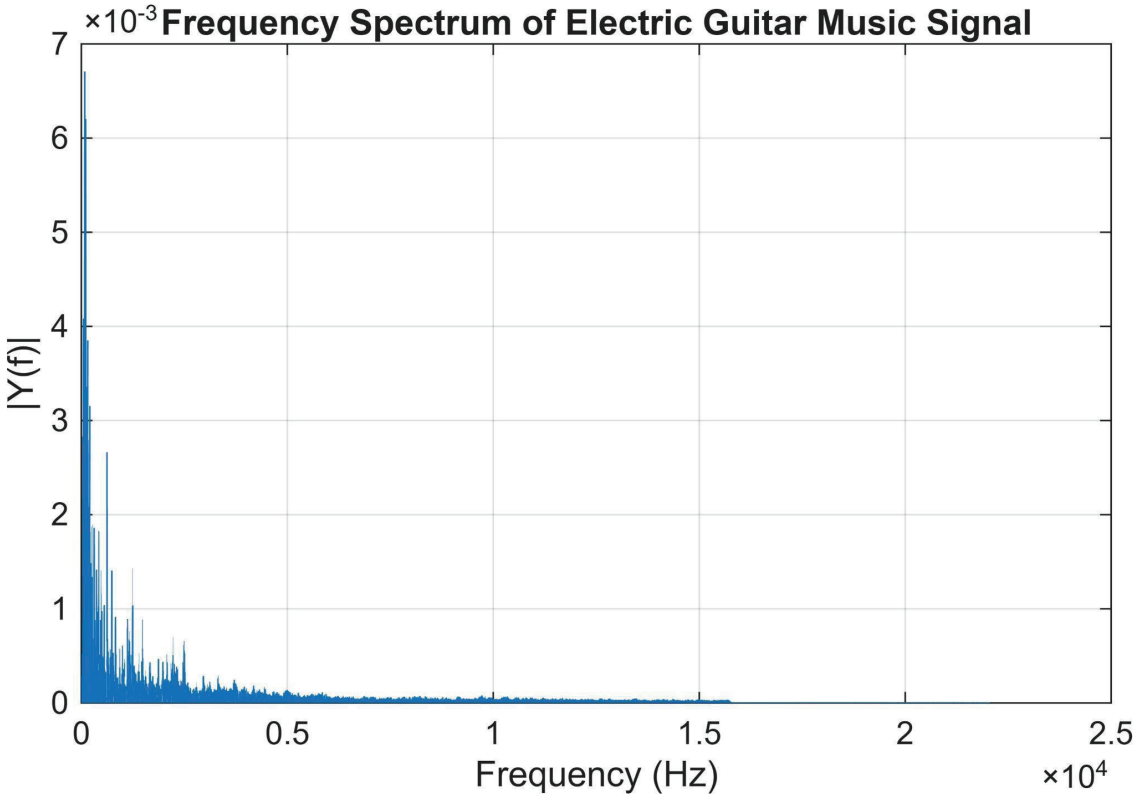
Figure 2 (a) and (b) show the frequency spectra of a recorded electric guitar music signal and a recorded *tabla* signal, respectively.

The frequency spectrum of the electric guitar music signal shows 1 or 2 spikes only whereas it includes frequencies spanning across a broad segment of the audible range.

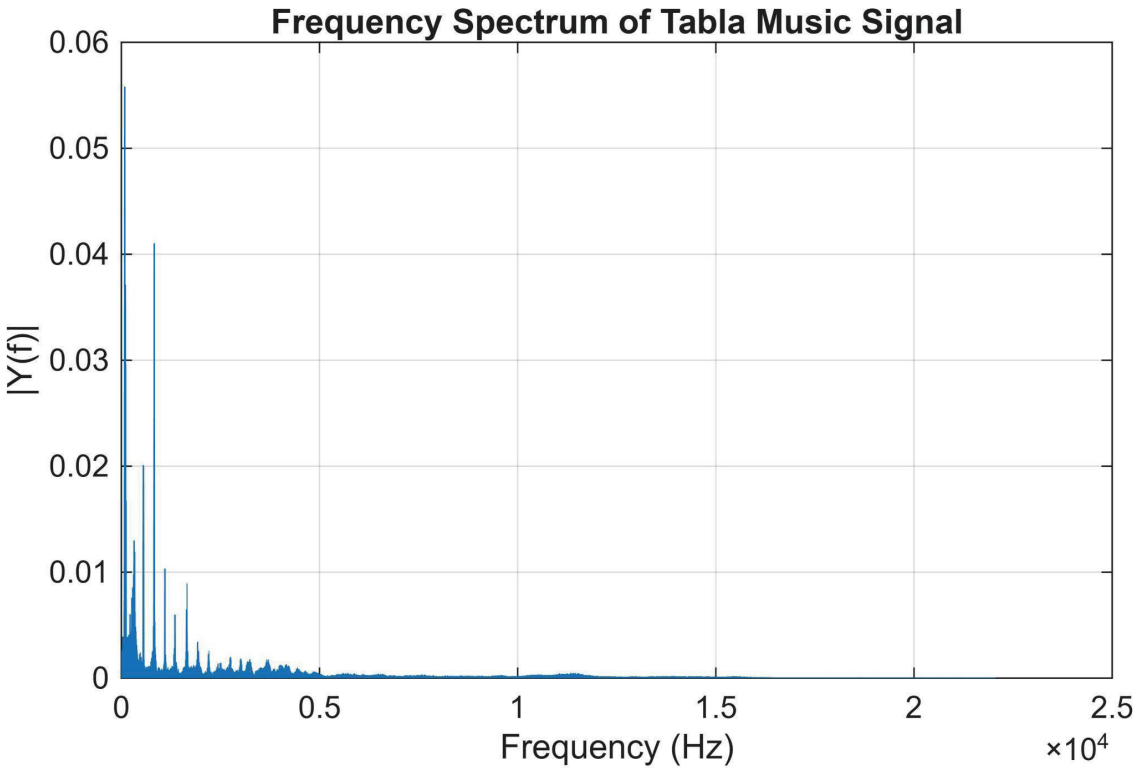
On the other hand, the *tabla* music signal shows spikes at specific frequencies which are the natural frequencies of the material of which the *tabla* is made. Such natural frequencies are found to trigger the formation of natural patterns in certain objects when they vibrate at their natural frequencies [45]. Whether it is possible to manufacture a fractal material having a number of natural frequencies is a question requiring exploration, which is beyond the scope of this research work.

Figure 3 shows the amplitude vs time plot of the continuous signal of an electric guitar. Since the signal's normalized amplitude fluctuates widely across the range [-1, +1], it indicates that the music is played at a loud and sustained volume. In the section from 110 to 220 seconds, since the amplitude reduces slightly, it indicates a part of the signal with lower volume.

Figure 4 shows a plot of amplitude vs time of a *tabla* music recording which has a consistent amplitude across



(a)



(b)

Figure 2. Frequency spectra of (a) a recorded Electric Guitar Music Signal (Non-fractal) and (b) a recorded *Tabla* Music Signal (Fractal).

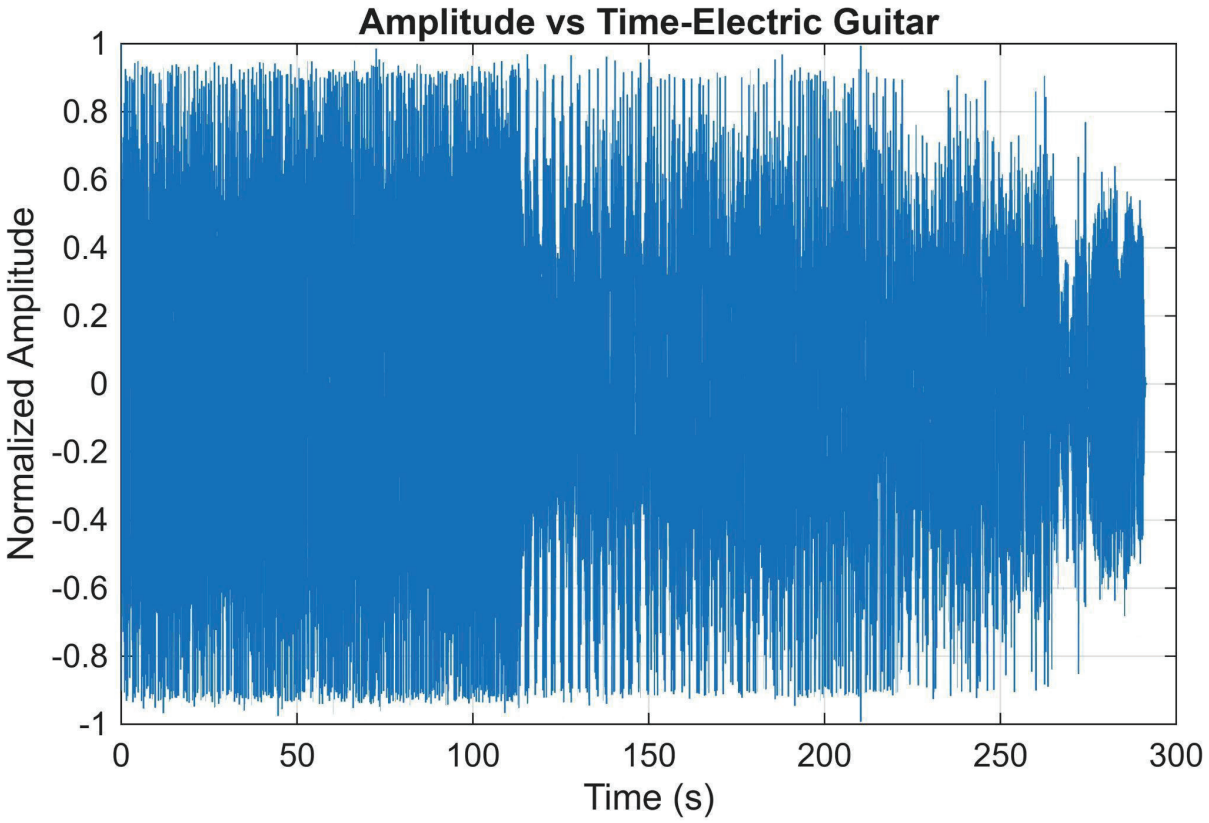


Figure 3. Amplitude vs. time plot of a recorded Electric Guitar Music Signal (Non-fractal).

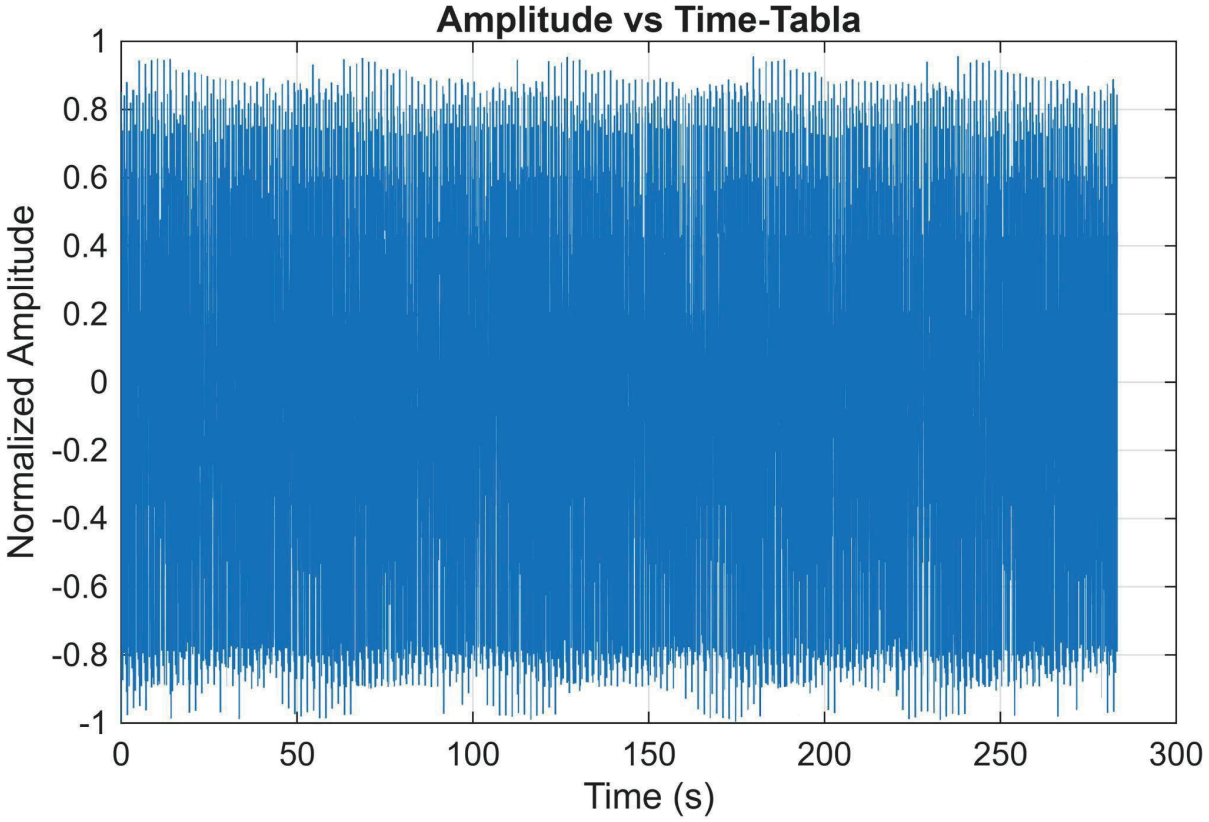


Figure 4. Amplitude vs. time plot of a recorded *Tabla* Music Signal (Fractal).

about 300 seconds of time. The amplitude is normalized and is in the range [-1, +1]. The amplitude shows variations according to the *taal* being played by the *tabla* player. Since the *taal* is a form of repetitive music, it leads to a waveform which has similarity across time.

Mathematical Analysis

Consider two real numbers, chosen at random, forming a real interval [0.3, 2.5]. It is possible to identify another interval within this interval.

0.3 2.5
0.3 0.32 2.34 2.5
0.3 0.32 0.327 1.69 2.34 2.5

This process can continue indefinitely for a real interval, theoretically, till infinity [46]. According to modern mathematical concepts, at -∞, the lower and upper bounds of the interval will become equal, thus forming a point. However, it is not possible to reach -∞ in the real world. Therefore, it is only possible to find one or more intervals within a given interval upto -∞. This implies that the concept of a point can not be found in nature, is imaginary and does not exist in nature.

Definition 6: A point interval $[a, a]$, $a \in R^+$ is an interval whose lower and upper bounds are the same number a.

Definition 7: A line interval $[a, b]$, $a, b \in R^+$ is an interval whose lower and upper bounds are not equal and $a < b$.

Theorem 1: A line interval $[a, b]$, *where* $a, b \in R^+$, $a < b$ is a zero-dimensional point a if it is a point interval $[a, a]$, $a \in R^+$.

Proof: A point interval $[a, a]$, by definition has equal lower and upper bounds. Therefore, if for the line interval , then $[a, b]$ becomes $[a, a]$. Also, the length of a point interval is equal to 0, since the difference *upper bound-lower bound* = $a - a = 0$. Therefore, a point interval with length 0 is a zero-dimensional point. *Hence, proved.*

Theorem 2: Within a line interval $[a, b]$, $a, b \in R^+$ there is always another line interval $[c, d]$, $c, d \in R^+ \mid c > a, d < b$ and $c < d$.

Proof: Let $[a, b]$ be an interval on the real number line as shown in Figure 5(a).

Consider a point c, $c > a$. Similarly, consider a point d, $c < d < b$. Then the interval $[c, d] \subset [a, b]$ as shown in Figure 5(b). Thus, $\forall a, b, c, d \in R$,where $a < c < d < b$, $[c, d] \subset [a, b]$. *Hence, proved.*

The visualizations of these concepts are shown in Figure 6.

A real-world analogy for a point interval is based on the concept of a point. Consider a point on a number line, such as the point for the number 0. A point interval for the number 0 will be [0, 0], which is something like attempting to hypothetically divide the point 0 into two points, a lower bound and an upper bound, such that an interval is created. Since this is physically impossible, the concept is only imaginary to support the analysis. Figure 7(a) shows an AI generated visualization of the point 0 being divided into 2 zeros to form a point interval.

A real-world analogy for a line interval being found within another line interval is simply the concept of Russian nested dolls (Fig. 7(b)), such that another smaller doll can be found nested inside any larger doll.

Theorem 3: A point interval $[a, a]$, $a \in R^+$ can never be obtained inside a line interval $[c, d]$, where $c,d \in R^+$, $c < d$, by repetitively finding any line interval $[c, d]$ inside another line interval $[a, b]$, *where* $a, b \in R^+$, $a < b$.

Proof by contradiction: Consider that a point interval $[a, a]$ can be found within the line interval $[c, d]$, where $a, c,d \in R^+$. $[c, d]$ will be found within $[a, b]$, only if the lower and upper bounds of $[c, d]$ are not equal, that is, $c \neq d$. Since a line interval is being found repetitively within another line interval, it implies that only an interval whose lower and upper bounds are not equal, can be found at every repetition. By Theorem 1, within a line interval there is always another line interval. However, by definition, a point interval must have its lower and upper bounds equal. This is a contradiction. Thus, a point interval can never be obtained within a line interval. *Hence proved.*

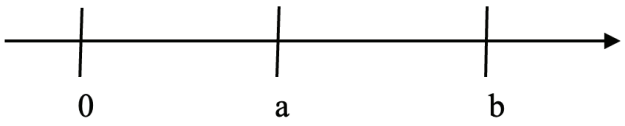


Figure 5(a). A line interval $[a, b]$ on the number line.

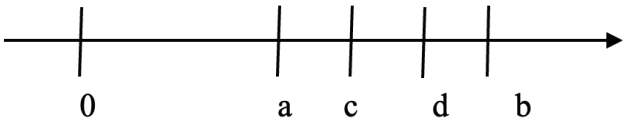


Figure 5(b). A line interval $[c, d]$, within another line interval $[a, b]$ on the number line.

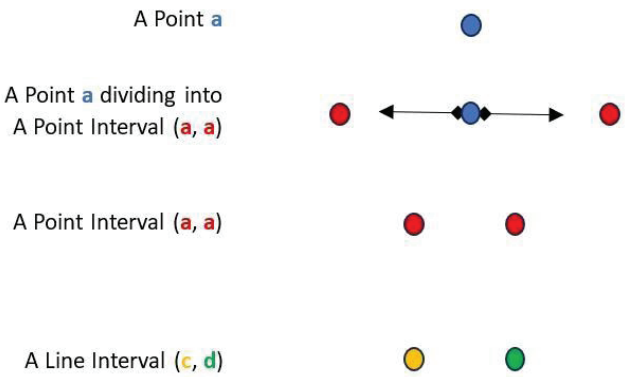


Figure 6. Visualizations of the concepts of a point, a point interval and a line interval.



(a) (b)

Figure 7. (a) An AI generated visualization of the point 0 being divided into 2 zeros to form a point interval. (b) A real-world analogy for a line interval being found within another line interval, which is simply the concept of Russian nested dolls [Open Access].

Theorem 3 presents mathematically that the concept of a fractal pattern, repeated infinitely within a fractal material, is fundamental to the unique acoustic properties of musical instruments made from natural materials. The infinitely repeating internal structure of these materials creates a phenomenon known as pattern coherence, where the regular patterns within the material produce music signals which resonate with the material due to the similarity of the patterns present in both the material as well as the music signal. This enhances those properties of the music signal, which make it likeable for a listener.

The above mathematical analysis is summarized as follows:

1. Theorem 1: Existence of a zero-dimensional point interval.
2. Theorem 2: Existence of another line interval within every line interval
3. Theorem 3: Within a line interval, another line interval can be found but a point interval can never be found. This permits a fractal to continue to repeat infinitely without end, since the fractal can stop repeating only when it reaches a point interval, which cannot be divided further into the non-zero-dimensional fundamental pattern of the fractal.
4. This is what makes the very existence of infinitely-repeating fractal patterns possible at smaller and smaller scales.
5. A musical instrument with an internal fractal structure will produce music signals which will have echo, timbre, melody, depth and harmony, and will sound "good" and "likable" to the listener. This is because the music signal as well as the human brain both have fractal patterns which when matched leads to a coherence between them.
6. A musical instrument with a non-fractal internal structure will produce music signals which are also without fractal patterns. This causes a mismatch between the non-fractal music and the fractal-pattern based human brain. As a result, the human being does not find this music "likeable".

Figure 8 gives a schematic view of the above mathematical analysis.

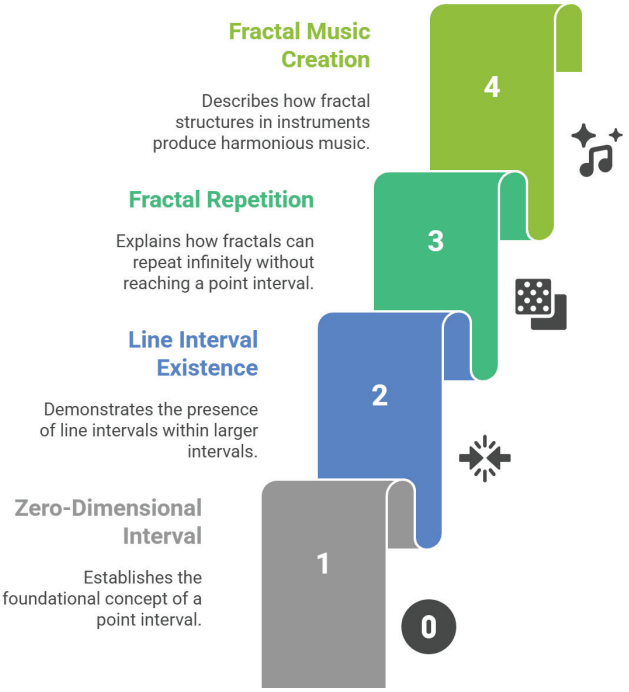


Figure 8. Flow of logic in the mathematical analysis [AI generated].

The music signals which are produced by instruments made of natural materials such as wood, are fractal pattern-based since the natural materials are themselves fractal structures. As a result, the music signals produced by them are relaxing to hear. On the other hand, non-natural materials such as wood are not having fractal patterns and therefore the music produced by them is without echo and melody, and sounds dull and non-relaxing.

Consequences of the Presence of Pattern Coherence

The presence of pattern coherence in fractal structure-based materials points in the direction of why such materials are suitable for producing musical instruments. The concept helps to understand the characteristics of echo and melody present in relaxing music. The pattern coherence of the fractal geometry with the fractal patterns in the music signals produced by them and with the fractal geometry of the human brain together are the basis for the presence of natural harmony in the music.

Thus, theorem 3 links together the infinite fractal patterns in natural materials with the natural acoustic properties of instruments made from these materials. It is able to do so using the concept of pattern coherence.

Applications of the Research Work

The application of the work is that the concept of pattern coherence leads to the conclusions that using natural materials, which are based on fractal patterns, will help to not only improve the ergonomics of musical instruments, but also make musicians use such materials if they wish their music to be liked by listeners through providing a relaxing experience.

Not only this, the findings of the work help to understand as to why natural materials are easier and quicker for nature to decompose, whereas non-natural materials take much longer to be decomposed by nature. Similarly, the idea of manufacturing devices made of natural fractal-pattern based materials will ultimately lead to a solution to the problem of overly-durable e-waste.

Pattern coherence between natural decomposers and the material of a waste product also helps to clarify as to what permits faster natural decomposition of the waste. For example, natural decomposers of fallen leaves are bacteria and fungi, which decompose fallen leaves in a few months. But natural decomposers take a much longer time to decompose plastics. A point of difference is that natural materials, as those in leaves, are fractal-pattern based and may exhibit a pattern coherence with natural decomposers, which also exhibit fractal patterns. Whereas, plastics are non-fractal pattern based and do not exhibit pattern coherence with natural decomposers, which therefore "dislike" the plastics, and take longer to decompose.

The research work is interdisciplinary in nature and links areas including materials science, signal processing, fractal geometry, ergonomics and green computing as shown in Figure 9.

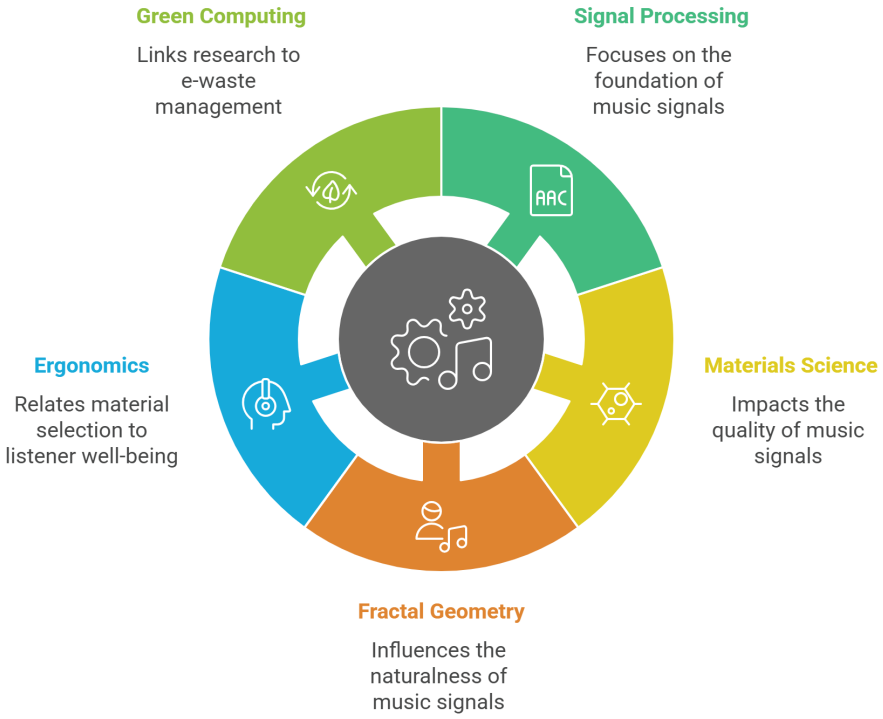


Figure 9. Interdisciplinarity in the research work [AI generated].

Fractal Internal Structure of Wood

Fractal theory has been used to study the internal structure of wood. Studies [47, 48] have shown that wood pore structures have self-similar fractal patterns across different sizes. In [49], the authors have used fractal models to predict equivalent thermal conductivity (ETC) from wood pore fractal geometry. Similarly, fractal geometry has been used to analyse the internal structure of many materials and structures [50, 51].

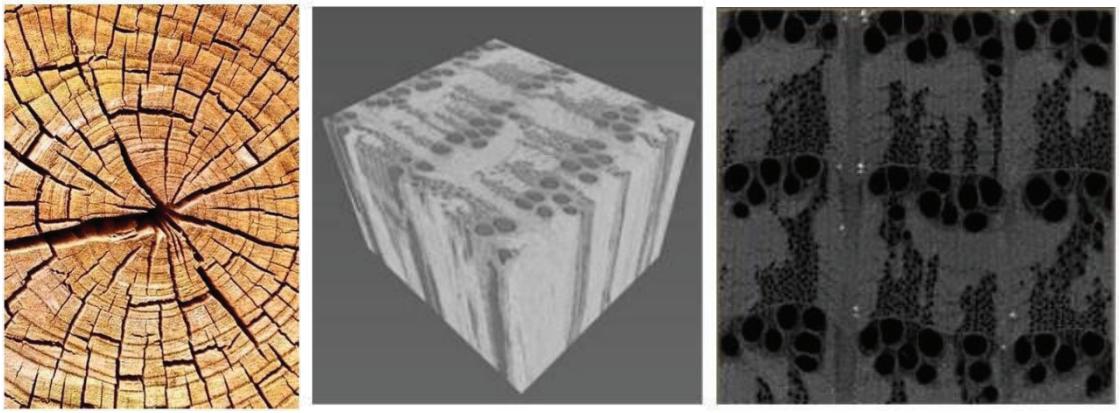
The trunk of a wooden tree exhibits fractal patterns at a large scale (Fig. 10(a)). Similarly, at a small scale the pores in the cell wall of wood also show fractal patterns (Fig. 10(b), (c)). Mathematically, the internal structure of wood has been found to be fractal pattern based [48]. According to [47], the fractal dimension of large wood pores is different from that of small pores. An understanding of the fractal patterns of wood is also essential to link the internal structure of wood with its strength [52-54].

RESULTS AND DISCUSSION

Modern mathematics is based on the concept of a point and it has led to the development of mathematics which does not include the concepts of fractals. Fractals are based on the concept of self-similarity of a non-point pattern across increasing or decreasing scales within intervals. Fractals exist in every natural material but traditional mathematics is based on the point as a fundamental concept, which is not suitable for representing fractals since they are based on non-point self-similar patterns. As a result, materials being manufactured artificially on the basis of point-based traditional mathematics are not natural and they are not possible to be assimilated easily by nature. For example, plastic is assimilated by nature in 100 to 1,000 years [55]. This is the reason why materials that are natural need to be used rather than using artificial materials. Also, rather than attempting to manufacture artificial materials in the lab, the need is to harness nature and use naturally available materials such as wood, ivory, etc. for musical instruments. This was the practice followed since ancient times in India, where artificial materials were never created.

Since the concept of a zero-dimensional point is imaginary, a one-dimensional fractal pattern which repeats within a fractal structure, repeats till -∞, without ever becoming a point. This is mathematically proven by Theorem 3. This self-similarity and repetition of a fractal pattern across scales within a fractal structure are not evident in a material with a non-fractal internal structure. The music produced by a musical instrument made out of a fractal pattern-based material such as wood will produce natural/green music signals exhibiting timbre, rhythm and harmony, whereas a musical instrument made out of a material, such as plastic, with a non-fractal pattern based internal structure, will not produce natural/green music signals. The reason behind this is that a material with a non-fractal internal structure cannot produce music which will possess the calming properties which natural fractal-pattern based music will exhibit. Natural/green music is produced by Indian musical instruments which use natural materials such as wood, for making them [9, 10]. On the other hand, the music produced by a musical instrument made out of a non-fractal internal structure will not exhibit these desirable and pleasing properties of music, instead it will be flat and will not please the mind of the listener.

The key findings of this paper include, firstly, that it becomes possible for infinitely repeating self-similar fractal patterns to be created at smaller and smaller scales since another line interval can be found within a given line interval, no matter how small it becomes. With decreasing scale, a line interval can approach infinitesimal scales but since a point interval is only a hypothetical concept, a line interval never becomes a point. Secondly, musical instruments made of fractal-pattern based materials, such as wood, produce a timbre and echo in their music signals,



(a) (b) (c)

Figure 10. Fractal patterns in wood (a) at the macroscopic level in the form of tree rings in a cross section of a trunk [53], and at the microscopic level in pores (b) in a 3-dimensional view and (c) in a 2-dimensional view [54] [Open Access].

Table 3. Generalization of the concepts of fractal materials and fractal music signals produced to musical instruments classified as per the Hornbostel-Sachs classification [66]

Musical Instrument Category	Mechanism of Producing Music	Examples of Musical Instruments	Type of material	Music Signals Produced are Fractal or Non-Fractal
Chordophones	Vibrations of a set of strings	Guitars, sitar, veena, violins, pianos	Natural, Fractal	Fractal
Aerophones	Causing a column of air to vibrate	Trumpets, flutes, clarinets	Natural, Fractal	Fractal
Mebranophones	Vibrations of a stretched membrane	Drums, *tabla*, timpani	Natural, Fractal	Fractal
Idiophones	Vibrations of the entire instrument itself	Bells, cymbals, xylophones	Natural, Fractal	Fractal
Electrophones	Produce synthetic sound using electronic methods	Synthesizers, electronic keyboards	Non-natural, non-fractal	Non-Fractal

whereas music signals from a musical instrument made of a non-fractal-pattern based material, such as plastic, lack echo and timbre. To a human listener, the former sound relaxing and soothing, whereas the latter do not lead to a relaxing experience. According to research studies [56, 57], it is clear that natural music leads to a soothing effect and relaxes a human listener, however, links between the fractality of materials and the ergonomics of music for humans have not been explored earlier.

Although there is no work to be found in literature linking together all the disciplines involved in this research work, one similar study exists which has related the fractal quality of Indian classical music on autonomic system parameters such as blood pressure, heart rate variability and ECG [58]. However, making a comparison will not be valid since the present research work is focussed on analysing the impact of fractality of materials on the ergonomics of listening to music produced by musical instruments on the human brain and not on the heart.

A few studies link fractals and psychoacoustics. In the research work in [59], the authors identified that the fractal dimension could be used as a quantifiable metric to categorize music signals. A recent study [60] found that fractal patterns were identifiable in brain activity indicating them to be linked to how music signals display fractality as well. Another study [61] spotted self-replicating patterns across time frames within music signals, emphasizing a connection between fractal patterns found in music and the human brain functions. A significant study [62] linking chemistry, music and psychoacoustics identified how certain connected chemicals were able to convey feelings to listeners through music performances, indicating that the selection of material for building a musical tool is the key to playing music which can, in turn, influence the listener emotionally. A paper focussed on material science [63] delves into how lightweight pipes should be designed so as to avoid resonance, since it can lead to structural breakdowns. The study used modal analysis based on finite element analysis using the ANSYS software.

Fractals play a crucial role in understanding wood formation by characterizing the complex pore structures and ultrastructure of wood at different scales. In [64], a study was conducted to study the pore structure of natural and heat-treated wood using fractal geometry. The study showed that there are differences in the fractal properties for different pore sizes between the two. Similarly, in [65], it was concluded that the small-scale structure of wood cells can be described using fractal geometry leading to a better understanding of wood anatomy.

Generalization to Musical Instrument Categories

The Hornbostel-Sachs classification of musical instruments [66] is presented in Table 3. The table maps the different categories of musical instruments to the type of material they are made of, as well as to the type of music signals they produce.

As summarized in Table 3, except for electrophones, all other categories of instruments produce fractal music signals. However, electrophones are based on synthesizing music signals using mathematical models, which produce non-fractal music signals.

However, generalizing to different categories of musical instruments experimentally will be possible only with the help of detailed acoustic response analysis of different categories of instruments, which is considered as a part of future work of this research.

CONCLUSION

The paper concludes that the choice of metal, wood and iron in musical instruments is a complex interplay of acoustics, material science, and craftsmanship. The key findings

of the paper are that natural materials are able to have a fractal pattern due to the fact that they are based on a non-point fractal pattern which repeats to an infinitesimal scale. Whereas, man-made materials are based on the imaginary concept of a point and are, therefore, not able to exhibit a fractal pattern. Thus, the analysis and mathematical rationale in this paper clearly indicate that materials based on fractal pattern-based internal structures are suitable to produce natural/green music signals. In contrast, non-fractal pattern-based materials are not suitable for producing natural music. This highlights the importance of material selection in the manufacturing of musical instruments which can produce pleasing and calming music.

The application of this conclusion is to give an understanding of why musical instruments should be made of natural materials. Natural materials have coherence with human listeners, who are also natural living systems. Natural systems will have coherence with natural/green music rather than with non-natural music.

Future Work

The authors will work on collecting evidence through a study based on EEG signals on human listeners, who will listen to Indian classical music instruments, as compared to Western music instruments made of plastic. The EEG-based experiments will consist of playing music of both types of signals, namely, fractal-pattern based and non-fractal-pattern based music signals. The musical instruments selected will, firstly, be the tabla, an Indian Classical Music instrument made of wood and skin playing a basic repeating taal over and over again. This instrument has been selected since it is made of natural materials and is already known to render a soothing effect on the listener. Secondly, the electric guitar, which is made of plastic and metal, has been selected since it is already known to have a non-soothing effect on listeners. Both the instruments will be played, one-by-one, and their music signals will be recorded. These recordings will then be used in experiments in which human volunteers will listen to each of the recordings one at a time, and their EEGs will be recorded while they listen to them.

Further experiments can be conducted on different categories of musical instruments using acoustic analysis techniques to relate them to the materials used for manufacturing them and to generalize the concepts presented in this research work.

AUTHORSHIP CONTRIBUTIONS

Ms. Neha Batra thought of the original idea and conducted the research. She further developed the mathematical reasoning and wrote the first draft of the manuscript. She also carried out the comparative analysis of fractal and non-fractal materials and their applications in musical instrument manufacturing.

Dr. Sachin Lakra provided guidance on the theoretical framework. He also critically reviewed and refined the mathematical proofs. He contributed to the development of the discussion and conclusions. He also assisted in structuring the paper and supervised the research activities throughout the project.

Both authors contributed to the final version of the manuscript and approved it for publication.

DATA AVAILABILITY STATEMENT

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

STATEMENT ON THE USE OF ARTIFICIAL INTELLIGENCE

During the preparation of this work, the authors used Gemini to generate the conceptual schematic in Figures 1(a), 7(a), 8 and 9.

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